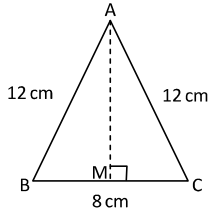
6. In ΔABC, AB = AC = 12cm and BC = 8cm. Express the length of the altitude from A to BC as a surd in its simplest form. [The line AM in the diagram]



7. An equilateral triangle has each of its sides measuring $2a$ metres.

(a) Find the exact length of an altitude of the triangle in terms of a.

(b) Hence find the exact area of the triangle in terms of a.

[Draw a diagram to help you with this question]

8. The exact **area** of a rectangle is $2(\sqrt{6} + \sqrt{3})$ square centimetres. Given that the breadth of the rectangle is $\sqrt{6}$ cm, show that the length is equal to $(2 + \sqrt{2})$ cm.

9. (a challenge) Given that $\tan 75^\circ = \dfrac{\sqrt{3}+1}{\sqrt{3}-1}$, show that $\tan 75^\circ = 2 + \sqrt{3}$.

1.2 INDICES

1. Write each of the following in its simplest index form.

(a)	$3^4 \times 3^2$	**(b)**	2×2^3	**(c)**	$10^5 \times 10^2$	**(d)**	$8^3 \times 8^5$
(e)	$7^6 \times 7$	**(f)**	$5^4 \times 5^4$	**(g)**	$9^6 \times 9^2$	**(h)**	$6^8 \times 6^5$
(i)	$x^3 \times x^5$	**(j)**	$c^2 \times c^9$	**(k)**	$a^2 \times a^{12}$	**(l)**	$y^5 \times y^5$
(m)	$b^{10} \times b^{30}$	**(n)**	$p \times p^9$	**(o)**	$d^2 \times d^4$	**(p)**	$q^{11} \times q^9$
(q)	$t^3 \times t^7$	**(r)**	$f^4 \times f^3$	**(s)**	$k \times k^{12}$	**(t)**	$z^{50} \times z^{50}$
(u)	$x^{30} \times x^{50}$	**(v)**	$y^{19} \times y$	**(w)**	$a^{25} \times a^{65}$	**(x)**	$b^1 \times b^0$

2. Write each of the following in its simplest index form.

(a)	$2^8 \div 2^3$	**(b)**	$5^4 \div 5^2$	**(c)**	$12^9 \div 1\,2^6$	**(d)**	$7^{11} \div 7^4$
(e)	$20^5 \div 20$	**(f)**	$8^8 \div 8^4$	**(g)**	$3^{18} \div 3^3$	**(h)**	$4^{15} \div 4^{13}$
(i)	$x^7 \div x^2$	**(j)**	$a^9 \div a^5$	**(k)**	$y^{20} \div y^{10}$	**(l)**	$b^4 \div b^1$
(m)	$p^{12} \div p^{11}$	**(n)**	$c^7 \div c^7$	**(o)**	$q^8 \div q^2$	**(p)**	$d^4 \div d$
(q)	$\dfrac{x^9}{x^3}$	**(r)**	$\dfrac{a^8}{a^2}$	**(s)**	$\dfrac{m^{14}}{m}$	**(t)**	$\dfrac{s^7}{s^7}$
(u)	$\dfrac{d^{20}}{d^{12}}$	**(v)**	$\dfrac{y^{100}}{y^{10}}$	**(w)**	$\dfrac{t^{100}}{t}$	**(x)**	$\dfrac{w^{10}}{w^0}$

3. Write each of the following in its simplest index form.

(a)	$(3^2)^4$	**(b)**	$(8^2)^2$	**(c)**	$(10^3)^2$	**(d)**	$(2^2)^5$
(e)	$(4^5)^3$	**(f)**	$(1^7)^2$	**(g)**	$(12^3)^3$	**(h)**	$(5^5)^5$
(i)	$(x^4)^2$	**(j)**	$(y^8)^5$	**(k)**	$(a^3)^7$	**(l)**	$(m^4)^4$
(m)	$(b^3)^6$	**(n)**	$(p^5)^3$	**(o)**	$(k^5)^{20}$	**(p)**	$(z^6)^0$

4. Write the following without brackets.

(a) $(2b)^2$ **(b)** $(7a)^3$ **(c)** $(3x)^4$ **(d)** $(2y)^5$

(e) $(ab)^4$ **(f)** $(xy)^7$ **(g)** $(wz)^5$ **(h)** $(st)^3$

(i) $(pq^2)^3$ **(j)** $(x^4y)^2$ **(k)** $(a^2b^3)^5$ **(l)** $(6a^5)^2$

(m) $(10x^2)^3$ **(n)** $(2c^4)^5$ **(o)** $(3ab^2)^3$ **(p)** $(4m^2k)^2$

5. Simplify these expressions.

(a) $2a^3 \times 5a^5$ **(b)** $7x \times 9x^8$ **(c)** $12p^7 \div 4p^4$ **(d)** $50b^{12} \div 10b^6$

(e) $3y \times (2y^2)^3$ **(f)** $(4q^3)^2 \times 5q^4$ **(g)** $(4c^3)^3 \div 8c^2$ **(h)** $72z^{12} \div (3z^4)^2$

(i) $k^2(k^3 + k^5)$ **(j)** $m^5(m^2 - m^3)$ **(k)** $2x^4(x^3 + 3x^2)$ **(l)** $5a^5(2a^2 - 3a^3)$

(m) $\dfrac{x^5 \times x^4}{x^6}$ **(n)** $\dfrac{(m^5)^4}{m^6}$ **(o)** $\dfrac{5c^3 \times 4c^7}{2c^6}$ **(p)** $\dfrac{(3q^3)^2 \times 4q^4}{6q^7}$

(q) $\dfrac{(3xy^5)^3}{9x^2y}$ **(r)** $\dfrac{(2a^2b^5)^6}{(4ab)^2}$ **(s)** $\dfrac{(4p^4)^3}{2p^3 \times 8p^6}$ **(t)** $\dfrac{(2ab^3)^5}{3a^2b \times 4ab^2}$

6. Write down the value of

(a) 5^0 **(b)** 2^0 **(c)** 100^0 **(d)** $(-3)^0$ **(e)** 25^0

(f) $\frac{1}{2}^0$ **(g)** a^0 **(h)** k^0 **(i)** $(mn)^0$ **(j)** $(ab^2)^0$

(k) $(10x^3)^0$ **(l)** $(16y^2z^3)^0$

7. Rewrite the following with positive indices.

(a) 3^{-2} **(b)** 5^{-4} **(c)** 2^{-6} **(d)** 10^{-3} **(e)** 4^{-5} **(f)** 200^{-7}

(g) a^{-5} **(h)** x^{-2} **(i)** p^{-7} **(j)** y^{-10} **(k)** $2b^{-3}$ **(l)** $10\,q^{-x}$

(m) $\dfrac{1}{x^{-3}}$ **(n)** $\dfrac{1}{w^{-5}}$ **(o)** $\dfrac{3}{a^{-2}}$ **(p)** $\dfrac{10}{c^{-8}}$ **(q)** $\dfrac{2}{3t^{-1}}$ **(r)** $\dfrac{5}{4y^{-3}}$

8. Rewrite the following with negative indices.

(a) $\dfrac{1}{3^2}$ **(b)** $\dfrac{1}{6^9}$ **(c)** $\dfrac{1}{5^4}$ **(d)** $\dfrac{1}{2^7}$ **(e)** $\dfrac{1}{10^3}$ **(f)** $\dfrac{1}{4^4}$

(g) $\dfrac{1}{x^3}$ **(h)** $\dfrac{1}{a^5}$ **(i)** $\dfrac{1}{p^4}$ **(j)** $\dfrac{1}{y^{10}}$ **(k)** $\dfrac{1}{q^6}$ **(l)** $\dfrac{1}{c^8}$

9. Simplify the following expressions.

(a) $m^3 \times m^{-5}$ **(b)** $x^7 \times x^{-2}$ **(c)** $p^{-8} \times p^5$ **(d)** $a^{-3} \times a^{-5}$

(e) $(y^3)^{-4}$ **(f)** $(c^{-5})^3$ **(g)** $(q^3)^{-5}$ **(h)** $(w^{-2})^{-4}$

(i) $4b^{-4} \times 5b^5$ **(j)** $3x^6 \times 9x^{-6}$ **(k)** $4k^3 \div 2k^{-2}$ **(l)** $18d \div 12d^4$

(m) $x^2(x^3 + x^{-1})$ **(n)** $p^{-3}(p^4 - p^{-8})$ **(o)** $3a^5(2a + 3a^{-2})$

(p) $\frac{1}{2}\, m^{-2}(4m^{-3} - 10m^6)$ **(q)** $\dfrac{v^3 \times v^5}{v^{-2}}$ **(r)** $\dfrac{4h^7 \times 3h^{-4}}{2h^4}$

(s) $\dfrac{4c^{-5} \times 9c^6}{6c^{-4}}$ **(t)** $\dfrac{5x^4 \times 6x^{-8}}{3x^{-4}}$

10. Find the value of

(a) $16^{\frac{1}{4}}$ **(b)** $8^{\frac{1}{3}}$ **(c)** $36^{\frac{1}{2}}$ **(d)** $27^{\frac{2}{3}}$ **(e)** $64^{\frac{1}{3}}$ **(f)** $1000^{\frac{1}{3}}$

(g) $25^{\frac{1}{2}}$ **(h)** $81^{\frac{3}{4}}$ **(i)** $125^{\frac{2}{3}}$ **(j)** $64^{\frac{1}{2}}$ **(k)** $216^{\frac{1}{3}}$ **(l)** $16^{-\frac{1}{4}}$

(m) $4^{-\frac{1}{2}}$ **(n)** $16^{-\frac{1}{2}}$ **(o)** $9^{-\frac{1}{2}}$ **(p)** $27^{-\frac{2}{3}}$ **(q)** $256^{-\frac{3}{4}}$ **(r)** $1000^{-\frac{2}{3}}$

(s) $16^{-\frac{3}{2}}$ **(t)** $8^{-\frac{4}{3}}$ **(u)** $8^{\frac{4}{3}}$ **(v)** $(-8)^{\frac{1}{3}}$ **(w)** $64^{\frac{2}{3}}$ **(x)** $100^{-\frac{3}{2}}$

(y) $(\frac{1}{2})^{-1}$ **(z)** $(\frac{1}{8})^{\frac{4}{3}}$

11. Simplify the following expressions, giving your answers with positive indices.

(a) $(x^{\frac{1}{2}})^6$ **(b)** $(p^{\frac{1}{3}})^6$ **(c)** $(a^{\frac{3}{4}})^8$ **(d)** $(y^{-\frac{2}{3}})^9$

(e) $(q^{-\frac{1}{5}})^{10}$ **(f)** $(k^{-\frac{2}{5}})^1$ **(g)** $(g^4)^{\frac{1}{2}}$ **(h)** $(m^{12})^{-\frac{2}{3}}$

(i) $(c^9)^{\frac{2}{3}}$ **(j)** $(h^5)^{-\frac{1}{2}}$ **(k)** $(z^4)^{-\frac{3}{4}}$ **(l)** $(b^{16})^{-\frac{3}{4}}$

(m) $x^{\frac{1}{2}} \times x^{-\frac{1}{2}}$ **(n)** $y^{\frac{1}{3}} \times y^{\frac{2}{3}}$ **(o)** $d^{-\frac{1}{4}} \times d^{\frac{9}{4}}$ **(p)** $s^{\frac{7}{2}} \times s^{-\frac{1}{2}}$

(q) $3x^{\frac{1}{2}} \times 4x^{\frac{1}{2}}$ **(r)** $6x^{\frac{1}{2}} \times 2x^{-\frac{1}{2}}$ **(s)** $2x^{\frac{1}{2}} \times 5x^{\frac{1}{2}}$ **(t)** $3x^{\frac{2}{3}} \times 2x^{-\frac{1}{3}}$

(u) $x^{\frac{1}{2}} \div x^{\frac{1}{2}}$ **(v)** $2x^{\frac{1}{2}} \div x^{-\frac{1}{2}}$ **(w)** $8x^{\frac{2}{3}} \div 2x^{\frac{1}{3}}$ **(x)** $6x^{\frac{1}{3}} \div 4x^{\frac{2}{3}}$

12. Write the following in surd form.

(a) $x^{\frac{1}{2}}$ **(b)** $y^{\frac{1}{3}}$ **(c)** $a^{\frac{1}{4}}$ **(d)** $y^{\frac{2}{3}}$

(e) $b^{\frac{3}{4}}$ **(f)** $x^{\frac{5}{3}}$ **(g)** $c^{\frac{3}{5}}$ **(h)** $a^{\frac{4}{5}}$

(i) $c^{-\frac{1}{3}}$ **(j)** $z^{-\frac{1}{2}}$ **(k)** $m^{-\frac{2}{3}}$ **(l)** $k^{-\frac{3}{5}}$

(m) $p^{-\frac{4}{3}}$ **(n)** $x^{-\frac{5}{3}}$ **(o)** $w^{-\frac{4}{5}}$ **(p)** $d^{-\frac{2}{7}}$

13. Write the following in index form.

(a) $\sqrt{x}$ **(b)** $\sqrt[3]{a}$ **(c)** $\sqrt{y^3}$ **(d)** $\sqrt[3]{z^2}$

(e) $\sqrt[3]{c^2}$ **(f)** $\sqrt[4]{x^3}$ **(g)** $\sqrt[3]{p^5}$ **(h)** $\sqrt[5]{m^2}$

(i) $\frac{1}{\sqrt{a}}$ **(j)** $\frac{1}{\sqrt[3]{z}}$ **(k)** $\frac{1}{\sqrt[3]{x^4}}$ **(l)** $\frac{1}{\sqrt{a^5}}$

(m) $\frac{1}{\sqrt[3]{b^2}}$ **(n)** $\frac{1}{\sqrt[5]{m^3}}$ **(o)** $\frac{1}{\sqrt[4]{y}}$ **(p)** $\frac{1}{\sqrt[3]{c^5}}$

14. Simplify each of the following by …..... (i) changing root signs to fractional powers;

(ii) moving x's onto the numerators;

(iii) expanding brackets …… where necessary.

(a) $x^{\frac{1}{2}}(x^4+1)$ **(b)** $x^{-\frac{1}{2}}(x^{\frac{3}{2}}-x^2)$ **(c)** $\frac{1}{x^2}(x^{\frac{1}{2}}+x)$

(d) $\frac{2}{x^{-3}}(x^2+\frac{1}{x})$ **(e)** $\frac{1}{\sqrt{x}}(x^2-\sqrt{x})$ **(f)** $\left(x^2+\frac{1}{x}\right)^2$

(g) $\frac{1}{x}(\sqrt{x}+x)$ **(h)** $\left(x+\frac{1}{\sqrt{x}}\right)^2$ **(i)** $x^{-2}\left(\frac{1}{x}-\sqrt[3]{x}\right)$

(j) $\frac{x^2+3}{x}$ **(k)** $\frac{\sqrt{x}-x}{x^2}$ **(l)** $\frac{(2x+1)^2}{x^{\frac{3}{2}}}$

INDICES EXAM QUESTIONS

1. **(a)** Simplify $\frac{7a^3b^2}{a\sqrt{b}}$

(b) If $a=-1$ and $b=4$, find the value of the expression in part (a).

2. Given that $y = 2x^{-2/3}$, find y when $x = 8$.

3. Simplify $x^{\frac{2}{3}}(x^{\frac{2}{3}}+x^{-\frac{2}{3}})$

4. **(a)** Simplify $\frac{m^5}{m^{-3}}$ **(b)** Evaluate $125^{-\frac{2}{3}}$

5. Express $\frac{p^5 \times 8p}{2p^{-3}}$ in its simplest form.

6. Simplify, writing your answer with a positive index: $3a^4 \times a^{-6}$

7. Simplify the fraction, giving your answer in positive index form: $\frac{x^3 \times x^4}{x^9}$

8. Simplify $\frac{a^2 \times a^5}{a^{-3}}$.

9. **(a)** Remove the brackets and simplify: $p^{\frac{1}{2}}(p^{\frac{5}{2}}-2)$.

(b) Hence, or otherwise, find the value of $p^{\frac{1}{2}}(p^{\frac{5}{2}}-2)$ when $p=4$.