Higher

Mathematics

Past Papers

2003 paper 1

ALL questions should be attempted.

Marks

1. Find the equation of the line which passes through the point (–1, 3) and is perpendicular to the line with equation $4x + y - 1 = 0$. **3**

2. (*a*) Write $f(x) = x^2 + 6x + 11$ in the form $(x + a)^2 + b$. **2**

(*b*) Hence or otherwise sketch the graph of $y = f(x)$. **2**

3. Vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ are defined by $\boldsymbol{u} = 3\boldsymbol{i} + 2\boldsymbol{j}$ and $\boldsymbol{v} = 2\boldsymbol{i} - 3\boldsymbol{j} + 4\boldsymbol{k}$.
Determine whether or not $\boldsymbol{u}$ and $\boldsymbol{v}$ are perpendicular to each other. **2**

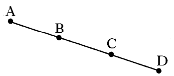
4. A recurrence relation is defined by $u_{n+1} = pu_n + q$, where $-1 < p < 1$ and $u_0 = 12$.

(*a*) If $u_1 = 15$ and $u_2 = 16$, find the values of p and q. **2**

(*b*) Find the limit of this recurrence relation as $n \to \infty$. **2**

5. Given that $f(x) = \sqrt{x} + \frac{2}{x^2}$, find $f'(4)$. **5**

6. A and B are the points (–1, –3, 2) and (2, –1, 1) respectively.
B and C are the points of trisection of AD, that is AB = BC = CD.
Find the coordinates of D. **3**



7. Show that the line with equation $y = 2x + 1$ does not intersect the parabola with equation $y = x^2 + 3x + 4$. **5**

8. Find $\int_0^1 \frac{dx}{(3x+1)^{\frac{1}{2}}}$. **4**

9. Functions $f(x) = \frac{1}{x-4}$ and $g(x) = 2x + 3$ are defined on suitable domains.

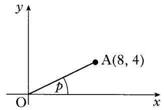
(*a*) Find an expression for $h(x)$ where $h(x) = f(g(x))$. **2**

(*b*) Write down any restriction on the domain of h. **1**

[Turn over for Questions 10 to 12 on *Page four*

Marks

10. A is the point (8, 4). The line OA is inclined at an angle p radians to the x-axis.

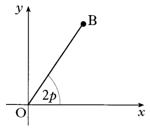


(*a*) Find the exact values of:

(i) $\sin(2p)$;

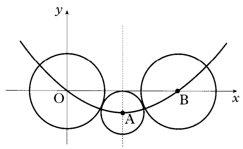
(ii) $\cos(2p)$. **5**

The line OB is inclined at an angle $2p$ radians to the x-axis.



(*b*) Write down the exact value of the gradient of OB. **1**

11.
- O, A and B are the centres of the three circles shown in the diagram below.
- The two outer circles are congruent and each touches the smallest circle.
- Circle centre A has equation $(x - 12)^2 + (y + 5)^2 = 25$.
- The three centres lie on a parabola whose axis of symmetry is shown by the broken line through A.



(*a*) (i) State the coordinates of A and find the length of the line OA. **2**

(ii) Hence find the equation of the circle with centre B. **3**

(*b*) The equation of the parabola can be written in the form $y = px(x + q)$.
Find the values of p and q. **2**

12. Simplify $3\log_e(2e) - 2\log_e(3e)$ expressing your answer in the form $A + \log_e B - \log_e C$ where A, B and C are whole numbers. **4**

[*END OF QUESTION PAPER*]

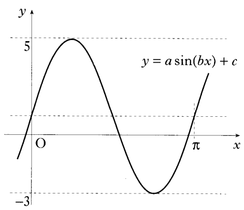
2003 paper 2

ALL questions should be attempted.

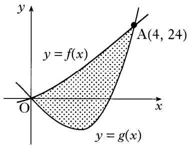
Marks

1. $f(x) = 6x^3 - 5x^2 - 17x + 6$.
   (*a*) Show that $(x - 2)$ is a factor of $f(x)$.
   (*b*) Express $f(x)$ in its fully factorised form. **4**

2. The diagram shows a sketch of part of the graph of a trigonometric function whose equation is of the form $y = a\sin(bx) + c$.
   Determine the values of a, b and c. **3**



3. The incomplete graphs of $f(x) = x^2 + 2x$ and $g(x) = x^3 - x^2 - 6x$ are shown in the diagram. The graphs intersect at A(4, 24) and the origin.
   Find the shaded area enclosed between the curves. **5**

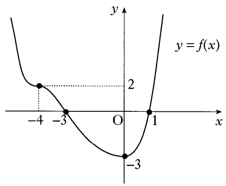


4. (*a*) Find the equation of the tangent to the curve with equation $y = x^3 + 2x^2 - 3x + 2$ at the point where $x = 1$. **5**
   (*b*) Show that this line is also a tangent to the circle with equation $x^2 + y^2 - 12x - 10y + 44 = 0$ and state the coordinates of the point of contact. **6**

[Turn over

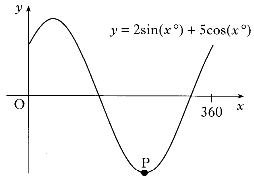
Marks

5. The diagram shows the graph of a function f.
   f has a minimum turning point at (0, −3) and a point of inflexion at (−4, 2).
   (*a*) Sketch the graph of $y = f(-x)$. **2**
   (*b*) On the same diagram, sketch the graph of $y = 2f(-x)$. **2**

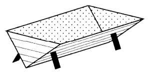


6. If $f(x) = \cos(2x) - 3\sin(4x)$, find the exact value of $f'\left(\frac{\pi}{6}\right)$. **4**

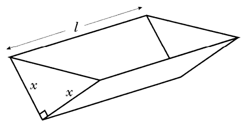
7. Part of the graph of $y = 2\sin(x°) + 5\cos(x°)$ is shown in the diagram.
   (*a*) Express $y = 2\sin(x°) + 5\cos(x°)$ in the form $k\sin(x° + a°)$ where $k > 0$ and $0 \leq a < 360$. **4**
   (*b*) Find the coordinates of the minimum turning point P. **3**



8. An open water tank, in the shape of a triangular prism, has a capacity of 108 litres. The tank is to be lined on the inside in order to make it watertight.



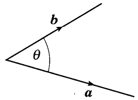
   The triangular cross-section of the tank is right-angled and isosceles, with equal sides of length x cm. The tank has a length of l cm.



   (*a*) Show that the surface area to be lined, A cm^2, is given by $A(x) = x^2 + \frac{432000}{x}$. **3**
   (*b*) Find the value of x which minimises this surface area. **5**

Marks

9. The diagram shows vectors $\boldsymbol{a}$ and $\boldsymbol{b}$.
If $|\boldsymbol{a}| = 5$, $|\boldsymbol{b}| = 4$ and $\boldsymbol{a}.(\boldsymbol{a} + \boldsymbol{b}) = 36$, find the size of the acute angle θ between $\boldsymbol{a}$ and $\boldsymbol{b}$. **4**



10. Solve the equation $3\cos(2x) + 10\cos(x) - 1 = 0$ for $0 \le x \le \pi$, correct to 2 decimal places. **5**

11. (*a*) (i) Sketch the graph of $y = a^x + 1$, $a > 2$.
(ii) On the same diagram, sketch the graph of $y = a^{x+1}$, $a > 2$. **2**

(*b*) Prove that the graphs intersect at a point where the x-coordinate is $\log_a\left(\frac{1}{a-1}\right)$. **3**

[*END OF QUESTION PAPER*]

ALL questions should be attempted.

Marks

1. The point A has coordinates (7, 4). The straight lines with equations $x + 3y + 1 = 0$ and $2x + 5y = 0$ intersect at B.

(*a*) Find the gradient of AB. **3**

(*b*) Hence show that AB is perpendicular to only one of these two lines. **5**

2. $f(x) = x^3 - x^2 - 5x - 3$.

(*a*) (i) Show that $(x + 1)$ is a factor of $f(x)$.
(ii) Hence or otherwise factorise $f(x)$ fully. **5**

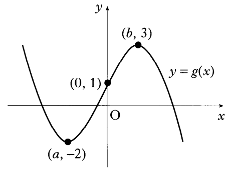
(*b*) One of the turning points of the graph of $y = f(x)$ lies on the x-axis. Write down the coordinates of this turning point. **1**

3. Find all the values of x in the interval $0 \le x \le 2\pi$ for which $\tan^2(x) = 3$. **4**

4. The diagram shows the graph of $y = g(x)$.

(*a*) Sketch the graph of $y = -g(x)$. **2**

(*b*) On the same diagram, sketch the graph of $y = 3 - g(x)$. **2**



5. A, B and C have coordinates (−3, 4, 7), (−1, 8, 3) and (0, 10, 1) respectively.

(*a*) Show that A, B and C are collinear. **3**

(*b*) Find the coordinates of D such that $\overrightarrow{AD} = 4\overrightarrow{AB}$. **2**

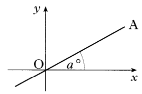
6. Given that $y = 3\sin(x) + \cos(2x)$, find $\frac{dy}{dx}$. **3**

[Turn over for Questions 7 to 11 on *Page four*

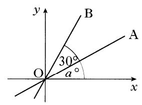
ALL questions should be attempted.

Marks

1. (*a*) The diagram shows line OA with equation $x - 2y = 0$.
The angle between OA and the x-axis is $a°$.
Find the value of a. **3**



(*b*) The second diagram shows lines OA and OB. The angle between these two lines is 30°.
Calculate the gradient of line OB correct to 1 decimal place. **1**



2. P, Q and R have coordinates (1, 3, –1), (2, 0, 1) and (–3, 1, 2) respectively.

(*a*) Express the vectors $\overrightarrow{QP}$ and $\overrightarrow{QR}$ in component form. **2**

(*b*) Hence or otherwise find the size of angle PQR. **5**

3. Prove that the roots of the equation $2x^2 + px - 3 = 0$ are real for all values of p. **4**

4. A sequence is defined by the recurrence relation $u_{n+1} = ku_n + 3$.

(*a*) Write down the condition on k for this sequence to have a limit. **1**

(*b*) The sequence tends to a limit of 5 as $n \to \infty$. Determine the value of k. **3**

5. The point P(x, y) lies on the curve with equation $y = 6x^2 - x^3$.

(*a*) Find the value of x for which the gradient of the tangent at P is 12. **5**

(*b*) Hence find the equation of the tangent at P. **2**

[Turn over

Marks

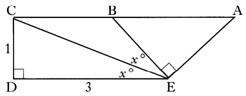
7. Find $\int_0^2 \sqrt{4x+1}\,dx$. **5**

8. (*a*) Write $x^2 - 10x + 27$ in the form $(x + b)^2 + c$. **2**

(*b*) Hence show that the function $g(x) = \frac{1}{3}x^3 - 5x^2 + 27x - 2$ is always increasing. **4**

9. Solve the equation $\log_2(x + 1) - 2\log_2(3) = 3$. **4**

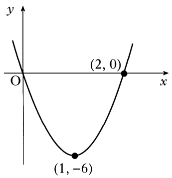
10. In the diagram
angle DEC = angle CEB = $x°$ and
angle CDE = angle BEA = 90°.
CD = 1 unit; DE = 3 units.
By writing angle DEA in terms of $x°$, find the exact value of $\cos(\hat{DEA})$. **7**



11. The diagram shows a parabola passing through the points (0, 0), (1, –6) and (2, 0).

(*a*) The equation of the parabola is of the form $y = ax(x - b)$.
Find the values of a and b. **3**

(*b*) This parabola is the graph of $y = f'(x)$.
Given that $f(1) = 4$, find the formula for $f(x)$. **5**



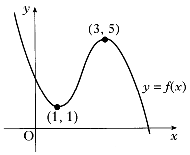
[*END OF QUESTION PAPER*]

6. (*a*) Express $3\cos(x°) + 5\sin(x°)$ in the form $k\cos(x° - a°)$ where $k > 0$ and $0 \leq a \leq 90$. **4**

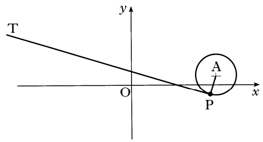
(*b*) Hence solve the equation $3\cos(x°) + 5\sin(x°) = 4$ for $0 \leq x \leq 90$. **3**

7. The graph of the cubic function $y = f(x)$ is shown in the diagram. There are turning points at (1, 1) and (3, 5).

Sketch the graph of $y = f'(x)$. **3**

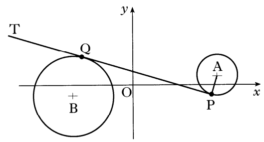


8. The circle with centre A has equation $x^2 + y^2 - 12x - 2y + 32 = 0$. The line PT is a tangent to this circle at the point P(5, –1).



(*a*) Show that the equation of this tangent is $x + 2y = 3$. **4**

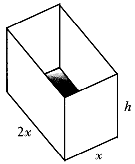
The circle with centre B has equation $x^2 + y^2 + 10x + 2y + 6 = 0$.



(*b*) Show that PT is also a tangent to this circle. **5**

(*c*) Q is the point of contact. Find the length of PQ. **2**

9. An open cuboid measures internally x units by $2x$ units by h units and has an inner surface area of 12 units^2.



(*a*) Show that the volume, V units^3, of the cuboid is given by $V(x) = \frac{2}{3}x(6 - x^2)$. **3**

(*b*) Find the exact value of x for which this volume is a maximum. **5**

10. The amount A_t micrograms of a certain radioactive substance remaining after t years decreases according to the formula $A_t = A_0e^{-0{\cdot}002t}$, where A_0 is the amount present initially.

(*a*) If 600 micrograms are left after 1000 years, how many micrograms were present initially? **3**

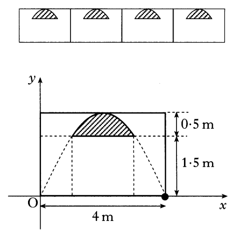
(*b*) The half-life of a substance is the time taken for the amount to decrease to half of its initial amount. What is the half-life of this substance? **4**

11. An architectural feature of a building is a wall with arched windows. The curved edge of each window is parabolic.

The second diagram shows one such window. The shaded part represents the glass.

The top edge of the window is part of the parabola with equation $y = 2x - \frac{1}{2}x^2$.

Find the area in square metres of the glass in one window. **8**

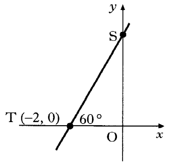


[*END OF QUESTION PAPER*]

ALL questions should be attempted.

Marks

1. Find the equation of the line ST, where T is the point (–2, 0) and angle STO is 60°. **3**



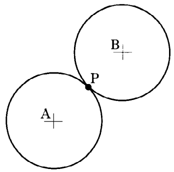
2. Two congruent circles, with centres A and B, touch at P.

   Relative to suitable axes, their equations are

   $x^2 + y^2 + 6x + 4y - 12 = 0$ and
   $x^2 + y^2 - 6x - 12y + 20 = 0$.

   (*a*) Find the coordinates of P. **3**

   (*b*) Find the length of AB. **2**

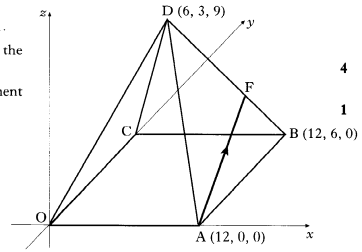


3. D,OABC is a pyramid. A is the point (12, 0, 0), B is (12, 6, 0) and D is (6, 3, 9).

   F divides DB in the ratio 2:1.

   (*a*) Find the coordinates of the point F. **4**

   (*b*) Express $\overrightarrow{AF}$ in component form. **1**



[Turn over

Marks

4. Functions $f(x) = 3x - 1$ and $g(x) = x^2 + 7$ are defined on the set of real numbers.

   (*a*) Find $h(x)$ where $h(x) = g(f(x))$. **2**

   (*b*) (i) Write down the coordinates of the minimum turning point of $y = h(x)$.

   (ii) Hence state the range of the function h. **2**

5. Differentiate $(1 + 2\sin x)^4$ with respect to x. **2**

6. (*a*) The terms of a sequence satisfy $u_{n+1} = ku_n + 5$. Find the value of k which produces a sequence with a limit of 4. **2**

   (*b*) A sequence satisfies the recurrence relation $u_{n+1} = mu_n + 5$, $u_0 = 3$.

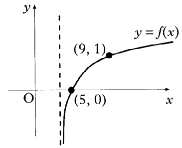
   (i) Express u_1 and u_2 in terms of m.

   (ii) Given that $u_2 = 7$, find the value of m which produces a sequence with no limit. **5**

7. The function f is of the form $f(x) = \log_b(x - a)$. The graph of $y = f(x)$ is shown in the diagram.

   (*a*) Write down the values of a and b. **2**

   (*b*) State the domain of f. **1**



8. A function f is defined by the formula $f(x) = 2x^3 - 7x^2 + 9$ where x is a real number.

   (*a*) Show that $(x - 3)$ is a factor of $f(x)$, and hence factorise $f(x)$ fully. **5**

   (*b*) Find the coordinates of the points where the curve with equation $y = f(x)$ crosses the x- and y-axes. **2**

   (*c*) Find the greatest and least values of f in the interval $-2 \le x \le 2$. **5**

9. If $\cos 2x = \frac{7}{25}$ and $0 < x < \frac{\pi}{2}$, find the exact values of $\cos x$ and $\sin x$. **4**

Marks

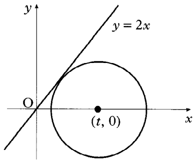
10. (*a*) Express $\sin x - \sqrt{3}\cos x$ in the form $k\sin(x - a)$ where $k > 0$ and $0 \le a \le 2\pi$. **4**

(*b*) Hence, or otherwise, sketch the curve with equation $y = 3 + \sin x - \sqrt{3}\cos x$ in the interval $0 \le x \le 2\pi$. **5**

11. (*a*) A circle has centre $(t, 0)$, $t > 0$, and radius 2 units.

Write down the equation of the circle. **1**

(*b*) Find the exact value of t such that the line $y = 2x$ is a tangent to the circle. **5**



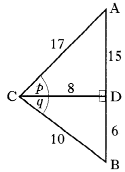
[*END OF QUESTION PAPER*]

ALL questions should be attempted.

Marks

1. Find $\int \frac{4x^3 - 1}{x^2}\,dx$, $x \ne 0$. **4**

2. Triangles ACD and BCD are right-angled at D with angles p and q and lengths as shown in the diagram.



(*a*) Show that the exact value of $\sin(p + q)$ is $\frac{84}{85}$. **4**

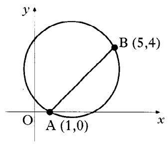
(*b*) Calculate the exact values of:

(i) $\cos(p + q)$;

(ii) $\tan(p + q)$. **3**

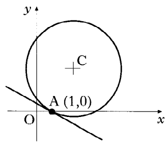
3. (*a*) A chord joins the points A(1,0) and B(5,4) on the circle as shown in the diagram.

Show that the equation of the perpendicular bisector of chord AB is $x + y = 5$. **4**



(*b*) The point C is the centre of this circle. The tangent at the point A on the circle has equation $x + 3y = 1$.

Find the equation of the radius CA. **4**



(*c*) (i) Determine the coordinates of the point C.

(ii) Find the equation of the circle. **4**

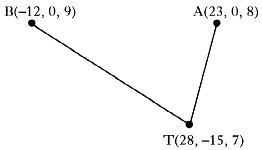
[Turn over

Marks

4. The sketch shows the positions of Andrew(A), Bob(B) and Tracy(T) on three hill-tops.

Relative to a suitable origin, the coordinates (in hundreds of metres) of the three people are A(23, 0, 8), B(−12, 0, 9) and T(28, −15, 7).

In the dark, Andrew and Bob locate Tracy using heat-seeking beams.

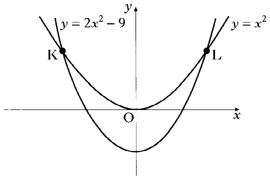


(*a*) Express the vectors $\overrightarrow{TA}$ and $\overrightarrow{TB}$ in component form. **2**

(*b*) Calculate the angle between these two beams. **5**

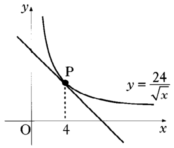
5. The curves with equations $y = x^2$ and $y = 2x^2 - 9$ intersect at K and L as shown.

Calculate the area enclosed between the curves. **8**



6. The diagram shows the graph of $y = \frac{24}{\sqrt{x}}$, $x > 0$.

Find the equation of the tangent at P, where $x = 4$. **6**



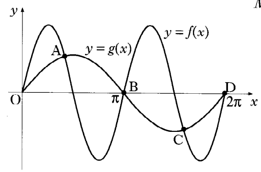
7. Solve the equation $\log_4(5 - x) - \log_4(3 - x) = 2$, $x < 3$. **4**

Marks

8. Two functions, f and g, are defined by $f(x) = k\sin 2x$ and $g(x) = \sin x$ where $k > 1$.

The diagram shows the graphs of $y = f(x)$ and $y = g(x)$ intersecting at O, A, B, C and D.

Show that, at A and C, $\cos x = \frac{1}{2k}$. **5**



9. The value V (in £ million) of a cruise ship t years after launch is given by the formula $V = 252e^{-0.06335t}$.

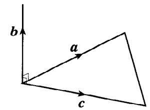
(*a*) What was its value when launched? **1**

(*b*) The owners decide to sell the ship once its value falls below £20 million. After how many years will it be sold? **4**

10. Vectors $\boldsymbol{a}$ and $\boldsymbol{c}$ are represented by two sides of an equilateral triangle with sides of length 3 units, as shown in the diagram.

Vector $\boldsymbol{b}$ is 2 units long and $\boldsymbol{b}$ is perpendicular to both $\boldsymbol{a}$ and $\boldsymbol{c}$.

Evaluate the scalar product $\boldsymbol{a}.(\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c})$. **4**



11. (*a*) Show that $x = -1$ is a solution of the cubic equation $x^3 + px^2 + px + 1 = 0$. **1**

(*b*) Hence find the range of values of p for which all the roots of the cubic equation are real. **7**

[END OF QUESTION PAPER]

2006 paper 1

ALL questions should be attempted.

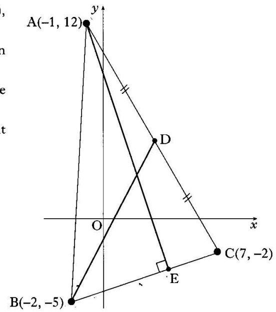
Marks

1. Triangle ABC has vertices A(–1, 12), B(–2, –5) and C(7, –2).

   (*a*) Find the equation of the median BD. **3**

   (*b*) Find the equation of the altitude AE. **3**

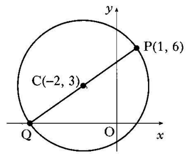
   (*c*) Find the coordinates of the point of intersection of BD and AE. **3**



2. A circle has centre C(–2, 3) and passes through P(1, 6).

   (*a*) Find the equation of the circle. **2**

   (*b*) PQ is a diameter of the circle. Find the equation of the tangent to this circle at Q. **4**



3. Two functions f and g are defined by $f(x) = 2x + 3$ and $g(x) = 2x - 3$, where x is a real number.

   (*a*) Find expressions for:

   (i) $f(g(x))$;

   (ii) $g(f(x))$. **3**

   (*b*) Determine the least possible value of the product $f(g(x)) \times g(f(x))$. **2**

[Turn over

Marks

4. A sequence is defined by the recurrence relation $u_{n+1} = 0{\cdot}8u_n + 12$, $u_0 = 4$.

   (*a*) State why this sequence has a limit. **1**

   (*b*) Find this limit. **2**

5. A function f is defined by $f(x) = (2x - 1)^5$.

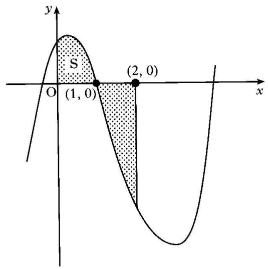
   Find the coordinates of the stationary point on the graph with equation $y = f(x)$ and determine its nature. **7**

6. The graph shown has equation $y = x^3 - 6x^2 + 4x + 1$.

   The total shaded area is bounded by the curve, the x-axis, the y-axis and the line $x = 2$.

   (*a*) Calculate the shaded area labelled S. **4**

   (*b*) Hence find the total shaded area. **3**



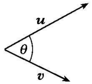
7. Solve the equation $\sin x° - \sin 2x° = 0$ in the interval $0 \le x \le 360$. **4**

8. (*a*) Express $2x^2 + 4x - 3$ in the form $a(x + b)^2 + c$. **3**

   (*b*) Write down the coordinates of the turning point on the parabola with equation $y = 2x^2 + 4x - 3$. **1**

Marks

9. $\boldsymbol{u}$ and $\boldsymbol{v}$ are vectors given by $\boldsymbol{u} = \begin{pmatrix} k^3 \\ 1 \\ k+2 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} 1 \\ 3k^2 \\ -1 \end{pmatrix}$, where $k > 0$.



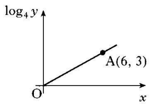
(*a*) If $\boldsymbol{u}.\boldsymbol{v} = 1$, show that $k^3 + 3k^2 - k - 3 = 0$. **2**

(*b*) Show that $(k + 3)$ is a factor of $k^3 + 3k^2 - k - 3$ and hence factorise $k^3 + 3k^2 - k - 3$ fully. **5**

(*c*) Deduce the only possible value of k. **1**

(*d*) The angle between $\boldsymbol{u}$ and $\boldsymbol{v}$ is θ. Find the exact value of $\cos\theta$. **3**

10. Two variables, x and y, are connected by the law $y = a^x$. The graph of $\log_4 y$ against x is a straight line passing through the origin and the point A(6, 3). Find the value of a. **4**



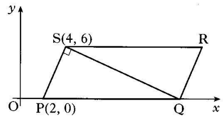
[*END OF QUESTION PAPER*]

ALL questions should be attempted.

Marks

1. PQRS is a parallelogram. P is the point (2, 0), S is (4, 6) and Q lies on the x-axis, as shown.

The diagonal QS is perpendicular to the side PS.



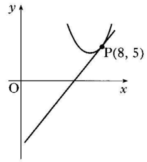
(*a*) Show that the equation of QS is $x + 3y = 22$. **4**

(*b*) Hence find the coordinates of Q and R. **2**

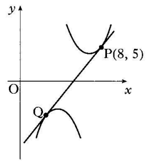
2. Find the value of k such that the equation $kx^2 + kx + 6 = 0$, $k \neq 0$, has equal roots. **4**

3. The parabola with equation $y = x^2 - 14x + 53$ has a tangent at the point P(8, 5).

(*a*) Find the equation of this tangent. **4**



(*b*) Show that the tangent found in (*a*) is also a tangent to the parabola with equation $y = -x^2 + 10x - 27$ and find the coordinates of the point of contact Q. **5**



4. The circles with equations $(x - 3)^2 + (y - 4)^2 = 25$ and $x^2 + y^2 - kx - 8y - 2k = 0$ have the same centre.

Determine the radius of the larger circle. **5**

Marks

5. The curve $y = f(x)$ is such that $\frac{dy}{dx} = 4x - 6x^2$. The curve passes through the point (–1, 9). Express y in terms of x. **4**

6. P is the point (–1, 2, –1) and Q is (3, 2, –4).

(*a*) Write down $\overrightarrow{PQ}$ in component form. **1**

(*b*) Calculate the length of $\overrightarrow{PQ}$. **1**

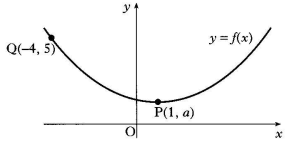
(*c*) Find the components of a unit vector which is parallel to $\overrightarrow{PQ}$. **1**

7. The diagram shows the graph of a function $y = f(x)$.

Copy the diagram and on it sketch the graphs of:

(*a*) $y = f(x - 4)$; **2**

(*b*) $y = 2 + f(x - 4)$. **2**

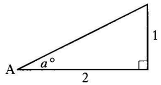


8. The diagram shows a right-angled triangle with height 1 unit, base 2 units and an angle of $a°$ at A.

(*a*) Find the exact values of:

(i) $\sin a°$;

(ii) $\sin 2a°$. **4**



(*b*) By expressing $\sin 3a°$ as $\sin(2a + a)°$, find the exact value of $\sin 3a°$. **4**

9. If $y = \frac{1}{x^3} - \cos 2x$, $x \neq 0$, find $\frac{dy}{dx}$. **4**

10. A curve has equation $y = 7\sin x - 24\cos x$.

(*a*) Express $7\sin x - 24\cos x$ in the form $k\sin(x - a)$ where $k > 0$ and $0 \leq a \leq \frac{\pi}{2}$. **4**

(*b*) Hence find, in the interval $0 \leq x \leq \pi$, the x-coordinate of the point on the curve where the gradient is 1. **3**

Marks

11. It is claimed that a wheel is made from wood which is over 1000 years old.

To test this claim, carbon dating is used.

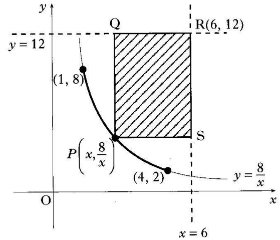
The formula $A(t) = A_0 e^{-0.000124t}$ is used to determine the age of the wood, where A_0 is the amount of carbon in any living tree, $A(t)$ is the amount of carbon in the wood being dated and t is the age of the wood in years.

For the wheel it was found that $A(t)$ was 88% of the amount of carbon in a living tree.

Is the claim true? **5**

12. PQRS is a rectangle formed according to the following conditions:

- it is bounded by the lines $x = 6$ and $y = 12$
- P lies on the curve with equation $y = \frac{8}{x}$ between (1, 8) and (4, 2)
- R is the point (6, 12).



(*a*) (i) Express the lengths of PS and RS in terms of x, the x-coordinate of P.

(ii) Hence show that the area, A square units, of PQRS is given by $A = 80 - 12x - \frac{48}{x}$. **3**

(*b*) Find the greatest and least possible values of A and the corresponding values of x for which they occur. **8**

[*END OF QUESTION PAPER*]

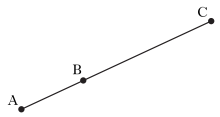
ALL questions should be attempted.

Marks

1. Find the equation of the line through the point (–1, 4) which is parallel to the line with equation $3x - y + 2 = 0$. **3**

2. Relative to a suitable coordinate system A and B are the points (–2, 1, –1) and (1, 3, 2) respectively.

A, B and C are collinear points and C is positioned such that BC = 2AB.



Find the coordinates of C. **4**

3. Functions f and g, defined on suitable domains, are given by $f(x) = x^2 + 1$ and $g(x) = 1 - 2x$.

Find:

(*a*) $g(f(x))$; **2**

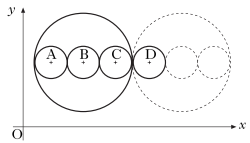
(*b*) $g(g(x))$. **2**

4. Find the range of values of k such that the equation $kx^2 - x - 1 = 0$ has no real roots. **4**

5. The large circle has equation $x^2 + y^2 - 14x - 16y + 77 = 0$.

Three congruent circles with centres A, B and C are drawn inside the large circle with the centres lying on a line parallel to the x-axis.

This pattern is continued, as shown in the diagram.



Find the equation of the circle with centre D. **5**

[Turn over

Marks

6. Solve the equation $\sin 2x° = 6\cos x°$ for $0 \leq x \leq 360$. **4**

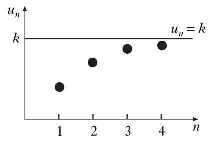
7. A sequence is defined by the recurrence relation

$$u_{n+1} = \frac{1}{4}u_n + 16, \; u_0 = 0.$$

(*a*) Calculate the values of u_1, u_2 and u_3. **3**

Four terms of this sequence, u_1, u_2, u_3 and u_4 are plotted as shown in the graph.

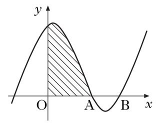
As $n \to \infty$, the points on the graph approach the line $u_n = k$, where k is the limit of this sequence.



(*b*) (i) Give a reason why this sequence has a limit.

(ii) Find the exact value of k. **3**

8. The diagram shows a sketch of the graph of $y = x^3 - 4x^2 + x + 6$.



(*a*) Show that the graph cuts the x-axis at (3, 0). **1**

(*b*) Hence or otherwise find the coordinates of A. **3**

(*c*) Find the shaded area. **5**

9. A function f is defined by the formula $f(x) = 3x - x^3$.

(*a*) Find the exact values where the graph of $y = f(x)$ meets the x- and y-axes. **2**

(*b*) Find the coordinates of the stationary points of the function and determine their nature. **7**

(*c*) Sketch the graph of $y = f(x)$. **1**

Marks

10. Given that $y = \sqrt{3x^2 + 2}$, find $\frac{dy}{dx}$. **3**

11. (*a*) Express $f(x) = \sqrt{3}\cos x + \sin x$ in the form $k\cos(x - a)$, where $k > 0$ and $0 < a < \frac{\pi}{2}$. **4**

(*b*) Hence or otherwise sketch the graph of $y = f(x)$ in the interval $0 \leq x \leq 2\pi$. **4**

[*END OF QUESTION PAPER*]

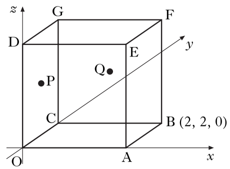
ALL questions should be attempted.

Marks

1. OABCDEFG is a cube with side 2 units, as shown in the diagram.

B has coordinates (2, 2, 0).

P is the centre of face OCGD and Q is the centre of face CBFG.

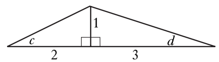


(*a*) Write down the coordinates of G. **1**

(*b*) Find $\boldsymbol{p}$ and $\boldsymbol{q}$, the position vectors of points P and Q. **2**

(*c*) Find the size of angle POQ. **5**

2. The diagram shows two right-angled triangles with angles c and d marked as shown.



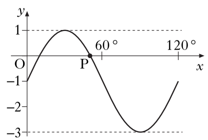
(*a*) Find the exact value of $\sin(c + d)$. **4**

(*b*) (i) Find the exact value of $\sin 2c$.

(ii) Show that $\cos 2d$ has the same exact value. **4**

3. Show that the line with equation $y = 6 - 2x$ is a tangent to the circle with equation $x^2 + y^2 + 6x - 4y - 7 = 0$ and find the coordinates of the point of contact of the tangent and the circle. **6**

4. The diagram shows part of the graph of a function whose equation is of the form $y = a\sin(bx°) + c$.



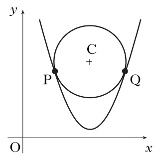
(*a*) Write down the values of a, b and c. **3**

(*b*) Determine the exact value of the x-coordinate of P, the point where the graph intersects the x-axis as shown in the diagram. **3**

[Turn over

Marks

5. A circle centre C is situated so that it touches the parabola with equation $y = \frac{1}{2}x^2 - 8x + 34$ at P and Q.



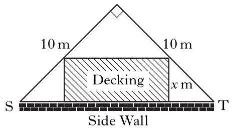
(*a*) The gradient of the tangent to the parabola at Q is 4. Find the coordinates of Q. **5**

(*b*) Find the coordinates of P. **2**

(*c*) Find the coordinates of C, the centre of the circle. **2**

6. A householder has a garden in the shape of a right-angled isosceles triangle.

It is intended to put down a section of rectangular wooden decking at the side of the house, as shown in the diagram.



(*a*) (i) Find the exact value of ST.

(ii) Given that the breadth of the decking is x metres, show that the area of the decking, A square metres, is given by

$$A = \left(10\sqrt{2}\right)x - 2x^2.$$ **3**

(*b*) Find the dimensions of the decking which maximises its area. **5**

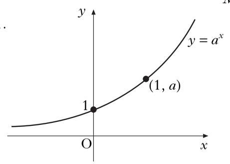
7. Find the value of $\int_0^2 \sin(4x+1)\,dx$. **4**

8. The curve with equation $y = \log_3(x-1) - 2{\cdot}2$, where $x > 1$, cuts the x-axis at the point $(a, 0)$.

Find the value of a. **4**

Marks

9. The diagram shows the graph of $y = a^x$, $a > 1$.

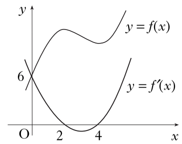


On separate diagrams, sketch the graphs of:

(*a*) $y = a^{-x}$; **2**

(*b*) $y = a^{1-x}$. **2**

10. The diagram shows the graphs of a cubic function $y = f(x)$ and its derived function $y = f'(x)$.



Both graphs pass through the point (0, 6).

The graph of $y = f'(x)$ also passes through the points (2, 0) and (4, 0).

(*a*) Given that $f'(x)$ is of the form $k(x-a)(x-b)$:

(i) write down the values of a and b;

(ii) find the value of k. **3**

(*b*) Find the equation of the graph of the cubic function $y = f(x)$. **4**

11. Two variables x and y satisfy the equation $y = 3 \times 4^x$.

(*a*) Find the value of a if $(a, 6)$ lies on the graph with equation $y = 3 \times 4^x$. **1**

(*b*) If $(-\frac{1}{2}, b)$ also lies on the graph, find b. **1**

(*c*) A graph is drawn of $\log_{10}y$ against x. Show that its equation will be of the form $\log_{10}y = Px + Q$ and state the gradient of this line. **4**

[*END OF QUESTION PAPER*]

2008 paper 1

SECTION A

ALL questions should be attempted.

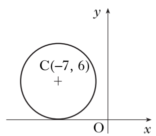
1. A sequence is defined by the recurrence relation

$$u_{n+1} = 0{\cdot}3u_n + 6 \text{ with } u_{10} = 10.$$

What is the value of u_{12}?

A 6·6
B 7·8
C 8·7
D 9·6

2. The x-axis is a tangent to a circle with centre (–7, 6) as shown in the diagram.



What is the equation of the circle?

A $(x+7)^2 + (y-6)^2 = 1$
B $(x+7)^2 + (y-6)^2 = 49$
C $(x-7)^2 + (y+6)^2 = 36$
D $(x+7)^2 + (y-6)^2 = 36$

3. The vectors $\boldsymbol{u} = \begin{pmatrix} k \\ -1 \\ 1 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} 0 \\ 4 \\ k \end{pmatrix}$ are perpendicular.

What is the value of k?

A 0
B 3
C 4
D 5

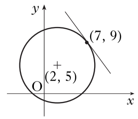
4. A sequence is generated by the recurrence relation $u_{n+1} = 0{\cdot}4u_n - 240$.

What is the limit of this sequence as $n \to \infty$?

A –800
B –400
C 200
D 400

5. The diagram shows a circle, centre (2, 5) and a tangent drawn at the point (7, 9).

What is the equation of this tangent?



A $y - 9 = -\frac{5}{4}(x-7)$

B $y + 9 = -\frac{4}{5}(x+7)$

C $y - 7 = \frac{4}{5}(x-9)$

D $y + 9 = \frac{5}{4}(x+7)$

[Turn over

6. What is the solution of the equation $2\sin x - \sqrt{3} = 0$ where $\frac{\pi}{2} \leq x \leq \pi$?

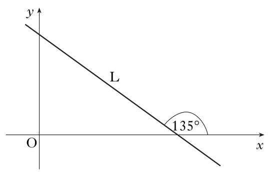
A $\frac{\pi}{6}$

B $\frac{2\pi}{3}$

C $\frac{3\pi}{4}$

D $\frac{5\pi}{6}$

7. The diagram shows a line L; the angle between L and the positive direction of the x-axis is 135°, as shown.



What is the gradient of line L?

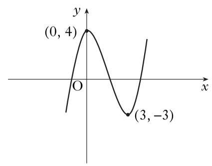
A $-\frac{1}{2}$

B $-\frac{\sqrt{3}}{2}$

C -1

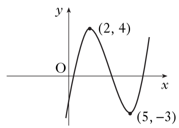
D $\frac{1}{2}$

8. The diagram shows part of the graph of a function with equation $y = f(x)$.

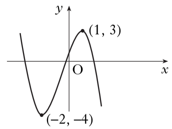


Which of the following diagrams shows the graph with equation $y = -f(x - 2)$?

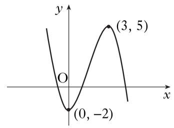
A



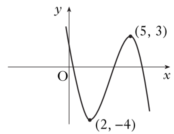
B



C



D



9. Given that $0 \le a \le \frac{\pi}{2}$ and $\sin a = \frac{3}{5}$, find an expression for $\sin(x + a)$.

A $\sin x + \frac{3}{5}$

B $\frac{4}{5}\sin x + \frac{3}{5}\cos x$

C $\frac{3}{5}\sin x - \frac{4}{5}\cos x$

D $\frac{2}{5}\sin x - \frac{3}{5}\cos x$

10. Here are two statements about the roots of the equation $x^2 + x + 1 = 0$:

(1) the roots are equal;
(2) the roots are real.

Which of the following is true?

A Neither statement is correct.
B Only statement (1) is correct.
C Only statement (2) is correct.
D Both statements are correct.

11. E(−2, −1, 4), P(1, 5, 7) and F(7, 17, 13) are three collinear points.

P lies between E and F.

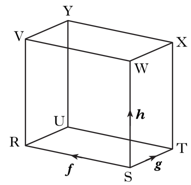
What is the ratio in which P divides EF?

A 1:1
B 1:2
C 1:4
D 1:6

12. In the diagram RSTU, VWXY represents a cuboid.

$\overrightarrow{SR}$ represents vector $\boldsymbol{f}$, $\overrightarrow{ST}$ represents vector $\boldsymbol{g}$ and $\overrightarrow{SW}$ represents vector $\boldsymbol{h}$.

Express $\overrightarrow{VT}$ in terms of $\boldsymbol{f}$, $\boldsymbol{g}$ and $\boldsymbol{h}$.



A $\overrightarrow{VT} = \boldsymbol{f} + \boldsymbol{g} + \boldsymbol{h}$

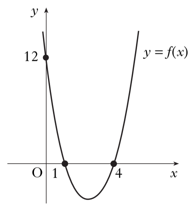
B $\overrightarrow{VT} = \boldsymbol{f} - \boldsymbol{g} + \boldsymbol{h}$

C $\overrightarrow{VT} = -\boldsymbol{f} + \boldsymbol{g} - \boldsymbol{h}$

D $\overrightarrow{VT} = -\boldsymbol{f} - \boldsymbol{g} + \boldsymbol{h}$

13. The diagram shows part of the graph of a quadratic function $y = f(x)$.

The graph has an equation of the form $y = k(x - a)(x - b)$.



What is the equation of the graph?

A $y = 3(x - 1)(x - 4)$
B $y = 3(x + 1)(x + 4)$
C $y = 12(x - 1)(x - 4)$
D $y = 12(x + 1)(x + 4)$

14. Find $\int 4\sin(2x+3)\,dx$.

A $-4\cos(2x+3)+c$

B $-2\cos(2x+3)+c$

C $4\cos(2x+3)+c$

D $8\cos(2x+3)+c$

15. What is the derivative of $(x^3+4)^2$?

A $(3x^2+4)^2$

B $\frac{1}{3}(x^3+4)^3$

C $6x^2(x^3+4)$

D $2(3x^2+4)^{-1}$

16. $2x^2+4x+7$ is expressed in the form $2(x+p)^2+q$.

What is the value of q?

A 5

B 7

C 9

D 11

17. A function f is given by $f(x)=\sqrt{9-x^2}$.

What is a suitable domain of f?

A $x \geq 3$

B $x \leq 3$

C $-3 \leq x \leq 3$

D $-9 \leq x \leq 9$

18. Vectors $\boldsymbol{p}$ and $\boldsymbol{q}$ are such that $|\boldsymbol{p}|=3$, $|\boldsymbol{q}|=4$ and $\boldsymbol{p}.\boldsymbol{q}=10$.

Find the value of $\boldsymbol{q}.(\boldsymbol{p}+\boldsymbol{q})$.

A 0

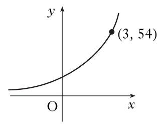
B 14

C 26

D 28

19. The diagram shows part of the graph whose equation is of the form $y=2m^x$.

What is the value of m?



A 2

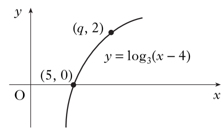
B 3

C 8

D 18

20. The diagram shows part of the graph of $y=\log_3(x-4)$.

The point $(q, 2)$ lies on the graph.



What is the value of q?

A 6

B 7

C 8

D 13

[*END OF SECTION A*]

SECTION B

ALL questions should be attempted.

Marks

21. A function f is defined on the set of real numbers by $f(x) = x^3 - 3x + 2$.

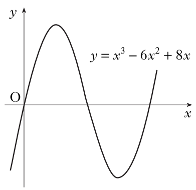
(*a*) Find the coordinates of the stationary points on the curve $y = f(x)$ and determine their nature. **6**

(*b*) (i) Show that $(x - 1)$ is a factor of $x^3 - 3x + 2$.

(ii) Hence or otherwise factorise $x^3 - 3x + 2$ fully. **5**

(*c*) State the coordinates of the points where the curve with equation $y = f(x)$ meets both the axes and hence sketch the curve. **4**

22. The diagram shows a sketch of the curve with equation $y = x^3 - 6x^2 + 8x$.



(*a*) Find the coordinates of the points on the curve where the gradient of the tangent is –1. **5**

(*b*) The line $y = 4 - x$ is a tangent to this curve at a point A. Find the coordinates of A. **2**

23. Functions f, g and h are defined on suitable domains by

$f(x) = x^2 - x + 10$, $g(x) = 5 - x$ and $h(x) = \log_2 x$.

(*a*) Find expressions for $h(f(x))$ and $h(g(x))$. **3**

(*b*) Hence solve $h(f(x)) - h(g(x)) = 3$. **5**

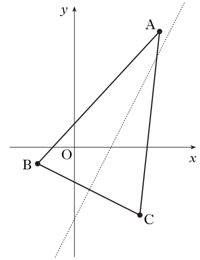
[*END OF SECTION B*]

[*END OF QUESTION PAPER*]

ALL questions should be attempted.

Marks

1. The vertices of triangle ABC are A(7, 9), B(–3, –1) and C(5, –5) as shown in the diagram.



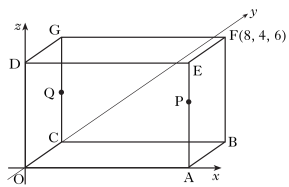
The broken line represents the perpendicular bisector of BC.

(*a*) Show that the equation of the perpendicular bisector of BC is $y = 2x - 5$. **4**

(*b*) Find the equation of the median from C. **3**

(*c*) Find the coordinates of the point of intersection of the perpendicular bisector of BC and the median from C. **3**

2. The diagram shows a cuboid OABC, DEFG.



F is the point (8, 4, 6).

P divides AE in the ratio 2:1.

Q is the midpoint of CG.

(*a*) State the coordinates of P and Q. **2**

(*b*) Write down the components of $\overrightarrow{PQ}$ and $\overrightarrow{PA}$. **2**

(*c*) Find the size of angle QPA. **5**

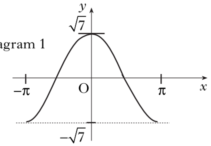
[Turn over

Marks

3. (*a*) (i) Diagram 1 shows part of the graph of $y = f(x)$, where $f(x) = p\cos x$.

Write down the value of p.

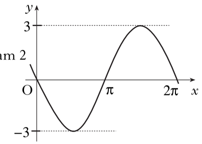
Diagram 1



(ii) Diagram 2 shows part of the graph of $y = g(x)$, where $g(x) = q\sin x$.

Write down the value of q. **2**

Diagram 2



(*b*) Write $f(x) + g(x)$ in the form $k\cos(x + a)$ where $k > 0$ and $0 < a < \frac{\pi}{2}$. **4**

(*c*) Hence find $f'(x) + g'(x)$ as a single trigonometric expression. **2**

4. (*a*) Write down the centre and calculate the radius of the circle with equation $x^2 + y^2 + 8x + 4y - 38 = 0$. **2**

(*b*) A second circle has equation $(x - 4)^2 + (y - 6)^2 = 26$.

Find the distance between the centres of these two circles and hence show that the circles intersect. **4**

(*c*) The line with equation $y = 4 - x$ is a common chord passing through the points of intersection of the two circles.

Find the coordinates of the points of intersection of the two circles. **5**

5. Solve the equation $\cos 2x° + 2\sin x° = \sin^2 x°$ in the interval $0 \leq x < 360$. **5**

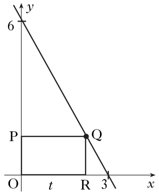
Marks

6. In the diagram, Q lies on the line joining (0, 6) and (3, 0).

OPQR is a rectangle, where P and R lie on the axes and OR = t.

(*a*) Show that QR = $6 - 2t$. **3**

(*b*) Find the coordinates of Q for which the rectangle has a maximum area. **6**

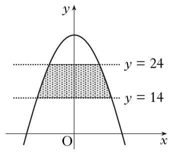


7. The parabola shown in the diagram has equation

$$y = 32 - 2x^2.$$

The shaded area lies between the lines $y = 14$ and $y = 24$.

Calculate the shaded area. **8**



[*END OF QUESTION PAPER*]

SECTION A

ALL questions should be attempted.

1. A sequence is defined by $u_{n+1} = 3u_n + 4$ with $u_1 = 2$.

What is the value of u_3?

A 34

B 21

C 18

D 13

2. A circle has equation $x^2 + y^2 + 8x + 6y - 75 = 0$.

What is the radius of this circle?

A 5

B 10

C $\sqrt{75}$

D $\sqrt{175}$

3. Triangle PQR has vertices at P(–3, –2), Q(–1, 4) and R(3, 6).

PS is a median. What is the gradient of PS?

A -2

B $-\frac{7}{4}$

C 1

D $\frac{7}{4}$

4. A curve has equation $y = 5x^3 - 12x$.

What is the gradient of the tangent at the point (1, –7)?

A –7

B –5

C 3

D 5

5. Here are two statements about the points S(2, 3) and T(5, –1):

(1) The length of ST = 5 units;

(2) The gradient of ST = $\frac{4}{3}$.

Which of the following is true?

A Neither statement is correct.

B Only statement (1) is correct.

C Only statement (2) is correct.

D Both statements are correct.

6. A sequence is generated by the recurrence relation $u_{n+1} = 0{\cdot}7u_n + 10$.

What is the limit of this sequence as $n \to \infty$?

A $\frac{100}{3}$

B $\frac{100}{7}$

C $\frac{17}{100}$

D $\frac{3}{10}$

7. If the exact value of $\cos x$ is $\frac{1}{\sqrt{5}}$, find the exact value of $\cos 2x$.

A $-\frac{3}{5}$

B $-\frac{2}{\sqrt{5}}$

C $\frac{2}{\sqrt{5}}$

D $\frac{3}{5}$

[Turn over

8. What is the derivative of $\frac{1}{4x^3}$, $x \neq 0$?

A $\frac{1}{12x^2}$

B $-\frac{1}{12x^2}$

C $\frac{4}{x^4}$

D $-\frac{3}{4x^4}$

9. The line with equation $y = 2x$ intersects the circle with equation $x^2 + y^2 = 5$ at the points J and K.

What are the x-coordinates of J and K?

A $x_J = 1$, $x_K = -1$

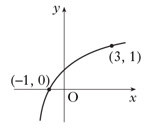
B $x_J = 2$, $x_K = -2$

C $x_J = 1$, $x_K = -2$

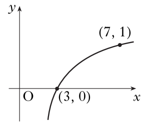
D $x_J = -1$, $x_K = 2$

10. Which of the following graphs has equation $y = \log_5(x - 2)$?

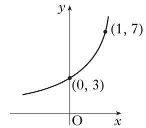
A



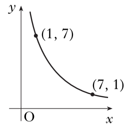
B



C



D



[Turn over

11. How many solutions does the equation

$$(4\sin x - \sqrt{5})(\sin x + 1) = 0$$

have in the interval $0 \le x < 2\pi$?

A 4

B 3

C 2

D 1

12. A function f is given by $f(x) = 2x^2 - x - 9$.

Which of the following describes the nature of the roots of $f(x) = 0$?

A No real roots

B Equal roots

C Real distinct roots

D Rational distinct roots

13. k and a are given by

$$k \sin a^\circ = 1$$
$$k \cos a^\circ = \sqrt{3}$$

where $k > 0$ and $0 \le a < 90$.

What are the values of k and a?

	k	a
A	2	60
B	2	30
C	$\sqrt{10}$	60
D	$\sqrt{10}$	30

14. If $f(x) = 2\sin\left(3x - \frac{\pi}{2}\right) + 5$, what is the range of values of $f(x)$?

A $-1 \le f(x) \le 11$

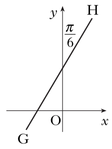
B $2 \le f(x) \le 8$

C $3 \le f(x) \le 7$

D $-3 \le f(x) \le 7$

15. The line GH makes an angle of $\frac{\pi}{6}$ radians with the y-axis, as shown in the diagram.

What is the gradient of GH?



A $\sqrt{3}$

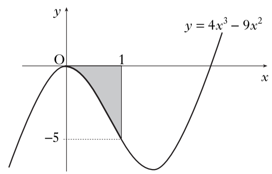
B $\frac{1}{2}$

C $\frac{1}{\sqrt{2}}$

D $\frac{\sqrt{3}}{2}$

16. The graph of $y = 4x^3 - 9x^2$ is shown in the diagram.

Which of the following gives the area of the shaded section?



A $\left[x^4 - 3x^3\right]_{-5}^{0}$

B $-\left[x^4 - 3x^3\right]_{0}^{1}$

C $\left[12x^2 - 18x\right]_{-5}^{0}$

D $-\left[12x^2 - 18x\right]_{0}^{1}$

17. The vector $\boldsymbol{u}$ has components $\begin{pmatrix} -3 \\ 0 \\ 4 \end{pmatrix}$.

Which of the following is a unit vector parallel to $\boldsymbol{u}$?

A $-\frac{3}{5}\boldsymbol{i} + \frac{4}{5}\boldsymbol{k}$

B $-3\boldsymbol{i} + 4\boldsymbol{k}$

C $-\frac{3}{\sqrt{7}}\boldsymbol{i} + \frac{4}{\sqrt{7}}\boldsymbol{k}$

D $-\frac{1}{3}\boldsymbol{i} + \frac{1}{4}\boldsymbol{k}$

18. Given that $f(x) = (4 - 3x^2)^{-\frac{1}{2}}$ on a suitable domain, find $f'(x)$.

A $-3x(4 - 3x^2)^{-\frac{1}{2}}$

B $-\frac{1}{2}(4 - 6x)^{-\frac{3}{2}}$

C $2(4 - 3x^3)^{\frac{1}{2}}$

D $3x(4 - 3x^2)^{-\frac{3}{2}}$

19. For what values of x is $6 + x - x^2 < 0$?

A $x > 3$ only

B $x < -2$ only

C $x < -2$, $x > 3$

D $-3 < x < 2$

20. $A = 2\pi r^2 + 6\pi r$.

What is the rate of change of A with respect to r when $r = 2$?

A 10π

B 12π

C 14π

D 20π

[*END OF SECTION A*]

SECTION B

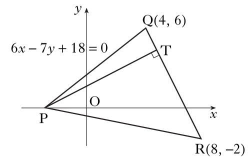
ALL questions should be attempted.

Marks

21. Triangle PQR has vertex P on the x-axis, as shown in the diagram.

Q and R are the points (4, 6) and (8, −2) respectively.

The equation of PQ is $6x - 7y + 18 = 0$.



(*a*) State the coordinates of P. **1**

(*b*) Find the equation of the altitude of the triangle from P. **3**

(*c*) The altitude from P meets the line QR at T. Find the coordinates of T. **4**

22. D, E and F have coordinates (10, −8, −15), (1, −2, −3) and (−2, 0, 1) respectively.

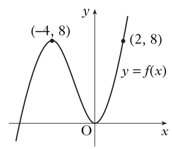
(*a*) (i) Show that D, E and F are collinear.

(ii) Find the ratio in which E divides DF. **4**

(*b*) G has coordinates (k, 1, 0).

Given that DE is perpendicular to GE, find the value of k. **4**

23. The diagram shows a sketch of the function $y = f(x)$.



(*a*) Copy the diagram and on it sketch the graph of $y = f(2x)$. **2**

(*b*) On a separate diagram sketch the graph of $y = 1 - f(2x)$. **3**

[Turn over for Question 24 on *Page twelve*

24. (*a*) Using the fact that $\frac{7\pi}{12} = \frac{\pi}{3} + \frac{\pi}{4}$, find the exact value of $\sin\left(\frac{7\pi}{12}\right)$. **3**

(*b*) Show that $\sin(A + B) + \sin(A - B) = 2\sin A \cos B$. **2**

(*c*) (i) Express $\frac{\pi}{12}$ in terms of $\frac{\pi}{3}$ and $\frac{\pi}{4}$.

(ii) Hence or otherwise find the exact value of $\sin\left(\frac{7\pi}{12}\right) + \sin\left(\frac{\pi}{12}\right)$. **4**

[*END OF SECTION B*]

[*END OF QUESTION PAPER*]

ALL questions should be attempted.

Marks

1. Find the coordinates of the turning points of the curve with equation $y = x^3 - 3x^2 - 9x + 12$ and determine their nature. **8**

2. Functions f and g are given by $f(x) = 3x + 1$ and $g(x) = x^2 - 2$.

(*a*) (i) Find $p(x)$ where $p(x) = f(g(x))$.

(ii) Find $q(x)$ where $q(x) = g(f(x))$. **3**

(*b*) Solve $p'(x) = q'(x)$. **3**

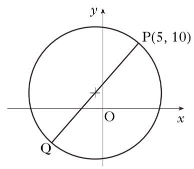
3. (*a*) (i) Show that $x = 1$ is a root of $x^3 + 8x^2 + 11x - 20 = 0$.

(ii) Hence factorise $x^3 + 8x^2 + 11x - 20$ fully. **4**

(*b*) Solve $\log_2(x + 3) + \log_2(x^2 + 5x - 4) = 3$. **5**

4. (*a*) Show that the point P(5, 10) lies on circle C_1 with equation $(x + 1)^2 + (y - 2)^2 = 100$. **1**

(*b*) PQ is a diameter of this circle as shown in the diagram. Find the equation of the tangent at Q. **5**



(*c*) Two circles, C_2 and C_3, touch circle C_1 at Q.

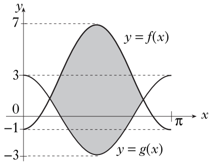
The radius of each of these circles is twice the radius of circle C_1.

Find the equations of circles C_2 and C_3. **4**

[Turn over

Marks

5. The graphs of $y = f(x)$ and $y = g(x)$ are shown in the diagram.



$f(x) = -4\cos(2x) + 3$ and $g(x)$ is of the form $g(x) = m\cos(nx)$.

(*a*) Write down the values of m and n. **1**

(*b*) Find, correct to one decimal place, the coordinates of the points of intersection of the two graphs in the interval $0 \le x \le \pi$. **5**

(*c*) Calculate the shaded area. **6**

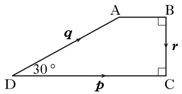
6. The size of the human population, N, can be modelled using the equation $N = N_0e^{rt}$ where N_0 is the population in 2006, t is the time in years since 2006, and r is the annual rate of increase in the population.

(*a*) In 2006 the population of the United Kingdom was approximately 61 million, with an annual rate of increase of 1·6%. Assuming this growth rate remains constant, what would be the population in 2020? **2**

(*b*) In 2006 the population of Scotland was approximately 5·1 million, with an annual rate of increase of 0·43%.

Assuming this growth rate remains constant, how long would it take for Scotland's population to double in size? **3**

7. Vectors $\boldsymbol{p}$, $\boldsymbol{q}$ and $\boldsymbol{r}$ are represented on the diagram shown where angle ADC = 30°.



It is also given that $|\boldsymbol{p}| = 4$ and $|\boldsymbol{q}| = 3$.

(*a*) Evaluate $\boldsymbol{p}.(\boldsymbol{q} + \boldsymbol{r})$ and $\boldsymbol{r}.(\boldsymbol{p} - \boldsymbol{q})$. **6**

(*b*) Find $|\boldsymbol{q} + \boldsymbol{r}|$ and $|\boldsymbol{p} - \boldsymbol{q}|$. **4**

[*END OF QUESTION PAPER*]

SECTION A

ALL questions should be attempted.

1. A line L is perpendicular to the line with equation $2x - 3y - 6 = 0$.

What is the gradient of the line L?

A $-\frac{3}{2}$

B $-\frac{1}{2}$

C $\frac{2}{3}$

D 2

2. A sequence is defined by the recurrence relation $u_{n+1} = 2u_n + 3$ and $u_0 = 1$.

What is the value of u_2?

A 7

B 10

C 13

D 16

3. Given that $\mathbf{u} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} -1 \\ 2 \\ 4 \end{pmatrix}$, find $3\mathbf{u} - 2\mathbf{v}$ in component form.

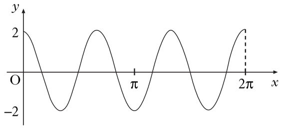
A $\begin{pmatrix} 4 \\ -1 \\ -5 \end{pmatrix}$

B $\begin{pmatrix} 4 \\ -4 \\ 11 \end{pmatrix}$

C $\begin{pmatrix} 8 \\ -1 \\ 5 \end{pmatrix}$

D $\begin{pmatrix} 8 \\ -4 \\ -5 \end{pmatrix}$

4. The diagram shows the graph with equation of the form $y = a\cos bx$ for $0 \leq x \leq 2\pi$.



What is the equation of this graph?

A $y = 2\cos 3x$

B $y = 2\cos 2x$

C $y = 3\cos 2x$

D $y = 4\cos 3x$

5. When $x^2 + 8x + 3$ is written in the form $(x + p)^2 + q$, what is the value of q?

A −19

B −13

C −5

D 19

[Turn over

6. The roots of the equation $kx^2 - 3x + 2 = 0$ are equal.

What is the value of k?

A $-\frac{9}{8}$

B $-\frac{8}{9}$

C $\frac{8}{9}$

D $\frac{9}{8}$

7. A sequence is generated by the recurrence relation $u_{n+1} = \frac{1}{4}u_n + 7$, with $u_0 = -2$.

What is the limit of this sequence as $n \to \infty$?

A $\frac{1}{28}$

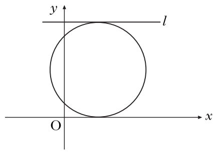
B $\frac{28}{5}$

C $\frac{28}{3}$

D 28

8. The equation of the circle shown in the diagram is $x^2 + y^2 - 6x - 10y + 9 = 0$.

The x-axis and the line l are parallel tangents to the circle.



What is the equation of line l?

A $y = 5$

B $y = 10$

C $y = 18$

D $y = 20$

9. Find $\int (2x^{-4} + \cos 5x)\,dx$.

A $-\frac{2}{5}x^{-5} - 5\sin 5x + c$

B $-\frac{2}{5}x^{-5} + \frac{1}{5}\sin 5x + c$

C $-\frac{2}{3}x^{-3} + \frac{1}{5}\sin 5x + c$

D $-\frac{2}{3}x^{-3} - 5\sin 5x + c$

10. The vectors $x\mathbf{i} + 5\mathbf{j} + 7\mathbf{k}$ and $-3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ are perpendicular.

What is the value of x?

A 0

B 1

C $\frac{4}{3}$

D $\frac{10}{3}$

[Turn over

11. Functions f and g are defined on suitable domains by $f(x) = \cos x$ and $g(x) = x + \frac{\pi}{6}$.

What is the value of $f\left(g\left(\frac{\pi}{6}\right)\right)$?

A $\frac{1}{2} + \frac{\pi}{6}$

B $\frac{\sqrt{3}}{2} + \frac{\pi}{6}$

C $\frac{\sqrt{3}}{2}$

D $\frac{1}{2}$

12. If $f(x) = \frac{1}{\sqrt[5]{x}},\ x \neq 0$, what is $f'(x)$?

A $-\frac{1}{5}x^{-\frac{6}{5}}$

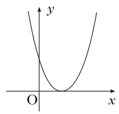
B $-\frac{1}{5}x^{-\frac{4}{5}}$

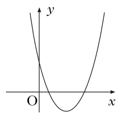
C $-\frac{5}{2}x^{-\frac{7}{2}}$

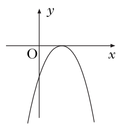
D $-\frac{5}{2}x^{-\frac{3}{2}}$

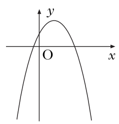
13. Which of the following diagrams shows a parabola with equation $y = ax^2 + bx + c$, where

- $a > 0$
- $b^2 - 4ac > 0$?

A


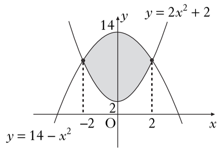
B


C


D


[Turn over

14. The diagram shows graphs with equations $y = 14 - x^2$ and $y = 2x^2 + 2$.



Which of the following represents the shaded area?

A $\int_{2}^{14} (12 - 3x^2)\,dx$

B $\int_{2}^{14} (3x^2 - 12)\,dx$

C $\int_{-2}^{2} (12 - 3x^2)\,dx$

D $\int_{-2}^{2} (3x^2 - 12)\,dx$

15. The derivative of a function f is given by $f'(x) = x^2 - 9$.

Here are two statements about f:

(1) f is increasing at $x = 1$;

(2) f is stationary at $x = -3$.

Which of the following is true?

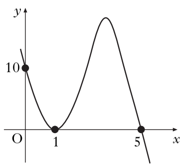
A Neither statement is correct.

B Only statement (1) is correct.

C Only statement (2) is correct.

D Both statements are correct.

16. The diagram shows the graph with equation $y = k(x-1)^2(x+t)$.



What are the values of k and t?

	k	t
A	−2	−5
B	−2	5
C	2	−5
D	2	5

17. If $s(t) = t^2 - 5t + 8$, what is the rate of change of s with respect to t when $t = 3$?

A −5

B 1

C 2

D 9

18. What is the solution of $x^2 + 4x > 0$, where x is a real number?

A $-4 < x < 0$

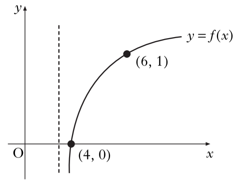
B $x < -4, x > 0$

C $0 < x < 4$

D $x < 0, x > 4$

[Turn over

19. The diagram shows the graph of $y = f(x)$ where f is a logarithmic function.



What is $f(x)$?

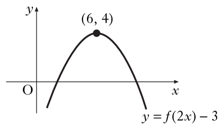
A $f(x) = \log_6(x-3)$

B $f(x) = \log_3(x+3)$

C $f(x) = \log_3(x-3)$

D $f(x) = \log_6(x+3)$

20. The diagram shows the graph of $y = f(2x) - 3$.



What are the coordinates of the turning point on the graph of $y = f(x)$?

A (12, 7)

B (12, 1)

C (3, 7)

D (3, 1)

[*END OF SECTION A*]

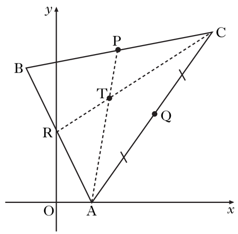
Marks

SECTION B

ALL questions should be attempted.

21. Triangle ABC has vertices A(4, 0), B(–4, 16) and C(18, 20), as shown in the diagram opposite.

Medians AP and CR intersect at the point T(6, 12).



(*a*) Find the equation of median BQ. **3**

(*b*) Verify that T lies on BQ. **1**

(*c*) Find the ratio in which T divides BQ. **2**

22. (*a*) (i) Show that $(x - 1)$ is a factor of $f(x) = 2x^3 + x^2 - 8x + 5$.

(ii) Hence factorise $f(x)$ fully. **5**

(*b*) Solve $2x^3 + x^2 - 8x + 5 = 0$. **1**

(*c*) The line with equation $y = 2x - 3$ is a tangent to the curve with equation $y = 2x^3 + x^2 - 6x + 2$ at the point G.

Find the coordinates of G. **5**

(*d*) This tangent meets the curve again at the point H.

Write down the coordinates of H. **1**

[Turn over for Question 23 on *Page fourteen*

Marks

23. (*a*) Diagram 1 shows a right angled triangle, where the line OA has equation $3x - 2y = 0$.

(i) Show that $\tan a = \frac{3}{2}$.

(ii) Find the value of $\sin a$. **4**

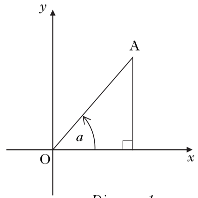


Diagram 1

(*b*) A second right angled triangle is added as shown in Diagram 2.

The line OB has equation $3x - 4y = 0$.

Find the values of $\sin b$ and $\cos b$. **4**

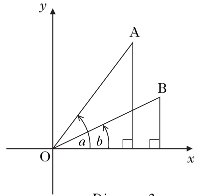


Diagram 2

(*c*) (i) Find the value of $\sin(a - b)$.

(ii) State the value of $\sin(b - a)$. **4**

[*END OF SECTION B*]

[*END OF QUESTION PAPER*]

2010 paper 2

Marks

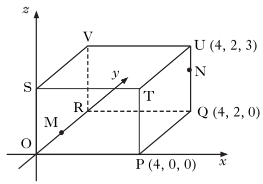
ALL questions should be attempted.

1. The diagram shows a cuboid OPQR,STUV relative to the coordinate axes.

P is the point (4, 0, 0),
Q is (4, 2, 0) and U is (4, 2, 3).

M is the midpoint of OR.

N is the point on UQ such that $UN = \frac{1}{3}UQ$.



(a) State the coordinates of M and N. **2**

(b) Express $\overrightarrow{VM}$ and $\overrightarrow{VN}$ in component form. **2**

(c) Calculate the size of angle MVN. **5**

2. (a) $12\cos x° - 5\sin x°$ can be expressed in the form $k\cos(x + a)°$, where $k > 0$ and $0 \le a < 360$.

Calculate the values of k and a. **4**

(b) (i) Hence state the maximum and minimum values of $12\cos x° - 5\sin x°$.

(ii) Determine the values of x, in the interval $0 \le x < 360$, at which these maximum and minimum values occur. **3**

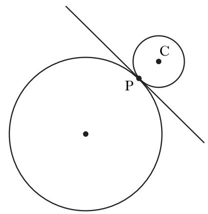
[Turn over

Marks

3. (a) (i) Show that the line with equation $y = 3 - x$ is a tangent to the circle with equation $x^2 + y^2 + 14x + 4y - 19 = 0$.

(ii) Find the coordinates of the point of contact, P. **5**

(b) Relative to a suitable set of coordinate axes, the diagram below shows the circle from (a) and a second smaller circle with centre C.



The line $y = 3 - x$ is a common tangent at the point P.

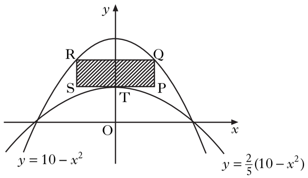
The radius of the larger circle is three times the radius of the smaller circle.

Find the equation of the smaller circle. **6**

4. Solve $2\cos 2x - 5\cos x - 4 = 0$ for $0 \le x < 2\pi$. **5**

Marks

5. The parabolas with equations $y = 10 - x^2$ and $y = \frac{2}{5}(10 - x^2)$ are shown in the diagram below.



A rectangle PQRS is placed between the two parabolas as shown, so that:

- Q and R lie on the upper parabola;
- RQ and SP are parallel to the x-axis;
- T, the turning point of the lower parabola, lies on SP.

(*a*) (i) If TP = x units, find an expression for the length of PQ.

(ii) Hence show that the area, A, of rectangle PQRS is given by

$$A(x) = 12x - 2x^3.$$ **3**

(*b*) Find the maximum area of this rectangle. **6**

[Turn over for Questions 6 and 7 on *Page six*

Marks

6. (*a*) A curve has equation $y = (2x - 9)^{\frac{1}{2}}$.

Show that the equation of the tangent to this curve at the point where $x = 9$ is $y = \frac{1}{3}x$. **5**

(*b*) Diagram 1 shows part of the curve and the tangent.

The curve cuts the x-axis at the point A.

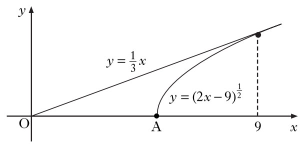


Diagram 1

Find the coordinates of point A. **1**

(*c*) Calculate the shaded area shown in diagram 2. **7**

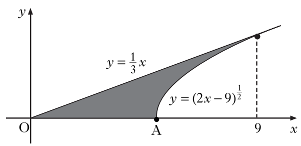


Diagram 2

7. (*a*) Given that $\log_4 x = P$, show that $\log_{16} x = \frac{1}{2}P$. **3**

(*b*) Solve $\log_3 x + \log_9 x = 12$. **3**

[*END OF QUESTION PAPER*]

2011 paper 1

SECTION A

ALL questions should be attempted.

1. Given that $\mathbf{p} = \begin{pmatrix} 2 \\ 5 \\ -7 \end{pmatrix}$, $\mathbf{q} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} -4 \\ 2 \\ 0 \end{pmatrix}$, express $2\mathbf{p} - \mathbf{q} - \frac{1}{2}\mathbf{r}$ in component form.

A $\begin{pmatrix} 1 \\ 9 \\ -15 \end{pmatrix}$

B $\begin{pmatrix} 1 \\ 11 \\ -13 \end{pmatrix}$

C $\begin{pmatrix} 5 \\ 9 \\ -13 \end{pmatrix}$

D $\begin{pmatrix} 5 \\ 11 \\ -15 \end{pmatrix}$

2. A line l has equation $3y + 2x = 6$.

What is the gradient of any line parallel to l?

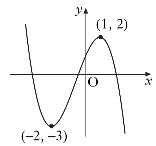
A -2

B $-\frac{2}{3}$

C $\frac{3}{2}$

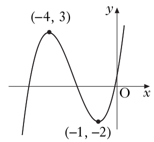
D 2

3. The diagram shows the graph of $y = f(x)$.

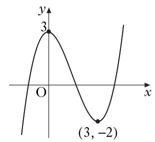


Which of the following shows the graph of $y = f(x + 2) - 1$?

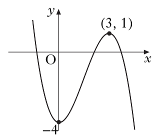
A



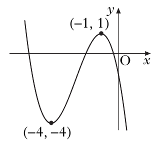
B



C



D



[Turn over

4. A tangent to the curve with equation $y = x^3 - 2x$ is drawn at the point (2, 4).

What is the gradient of this tangent?

A 2

B 3

C 4

D 10

5. If $x^2 - 8x + 7$ is written in the form $(x - p)^2 + q$, what is the value of q?

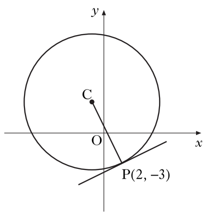
A –9

B –1

C 7

D 23

6. The point P(2, –3) lies on the circle with centre C as shown.



The gradient of CP is –2.

What is the equation of the tangent at P?

A $y + 3 = -2(x - 2)$

B $y - 3 = -2(x + 2)$

C $y + 3 = \frac{1}{2}(x - 2)$

D $y - 3 = \frac{1}{2}(x + 2)$

7. A function f is defined on the set of real numbers by $f(x) = x^3 - x^2 + x + 3$.

What is the remainder when $f(x)$ is divided by $(x - 1)$?

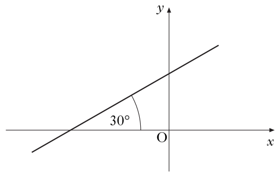
A 0

B 2

C 3

D 4

8. A line makes an angle of 30° with the positive direction of the x-axis as shown.



What is the gradient of the line?

A $\frac{1}{\sqrt{3}}$

B $\frac{1}{\sqrt{2}}$

C $\frac{1}{2}$

D $\frac{\sqrt{3}}{2}$

9. The discriminant of a quadratic equation is 23.

Here are two statements about this quadratic equation:

(1) the roots are real;

(2) the roots are rational.

Which of the following is true?

A Neither statement is correct.

B Only statement (1) is correct.

C Only statement (2) is correct.

D Both statements are correct.

[Turn over

10. Solve $2\cos x = \sqrt{3}$ for x, where $0 \leq x < 2\pi$.

A $\frac{\pi}{3}$ and $\frac{5\pi}{3}$

B $\frac{\pi}{3}$ and $\frac{2\pi}{3}$

C $\frac{\pi}{6}$ and $\frac{5\pi}{6}$

D $\frac{\pi}{6}$ and $\frac{11\pi}{6}$

11. Find $\int \left(4x^{\frac{1}{2}} + x^{-3}\right) dx$, where $x > 0$.

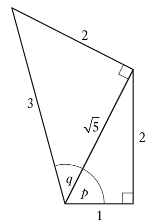
A $2x^{-\frac{1}{2}} - 3x^{-4} + c$

B $2x^{-\frac{1}{2}} - \frac{1}{2}x^{-2} + c$

C $\frac{8}{3}x^{\frac{3}{2}} - 3x^{-4} + c$

D $\frac{8}{3}x^{\frac{3}{2}} - \frac{1}{2}x^{-2} + c$

12. The diagram shows two right-angled triangles with sides and angles as given.



What is the value of $\sin(p + q)$?

A $\frac{2}{\sqrt{5}} + \frac{2}{3}$

B $\frac{2}{\sqrt{5}} + \frac{\sqrt{5}}{3}$

C $\frac{2}{3} + \frac{2}{3\sqrt{5}}$

D $\frac{4}{3\sqrt{5}} + \frac{1}{3}$

13. Given that $f(x) = 4\sin 3x$, find $f'(0)$.

A 0

B 1

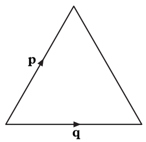
C 12

D 36

[Turn over

14. An equilateral triangle of side 3 units is shown.

The vectors **p** and **q** are as represented in the diagram.



What is the value of **p**.**q**?

A 9

B $\frac{9}{2}$

C $\frac{9}{\sqrt{2}}$

D 0

15. Given that the points S(–4, 5, 1), T(–16, –4, 16) and U(–24, –10, 26) are collinear, calculate the ratio in which T divides SU.

A 2 : 3

B 3 : 2

C 2 : 5

D 3 : 5

16. Find $\int \frac{1}{3x^4}\,dx$, where $x \neq 0$.

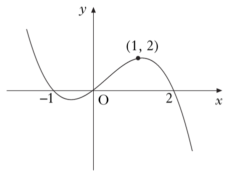
A $-\frac{1}{9x^3} + c$

B $-\frac{1}{x^3} + c$

C $\frac{1}{x^3} + c$

D $\frac{1}{12x^3} + c$

17. The diagram shows the graph of a cubic.



What is the equation of this cubic?

A $y = -x(x + 1)(x - 2)$

B $y = -x(x - 1)(x + 2)$

C $y = x(x + 1)(x - 2)$

D $y = x(x - 1)(x + 2)$

18. If $f(x) = (x - 3)(x + 5)$, for what values of x is the graph of $y = f(x)$ above the x-axis?

A $-5 < x < 3$

B $-3 < x < 5$

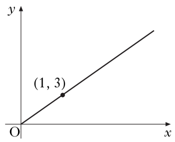
C $x < -5, x > 3$

D $x < -3, x > 5$

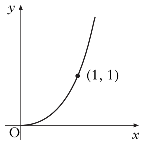
[Turn over

19. Which of the following diagrams represents the graph with equation $\log_3 y = x$?

A



B



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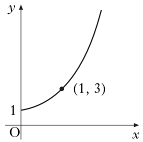

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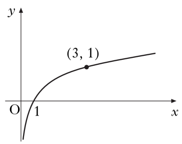


20. On a suitable domain, D, a function g is defined by $g(x) = \sin^2 \sqrt{x-2}$.

Which of the following gives the real values of x in D and the corresponding values of $g(x)$?

A $x \geq 0$ and $-1 \leq g(x) \leq 1$

B $x \geq 0$ and $0 \leq g(x) \leq 1$

C $x \geq 2$ and $-1 \leq g(x) \leq 1$

D $x \geq 2$ and $0 \leq g(x) \leq 1$

[*END OF SECTION A*]

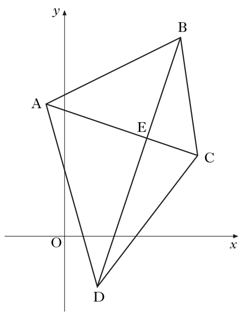
[Turn over for SECTION B

Marks

SECTION B

ALL questions should be attempted.

21. A quadrilateral has vertices A(–1, 8), B(7, 12), C(8, 5) and D(2, –3) as shown in the diagram.



(*a*) Find the equation of diagonal BD. **2**

(*b*) The equation of diagonal AC is $x + 3y = 23$.
Find the coordinates of E, the point of intersection of the diagonals. **3**

(*c*) (i) Find the equation of the perpendicular bisector of AB.
(ii) Show that this line passes through E. **5**

Marks

22. A function f is defined on the set of real numbers by $f(x) = (x - 2)(x^2 + 1)$.

(*a*) Find where the graph of $y = f(x)$ cuts:
(i) the x-axis;
(ii) the y-axis. **2**

(*b*) Find the coordinates of the stationary points on the curve with equation $y = f(x)$ and determine their nature. **8**

(*c*) On separate diagrams sketch the graphs of:
(i) $y = f(x)$;
(ii) $y = -f(x)$. **3**

23. (*a*) Solve $\cos 2x° - 3\cos x° + 2 = 0$ for $0 \leq x < 360$. **5**

(*b*) Hence solve $\cos 4x° - 3\cos 2x° + 2 = 0$ for $0 \leq x < 360$. **2**

[*END OF SECTION B*]

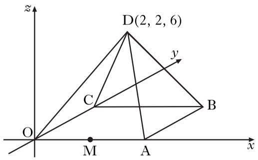
[*END OF QUESTION PAPER*]

2011 paper 2

ALL questions should be attempted.

Marks

1. D,OABC is a square based pyramid as shown in the diagram below.



O is the origin, D is the point (2, 2, 6) and OA = 4 units.

M is the mid-point of OA.

(*a*) State the coordinates of B. **1**

(*b*) Express $\overrightarrow{DB}$ and $\overrightarrow{DM}$ in component form. **3**

(*c*) Find the size of angle BDM. **5**

2. Functions f, g and h are defined on the set of real numbers by

- $f(x) = x^3 - 1$
- $g(x) = 3x + 1$
- $h(x) = 4x - 5$.

(*a*) Find $g(f(x))$. **2**

(*b*) Show that $g(f(x)) + xh(x) = 3x^3 + 4x^2 - 5x - 2$. **1**

(*c*) (i) Show that $(x - 1)$ is a factor of $3x^3 + 4x^2 - 5x - 2$.

(ii) Factorise $3x^3 + 4x^2 - 5x - 2$ fully. **5**

(*d*) Hence solve $g(f(x)) + xh(x) = 0$. **1**

[Turn over

Marks

3. (*a*) A sequence is defined by $u_{n+1} = -\frac{1}{2}u_n$ with $u_0 = -16$.

Write down the values of u_1 and u_2. **1**

(*b*) A second sequence is given by 4, 5, 7, 11, ...

It is generated by the recurrence relation $v_{n+1} = pv_n + q$ with $v_1 = 4$.

Find the values of p and q. **3**

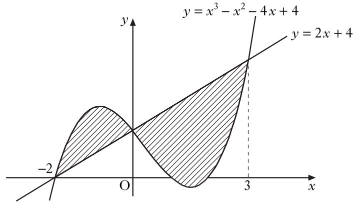
(*c*) Either the sequence in (*a*) or the sequence in (*b*) has a limit.

(i) Calculate this limit.

(ii) Why does the other sequence not have a limit? **3**

4. The diagram shows the curve with equation $y = x^3 - x^2 - 4x + 4$ and the line with equation $y = 2x + 4$.

The curve and the line intersect at the points (–2, 0), (0, 4) and (3, 10).

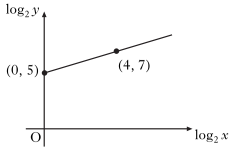


Calculate the total shaded area. **10**

Marks

5. Variables x and y are related by the equation $y = kx^n$.

The graph of $\log_2 y$ against $\log_2 x$ is a straight line through the points (0, 5) and (4, 7), as shown in the diagram.



Find the values of k and n. **5**

6. (*a*) The expression $3\sin x - 5\cos x$ can be written in the form $R\sin(x + a)$ where $R > 0$ and $0 \leq a < 2\pi$.

Calculate the values of R and a. **4**

(*b*) Hence find the value of t, where $0 \leq t \leq 2$, for which

$$\int_0^t (3\cos x + 5\sin x)\, dx = 3.$$ **7**

7. Circle C_1 has equation $(x + 1)^2 + (y - 1)^2 = 121$.

A circle C_2 with equation $x^2 + y^2 - 4x + 6y + p = 0$ is drawn inside C_1.

The circles have no points of contact.

What is the range of values of p? **9**

[*END OF QUESTION PAPER*]

SECTION A

ALL questions should be attempted.

1. A sequence is defined by the recurrence relation $u_{n+1} = 3u_n + 4$, with $u_0 = 1$.

Find the value of u_2.

A 7

B 10

C 25

D 35

2. What is the gradient of the tangent to the curve with equation $y = x^3 - 6x + 1$ at the point where $x = -2$?

A −24

B 3

C 5

D 6

3. If $x^2 - 6x + 14$ is written in the form $(x - p)^2 + q$, what is the value of q?

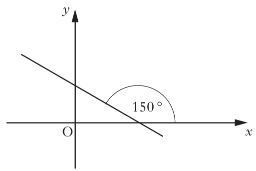
A −22

B 5

C 14

D 50

4. What is the gradient of the line shown in the diagram?



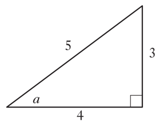
A $-\sqrt{3}$

B $-\frac{1}{\sqrt{3}}$

C $-\frac{1}{2}$

D $-\frac{\sqrt{3}}{2}$

5. The diagram shows a right-angled triangle with sides and angles as marked.



What is the value of $\cos 2a$?

A $\frac{7}{25}$

B $\frac{3}{5}$

C $\frac{24}{25}$

D $\frac{6}{5}$

6. If $y = 3x^{-2} + 2x^{\frac{3}{2}}$, $x > 0$, determine $\frac{dy}{dx}$.

A $-6x^{-3} + \frac{4}{5}x^{\frac{5}{2}}$

B $-3x^{-1} + 3x^{\frac{1}{2}}$

C $-6x^{-3} + 3x^{\frac{1}{2}}$

D $-3x^{-1} + \frac{4}{5}x^{\frac{5}{2}}$

7. If $\mathbf{u} = \begin{pmatrix} -3 \\ 1 \\ 2t \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 1 \\ t \\ -1 \end{pmatrix}$ are perpendicular, what is the value of t?

A -3

B -2

C $\frac{2}{3}$

D 1

[Turn over

8. The volume of a sphere is given by the formula $V = \frac{4}{3}\pi r^3$.
What is the rate of change of V with respect to r, at $r = 2$?

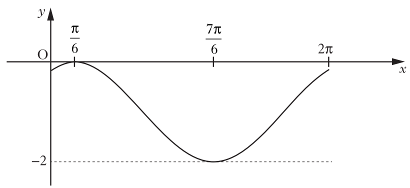
A $\frac{16\pi}{3}$

B $\frac{32\pi}{3}$

C 16π

D 32π

9. The diagram shows the curve with equation of the form $y = \cos(x + a) + b$ for $0 \leq x \leq 2\pi$.



What is the equation of this curve?

A $y = \cos\left(x - \frac{\pi}{6}\right) - 1$

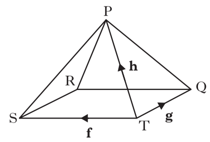
B $y = \cos\left(x - \frac{\pi}{6}\right) + 1$

C $y = \cos\left(x + \frac{\pi}{6}\right) - 1$

D $y = \cos\left(x + \frac{\pi}{6}\right) + 1$

10. The diagram shows a square-based pyramid P,QRST.

$\overrightarrow{TS}$, $\overrightarrow{TQ}$ and $\overrightarrow{TP}$ represent **f**, **g** and **h** respectively.



Express $\overrightarrow{RP}$ in terms of **f**, **g** and **h**.

A $-\mathbf{f}+\mathbf{g}-\mathbf{h}$

B $-\mathbf{f}-\mathbf{g}+\mathbf{h}$

C $\mathbf{f}-\mathbf{g}-\mathbf{h}$

D $\mathbf{f}+\mathbf{g}+\mathbf{h}$

11. Find $\int\left(\frac{1}{6x^2}\right)dx,\ x \neq 0$.

A $-12x^{-3}+c$

B $-6x^{-1}+c$

C $-\frac{1}{3}x^{-3}+c$

D $-\frac{1}{6}x^{-1}+c$

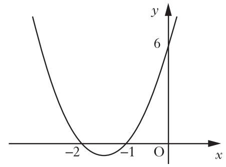
12. Find the maximum value of

$$2-3\sin\left(x-\frac{\pi}{3}\right)$$

and the value of x where this occurs in the interval $0 \leq x \leq 2\pi$.

	max value	x
A	-1	$\frac{11\pi}{6}$
B	5	$\frac{11\pi}{6}$
C	-1	$\frac{5\pi}{6}$
D	5	$\frac{5\pi}{6}$

13. A parabola intersects the axes at $x=-2$, $x=-1$ and $y=6$, as shown in the diagram.



What is the equation of the parabola?

A $y=6(x-1)(x-2)$

B $y=6(x+1)(x+2)$

C $y=3(x-1)(x-2)$

D $y=3(x+1)(x+2)$

14. Find $\int(2x-1)^{\frac{1}{2}}\,dx$ where $x>\frac{1}{2}$.

A $\frac{1}{3}(2x-1)^{\frac{3}{2}}+c$

B $\frac{1}{2}(2x-1)^{-\frac{1}{2}}+c$

C $\frac{1}{2}(2x-1)^{\frac{3}{2}}+c$

D $\frac{1}{3}(2x-1)^{-\frac{1}{2}}+c$

15. If $\mathbf{u}=k\begin{pmatrix}3\\-1\\0\end{pmatrix}$, where $k>0$ and $\mathbf{u}$ is a unit vector, determine the value of k.

A $\frac{1}{2}$

B $\frac{1}{8}$

C $\frac{1}{\sqrt{2}}$

D $\frac{1}{\sqrt{10}}$

16. If $y = 3\cos^4 x$, find $\frac{dy}{dx}$.

A $12\cos^3 x \sin x$

B $12\cos^3 x$

C $-12\cos^3 x \sin x$

D $-12\sin^3 x$

17. Given that $\mathbf{a} = \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}$ and $\mathbf{a}.(\mathbf{a}+\mathbf{b}) = 7$, what is the value of $\mathbf{a}.\mathbf{b}$?

A $\frac{7}{25}$

B $-\frac{18}{5}$

C -6

D -18

18. The graph of $y = f(x)$ shown has stationary points at $(0, p)$ and (q, r).

Here are two statements about $f(x)$:

(1) $f(x) < 0$ for $s < x < t$;
(2) $f'(x) < 0$ for $x < q$.

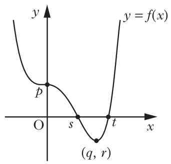
Which of the following is true?

A Neither statement is correct.

B Only statement (1) is correct.

C Only statement (2) is correct.

D Both statements are correct.



[Turn over

19. Solve $6 - x - x^2 < 0$.

A $-3 < x < 2$

B $x < -3, x > 2$

C $-2 < x < 3$

D $x < -2, x > 3$

20. Simplify $\frac{\log_b 9a^2}{\log_b 3a}$, where $a > 0$ and $b > 0$.

A 2

B $3a$

C $\log_b 3a$

D $\log_b (9a^2 - 3a)$

[END OF SECTION A]

SECTION B

ALL questions should be attempted.

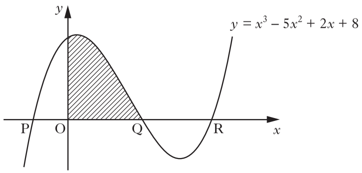
Marks

21. (*a*) (i) Show that $(x - 4)$ is a factor of $x^3 - 5x^2 + 2x + 8$.

(ii) Factorise $x^3 - 5x^2 + 2x + 8$ fully.

(iii) Solve $x^3 - 5x^2 + 2x + 8 = 0$. **6**

(*b*) The diagram shows the curve with equation $y = x^3 - 5x^2 + 2x + 8$.



The curve crosses the x-axis at P, Q and R.

Determine the shaded area. **6**

22. (*a*) The expression $\cos x - \sqrt{3}\sin x$ can be written in the form $k\cos(x + a)$ where $k > 0$ and $0 \le a < 2\pi$.

Calculate the values of k and a. **4**

(*b*) Find the points of intersection of the graph of $y = \cos x - \sqrt{3}\sin x$ with the x and y axes, in the interval $0 \le x \le 2\pi$. **3**

[Turn over for Question 23 on *Page twelve*

Marks

23. (*a*) Find the equation of ℓ_1, the perpendicular bisector of the line joining P(3, –3) to Q(–1, 9). **4**

(*b*) Find the equation of ℓ_2 which is parallel to PQ and passes through R(1, –2). **2**

(*c*) Find the point of intersection of ℓ_1 and ℓ_2. **3**

(*d*) Hence find the shortest distance between PQ and ℓ_2. **2**

[*END OF SECTION B*]

[*END OF QUESTION PAPER*]

2012 paper 2

ALL questions should be attempted. *Marks*

1. Functions f and g are defined on the set of real numbers by

- $f(x) = x^2 + 3$
- $g(x) = x + 4$.

(*a*) Find expressions for:

(i) $f(g(x))$;

(ii) $g(f(x))$. **3**

(*b*) Show that $f(g(x)) + g(f(x)) = 0$ has no real roots. **3**

2. (*a*) Relative to a suitable set of coordinate axes, Diagram 1 shows the line $2x - y + 5 = 0$ intersecting the circle $x^2 + y^2 - 6x - 2y - 30 = 0$ at the points P and Q.

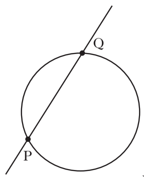


Diagram 1

Find the coordinates of P and Q. **6**

(*b*) Diagram 2 shows the circle from (*a*) and a second congruent circle, which also passes through P and Q.

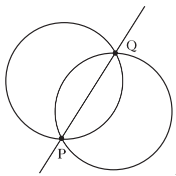


Diagram 2

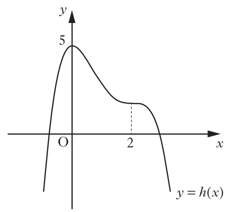
Determine the equation of this second circle. **6**

Marks

3. A function f is defined on the domain $0 \leq x \leq 3$ by $f(x) = x^3 - 2x^2 - 4x + 6$.

Determine the maximum and minimum values of f. **7**

4. The diagram below shows the graph of a quartic $y = h(x)$, with stationary points at $x = 0$ and $x = 2$.



On separate diagrams sketch the graphs of:

(*a*) $y = h'(x)$; **3**

(*b*) $y = 2 - h'(x)$. **3**

5. A is the point (3, –3, 0), B is (2, –3, 1) and C is (4, k, 0).

(*a*) (i) Express $\overrightarrow{BA}$ and $\overrightarrow{BC}$ in component form.

(ii) Show that $\cos A\hat{B}C = \dfrac{3}{\sqrt{2(k^2 + 6k + 14)}}$. **7**

(*b*) If angle ABC = 30°, find the possible values of k. **5**

Marks

6. For $0 < x < \frac{\pi}{2}$, sequences can be generated using the recurrence relation

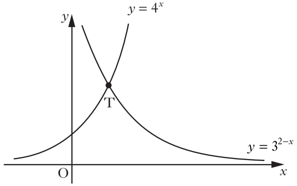
$$u_{n+1} = (\sin x)u_n + \cos 2x, \text{ with } u_0 = 1.$$

(*a*) Why do these sequences have a limit? **2**

(*b*) The limit of one sequence generated by this recurrence relation is $\frac{1}{2}\sin x$.

Find the value(s) of x. **7**

7. The diagram shows the curves with equations $y = 4^x$ and $y = 3^{2-x}$.



The graphs intersect at the point T.

(*a*) Show that the x – coordinate of T can be written in the form $\frac{\log_a p}{\log_a q}$, for all $a > 1$. **6**

(*b*) Calculate the y – coordinate of T. **2**

[*END OF QUESTION PAPER*]

SECTION A

ALL questions should be attempted.

1. The functions f and g are defined by $f(x) = x^2 + 1$ and $g(x) = 3x - 4$, on the set of real numbers.

Find $g(f(x))$.

A $3x^2 - 1$

B $9x^2 - 15$

C $9x^2 + 17$

D $3x^3 - 4x^2 + 3x - 4$

2. The point P (5, 12) lies on the curve with equation $y = x^2 - 4x + 7$.

What is the gradient of the tangent to this curve at P?

A 2

B 6

C 12

D 13

3. Calculate the discriminant of the quadratic equation $2x^2 + 4x + 5 = 0$.

A –32

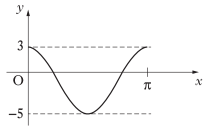
B –24

C 48

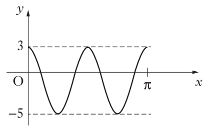
D 56

4. Which of the following shows the graph of $y = 4\cos 2x - 1$, for $0 \le x \le \pi$?

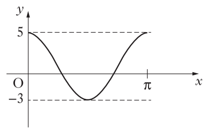
A



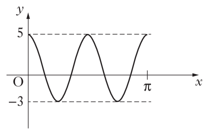
B



C



D



[Turn over

5. The line L passes through the point (–2, –1) and is parallel to the line with equation $5x + 3y - 6 = 0$.

What is the equation of L?

A $3x + 5y - 11 = 0$

B $3x + 5y + 11 = 0$

C $5x + 3y - 13 = 0$

D $5x + 3y + 13 = 0$

6. What is the remainder when $x^3 + 3x^2 - 5x - 6$ is divided by $(x - 2)$?

A 0

B 3

C 4

D 8

7. Find $\int x(3x + 2)\, dx$.

A $x^3 + c$

B $x^3 + x^2 + c$

C $\frac{1}{2}x^2\left(\frac{3}{2}x^2 + 2x\right) + c$

D $3x^2 + 2x + c$

8. A sequence is defined by the recurrence relation $u_{n+1} = 0{\cdot}1u_n + 8$, with $u_1 = 11$.

Here are two statements about this sequence:

(1) $u_0 = 9{\cdot}1$;

(2) The sequence has a limit as $n \longrightarrow \infty$.

Which of the following is true?

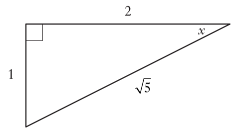
A Neither statement is correct.

B Only statement (1) is correct.

C Only statement (2) is correct.

D Both statements are correct.

9. The diagram shows a right-angled triangle with sides and angles as marked.



Find the value of $\sin 2x$.

A $\frac{4}{5}$

B $\frac{2}{5}$

C $\frac{2}{\sqrt{5}}$

D $\frac{1}{\sqrt{5}}$

10. If $0 < a < 90$, which of the following is equivalent to $\cos(270 - a)°$?

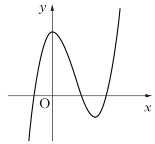
A $\cos a°$

B $\sin a°$

C $-\cos a°$

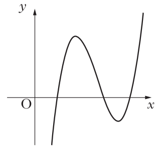
D $-\sin a°$

11. The diagram shows a cubic curve with equation $y = f(x)$.

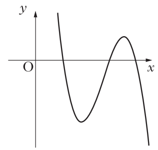


Which of the following diagrams could show the curve with equation $y = -f(x - k)$, $k > 0$?

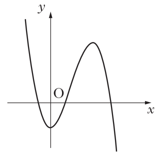
A



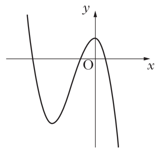
B



C



D



12. If $\mathbf{f} = 3\mathbf{i} + 2\mathbf{k}$ and $\mathbf{g} = 2\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$, find $|\mathbf{f} + \mathbf{g}|$.

A $\sqrt{14}$ units

B $\sqrt{42}$ units

C $\sqrt{66}$ units

D $\sqrt{70}$ units

13. A function f is defined on a suitable domain by $f(x) = \dfrac{x+2}{x^2 - 7x + 12}$.

What value(s) of x cannot be in this domain?

A 3 and 4

B −3 and −4

C −2

D 0

14. Given that $|\mathbf{a}| = 3$, $|\mathbf{b}| = 2$ and $\mathbf{a.b} = 5$, what is the value of $\mathbf{a.(a + b)}$?

A 11

B 14

C 15

D 21

15. Solve $\tan\left(\dfrac{x}{2}\right) = -1$ for $0 \le x < 2\pi$.

A $\dfrac{\pi}{2}$

B $\dfrac{7\pi}{8}$

C $\dfrac{3\pi}{2}$

D $\dfrac{15\pi}{8}$

[Turn over

16. Find $\int (1-6x)^{-\frac{1}{2}}dx$ where $x < \frac{1}{6}$.

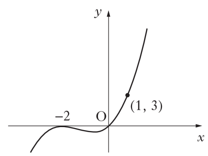
A $\frac{1}{9}(1-6x)^{-\frac{3}{2}} + c$

B $3(1-6x)^{-\frac{3}{2}} + c$

C $-\frac{1}{3}(1-6x)^{\frac{1}{2}} + c$

D $-3(1-6x)^{\frac{1}{2}} + c$

17. The diagram shows a curve with equation of the form $y = kx(x + a)^2$, which passes through the points (−2, 0), (0, 0) and (1, 3).



What are the values of a and k?

	a	k
A	−2	$\frac{1}{3}$
B	−2	3
C	2	$\frac{1}{3}$
D	2	3

18. Given that $y = \sin(x^2 - 3)$, find $\frac{dy}{dx}$.

A $\sin 2x$

B $\cos 2x$

C $2x \sin(x^2 - 3)$

D $2x \cos(x^2 - 3)$

19. Solve $1 - 2x - 3x^2 > 0$, where x is a real number.

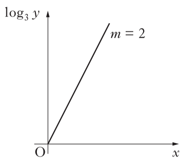
A $x < -1$ or $x > \frac{1}{3}$

B $-1 < x < \frac{1}{3}$

C $x < -\frac{1}{3}$ or $x > 1$

D $-\frac{1}{3} < x < 1$

20. The graph of $\log_3 y$ plotted against x is a line through the origin with gradient 2, as shown.



Express y in terms of x.

A $y = 2x$

B $y = 9x$

C $y = 6^x$

D $y = 9^x$

[END OF SECTION A]

[Turn over for SECTION B on *Page twelve*

SECTION B

Marks

ALL questions should be attempted.

21. Express $2x^2 + 12x + 1$ in the form $a(x + b)^2 + c$. **3**

22. A circle C_1 has equation $x^2 + y^2 + 2x + 4y - 27 = 0$.

(*a*) Write down the centre and calculate the radius of C_1. **2**

(*b*) The point P(3, 2) lies on the circle C_1.

Find the equation of the tangent at P. **3**

(*c*) A second circle C_2 has centre (10, –1). The radius of C_2 is half of the radius of C_1.

Show that the equation of C_2 is $x^2 + y^2 - 20x + 2y + 93 = 0$. **3**

(*d*) Show that the tangent found in part (*b*) is also a tangent to circle C_2. **4**

23. (*a*) The expression $\sqrt{3}\sin x° - \cos x°$ can be written in the form $k\sin(x - a)°$, where $k > 0$ and $0 \le a < 360$.

Calculate the values of k and a. **4**

(*b*) Determine the maximum value of $4 + 5\cos x° - 5\sqrt{3}\sin x°$, where $0 \le x < 360$. **2**

24. (*a*) (i) Show that the points A(–7, –8, 1), T(3, 2, 5) and B(18, 17, 11) are collinear.

(ii) Find the ratio in which T divides AB. **4**

(*b*) The point C lies on the x-axis.

If TB and TC are perpendicular, find the coordinates of C. **5**

[END OF SECTION B]

[END OF QUESTION PAPER]

ALL questions should be attempted. *Marks*

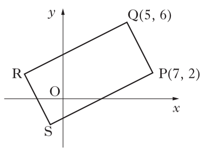
1. The first three terms of a sequence are 4, 7 and 16.

The sequence is generated by the recurrence relation

$$u_{n+1} = mu_n + c, \text{ with } u_1 = 4.$$

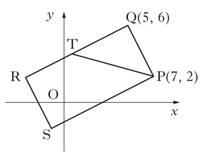
Find the values of m and c. **4**

2. The diagram shows rectangle PQRS with P(7, 2) and Q(5, 6).



(*a*) Find the equation of QR. **3**

(*b*) The line from P with the equation $x + 3y = 13$ intersects QR at T.



Find the coordinates of T. **3**

(*c*) Given that T is the midpoint of QR, find the coordinates of R and S. **3**

[Turn over

Marks

3. (*a*) Given that $(x - 1)$ is a factor of $x^3 + 3x^2 + x - 5$, factorise this cubic fully. **4**

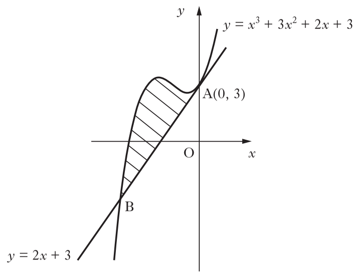
(*b*) Show that the curve with equation

$$y = x^4 + 4x^3 + 2x^2 - 20x + 3$$

has only one stationary point.

Find the x-coordinate and determine the nature of this point. **5**

4. The line with equation $y = 2x + 3$ is a tangent to the curve with equation $y = x^3 + 3x^2 + 2x + 3$ at A(0, 3), as shown in the diagram.



The line meets the curve again at B.

Show that B is the point (–3, –3) and find the area enclosed by the line and the curve. **6**

5. Solve the equation

$$\log_5(3 - 2x) + \log_5(2 + x) = 1, \text{ where } x \text{ is a real number.}$$ **4**

Marks

6. Given that $\int_0^a 5\sin 3x \, dx = \frac{10}{3}$, $0 \le a < \pi$,

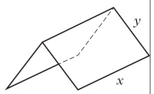
calculate the value of a. **5**

7. A manufacturer is asked to design an open-ended shelter, as shown, subject to the following conditions.

Condition 1

The frame of a shelter is to be made of rods of two different lengths:

- x metres for top and bottom edges;
- y metres for each sloping edge.



Condition 2

The frame is to be covered by a rectangular sheet of material.

The total area of the sheet is $24\,m^2$.

(*a*) Show that the total length, L metres, of the rods used in a shelter is given by

$$L = 3x + \frac{48}{x}.$$ **3**

(*b*) These rods cost £8·25 per metre.

To minimise production costs, the total length of rods used for a frame should be as small as possible.

(i) Find the value of x for which L is a minimum.

(ii) Calculate the minimum cost of a frame. **7**

8. Solve algebraically the equation

$$\sin 2x = 2\cos^2 x \qquad \text{for } 0 \le x < 2\pi$$ **6**

[Turn over for Question 9 on *Page six*

Marks

9. The concentration of the pesticide, *Xpesto*, in soil can be modelled by the equation

$$P_t = P_0 e^{-kt}$$

where:

- P_0 is the initial concentration;
- P_t is the concentration at time t;
- t is the time, in days, after the application of the pesticide.

(*a*) Once in the soil, the half-life of a pesticide is the time taken for its concentration to be reduced to one half of its initial value.

If the half-life of *Xpesto* is 25 days, find the value of k to 2 significant figures. **4**

(*b*) Eighty days after the initial application, what is the percentage decrease in concentration of *Xpesto*? **3**

[*END OF QUESTION PAPER*]

2014 paper 1

SECTION A

ALL questions should be attempted.

1. A sequence is defined by the recurrence relation $u_{n+1} = \frac{1}{3}u_n + 1$, with $u_2 = 15$.
What is the value of u_4?

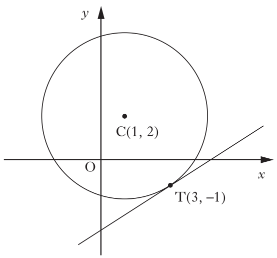
A $2\frac{1}{9}$

B $2\frac{1}{3}$

C 3

D 30

2. The diagram shows a circle with centre C(1, 2) and the tangent at T(3, –1).



What is the gradient of this tangent?

A $\frac{1}{4}$

B $\frac{2}{3}$

C $\frac{3}{2}$

D 4

3. If $\log_4 12 - \log_4 x = \log_4 6$, what is the value of x?

A 2

B 6

C 18

D 72

4. If $3\sin x - 4\cos x$ is written in the form $k\cos(x - a)$, what are the values of $k\cos a$ and $k\sin a$?

	$k\cos a$	$k\sin a$
A	–3	4
B	3	–4
C	4	–3
D	–4	3

5. Find $\int (2x+9)^5\,dx$.

A $10(2x + 9)^4 + c$

B $\frac{1}{4}(2x + 9)^4 + c$

C $10(2x + 9)^6 + c$

D $\frac{1}{12}(2x + 9)^6 + c$

[Turn over

6. Given that $\boldsymbol{u} = \begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$, find $2\boldsymbol{u} - 3\boldsymbol{v}$ in component form.

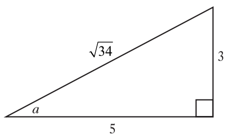
A $\begin{pmatrix} -9 \\ 5 \\ -6 \end{pmatrix}$

B $\begin{pmatrix} -9 \\ -1 \\ -4 \end{pmatrix}$

C $\begin{pmatrix} -3 \\ -1 \\ 6 \end{pmatrix}$

D $\begin{pmatrix} 11 \\ -5 \\ 4 \end{pmatrix}$

7. A right-angled triangle has sides and angles as shown in the diagram.



What is the value of $\sin 2a$?

A $\frac{8}{17}$

B $\frac{3}{\sqrt{34}}$

C $\frac{15}{17}$

D $\frac{6}{\sqrt{34}}$

8. What is the derivative of $\left(4 - 9x^4\right)^{\frac{1}{2}}$?

A $-\frac{9}{2}\left(4 - 9x^4\right)^{-\frac{1}{2}}$

B $\frac{1}{2}\left(4 - 9x^{-4}\right)^{-\frac{1}{2}}$

C $2\left(4 - 9x^4\right)^{-\frac{1}{2}}$

D $-18x^3\left(4 - 9x^4\right)^{-\frac{1}{2}}$

9. $\sin x + \sqrt{3}\cos x$ can be written as $2\cos\left(x - \frac{\pi}{6}\right)$.

The maximum value of $\sin x + \sqrt{3}\cos x$ is 2.

What is the maximum value of $5\sin 2x + 5\sqrt{3}\cos 2x$?

A 20

B 10

C 5

D 2

10. A sequence is defined by the recurrence relation

$u_{n+1} = (k-2)u_n + 5$ with $u_0 = 3$.

For what values of k does this sequence have a limit as $n \rightarrow \infty$?

A $-3 < k < -1$

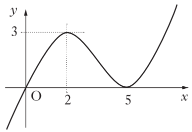
B $-1 < k < 1$

C $1 < k < 3$

D $k < 3$

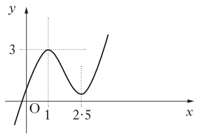
[Turn over

11. The diagram shows part of the graph of $y = f(x)$.

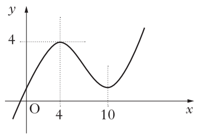


Which of the following diagrams could be the graph of $y = 2f(x) + 1$?

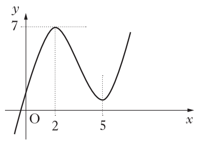
A



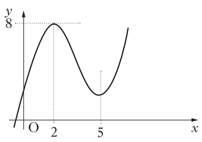
B



C



D



12. A function f, defined on a suitable domain, is given by $f(x) = \frac{6x}{x^2 + 6x - 16}$.

What restrictions are there on the domain of f?

A $x \neq -8$ or $x \neq 2$

B $x \neq -4$ or $x \neq 4$

C $x \neq 0$

D $x \neq 10$ or $x \neq 16$

13. What is the value of $\sin\left(\frac{\pi}{3}\right) - \cos\left(\frac{5\pi}{4}\right)$?

A $\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}$

B $\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}$

C $\frac{1}{2} - \frac{1}{\sqrt{2}}$

D $\frac{1}{2} + \frac{1}{\sqrt{2}}$

14. The vectors $\boldsymbol{u} = \begin{pmatrix} 1 \\ k \\ k \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} -6 \\ 2 \\ 5 \end{pmatrix}$ are perpendicular.

What is the value of k?

A $\frac{-6}{7}$

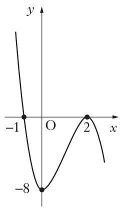
B -1

C 1

D $\frac{6}{7}$

[Turn over

15. The diagram shows a cubic curve passing through (–1, 0), (2, 0) and (0, –8).



What is the equation of the curve?

A $y = -2(x + 1)^2(x + 2)$

B $y = -2(x + 1)(x - 2)^2$

C $y = 4(x + 1)(x - 2)$

D $y = -8(x + 1)(x - 2)^2$

16. The unit vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are such that $\boldsymbol{a}.\boldsymbol{b} = \frac{2}{3}$. Determine the value of $\boldsymbol{a}.(\boldsymbol{a} + 2\boldsymbol{b})$.

A $\frac{2}{3}$

B $\frac{4}{3}$

C $\frac{7}{3}$

D 3

17. $3x^2 + 12x + 17$ is expressed in the form $3(x + p)^2 + q$.

What is the value of q?

A 1

B 5

C 17

D –19

18. What is the value of $1 - 2\sin^2 15°$?

A $\frac{1}{2}$

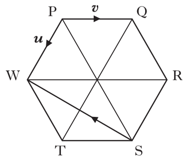
B $\frac{3}{4}$

C $\frac{\sqrt{3}}{2}$

D $\frac{7}{8}$

19. The diagram shows a regular hexagon PQRSTW.

$\overrightarrow{PW}$ and $\overrightarrow{PQ}$ represent vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ respectively.



What is $\overrightarrow{SW}$ in terms of $\boldsymbol{u}$ and $\boldsymbol{v}$?

A $-\boldsymbol{u} - 2\boldsymbol{v}$

B $-\boldsymbol{u} - \boldsymbol{v}$

C $\boldsymbol{u} - \boldsymbol{v}$

D $\boldsymbol{u} + 2\boldsymbol{v}$

20. Evaluate $2 - \log_5 \frac{1}{25}$.

A –3

B 0

C $\frac{3}{2}$

D 4

[END OF SECTION A]

SECTION B

Marks

ALL questions should be attempted.

21. A curve has equation $y = 3x^2 - x^3$.

(*a*) Find the coordinates of the stationary points on this curve and determine their nature. **6**

(*b*) State the coordinates of the points where the curve meets the coordinate axes and sketch the curve. **2**

22. For the polynomial $6x^3 + 7x^2 + ax + b$,

- $x + 1$ is a factor
- 72 is the remainder when it is divided by $x - 2$.

(*a*) Determine the values of a and b. **4**

(*b*) Hence factorise the polynomial completely. **3**

23. (*a*) Find P and Q, the points of intersection of the line $y = 3x - 5$ and the circle C_1 with equation $x^2 + y^2 + 2x - 4y - 15 = 0$. **4**

(*b*) T is the centre of C_1.

Show that PT and QT are perpendicular. **3**

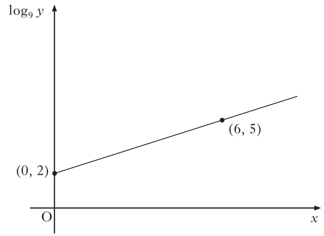
(*c*) A second circle C_2 passes through P, Q and T.

Find the equation of C_2. **3**

24. Two variables, x and y, are related by the equation *Marks*

$$y = ka^x.$$

When $\log_9 y$ is plotted against x, a straight line passing through the points (0, 2) and (6, 5) is obtained, as shown in the diagram.



Find the values of k and a. **5**

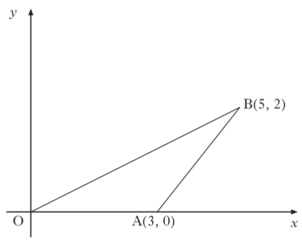
[END OF SECTION B]

[END OF QUESTION PAPER]

2014 paper 2

ALL questions should be attempted. *Marks*

1. A(3, 0), B(5, 2) and the origin are the vertices of a triangle as shown in the diagram.



(*a*) Obtain the equation of the perpendicular bisector of AB. **4**

(*b*) The median from A has equation $y + 2x = 6$.

Find T, the point of intersection of this median and the perpendicular bisector of AB. **2**

(*c*) Calculate the angle that AT makes with the positive direction of the x-axis. **2**

2. A curve has equation $y = x^4 - 2x^3 + 5$.

Find the equation of the tangent to this curve at the point where $x = 2$. **4**

3. Functions f and g are defined on suitable domains by

$$f(x) = x(x-1) + q \quad \text{and} \quad g(x) = x + 3.$$

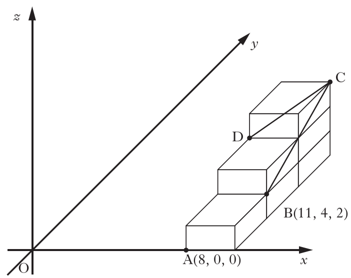
(*a*) Find an expression for $f(g(x))$. **2**

(*b*) Hence, find the value of q such that the equation $f(g(x)) = 0$ has equal roots. **4**

[Turn over

Marks

4. Six identical cuboids are placed with their edges parallel to the coordinate axes as shown in the diagram.



A and B are the points (8, 0, 0) and (11, 4, 2) respectively.

(*a*) State the coordinates of C and D. **2**

(*b*) Determine the components of $\overrightarrow{CB}$ and $\overrightarrow{CD}$. **2**

(*c*) Find the size of the angle BCD. **5**

5. Given that $\int_4^t (3x+4)^{-\frac{1}{2}}\,dx = 2$, find the value of t. **5**

6. Solve the equation

$$\sin x - 2\cos 2x = 1 \qquad \text{for } 0 \le x < 2\pi.$$ **5**

Marks

7. Land enclosed between a path and a railway line is being developed for housing.

This land is represented by the shaded area shown in Diagram 1.

- The path is represented by a parabola with equation $y = 6x - x^2$.
- The railway is represented by a line with equation $y = 2x$.
- One square unit in the diagram represents $300\,m^2$ of land.

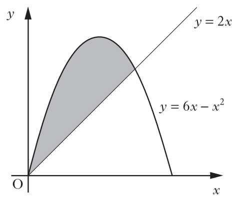


Diagram 1

(*a*) Calculate the area of land being developed. **5**

(*b*) A road is built parallel to the railway line and is a tangent to the path as shown in Diagram 2.

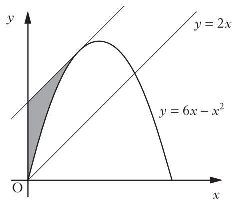


Diagram 2

It is decided that the land, represented by the shaded area in Diagram 2, will become a car park.

Calculate the area of the car park. **5**

[Turn over

Marks

8. Given that the equation

$$x^2 + y^2 - 2px - 4py + 3p + 2 = 0$$

represents a circle, determine the range of values of p. **5**

9. Acceleration is defined as the rate of change of velocity.

An object is travelling in a straight line. The velocity, v m/s, of this object, t seconds after the start of the motion, is given by $v(t) = 8\cos(2t - \frac{\pi}{2})$.

(*a*) Find a formula for $a(t)$, the acceleration of this object, t seconds after the start of the motion. **3**

(*b*) Determine whether the velocity of the object is increasing or decreasing when $t = 10$. **2**

(*c*) Velocity is defined as the rate of change of displacement.

Determine a formula for $s(t)$, the displacement of the object, given that $s(t) = 4$ when $t = 0$. **3**

[*END OF QUESTION PAPER*]

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\mathbf{a.b} = |\mathbf{a}||\mathbf{b}| \cos \theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a.b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + c$
$\cos ax$	$\frac{1}{a}\sin ax + c$

MARKS

Attempt ALL questions

Total marks – 60

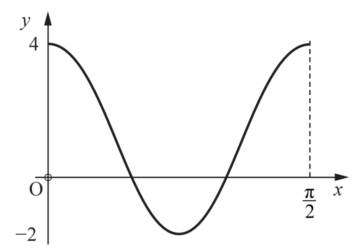
1. Vectors $\mathbf{u} = 8\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $\mathbf{v} = -3\mathbf{i} + t\mathbf{j} - 6\mathbf{k}$ are perpendicular.

Determine the value of t. **2**

2. Find the equation of the tangent to the curve $y = 2x^3 + 3$ at the point where $x = -2$. **4**

3. Show that $(x + 3)$ is a factor of $x^3 - 3x^2 - 10x + 24$ and hence factorise $x^3 - 3x^2 - 10x + 24$ fully. **4**

4. The diagram shows part of the graph of the function $y = p\cos qx + r$.



Write down the values of p, q and r. **3**

5. A function g is defined on $\mathbb{R}$, the set of real numbers, by $g(x) = 6 - 2x$.

(a) Determine an expression for $g^{-1}(x)$. **2**

(b) Write down an expression for $g(g^{-1}(x))$. **1**

6. Evaluate $\log_6 12 + \frac{1}{3}\log_6 27$. **3**

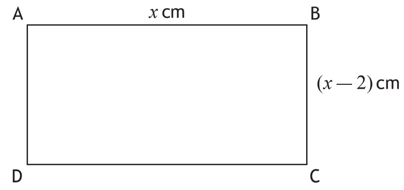
7. A function f is defined on a suitable domain by $f(x) = \sqrt{x}\left(3x - \frac{2}{x\sqrt{x}}\right)$.

Find $f'(4)$. **4**

[Turn over

MARKS

8. ABCD is a rectangle with sides of lengths x centimetres and $(x - 2)$ centimetres, as shown.



If the area of ABCD is less than $15\,\text{cm}^2$, determine the range of possible values of x. **4**

9. A, B and C are points such that AB is parallel to the line with equation $y + \sqrt{3}\,x = 0$ and BC makes an angle of $150°$ with the positive direction of the x-axis.

Are the points A, B and C collinear? **3**

10. Given that $\tan 2x = \frac{3}{4},\ 0 < x < \frac{\pi}{4}$, find the exact value of

(a) $\cos 2x$ **1**

(b) $\cos x$. **2**

11. T$(-2, -5)$ lies on the circumference of the circle with equation

$$(x+8)^2 + (y+2)^2 = 45.$$

(a) Find the equation of the tangent to the circle passing through T. **4**

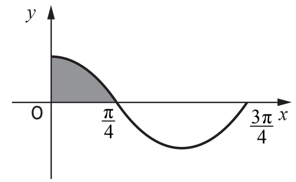
(b) This tangent is also a tangent to a parabola with equation $y = -2x^2 + px + 1 - p$, where $p > 3$.

Determine the value of p. **6**

MARKS

12. The diagram shows part of the graph of $y = a\cos bx$.

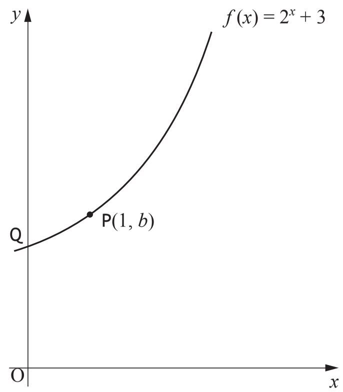
The shaded area is $\frac{1}{2}$ unit^2.



What is the value of $\int_0^{\frac{3\pi}{4}} (a\cos bx)\,dx$? **2**

13. The function $f(x) = 2^x + 3$ is defined on $\mathbb{R}$, the set of real numbers.

The graph with equation $y = f(x)$ passes through the point P$(1, b)$ and cuts the y-axis at Q as shown in the diagram.



(a) What is the value of b? **1**

(b) (i) Copy the above diagram.

On the same diagram, sketch the graph with equation $y = f^{-1}(x)$. **1**

(ii) Write down the coordinates of the images of P and Q. **3**

(c) R $(3, 11)$ also lies on the graph with equation $y = f(x)$.

Find the coordinates of the image of R on the graph with equation $y = 4 - f(x+1)$. **2**

MARKS

14. The circle with equation $x^2 + y^2 - 12x - 10y + k = 0$ meets the coordinate axes at exactly three points.

What is the value of k? **2**

15. The rate of change of the temperature, T °C of a mug of coffee is given by

$$\frac{dT}{dt} = \frac{1}{25}t - k\ ,\ 0 \leq t \leq 50$$

- t is the elapsed time, in minutes, after the coffee is poured into the mug
- k is a constant
- initially, the temperature of the coffee is 100 °C
- 10 minutes later the temperature has fallen to 82 °C.

Express T in terms of t. **6**

[END OF QUESTION PAPER]

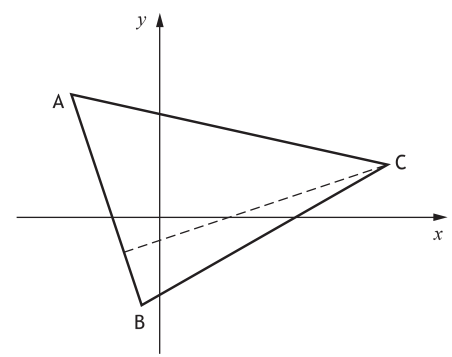
MARKS

Attempt ALL questions

Total marks – 70

1. The vertices of triangle ABC are A$(-5, 7)$, B$(-1, -5)$ and C$(13, 3)$ as shown in the diagram.

The broken line represents the altitude from C.



(a) Show that the equation of the altitude from C is $x - 3y = 4$. **4**

(b) Find the equation of the median from B. **3**

(c) Find the coordinates of the point of intersection of the altitude from C and the median from B. **2**

2. Functions f and g are defined on suitable domains by

$f(x) = 10 + x$ and $g(x) = (1 + x)(3 - x) + 2$.

(a) Find an expression for $f(g(x))$. **2**

(b) Express $f(g(x))$ in the form $p(x + q)^2 + r$. **3**

(c) Another function h is given by $h(x) = \dfrac{1}{f(g(x))}$.

What values of x cannot be in the domain of h? **2**

[Turn over

MARKS

3. A version of the following problem first appeared in print in the 16th Century.

A frog and a toad fall to the bottom of a well that is 50 feet deep.

Each day, the frog climbs 32 feet and then rests overnight. During the night, it slides down $\frac{2}{3}$ of its height above the floor of the well.

The toad climbs 13 feet each day before resting.

Overnight, it slides down $\frac{1}{4}$ of its height above the floor of the well.

Their progress can be modelled by the recurrence relations:

- $f_{n+1} = \frac{1}{3}f_n + 32, \qquad f_1 = 32$
- $t_{n+1} = \frac{3}{4}t_n + 13, \qquad t_1 = 13$

where f_n and t_n are the heights reached by the frog and the toad at the end of the nth day after falling in.

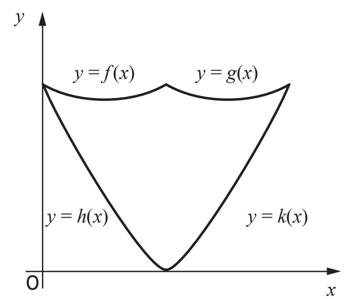
(a) Calculate t_2, the height of the toad at the end of the second day. **1**

(b) Determine whether or not either of them will eventually escape from the well. **5**

MARKS

4. A wall plaque is to be made to commemorate the 150th anniversary of the publication of "*Alice's Adventures in Wonderland*".

The edges of the wall plaque can be modelled by parts of the graphs of four quadratic functions as shown in the sketch.



- $f(x) = \frac{1}{4}x^2 - \frac{1}{2}x + 3$
- $g(x) = \frac{1}{4}x^2 - \frac{3}{2}x + 5$
- $h(x) = \frac{3}{8}x^2 - \frac{9}{4}x + 3$
- $k(x) = \frac{3}{8}x^2 - \frac{3}{4}x$

(a) Find the x-coordinate of the point of intersection of the graphs with equations $y = f(x)$ and $y = g(x)$. **2**

The graphs of the functions $f(x)$ and $h(x)$ intersect on the y-axis.

The plaque has a vertical line of symmetry.

(b) Calculate the area of the wall plaque. **7**

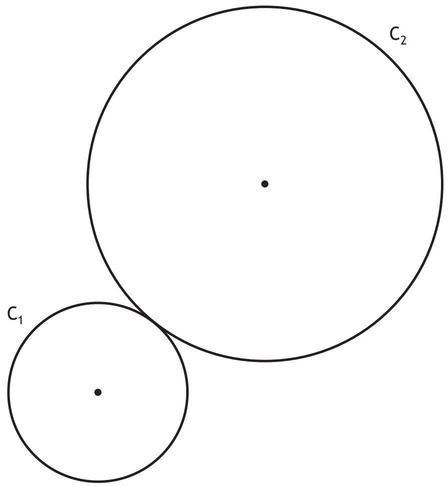
[Turn over

5. Circle C_1 has equation $x^2 + y^2 + 6x + 10y + 9 = 0$.

The centre of circle C_2 is $(9, 11)$.

Circles C_1 and C_2 touch externally.

MARKS



(a) Determine the radius of C_2. **4**

A third circle, C_3, is drawn such that:

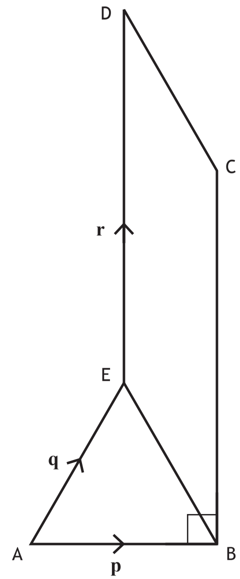
- both C_1 and C_2 touch C_3 internally
- the centres of C_1, C_2 and C_3 are collinear.

(b) Determine the equation of C_3. **4**

6. Vectors $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$ are represented on the diagram as shown.

- BCDE is a parallelogram
- ABE is an equilateral triangle
- $|\mathbf{p}| = 3$
- Angle ABC $= 90°$

MARKS



(a) Evaluate $\mathbf{p}.(\mathbf{q}+\mathbf{r})$. **3**

(b) Express $\overrightarrow{EC}$ in terms of $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$. **1**

(c) Given that $\overrightarrow{AE}.\overrightarrow{EC} = 9\sqrt{3} - \frac{9}{2}$, find $|\mathbf{r}|$. **3**

[Turn over

MARKS

7. (a) Find $\int (3\cos 2x + 1)\,dx$. **2**

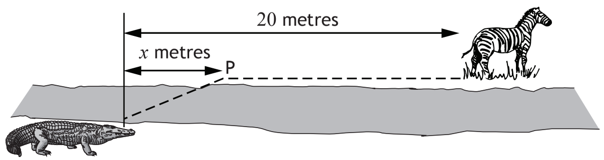
(b) Show that $3\cos 2x + 1 = 4\cos^2 x - 2\sin^2 x$. **2**

(c) Hence, or otherwise, find $\int (\sin^2 x - 2\cos^2 x)\,dx$. **2**

8. A crocodile is stalking prey located 20 metres further upstream on the opposite bank of a river.

Crocodiles travel at different speeds on land and in water.

The time taken for the crocodile to reach its prey can be minimised if it swims to a particular point, P, x metres upstream on the other side of the river as shown in the diagram.



The time taken, T, measured in tenths of a second, is given by

$$T(x) = 5\sqrt{36 + x^2} + 4(20 - x)$$

(a) (i) Calculate the time taken if the crocodile does not travel on land. **1**

(ii) Calculate the time taken if the crocodile swims the shortest distance possible. **1**

(b) Between these two extremes there is one value of x which minimises the time taken. Find this value of x and hence calculate the minimum possible time. **8**

MARKS

9. The blades of a wind turbine are turning at a steady rate.

The height, h metres, of the tip of one of the blades above the ground at time, t seconds, is given by the formula

$$h = 36\sin(1{\cdot}5t) - 15\cos(1{\cdot}5t) + 65.$$

Express $36\sin(1{\cdot}5t) - 15\cos(1{\cdot}5t)$ in the form

$$k\sin(1{\cdot}5t - a), \text{ where } k > 0 \text{ and } 0 < a < \frac{\pi}{2},$$

and hence find the **two** values of t for which the tip of this blade is at a height of 100 metres above the ground during the first turn. **8**

[END OF QUESTION PAPER]

2016 paper 1

MARKS

Attempt ALL questions

Total marks – 60

1. Find the equation of the line passing through the point $(-2, 3)$ which is parallel to the line with equation $y + 4x = 7$. **2**

2. Given that $y = 12x^3 + 8\sqrt{x}$, where $x > 0$, find $\frac{dy}{dx}$. **3**

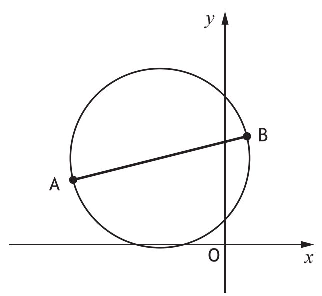
3. A sequence is defined by the recurrence relation $u_{n+1} = \frac{1}{3}u_n + 10$ with $u_3 = 6$.

(a) Find the value of u_4. **1**

(b) Explain why this sequence approaches a limit as $n \to \infty$. **1**

(c) Calculate this limit. **2**

4. A and B are the points $(-7, 3)$ and $(1, 5)$.
AB is a diameter of a circle.



Find the equation of this circle. **3**

[Turn over

MARKS

5. Find $\int 8\cos(4x+1)\,dx$. **2**

6. Functions f and g are defined on $\mathbb{R}$, the set of real numbers.
The inverse functions f^{-1} and g^{-1} both exist.

(a) Given $f(x) = 3x + 5$, find $f^{-1}(x)$. **3**

(b) If $g(2) = 7$, write down the value of $g^{-1}(7)$. **1**

7. Three vectors can be expressed as follows:

$\overrightarrow{FG} = -2\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}$

$\overrightarrow{GH} = 3\mathbf{i} + 9\mathbf{j} - 7\mathbf{k}$

$\overrightarrow{EH} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$

(a) Find $\overrightarrow{FH}$. **2**

(b) Hence, or otherwise, find $\overrightarrow{FE}$. **2**

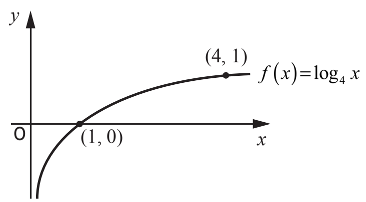
8. Show that the line with equation $y = 3x - 5$ is a tangent to the circle with equation $x^2 + y^2 + 2x - 4y - 5 = 0$ and find the coordinates of the point of contact. **5**

MARKS

9. (a) Find the x-coordinates of the stationary points on the graph with equation $y = f(x)$, where $f(x) = x^3 + 3x^2 - 24x$. **4**

(b) Hence determine the range of values of x for which the function f is strictly increasing. **2**

10. The diagram below shows the graph of the function $f(x) = \log_4 x$, where $x > 0$.



The inverse function, f^{-1}, exists.

On the diagram in your answer booklet, sketch the graph of the inverse function. **2**

11. (a) A and C are the points $(1,\ 3,\ -2)$ and $(4,\ -3,\ 4)$ respectively.

Point B divides AC in the ratio $1 : 2$.

Find the coordinates of B. **2**

(b) $k\overrightarrow{\mathrm{AC}}$ is a vector of magnitude 1, where $k > 0$.

Determine the value of k. **3**

[Turn over

MARKS

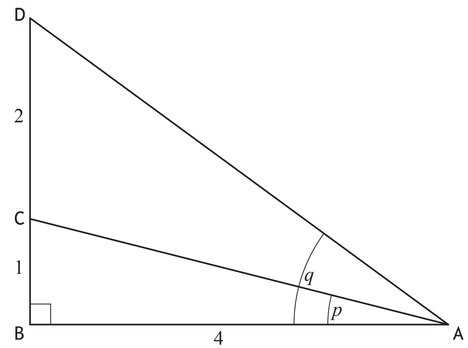
12. The functions f and g are defined on $\mathbb{R}$, the set of real numbers by

$f(x) = 2x^2 - 4x + 5$ and $g(x) = 3 - x$.

(a) Given $h(x) = f(g(x))$, show that $h(x) = 2x^2 - 8x + 11$. **2**

(b) Express $h(x)$ in the form $p(x+q)^2 + r$. **3**

13. Triangle ABD is right-angled at B with angles BAC = p and BAD = q and lengths as shown in the diagram below.



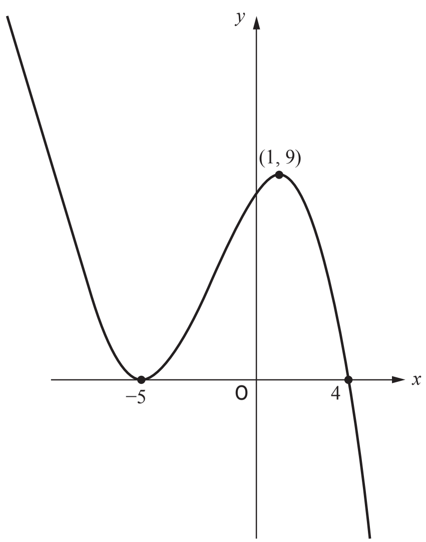
Show that the exact value of $\cos(q - p)$ is $\dfrac{19\sqrt{17}}{85}$. **5**

MARKS

14. (a) Evaluate $\log_5 25$. **1**

(b) Hence solve $\log_4 x + \log_4 (x-6) = \log_5 25$, where $x > 6$. **5**

15. The diagram below shows the graph with equation $y = f(x)$, where $f(x) = k(x-a)(x-b)^2$.



(a) Find the values of a, b and k. **3**

(b) For the function $g(x) = f(x) - d$, where d is positive, determine the range of values of d for which $g(x)$ has exactly one real root. **1**

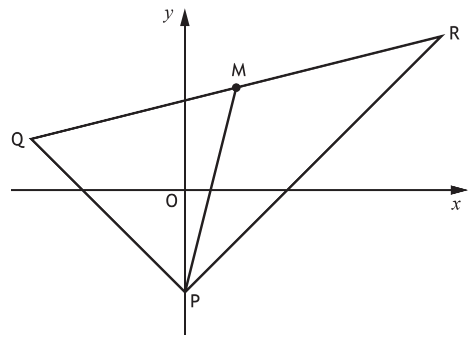
[END OF QUESTION PAPER]

MARKS

Attempt ALL questions

Total marks — 70

1. PQR is a triangle with vertices P$(0,-4)$, Q$(-6,2)$ and R$(10,6)$.



(a) (i) State the coordinates of M, the midpoint of QR. **1**

(ii) Hence find the equation of PM, the median through P. **2**

(b) Find the equation of the line, *L*, passing through M and perpendicular to PR. **3**

(c) Show that line *L* passes through the midpoint of PR. **3**

2. Find the range of values for p such that $x^2 - 2x + 3 - p = 0$ has no real roots. **3**

[Turn over

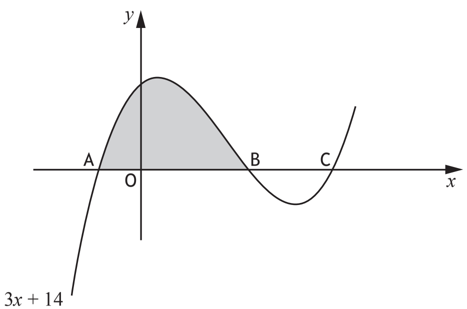
MARKS

3. (a) (i) Show that $(x+1)$ is a factor of $2x^3 - 9x^2 + 3x + 14$. **2**

(ii) Hence solve the equation $2x^3 - 9x^2 + 3x + 14 = 0$. **3**

(b) The diagram below shows the graph with equation $y = 2x^3 - 9x^2 + 3x + 14$.

The curve cuts the x-axis at A, B and C.



$y = 2x^3 - 9x^2 + 3x + 14$

(i) Write down the coordinates of the points A and B. **1**

(ii) Hence calculate the shaded area in the diagram. **4**

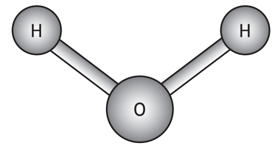
4. Circles C_1 and C_2 have equations $(x+5)^2 + (y-6)^2 = 9$ and $x^2 + y^2 - 6x - 16 = 0$ respectively.

(a) Write down the centres and radii of C_1 and C_2. **4**

(b) Show that C_1 and C_2 do not intersect. **3**

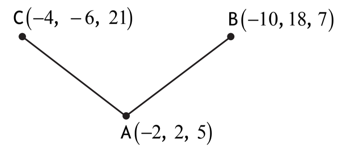
MARKS

5. The picture shows a model of a water molecule.



Relative to suitable coordinate axes, the oxygen atom is positioned at point A$(-2, 2, 5)$.

The two hydrogen atoms are positioned at points B$(-10, 18, 7)$ and C$(-4, -6, 21)$ as shown in the diagram below.



(a) Express $\overrightarrow{AB}$ and $\overrightarrow{AC}$ in component form. **2**

(b) Hence, or otherwise, find the size of angle BAC. **4**

6. Scientists are studying the growth of a strain of bacteria. The number of bacteria present is given by the formula

$$B(t) = 200e^{0\cdot107t},$$

where t represents the number of hours since the study began.

(a) State the number of bacteria present at the start of the study. **1**

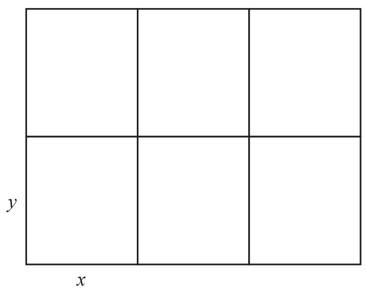
(b) Calculate the time taken for the number of bacteria to double. **4**

[Turn over

MARKS

7. A council is setting aside an area of land to create six fenced plots where local residents can grow their own food.

Each plot will be a rectangle measuring x metres by y metres as shown in the diagram.



(a) The area of land being set aside is 108 m^2.

Show that the total length of fencing, L metres, is given by

$$L(x) = 9x + \frac{144}{x}.$$ **3**

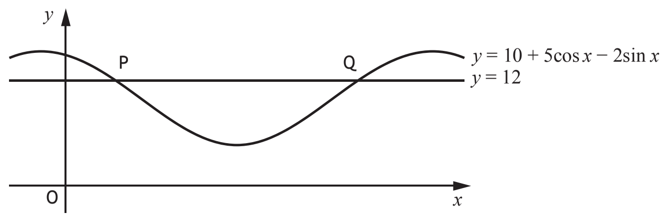
(b) Find the value of x that minimises the length of fencing required. **6**

MARKS

8. (a) Express $5\cos x - 2\sin x$ in the form $k\cos(x + a)$, where $k > 0$ and $0 < a < 2\pi$. **4**

(b) The diagram shows a sketch of part of the graph of $y = 10 + 5\cos x - 2\sin x$ and the line with equation $y = 12$.

The line cuts the curve at the points P and Q.



Find the x-coordinates of P and Q. **4**

9. For a function f, defined on a suitable domain, it is known that:

- $f'(x) = \frac{2x+1}{\sqrt{x}}$
- $f(9) = 40$

Express $f(x)$ in terms of x. **4**

[Turn over for next question

MARKS

10. (a) Given that $y=\left(x^2+7\right)^{\frac{1}{2}}$, find $\frac{dy}{dx}$. **2**

(b) Hence find $\int \frac{4x}{\sqrt{x^2+7}}\,dx$. **1**

11. (a) Show that $\sin 2x \tan x = 1 - \cos 2x$, where $\frac{\pi}{2} < x < \frac{3\pi}{2}$. **4**

(b) Given that $f(x)=\sin 2x \tan x$, find $f'(x)$. **2**

[END OF QUESTION PAPER]

ATTEMPT ALL QUESTIONS

Total marks — 60

MARKS

1. Functions f and g are defined on suitable domains by $f(x)=5x$ and $g(x)=2\cos x$.

(a) Evaluate $f\left(g(0)\right)$. **1**

(b) Find an expression for $g\left(f(x)\right)$. **2**

2. The point P $(-2, 1)$ lies on the circle $x^2+y^2-8x-6y-15=0$.

Find the equation of the tangent to the circle at P. **4**

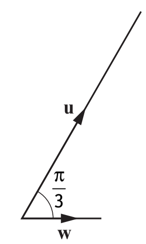
3. Given $y=(4x-1)^{12}$, find $\frac{dy}{dx}$. **2**

4. Find the value of k for which the equation $x^2+4x+(k-5)=0$ has equal roots. **3**

5. Vectors $\mathbf{u}$ and $\mathbf{v}$ are $\begin{pmatrix}5\\1\\-1\end{pmatrix}$ and $\begin{pmatrix}3\\-8\\6\end{pmatrix}$ respectively.

(a) Evaluate $\mathbf{u}.\mathbf{v}$. **1**

(b)



Vector $\mathbf{w}$ makes an angle of $\frac{\pi}{3}$ with $\mathbf{u}$ and $|\mathbf{w}|=\sqrt{3}$.

Calculate $\mathbf{u}.\mathbf{w}$. **3**

MARKS

6. A function, h, is defined by $h(x) = x^3 + 7$, where $x \in \mathbb{R}$.

Determine an expression for $h^{-1}(x)$. **3**

7. A $(-3, 5)$, B $(7, 9)$ and C $(2, 11)$ are the vertices of a triangle.

Find the equation of the median through C. **3**

8. Calculate the rate of change of $d(t) = \frac{1}{2t}$, $t \neq 0$, when $t = 5$. **3**

9. A sequence is generated by the recurrence relation $u_{n+1} = mu_n + 6$ where m is a constant.

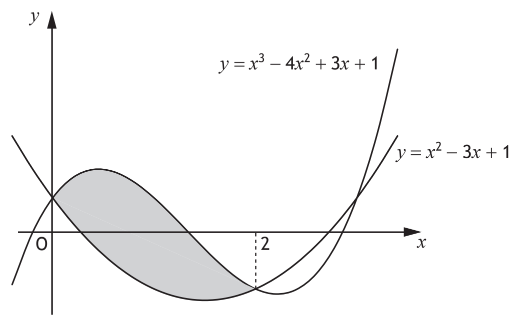
(a) Given $u_1 = 28$ and $u_2 = 13$, find the value of m. **2**

(b) (i) Explain why this sequence approaches a limit as $n \to \infty$. **1**

(ii) Calculate this limit. **2**

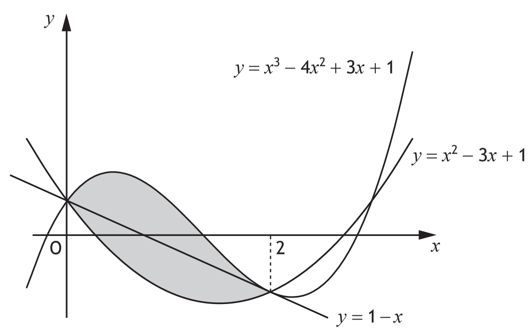
MARKS

10. Two curves with equations $y = x^3 - 4x^2 + 3x + 1$ and $y = x^2 - 3x + 1$ intersect as shown in the diagram.



(a) Calculate the shaded area. **5**

The line passing through the points of intersection of the curves has equation $y = 1 - x$.



(b) Determine the fraction of the shaded area which lies below the line $y = 1 - x$. **4**

[Turn over

MARKS

11. A and B are the points (−7, 2) and (5, a).

AB is parallel to the line with equation $3y - 2x = 4$.

Determine the value of a. **3**

12. Given that $\log_a 36 - \log_a 4 = \frac{1}{2}$, find the value of a. **3**

13. Find $\int \frac{1}{(5-4x)^{\frac{1}{2}}}\,dx,\ \ x < \frac{5}{4}$. **4**

14. (a) Express $\sqrt{3}\sin x° - \cos x°$ in the form $k\sin(x-a)°$, where $k > 0$ and $0 < a < 360$. **4**

(b) Hence, or otherwise, sketch the graph with equation $y = \sqrt{3}\sin x° - \cos x°$, $0 \le x \le 360$. **3**

Use the diagram provided in the answer booklet.

MARKS

15. A quadratic function, f, is defined on $\mathbb{R}$, the set of real numbers.

Diagram 1 shows part of the graph with equation $y = f(x)$.
The turning point is (2, 3).

Diagram 2 shows part of the graph with equation $y = h(x)$.
The turning point is (7, 6).

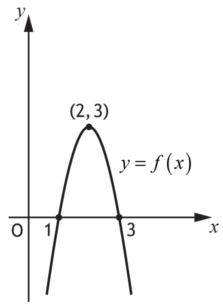


Diagram 1

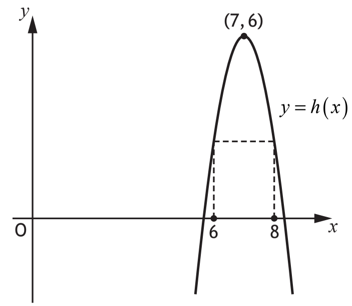


Diagram 2

(a) Given that $h(x) = f(x+a) + b$.

Write down the values of a and b. **2**

(b) It is known that $\int_1^3 f(x)\,dx = 4$.

Determine the value of $\int_6^8 h(x)\,dx$. **1**

(c) Given $f'(1) = 6$, state the value of $h'(8)$. **1**

[END OF QUESTION PAPER]

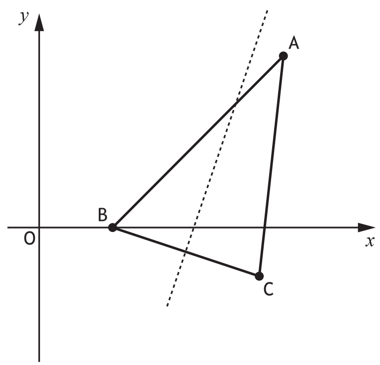
2017 paper 2

MARKS

Attempt ALL questions

Total marks — 70

1. Triangle ABC is shown in the diagram below.
   The coordinates of B are (3,0) and the coordinates of C are (9,−2).
   The broken line is the perpendicular bisector of BC.



(a) Find the equation of the perpendicular bisector of BC. **4**

(b) The line AB makes an angle of 45° with the positive direction of the x-axis.
Find the equation of AB. **2**

(c) Find the coordinates of the point of intersection of AB and the perpendicular bisector of BC. **2**

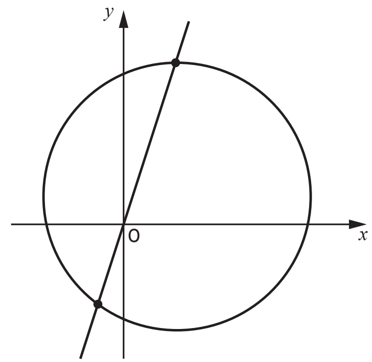
2. (a) Show that $(x-1)$ is a factor of $f(x)=2x^3-5x^2+x+2$. **2**

(b) Hence, or otherwise, solve $f(x)=0$. **3**

[Turn over

MARKS

3. The line $y=3x$ intersects the circle with equation $(x-2)^2+(y-1)^2=25$.



Find the coordinates of the points of intersection. **5**

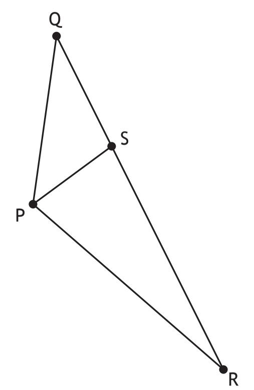
4. (a) Express $3x^2+24x+50$ in the form $a(x+b)^2+c$. **3**

(b) Given that $f(x)=x^3+12x^2+50x-11$, find $f'(x)$. **2**

(c) Hence, or otherwise, explain why the curve with equation $y=f(x)$ is strictly increasing for all values of x. **2**

MARKS

5. In the diagram, $\overrightarrow{PR} = 9\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$ and $\overrightarrow{RQ} = -12\mathbf{i} - 9\mathbf{j} + 3\mathbf{k}$.



(a) Express $\overrightarrow{PQ}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. **2**

The point S divides QR in the ratio 1:2.

(b) Show that $\overrightarrow{PS} = \mathbf{i} - \mathbf{j} + 4\mathbf{k}$. **2**

(c) Hence, find the size of angle QPS. **5**

6. Solve $5\sin x - 4 = 2\cos 2x$ for $0 \le x < 2\pi$. **5**

7. (a) Find the x-coordinate of the stationary point on the curve with equation $y = 6x - 2\sqrt{x^3}$. **4**

(b) Hence, determine the greatest and least values of y in the interval $1 \le x \le 9$. **3**

[Turn over

MARKS

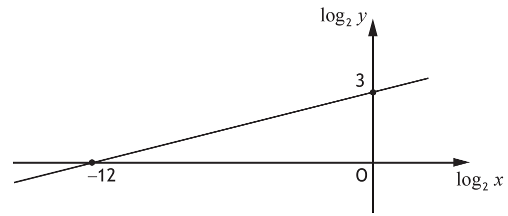
8. Sequences may be generated by recurrence relations of the form $u_{n+1} = k u_n - 20$, $u_0 = 5$ where $k \in \mathbb{R}$.

(a) Show that $u_2 = 5k^2 - 20k - 20$. **2**

(b) Determine the range of values of k for which $u_2 < u_0$. **4**

9. Two variables, x and y, are connected by the equation $y = kx^n$.

The graph of $\log_2 y$ against $\log_2 x$ is a straight line as shown.

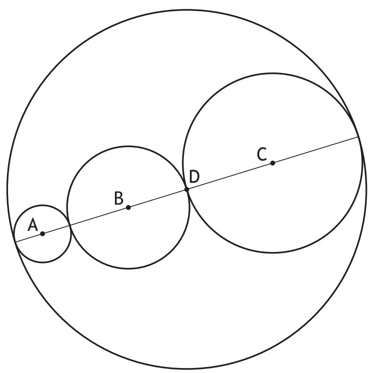


Find the values of k and n. **5**

MARKS

10. (a) Show that the points A(−7, −2), B(2, 1) and C(17, 6) are collinear. **3**

Three circles with centres A, B and C are drawn inside a circle with centre D as shown.



The circles with centres A, B and C have radii r_A, r_B and r_C respectively.

- $r_A = \sqrt{10}$
- $r_B = 2r_A$
- $r_C = r_A + r_B$

(b) Determine the equation of the circle with centre D. **4**

11. (a) Show that $\frac{\sin 2x}{2\cos x} - \sin x\cos^2 x = \sin^3 x$, where $0 < x < \frac{\pi}{2}$. **3**

(b) Hence, differentiate $\frac{\sin 2x}{2\cos x} - \sin x\cos^2 x$, where $0 < x < \frac{\pi}{2}$. **3**

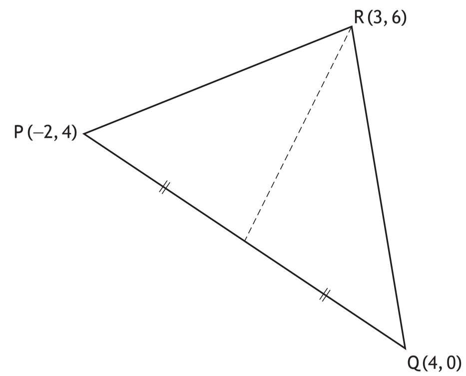
[END OF QUESTION PAPER]

Attempt ALL questions

Total marks — 60

MARKS

1. PQR is a triangle with vertices P(−2, 4), Q(4, 0) and R(3, 6).



Find the equation of the median through R. **3**

2. A function $g(x)$ is defined on $\mathbb{R}$, the set of real numbers, by

$$g(x) = \frac{1}{5}x - 4.$$

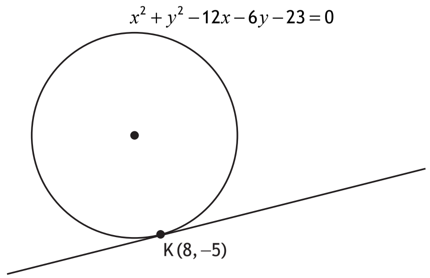
Find the inverse function, $g^{-1}(x)$. **3**

3. Given $h(x) = 3\cos 2x$, find the value of $h'\left(\frac{\pi}{6}\right)$. **3**

[Turn over

MARKS

4. The point K (8, −5) lies on the circle with equation $x^2 + y^2 - 12x - 6y - 23 = 0$.



Find the equation of the tangent to the circle at K. **4**

5. A (−3, 4, −7), B (5, t, 5) and C (7, 9, 8) are collinear.

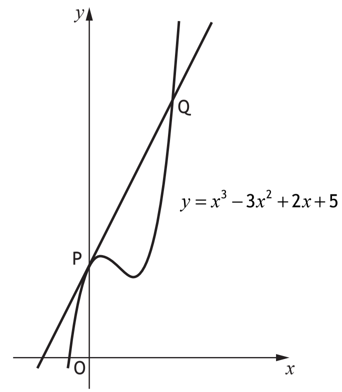
(a) State the ratio in which B divides AC. **1**

(b) State the value of t. **1**

6. Find the value of $\log_5 250 - \frac{1}{3}\log_5 8$. **3**

MARKS

7. The curve with equation $y = x^3 - 3x^2 + 2x + 5$ is shown on the diagram.



(a) Write down the coordinates of P, the point where the curve crosses the y-axis . **1**

(b) Determine the equation of the tangent to the curve at P. **3**

(c) Find the coordinates of Q, the point where this tangent meets the curve again. **4**

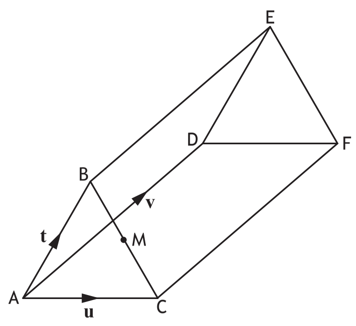
8. A line has equation $y - \sqrt{3}x + 5 = 0$.

Determine the angle this line makes with the positive direction of the x-axis. **2**

[Turn over

9. The diagram shows a triangular prism ABC,DEF.

$\overrightarrow{AB} = \mathbf{t}$, $\overrightarrow{AC} = \mathbf{u}$ and $\overrightarrow{AD} = \mathbf{v}$.



(a) Express $\overrightarrow{BC}$ in terms of $\mathbf{u}$ and $\mathbf{t}$. **1**

M is the midpoint of BC.

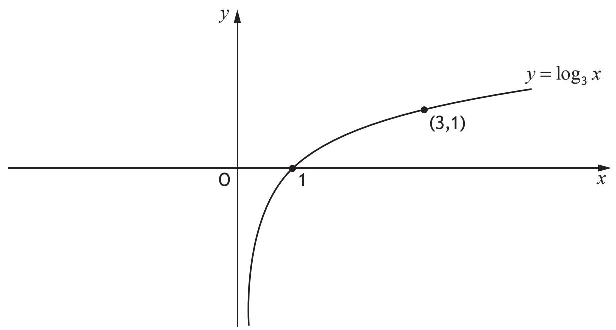
(b) Express $\overrightarrow{MD}$ in terms of $\mathbf{t}$, $\mathbf{u}$ and $\mathbf{v}$. **2**

10. Given that

- $\frac{dy}{dx} = 6x^2 - 3x + 4$, and
- $y = 14$ when $x = 2$,

express y in terms of x. **4**

11. The diagram shows the curve with equation $y = \log_3 x$.



(a) On the diagram in your answer booklet, sketch the curve with equation $y = 1 - \log_3 x$. **2**

(b) Determine the exact value of the x-coordinate of the point of intersection of the two curves. **3**

12. Vectors $\mathbf{a}$ and $\mathbf{b}$ are such that $\mathbf{a} = 4\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ and $\mathbf{b} = -2\mathbf{i} + \mathbf{j} + p\mathbf{k}$.

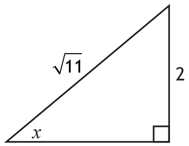
(a) Express $2\mathbf{a} + \mathbf{b}$ in component form. **1**

(b) Hence find the values of p for which $|2\mathbf{a} + \mathbf{b}| = 7$. **3**

[Turn over for next question

MARKS

13. The right-angled triangle in the diagram is such that $\sin x = \frac{2}{\sqrt{11}}$ and $0 < x < \frac{\pi}{4}$.



(a) Find the exact value of:

(i) $\sin 2x$ **3**

(ii) $\cos 2x$. **1**

(b) By expressing $\sin 3x$ as $\sin(2x + x)$, find the exact value of $\sin 3x$. **3**

14. Evaluate $\int_{-4}^{9} \frac{1}{\sqrt[3]{(2x+9)^2}}\,dx$. **5**

15. A cubic function, f, is defined on the set of real numbers.

- $(x+4)$ is a factor of $f(x)$
- $x = 2$ is a repeated root of $f(x)$
- $f'(-2) = 0$
- $f'(x) > 0$ where the graph with equation $y = f(x)$ crosses the y-axis

Sketch a possible graph of $y = f(x)$ on the diagram in your answer booklet. **4**

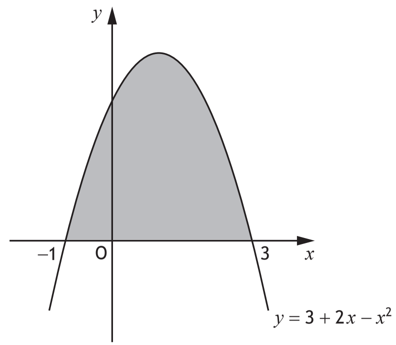
[END OF QUESTION PAPER]

MARKS

Attempt ALL questions

Total marks — 70

1. The diagram shows the curve with equation $y = 3 + 2x - x^2$.



Calculate the shaded area. **4**

2. Vectors **u** and **v** are defined by $\mathbf{u} = \begin{pmatrix} -1 \\ 4 \\ -3 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} -7 \\ 8 \\ 5 \end{pmatrix}$.

(a) Find $\mathbf{u}.\mathbf{v}$. **1**

(b) Calculate the acute angle between **u** and **v**. **4**

3. A function, f, is defined on the set of real numbers by $f(x) = x^3 - 7x - 6$.

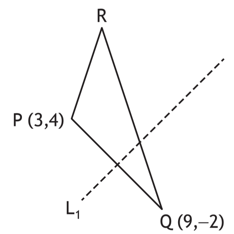
Determine whether f is increasing or decreasing when $x = 2$. **3**

4. Express $-3x^2 - 6x + 7$ in the form $a(x+b)^2 + c$. **3**

[Turn over

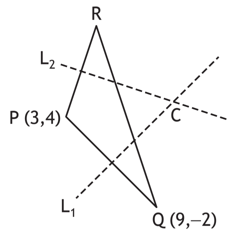
MARKS

5. PQR is a triangle with P(3,4) and Q(9,–2).



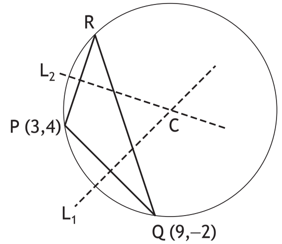
(a) Find the equation of L_1, the perpendicular bisector of PQ. **3**

The equation of L_2, the perpendicular bisector of PR is $3y + x = 25$.



(b) Calculate the coordinates of C, the point of intersection of L_1 and L_2. **2**

C is the centre of the circle which passes through the vertices of triangle PQR.



(c) Determine the equation of this circle. **2**

MARKS

6. Functions, f and g, are given by $f(x) = 3 + \cos x$ and $g(x) = 2x$, $x \in \mathbb{R}$.

(a) Find expressions for

(i) $f(g(x))$ and **2**

(ii) $g(f(x))$. **1**

(b) Determine the value(s) of x for which $f(g(x)) = g(f(x))$ where $0 \leq x < 2\pi$. **6**

7. (a) (i) Show that $(x-2)$ is a factor of $2x^3 - 3x^2 - 3x + 2$. **2**

(ii) Hence, factorise $2x^3 - 3x^2 - 3x + 2$ fully. **2**

The fifth term, u_5, of a sequence is $u_5 = 2a - 3$.

The terms of the sequence satisfy the recurrence relation $u_{n+1} = au_n - 1$.

(b) Show that $u_7 = 2a^3 - 3a^2 - a - 1$. **1**

For this sequence, it is known that

- $u_7 = u_5$
- a limit exists.

(c) (i) Determine the value of a. **3**

(ii) Calculate the limit. **1**

[Turn over

MARKS

8. (a) Express $2\cos x° - \sin x°$ in the form $k\cos(x-a)°$, $k>0$, $0<a<360$. **4**

(b) Hence, or otherwise, find

(i) the minimum value of $6\cos x° - 3\sin x°$ and **1**

(ii) the value of x for which it occurs where $0 \le x < 360$. **2**

9. A sector with a particular fixed area has radius x cm.

The perimeter, P cm, of the sector is given by

$$P = 2x + \frac{128}{x}.$$

Find the minimum value of P. **6**

10. The equation $x^2 + (m-3)x + m = 0$ has two real and distinct roots.

Determine the range of values for m. **4**

11. A supermarket has been investigating how long customers have to wait at the checkout.

During any half hour period, the percentage, P%, of customers who wait for less than t minutes, can be modelled by

$P = 100(1 - e^{kt})$, where k is a constant.

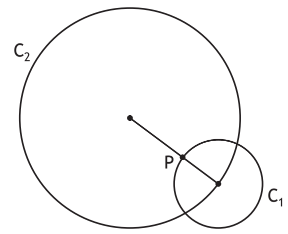
(a) If 50% of customers wait for less than 3 minutes, determine the value of k. **4**

(b) Calculate the percentage of customers who wait for 5 minutes or longer. **2**

MARKS

12. Circle C_1 has equation $(x-13)^2 + (y+4)^2 = 100$.

Circle C_2 has equation $x^2 + y^2 + 14x - 22y + c = 0$.



(a) (i) Write down the coordinates of the centre of C_1. **1**

(ii) The centre of C_1 lies on the circumference of C_2.

Show that $c = -455$. **1**

The line joining the centres of the circles intersects C_1 at P.

(b) (i) Determine the ratio in which P divides the line joining the centres of the circles. **2**

(ii) Hence, or otherwise, determine the coordinates of P. **2**

P is the centre of a third circle, C_3.

C_2 touches C_3 internally.

(c) Determine the equation of C_3. **1**

[END OF QUESTION PAPER]

2019

Attempt ALL questions

Total marks — 70

MARKS

1. Find the x-coordinates of the stationary points on the curve with equation $y=\frac{1}{2}x^4-2x^3+6$. **4**

2. The equation $x^2+(k-5)x+1=0$ has equal roots.
   Determine the possible values of k. **3**

3. Circle C_1 has equation $x^2+y^2-6x-2y-26=0$.
   Circle C_2 has centre (4,–2).
   The radius of C_2 is equal to the radius of C_1.
   Find the equation of circle C_2. **2**

4. A sequence is generated by the recurrence relation

$$u_{n+1}=mu_n+c,$$

   where the first three terms of the sequence are 6, 9 and 11.

   (a) Find the values of m and c. **3**

   (b) Hence, calculate the fourth term of the sequence. **1**

5. (a) Show that the points A(1,5,–3), B(4,–1,0) and C(8,–9,4) are collinear. **3**

   (b) State the ratio in which B divides AC. **1**

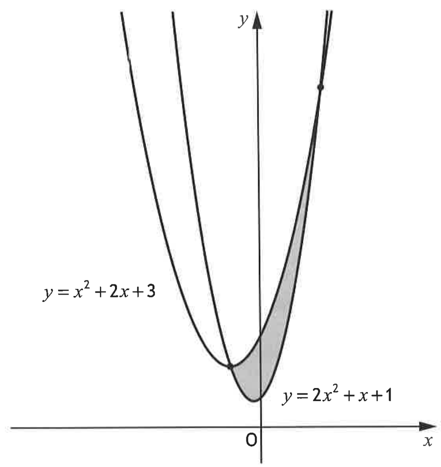
[Turn over

MARKS

6. Given that $y=\frac{1}{(1-3x)^5}$, $x\neq\frac{1}{3}$, find $\frac{dy}{dx}$. **3**

7. The line, L, makes an angle of 30° with the positive direction of the x-axis.
   Find the equation of the line perpendicular to L, passing through (0,–4). **4**

8. The graphs of $y=x^2+2x+3$ and $y=2x^2+x+1$ are shown below.



   The graphs intersect at the points where $x=-1$ and $x=2$.

   (a) Express the shaded area, enclosed between the curves, as an integral. **1**

   (b) Evaluate the shaded area. **3**

MARKS

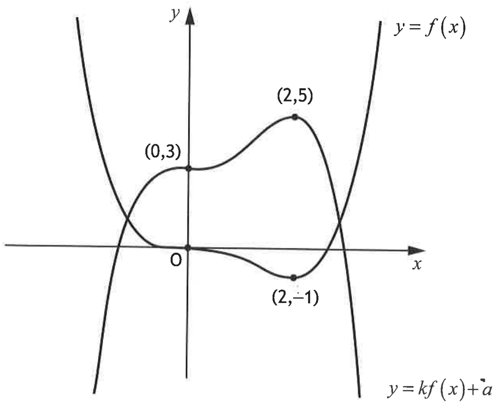
9. Vectors **u** and **v** have components $\begin{pmatrix} p \\ -2 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} 2p+16 \\ -3 \\ 6 \end{pmatrix}$, $p \in \mathbb{R}$.

(a) (i) Find an expression for **u.v**. **1**

(ii) Determine the values of p for which **u** and **v** are perpendicular. **3**

(b) Determine the value of p for which **u** and **v** are parallel. **2**

10. The diagram shows the graphs with equations $y = f(x)$ and $y = kf(x) + a$.



(a) State the value of a. **1**

(b) Find the value of k. **1**

11. Evaluate $\int_0^{\frac{\pi}{9}} \cos\left(3x - \frac{\pi}{6}\right) dx$. **4**

[Turn over

MARKS

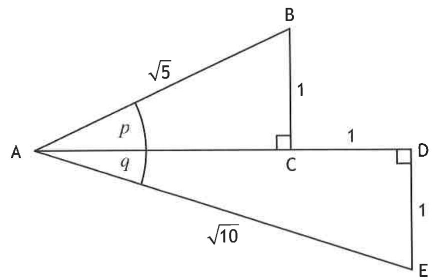
12. Functions f and g are defined by

- $f(x) = \frac{1}{\sqrt{x}}$, where $x > 0$
- $g(x) = 5 - x$, where $x \in \mathbb{R}$.

(a) Determine an expression for $f(g(x))$. **2**

(b) State the range of values of x for which $f(g(x))$ is undefined. **1**

13. Triangles ABC and ADE are both right angled.
Angles p and q are as shown in the diagram.



(a) Determine the value of

(i) $\cos p$ **1**

(ii) $\cos q$. **1**

(b) Hence determine the value of $\sin(p+q)$. **3**

14. (a) Evaluate $\log_{10} 4 + 2\log_{10} 5$. **3**

(b) Solve $\log_2(7x-2) - \log_2 3 = 5$, $x \ge 1$. **3**

MARKS

15. (a) Solve the equation $\sin 2x° + 6\cos x° = 0$ for $0 \le x < 360$. **4**

(b) Hence solve $\sin 4x° + 6\cos 2x° = 0$ for $0 \le x < 360$. **1**

16. The point P has coordinates $(4,k)$.

C is the centre of the circle with equation $(x-1)^2 + (y+2)^2 = 25$.

(a) Show that the distance between the points P and C is given by $\sqrt{k^2+4k+13}$. **2**

(b) Hence, or otherwise, find the range of values of k such that P lies outside the circle. **4**

17. (a) Express $(\sin x - \cos x)^2$ in the form $p + q\sin rx$ where p, q and r are integers. **3**

(b) Hence, find $\int (\sin x - \cos x)^2\, dx$. **2**

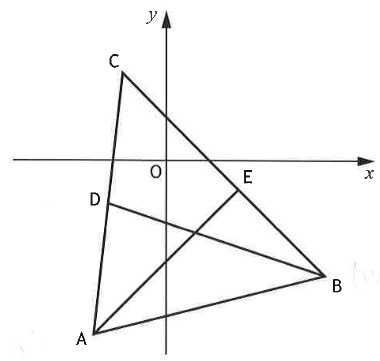
[END OF QUESTION PAPER]

MARKS

Attempt ALL questions

Total marks — 80

1. Triangle ABC has vertices A(–5,–12), B(11,–8) and C(–3,6).



(a) Find the equation of the median BD. **3**

(b) Find the equation of the altitude AE. **3**

(c) Find the coordinates of the point of intersection of BD and AE. **2**

2. Find $\int \left(6\sqrt{x} - 4x^{-3} + 5\right) dx$. **4**

[Turn over

MARKS

8. A function, f, is given by $f(x)=\sqrt[3]{x}+8$.

The domain of f is $1\le x\le 1000,\ x\in\mathbb{R}$.

The inverse function, f^{-1}, exists.

(a) Find $f^{-1}(x)$. **3**

(b) State the domain of f^{-1}. **1**

9. Electricity on a spacecraft can be produced by a type of nuclear generator.

The electrical power produced by this generator can be modelled by

$$P_t=120e^{-0.0079t}$$

where P_t is the electrical power produced, in watts, after t years.

(a) Determine the electrical power initially produced by the generator. **1**

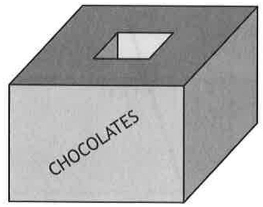
(b) Calculate how long it takes for the electrical power produced by the generator to reduce by 15%. **4**

10. (a) Show that $(x+3)$ is a factor of $3x^4+10x^3+x^2-8x-6$. **2**

(b) Hence, or otherwise, factorise $3x^4+10x^3+x^2-8x-6$ fully. **5**

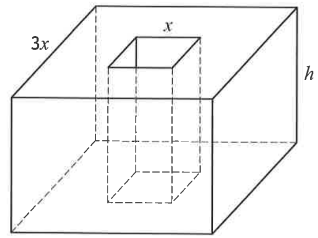
MARKS

11. A manufacturer of chocolates is launching a new product in novelty shaped cardboard boxes.



The box is a cuboid with a cuboid shaped tunnel through it.

- The height of the box is h centimetres
- The top of the box is a square of side $3x$ centimetres
- The end of the tunnel is a square of side x centimetres
- The volume of the box is 2000 cm^3



(a) Show that the total surface area, A cm^2, of the box is given by

$$A=16x^2+\frac{4000}{x}.$$ **3**

(b) To minimise the cost of production, the surface area, A, of the box should be as small as possible.

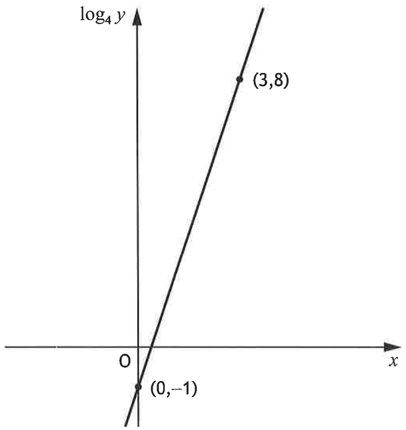
Find the minimum value of A. **6**

[Turn over

MARKS

12. Two variables, x and y, are connected by the equation $y = ab^x$.

The graph of $\log_4 y$ against x is a straight line as shown.



Find the values of a and b. **5**

13. For a function, f, defined on the set of real numbers, $\mathbb{R}$, it is known that

- the rate of change of f with respect to x is given by $3x^2 - 16x + 11$
- the graph with equation $y = f(x)$ crosses the x-axis at (7,0).

Express $f(x)$ in terms of x. **5**

14. The vectors $\mathbf{u}$ and $\mathbf{v}$ are such that

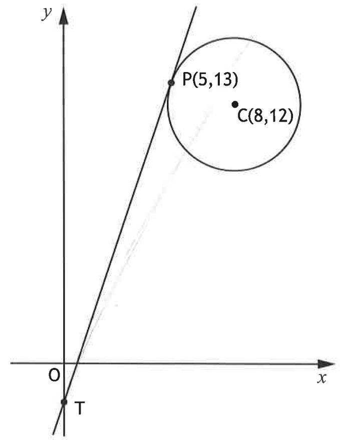
- $|\mathbf{u}| = 4$
- $|\mathbf{v}| = 5$
- $\mathbf{u}.(\mathbf{u}+\mathbf{v}) = 21$

Determine the size of the angle between the vectors $\mathbf{u}$ and $\mathbf{v}$. **4**

MARKS

15. A circle has centre C(8,12).

The point P(5,13) lies on the circle as shown.



(a) Find the equation of the tangent at P. **3**

The tangent from P meets the y-axis at the point T.

(b) (i) State the coordinates of T. **1**

(ii) Find the equation of the circle that passes through the points C, P and T. **3**

[END OF QUESTION PAPER]