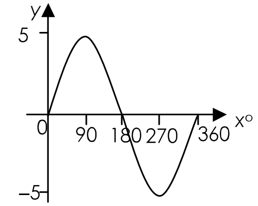
4.1 WORKING with the GRAPHS of TRIGONOMETRIC FUNCTIONS (1)

1. The graphs represent the functions $a \sin x^o$ and $a \cos x^o$. Write down the equation for each.

(a)


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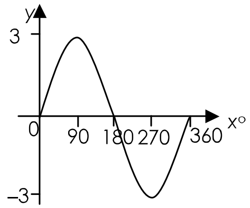

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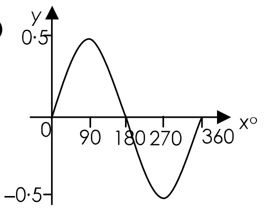

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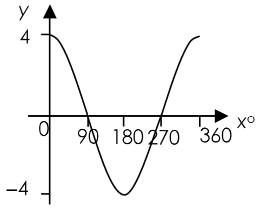

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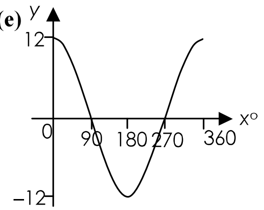

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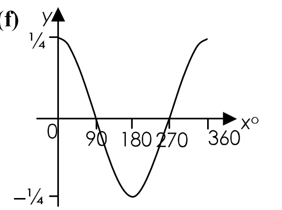

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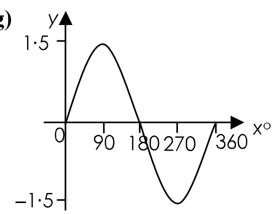

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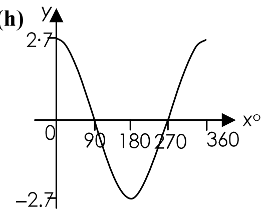

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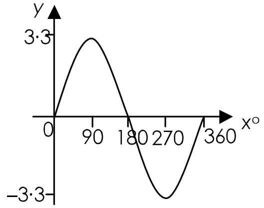

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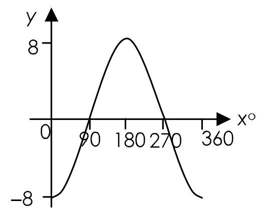

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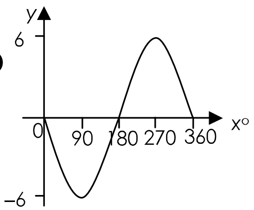

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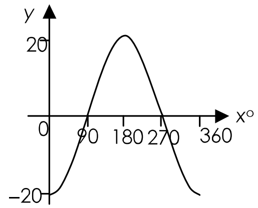

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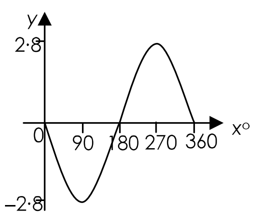

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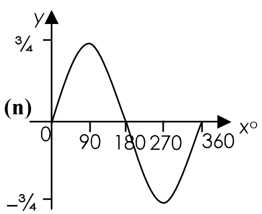

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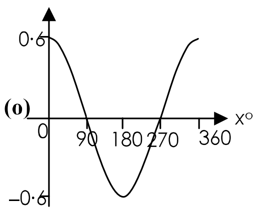
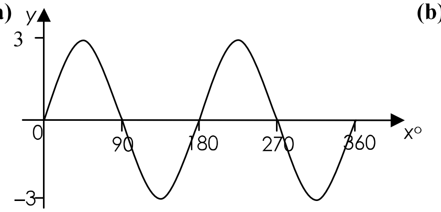


2. The graphs represent trigonometric functions. Write down the equation for each.

(a)


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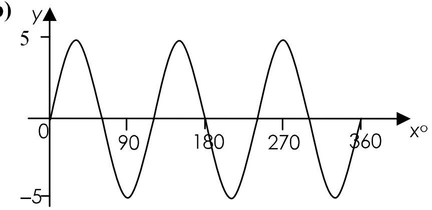

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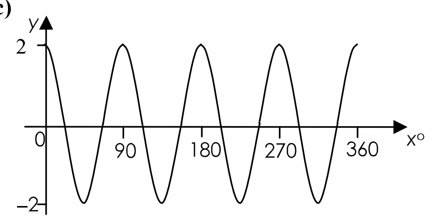

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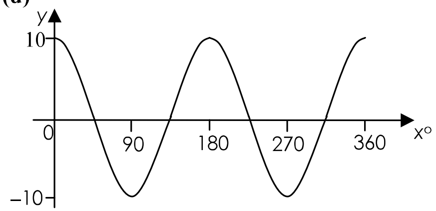

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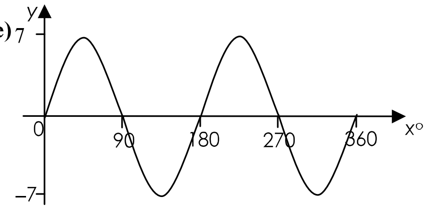

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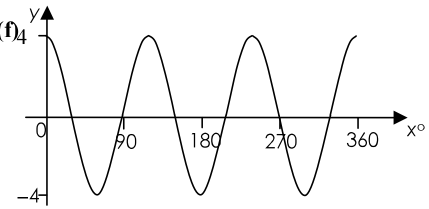

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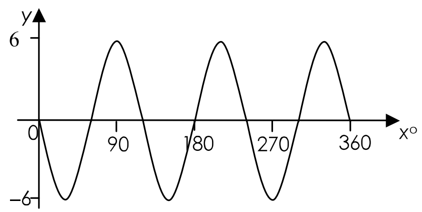

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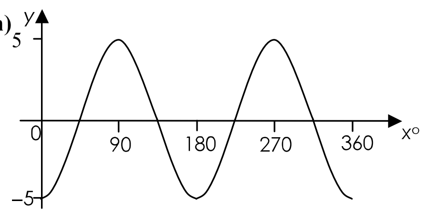

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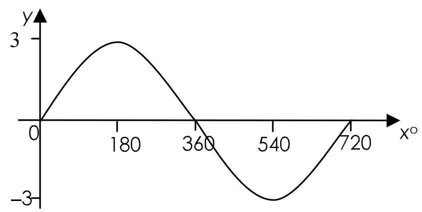

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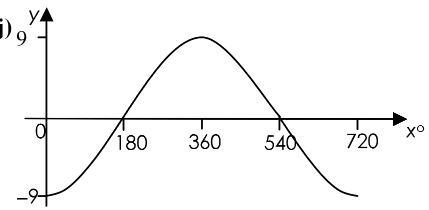

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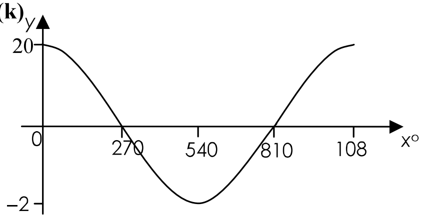

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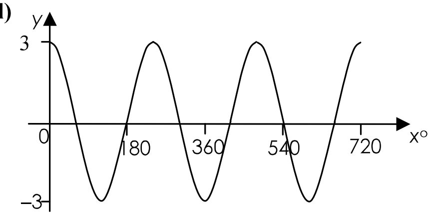

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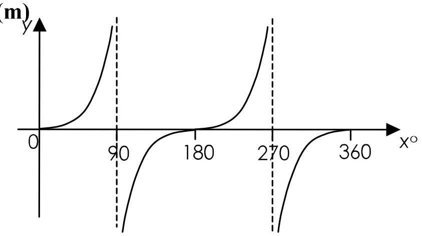

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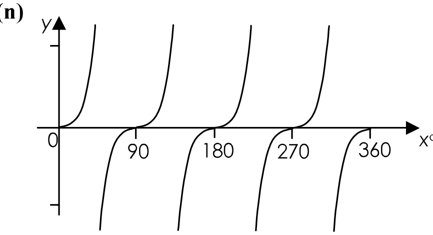

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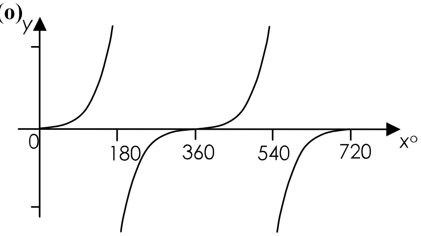

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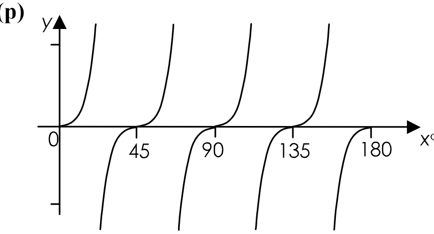
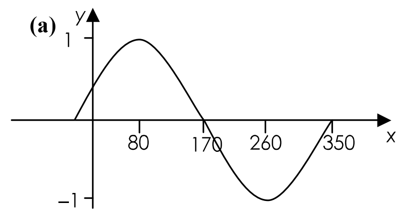


3. Make sketches of the following functions, $0 \le x < 360$, clearly marking any important points.

(a)	$y = \cos x^o$	**(b)**	$y = \sin x^o$	**(c)**	$y = \tan x^o$
(d)	$y = 3 \sin x^o$	**(e)**	$y = 2 \cos x^o$	**(f)**	$y = \sin 2x^o$
(g)	$y = \cos 3x^o$	**(h)**	$y = 2 \sin 3x^o$	**(i)**	$y = 3 \cos 2x^o$
(j)	$y = 4 \cos 3x^o$	**(k)**	$y = 3 \sin \tfrac{1}{2}x^o$	**(l)**	$y = 5 \cos {}^3/_2\, x^o$
(m)	$y = \tan 2x^o$	**(n)**	$y = -2 \sin 3x^o$	**(o)**	$y = -8 \cos 4x^o$

4. The graphs represent the functions $\sin (x \pm a)^o$ and $\cos (x \pm a)^o$. Write down the equation for each.

(a)



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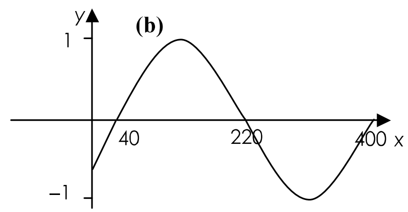

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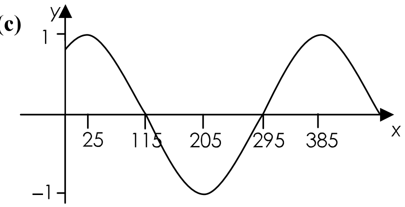

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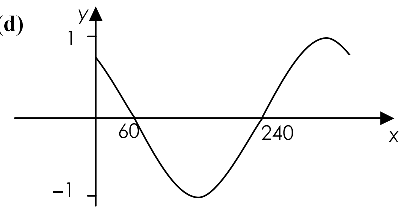

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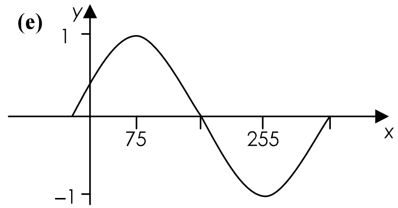

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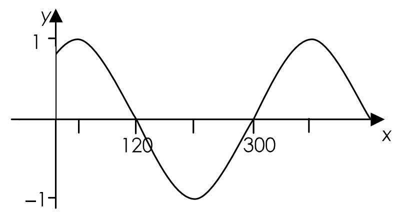
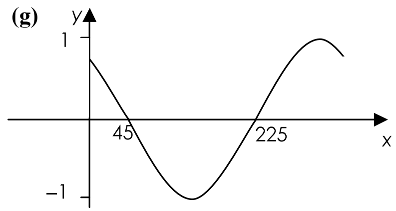
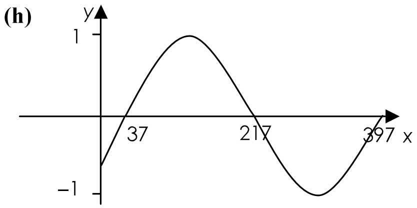


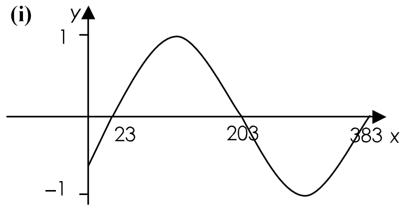
(g)



(h)



(i)



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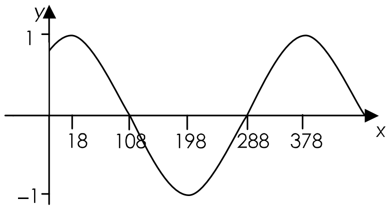
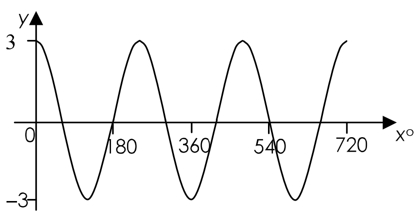


(i)



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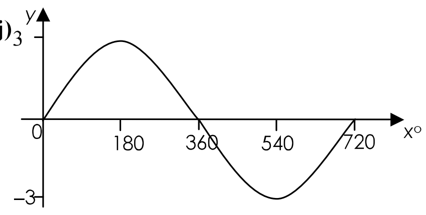


2. Write down the period of each of the following trigonometrical functions.

(a)	$y = \sin 2x^o$	**(b)**	$y = \tan 2x^o$	**(c)**	$y = \cos 2x^o$
(d)	$y = \tan 3x^o$	**(e)**	$y = \cos 4x^o$	**(f)**	$y = \sin 3x^o$
(g)	$y = \cos 1{\cdot}5x^o$	**(h)**	$y = \sin 4{\cdot}5x^o$	**(i)**	$y = \tan 0{\cdot}5x^o$
(j)	$y = \sin 8x^o$	**(k)**	$y = \tan 6x^o$	**(l)**	$y = \cos 12x^o$
(m)	$y = \tan 18x^o$	**(n)**	$y = \cos 9x^o$	**(o)**	$y = \sin 30x^o$
(p)	$y = \cos 15x^o$	**(q)**	$y = \sin 10x^o$	**(r)**	$y = \tan 4x^o$

3. Write down the period of each of the following trigonometrical functions.

(a)	$y = \sin \frac{1}{2} x^o$	**(b)**	$y = \tan {}^1/_3x^o$	**(c)**	$y = \cos \frac{1}{4} x^o$
(d)	$y = \tan {}^1/_5x^o$	**(e)**	$y = \cos {}^1/_6x^o$	**(f)**	$y = \sin {}^2/_3x^o$
(g)	$y = 3\cos 2x^o$	**(h)**	$y = 4\sin 3x^o$	**(i)**	$y = 2\tan 2x^o$
(j)	$y = 5\sin 2x^o$	**(k)**	$y = 4\tan x^o$	**(l)**	$y = 2\cos 4x^o$

4.2 WORKING with TRIGONOMETRIC RELATIONSHIPS in DEGREES

SOLVING BASIC EQUATIONS

1. Solve the following equations where $0 \leq x \leq 360$

(a)	$\sin x^o = 0{\cdot}5$	**(b)**	$\cos x^o = 0{\cdot}866$	**(c)**	$\tan x^o = 1$
(d)	$\cos x^o = -0{\cdot}5$	**(e)**	$\tan x^o = -0{\cdot}577$	**(f)**	$\sin x^o = -0{\cdot}866$
(g)	$\tan x^o = 1{\cdot}732$	**(h)**	$\sin x^o = 0{\cdot}707$	**(i)**	$\cos x^o = 0{\cdot}707$
(j)	$\sin x^o = -0{\cdot}707$	**(k)**	$\cos x^o = -0{\cdot}866$	**(l)**	$\tan x^o = -1{\cdot}732$

2. Solve the following equations where $0 \leq x \leq 360$

(a)	$\sin x^o = 0{\cdot}313$	**(b)**	$\cos x^o = 0{\cdot}425$	**(c)**	$\tan x^o = 5{\cdot}145$
(d)	$\cos x^o = -0{\cdot}087$	**(e)**	$\tan x^o = -0{\cdot}869$	**(f)**	$\sin x^o = -0{\cdot}191$
(g)	$\tan x^o = 11{\cdot}43$	**(h)**	$\sin x^o = 0{\cdot}695$	**(i)**	$\cos x^o = 0{\cdot}755$
(j)	$\sin x^o = -0{\cdot}358$	**(k)**	$\cos x^o = -0{\cdot}682$	**(l)**	$\tan x^o = -0{\cdot}268$

3. Solve the following equations where $0 \leq x \leq 360$

(a)	$2 \sin x^o = 1$	**(b)**	$3 \cos x^o = 2$	**(c)**	$3 \tan x^o = 5$
(d)	$2 \cos x^o = -1$	**(e)**	$2 \tan x^o = -8$	**(f)**	$4 \sin x^o = -3$
(g)	$5 \tan x^o = 23{\cdot}5$	**(h)**	$5 \sin x^o = 2$	**(i)**	$6 \cos x^o = 1$
(j)	$8 \sin x^o = -3$	**(k)**	$11 \cos x^o = -9$	**(l)**	$10 \tan x^o = -9$

4. Solve the following equations where $0 \leq x \leq 360$

(a)	$\sin x^o - 1 = 0$	**(b)**	$\cos x^o + 1 = 0$	**(c)**	$\tan x^o - 1 = 0$
(d)	$2 \sin x^o + 1 = 0$	**(e)**	$2 \cos x^o - 1 = 0$	**(f)**	$2 \tan x^o - 1 = 0$
(g)	$4 \cos x^o - 3 = 0$	**(h)**	$3 \sin x^o - 2 = 0$	**(i)**	$5 \cos x^o + 2 = 0$
(j)	$3 \tan x^o - 2 = 0$	**(k)**	$3 \cos x^o + 1 = 0$	**(l)**	$7 \sin x^o + 3 = 0$

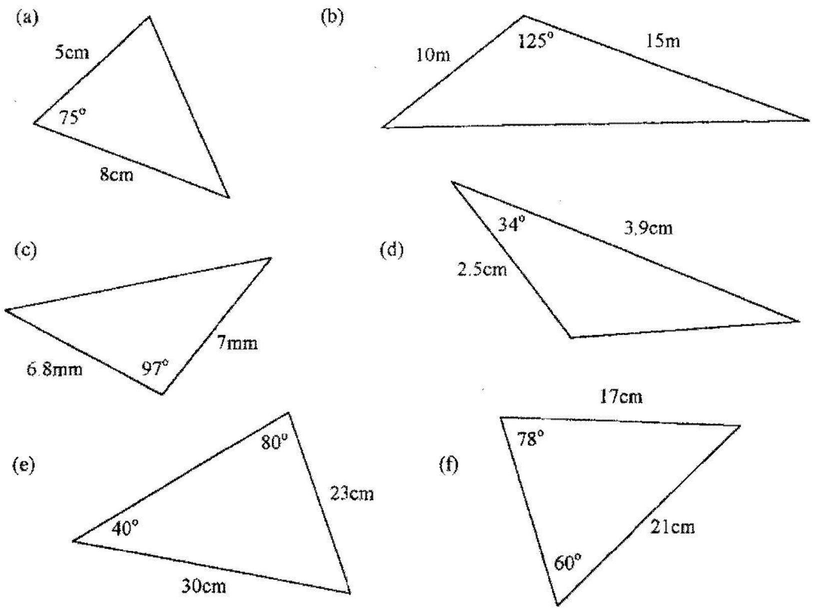
5. Solve the following equations where $0 \leq x \leq 360$

(a)	$4 \cos x^o + 3 = 2$	**(b)**	$10 \sin x^o - 4 = 3$	**(c)**	$2 \tan x^o - 3 = 17$
(d)	$7 + 10 \cos x^o = 12$	**(e)**	$2 \tan x^o + 3 = 5$	**(f)**	$17 - 5 \cos x^o = 20$
(g)	$5 \sin x^o + 3 = 5$	**(h)**	$21 + 2 \cos x^o = 20$	**(i)**	$2 \sin x^o - 1{\cdot}6 = 0$
(j)	$3 \cos x^o + \sqrt{2} = 0$	**(k)**	$7 \sin x^o - 1 = 4$	**(l)**	$2 \sin x^o + \sqrt{3} = 2\sqrt{2}$

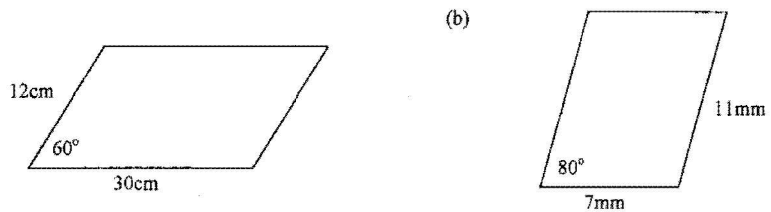
1.1 CALCULATING the AREA of a TRIANGLE using TRIGONOMETRY

The area of a triangle : $A = \frac{1}{2}ab\sin C$

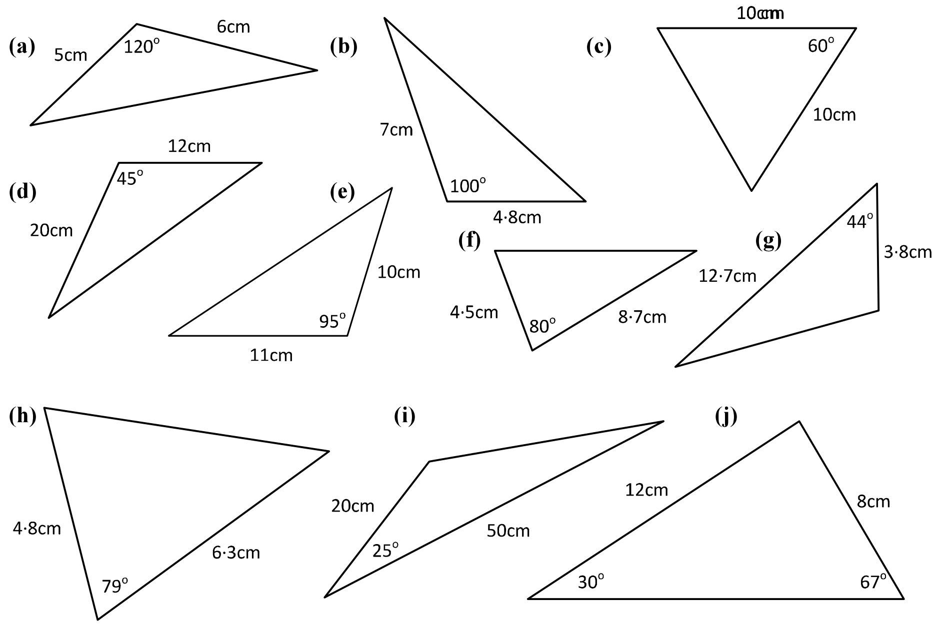
1. Use trigonometry to calculate the area of each triangle below.



2. Calculate the area of each parallelogram below.



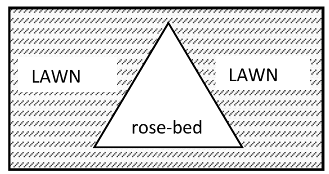
3. Find the area of the following triangles :



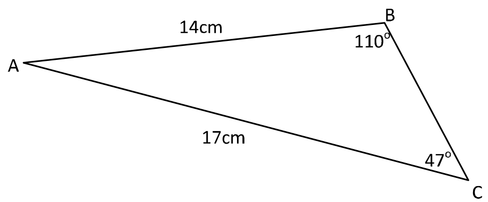
4. Mr. Fields is planting a rose-bed in his garden.

It is to be in the shape of an **equilateral** triangle of side 2m.

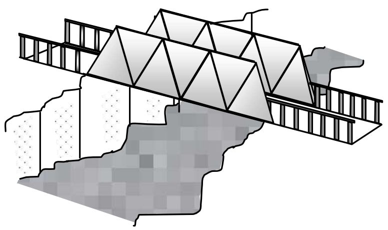
What area of lawn will he need to remove to plant his rose-bed?



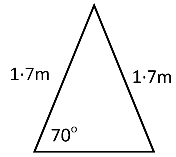
5. Calculate the area of triangle ABC where AB = 14cm, AC = 17cm, $\angle ABC = 110^\circ$ and $\angle BCA = 47^\circ$.



6. For safety reasons the sides of a footbridge are to be covered with triangular panels.



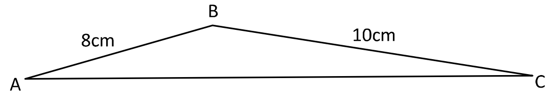
Each panel is an isosceles triangle as shown.



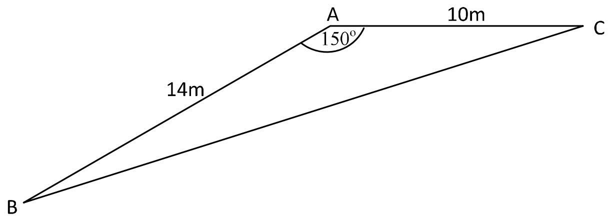
(a) Find the area of each panel.

(b) If there are 7 panels on each side of the bridge, find the total area of material required to cover the bridge.

7. Given that the area of this triangle is $20cm^2$, calculate the size of the **obtuse** angle ABC.



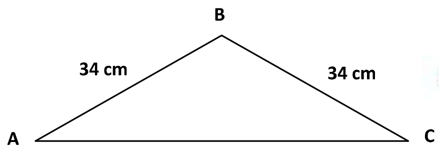
8. In triangle ABC, AB = 14m and AC = 10m. Angle BAC = 150°.



Given that $\sin 150^\circ = 0{\cdot}5$, calculate the area of triangle ABC.

9. The area of a triangular flag is $429{\cdot}5cm^2$.

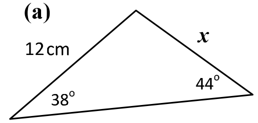
Calculate the size of the obtuse angle ABC.



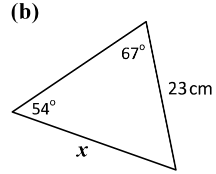
1.2 USING the SINE RULE to CALCULATE a SIDE

1. Use the *sine rule* to calculate the side marked x in each triangle below.

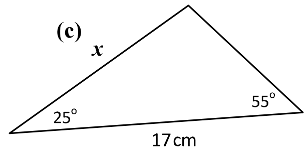
(a)



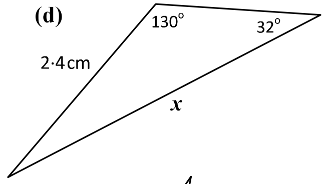
(b)



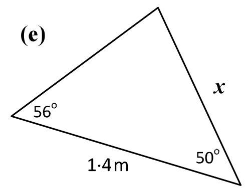
(c)



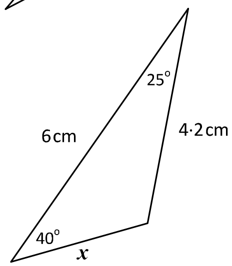
(d)



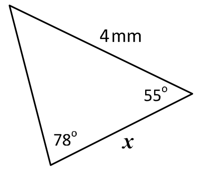
(e)



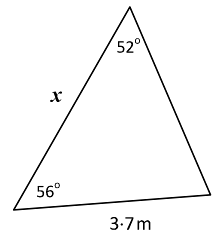
(f)



(g)

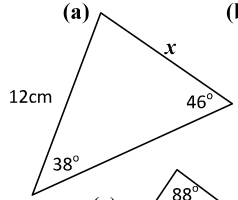


(h)

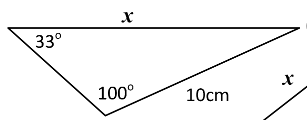


2. Use the sine rule to calculate the length of the side marked x in each of the triangles below.

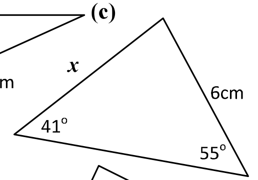
(a)



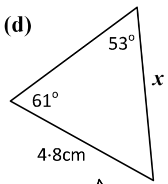
(b)



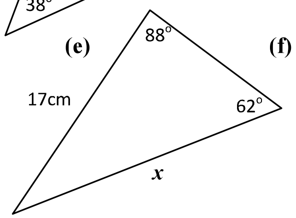
(c)



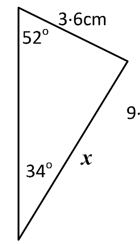
(d)



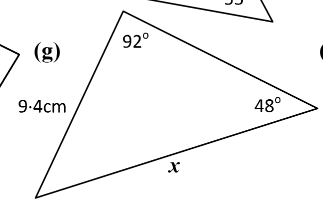
(e)



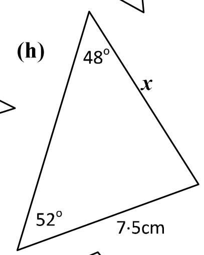
(f)



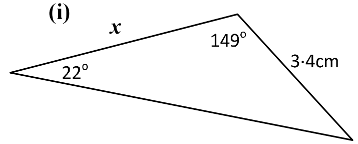
(g)



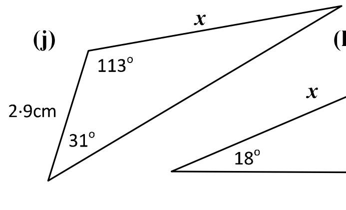
(h)



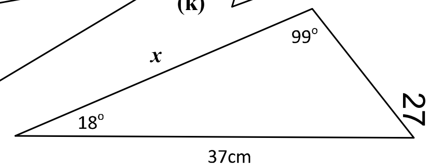
(i)



(j)

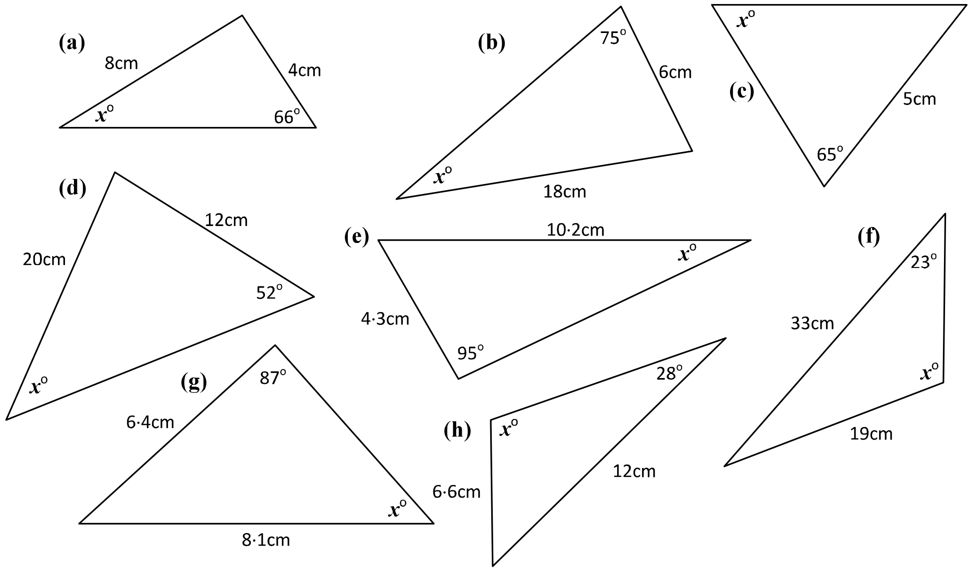


(k)

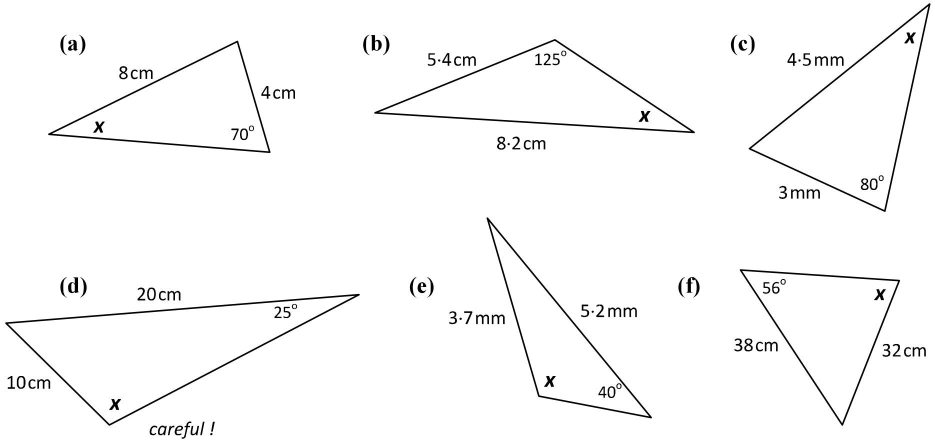


1.2 USING the SINE RULE to CALCULATE an ANGLE

1. Use the sine rule to calculate the length of the angle marked x° in each of the triangles below.

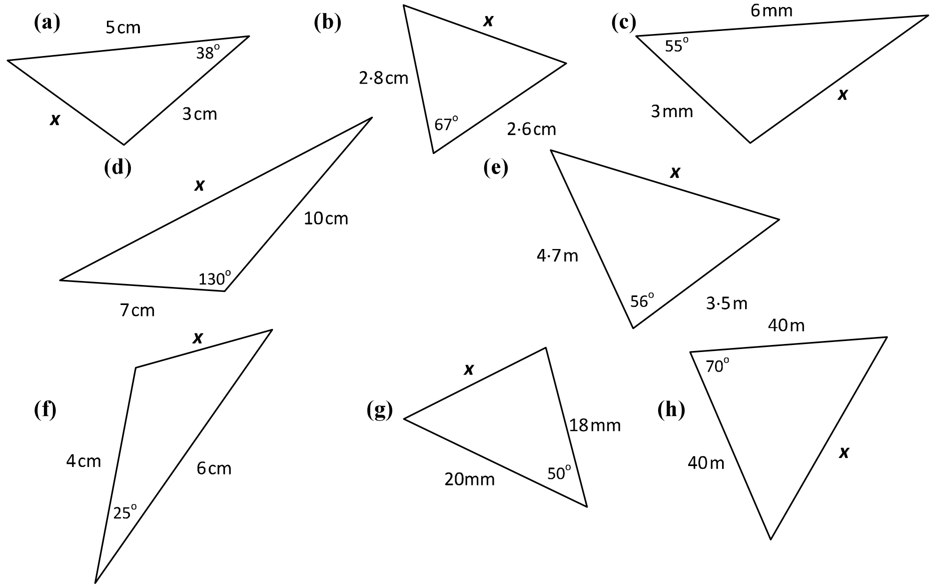


2. Use the *sine rule* to calculate the size of the angle marked ***x*** in each triangle below.

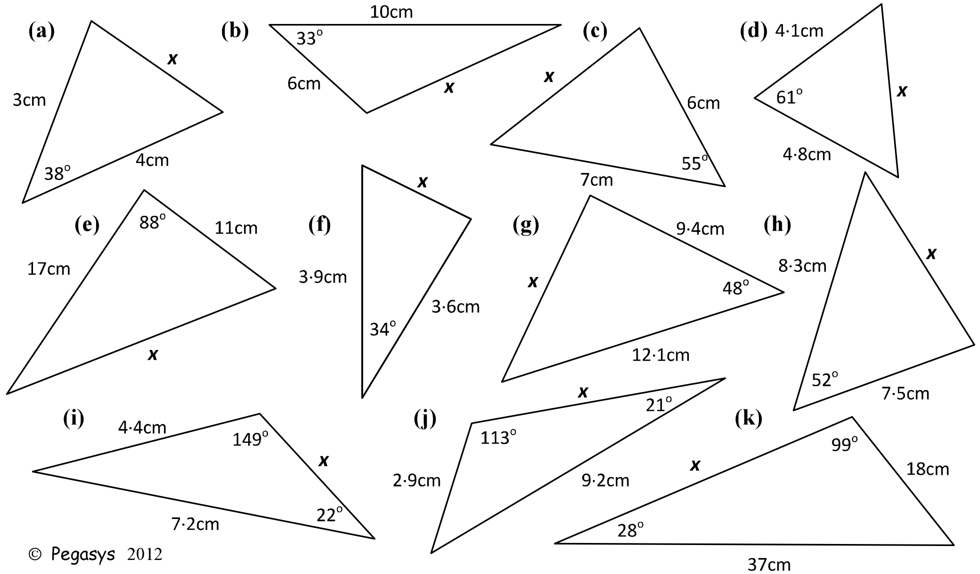


1.2 USING the COSINE RULE to CALCULATE a SIDE

1. Use the *cosine rule* to calculate the side marked ***x*** in each triangle below.

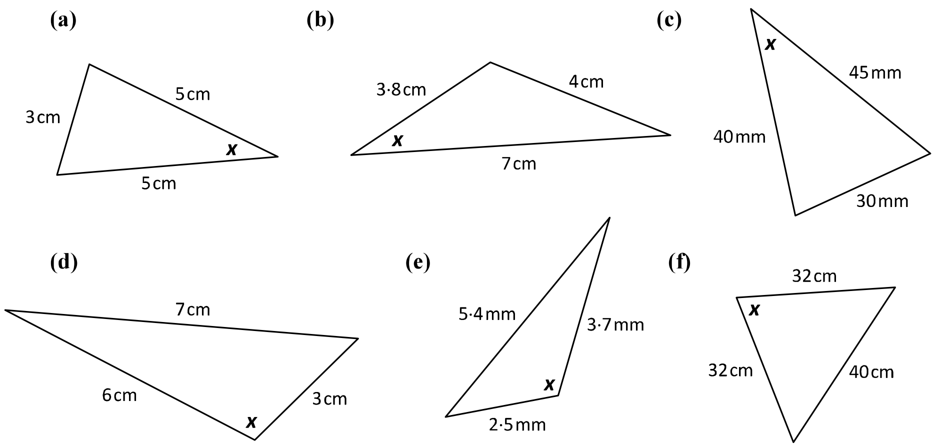


2. Use the cosine rule to calculate the length of the side marked ***x*** in each of the triangles below.

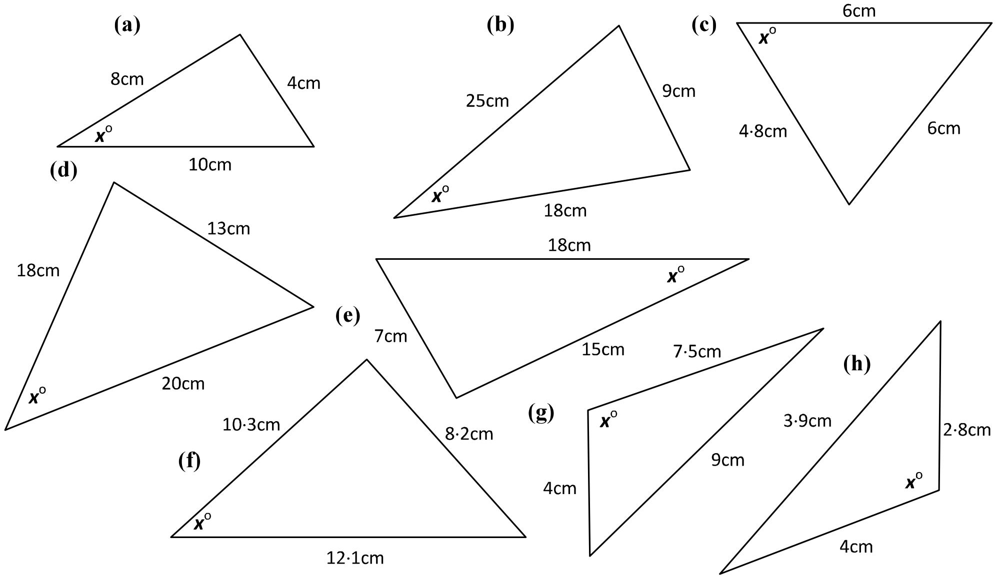


1.2 USING the COSINE RULE to CALCULATE an ANGLE

1. Use the 2nd form of the *cosine rule* to calculate the size of the <u>angle</u> marked ***x*** in each triangle below.

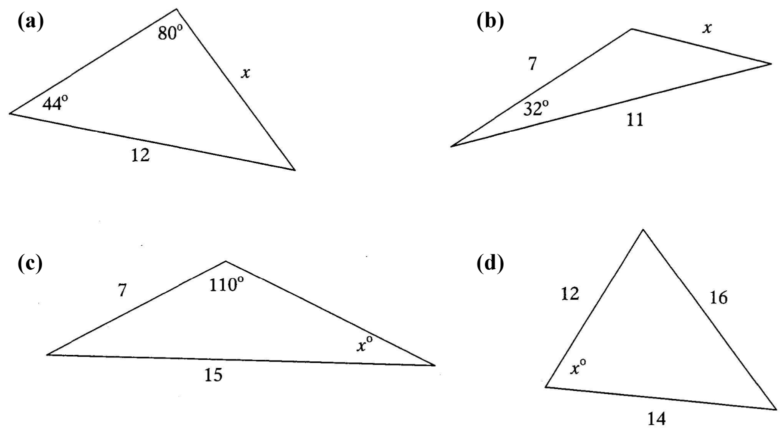


2. Use the cosine rule to calculate the angle marked ***x***° in each of the triangles below.



MIXED EXERCISE using TRIGONOMETRY RULES

1. Calculate the value of x in each triangle below.

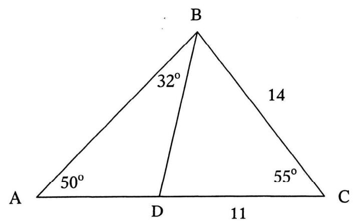


2. Calculate the area of the triangle with sides measuring 12 cm, 14 cm and 20 cm.

3. **(a)** Calculate the length of BD.

(b) Calculate the length of AD.

(c) Calculate the area of triangle ABC



4. From the framework opposite:

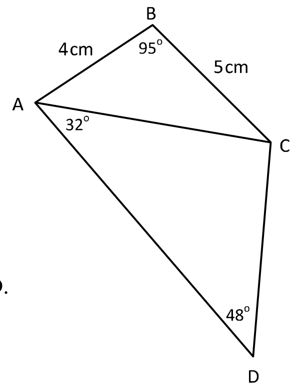
(a) Calculate the length of AC.

(b) Calculate the size of $\angle BAC$.

(c) Write down the size of $\angle ACD$.

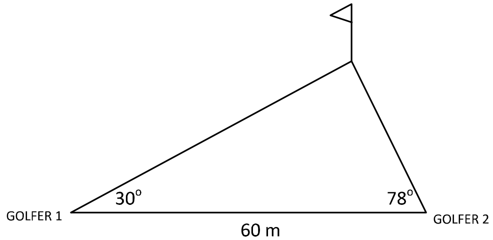
(d) Calculate the length of AD.

(e) Calculate the area of the quadrilateral ABCD.



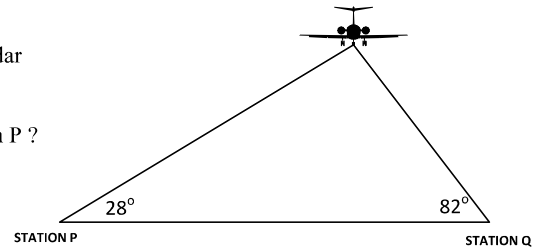
5. Two golfers are aiming for the green. The golfers are 60 m apart and the angles are as shown in the diagram.

What distance will each golfer have to hit the ball in order to reach the pin?

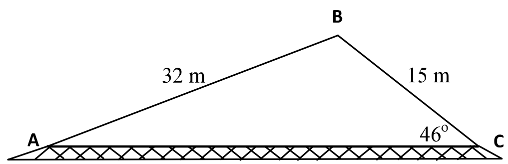


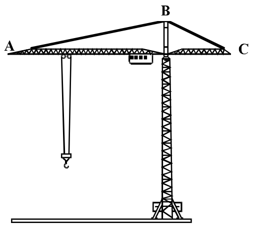
6. An aircraft is picked up by two radar stations, P and Q, 120 km apart.

How far is the aircraft from station P ?



7. A large crane is being used in the construction of a block of flats. The crossbeam is supported by two metal stays.

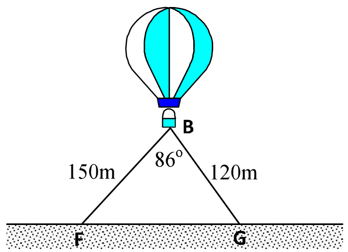




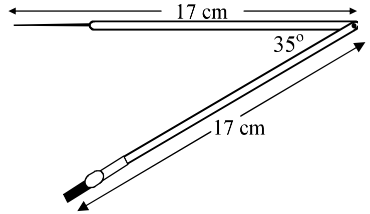
The length of AB is 32 m and the length of BC is 15 m. ∠BCA is 46°.
Calculate the size of ∠BAC and the length of the crossbeam AC.

8. A hot air balloon B is fixed to the ground at F and G by 2 ropes 120m and 150 m long.

If ∠FBG is 86°, how far apart are F and G?



9.



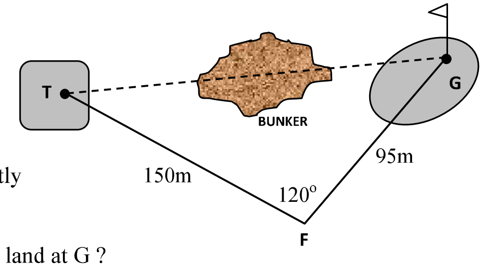
A set of compasses is shown where the angle between the arms is set at 35°

Calculate the diameter of the circle which could be drawn with the arms in this position.

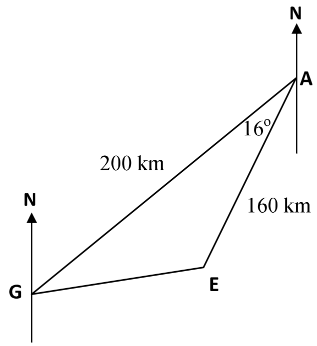
10. During a golf match, Ian discovers that he has forgotten his sand wedge, so to avoid the bunker he plays a shot from T to F and then from F to G.

His opponent Fred decides to play directly from T to G.

How far will Fred need to hit his shot to land at G ?



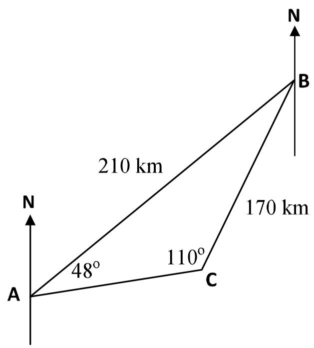
11.



The diagram shows the path of an aircraft from Glasgow to Aberdeen, a distance of 200 km and then from Aberdeen to Edinburgh, a distance of 160 km.

Calculate the distance from Glasgow to Edinburgh.

12.



The diagram shows the path of an aircraft from A to B to C.

(a) Write down the size of ∠ABC.

(b) Calculate the distance AC.

4.2 WORKING with TRIGONOMETRIC RELATIONSHIPS in DEGREES

IDENTITIES INVOLVING $\cos^2 x + \sin^2 x = 1$ and $\tan x = \sin x/\cos x$

1. Simplify

(a) $3\cos^2 x + 3\sin^2 x$ **(b)** $1 - \cos^2 x$ **(c)** $\cos A \tan A$

(d) $5 - 5\sin^2 B^o$ **(e)** $\frac{4\sin a^o}{4\cos a^o}$ **(f)** $\frac{4\tan x^o}{2\cos x^o}$

(g) $\frac{(1-\sin^2 x)}{2\cos x}$ **(h)** $\frac{8-8\cos^2 x}{2\sin x}$ **(i)** $\frac{3\sin x\cos x}{6\tan x}$

(j) $4\sin^2 A + 3\cos^2 A - 3$ **(k)** $4\cos^2 B - 2\sin^2 B + 2$

(l) $(\cos x + \sin x)^2 - 2\sin x\cos x$ **(m)** $\tan^2 a(1 - \sin^2 a)$

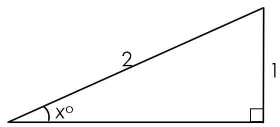
2. Prove that

(a) $2\cos^2 A + 3\sin^2 A = 3 - \cos^2 A$ **(b)** $\frac{1}{\tan x} + \tan x = \frac{1}{\sin x\cos x}$

(c) $(2\cos B + 3\sin B)^2 + (3\cos B - 2\sin B)^2 = 13$

(d) $(1 + \sin x)(1 - \sin x) = \cos^2 x$ **(e)** $\sin\theta.\tan\theta = \frac{1-\cos^2\theta}{\cos\theta}$

3. Consider the diagram opposite :



(a) Write down the **exact** values of .....

$\sin x^\circ$, $\cos x^\circ$ **and** $\tan x^\circ$.

(b) Prove that $\sin^2 x^\circ + \cos^2 x^\circ = 1$.

(c) Show that $\frac{\sin x^\circ}{\cos x^\circ} = \tan x^\circ$.

4. **(a)** If $\cos a^\circ = \frac{1}{\sqrt{5}}$, find the **exact** values of $\sin a^\circ$ **and** $\tan a^\circ$ where $0 < a < 90$.

(b) Prove that $\cos^2 a^\circ = 1 - \sin^2 a^\circ$.

(c) Show that $\frac{\sin^2 a^\circ}{\cos^2 a^\circ} = \tan^2 a^\circ$.

(d) Show that $2(3\sin a^\circ + 4\cos a^\circ) = 4\sqrt{5}$.

5. Prove that

(a) $3\cos^2 a + 3\sin^2 a = 3$

(b) $(\cos x + \sin x)^2 = 1 + 2\sin x\cos x$

(c) $(\cos x + \sin x)(\cos x - \sin x) = 2\cos^2 x - 1$

(d) $\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{1}{\cos x\sin x}$

(e) $\tan^2 p - \tan^2 p\sin^2 p = \sin^2 p$

(f) $\cos^4 x - \sin^4 x = 2\cos^2 x - 1$

(g) $3\sin^2\theta + 2\cos^2\theta = 2 + \sin^2\theta$

6. Show that $(2\cos x + 5\sin x)^2 + (5\cos x - 2\sin x)^2 = 29$

7. Given that $p = \cos\theta + \sin\theta$ and $q = \cos\theta - \sin\theta$, prove that :

(a) $pq = 2\cos^2\theta - 1$ **(b)** $\sin^2\theta = \frac{1}{2}(1 - pq)$

EXAM QUESTIONS

1. Solve the following equation

$3 - 5\sin x^\circ = -1$ where $0 \le x \le 360$

2. Sketch the graph of $f(x) = \cos(x - 30)^o$ for $0 \le x \le 360$

3. Simplify

$$\frac{1-\cos^2 x}{\cos^2 x}$$