

Statistics

Comparing the mean

1. On average, Paul rolled a higher number because his mean is higher.
Paul's roll was more consistent because his range is smaller.
2. On average, Jake's weight was higher over the month because his mean is higher.
Sarah's weight was more consistent over the month because her range is smaller.

Calculating Standard Deviation

1. 20, 21, 19, 22, 18

$$\bar{x} = \frac{100}{5} = 20$$

$$s = \sqrt{\frac{\sum (x - \bar{x})^2}{n-1}}$$
$$= \sqrt{\frac{10}{4}}$$
$$= 1.58$$

x	$x - \bar{x}$	$(x - \bar{x})^2$
20	0	0
21	1	1
19	-1	1
22	2	4
18	-2	4

$$\sum (x - \bar{x})^2 = 10$$
$$n = 5$$

2. 18, 6, 9, 30, 4, 28, 78

$$\bar{x} = \frac{173}{7} = 24.71$$

$$s = \sqrt{\frac{\sum (x - \bar{x})^2}{n-1}}$$
$$= \sqrt{\frac{3949.4}{6}}$$
$$= 25.66$$

x	$x - \bar{x}$	$(x - \bar{x})^2$
18	-6.71	45.02
6	-18.71	350.06
9	-15.71	246.80
30	5.29	27.98
4	-20.71	428.90
28	3.29	10.82
78	53.29	2839.82

$$\sum (x - \bar{x})^2 = 3949.4$$
$$n = 7$$

3a) 30 25 38 45 36 40 27 43 39

$$\bar{x} = \frac{323}{9} = 35.8$$

b)

$$s = \sqrt{\frac{\sum (x - \bar{x})^2}{n-1}}$$

$$= \sqrt{\frac{426.96}{8}}$$

$$= 7.3$$

x	$x - \bar{x}$	$(x - \bar{x})^2$
30	-5.8	33.64
25	-10.8	116.64
38	2.2	4.84
45	9.2	84.64
36	-0.2	0.04
40	4.2	17.64
27	-8.8	77.44
43	7.2	51.84
39	3.2	10.24

$$\sum (x - \bar{x})^2 = 426.96$$

$$n = 9$$

4. £1.20 £1.25 £1.40 £1.28 £1.35

$$\bar{x} = \frac{6.48}{5} = £1.30$$

$$s = \sqrt{\frac{\sum (x - \bar{x})^2}{n-1}}$$

$$= \sqrt{\frac{0.0254}{4}}$$

$$= 0.08$$

x	$x - \bar{x}$	$(x - \bar{x})^2$
1.2	-0.1	0.01
1.25	-0.05	0.0025
1.4	0.1	0.01
1.28	-0.02	0.0006
1.35	0.05	0.0025

$$\sum (x - \bar{x})^2 = 0.0254$$

$$n = 5$$

Comparing and Calculating mean / SD.

1a) Boys: 28.3 25.6 29.4 26.5 32.7 27.3 26.2 24.8

$$\bar{x} = 27.65$$

$$s = \sqrt{\frac{\sum (x - \bar{x})^2}{n-1}}$$

$$= \sqrt{\frac{44.84}{7}}$$

$$= 2.53$$

x	$x - \bar{x}$	$(x - \bar{x})^2$
28.3	0.7	0.49
25.6	-2	4
29.4	1.8	3.24
26.5	-1.1	1.21
32.7	5.1	26.01
27.3	-0.3	0.09
26.2	-1.4	1.96
24.8	-2.8	7.84

$$\sum (x - \bar{x})^2 = 44.84$$

$$n = 8$$

b) Girls: 33.3 29.7 32.5 29.4 30.6 33.2

$$\bar{x} = \frac{188.7}{6} = 31.45$$

$$s = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}}$$

$$= \sqrt{\frac{15.56}{5}}$$

$$= 1.76$$

x	$x - \bar{x}$	$(x - \bar{x})^2$
33.3	1.85	3.42
29.7	-1.75	3.06
32.5	1.05	1.10
29.4	-2.05	4.20
30.6	-0.85	0.72
33.2	1.75	3.06

$$\sum(x-\bar{x})^2 = 15.56$$

$$n = 6$$

c) On average, the boys swam faster than the girls because their mean is smaller.
The girls' times were more consistent because their standard deviation is smaller.

2a) 61 45 48 52 53 49

$$\bar{x} = \frac{308}{6} = 51.3 \text{ beats per minute}$$

$$s = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}}$$

x	$x - \bar{x}$	$(x - \bar{x})^2$
61	9.7	94.09
45	-6.3	39.69
48	-3.3	10.89
52	0.7	0.49
53	1.7	2.89
49	-2.3	5.29

$$\sum(x-\bar{x})^2 = 153.34$$

$$n = 6$$

$$= \sqrt{\frac{153.34}{5}}$$

$$= 5.54$$

b) On average, the athlete had ~~lower~~ a lower heart rate than the sedentary adults because the mean is smaller.
The athlete had more consistent heart rates because the standard deviation is smaller.

Creating a 5 figure summary.

1. 1 (3) 4 (7) 7 (9) 13

$$L = 1 \quad Q_1 = 3 \quad Q_2 = 7 \quad Q_3 = 9 \quad H = 13$$

2. 13 13 | 15 16 (21) 23 24 | 28 29

$$L = 13 \quad Q_1 = \frac{13+15}{2} = 14 \quad Q_2 = 21 \quad Q_3 = \frac{24+28}{2} = 26 \quad H = 29$$

3. 3.2 3.5 3.6 | 3.8 3.8 4.0 | 4.4 4.4 4.7 | 5.3 5.4 5.9

$$L = 3.2 \quad Q_1 = 3.7 \quad Q_2 = \frac{4+4.4}{2} = 4.2 \quad Q_3 = 5.0 \quad H = 5.9$$

4. 48 51 (54) 54 58 | 64 67 (71) 73 78

$$L = 48 \quad Q_1 = 54 \quad Q_2 = \frac{58+64}{2} = 61 \quad Q_3 = 71 \quad H = 78$$

Determining median, IQR and SIQR

a) 42 43 43 (43) 48 48 51 (54) 58⁵⁹ 61 (62) 62 65 70

$$\text{Median} = Q_2 = 54$$

$$Q_1 = 43$$

$$Q_3 = 62$$

$$\text{SIQR} = \frac{62-43}{2} = 9.5$$

$$\text{IQR} = 62 - 43 = 19$$

b) 38 39 41 (43) 45 49 49 (49) 53 54 56 (62) 70 83 87

$$Q_1 = 43 \quad Q_2 = 49 \quad Q_3 = 62$$

$$\text{IQR} = 62 - 43 = 19$$

$$\text{SIQR} = \frac{19}{2} = 9.5$$

c) 17 18 18 (19) 19 19 20 | 21 23 24 (25) 25 25 28

$$Q_1 = 19$$

$$Q_2 = \frac{41}{2} = 20.5 \quad Q_3 = 25$$

$$\text{IQR} = 25 - 19 = 6$$

$$\text{SIQR} = \frac{6}{2} = 3$$

Comparing median, IQR and SIQR

1. On average, Team B scored higher because the median is higher.

Team B was more consistent because the SIQR is smaller.

2a) Men: 2, 2, 2, 3, 3, 3, 5, 5, 5, 6, 7, 7, 7, 7, 8, 8, 8, 10, 11, 11, 12, 12, 13, 17

$$Q_1 = \frac{3+5}{2} = 4$$

$$Q_2 = 7$$

$$Q_3 = \frac{10+11}{2} = 10.5$$

$$SIQR = \frac{6.5-3.25}{2}$$

Women: 0, 0, 0, 0, 1, 1, 1, 2, 2, 2, 2, 3, 4, 4, 6, 6, 7, 8, 8, 8, 8, 8, 9, 9

$$Q_1 = 1$$

$$Q_2 = 4$$

$$Q_3 = 8$$

$$SIQR = \frac{7}{2} = 3.5$$

b) On average, the men were absent more because the median is higher.

The men's absences were more consistent because the their SIQR is smaller.

3a) With fertiliser: 5, 6, 7, 10, 15, 16, 18, 22, 23, 28, 29, 30

$$Q_1 = \frac{7+10}{2} = 8.5$$

$$Q_2 = 17$$

$$Q_3 = \frac{23+28}{2} = 25.5$$

$$SIQR = \frac{25.5-8.5}{2} = 7$$

Without fertiliser: 3, 4, 4, 7, 11, 12, 13, 17, 17, 21, 24, 25

$$Q_1 = \frac{4+7}{2} = 5.5$$

$$Q_2 = 12.5$$

$$Q_3 = \frac{17+21}{2} = 19$$

$$SIQR = \frac{19-5.5}{2} = 6.75$$

b) On average, the plants with fertiliser were taller because the median is higher.

The plants without fertiliser had more consistent height because the SIQR is smaller.

Comparing Distributions

a) $(40, 132)$ $(42.5, 140)$

$$m = \frac{140 - 132}{42.5 - 40} = \frac{8}{2.5} = 3.2$$

$$H - b = m(w - a)$$

$$H - 132 = 3.2(w - 40)$$

$$H = 3.2w - 128 + 132$$

$$H = 3.2w + 4$$

b) $w = 42 \text{ kg}$

$$H = 3.2 \times 42 + 4$$

$$= 134.4 + 4$$

$$= 138.4 \text{ cm}$$

a) $(0, 20)$ $(40, 30)$

$$m = \frac{30 - 20}{40 - 0} = \frac{10}{40} = \frac{1}{4}$$

$$C - b = m(t - a)$$

$$C - 20 = \frac{1}{4}(t - 0)$$

$$C - 20 = \frac{1}{4}t$$

$$4C - 80 = t$$

$$4C = t + 80$$

$$C = \frac{t}{4} + 20$$

b) $t = 52 \text{ minutes}$

$$C = \frac{52}{4} + 20$$

$$= 13 + 20$$

$$= 33$$