

Brackets and Factorising

Expanding single and multiple brackets.

$$1. \quad 9(1-y) \\ = 9 - 9y$$

$$2. \quad -x(7y-11x) \\ = -7xy + 11x^2$$

$$3. \quad 6(z+2) - 2z \\ = 6z + 12 - 2z \\ = 4z + 12$$

$$4. \quad 12 - 2(x+5) \\ = 12 - 2x - 10 \\ = 2 - 2x$$

$$5. \quad 50 - 13(3x-2) \\ = 50 - 39x + 26 \\ = 76 - 39x$$

$$6. \quad 3(m+2) + 4(m+1) \\ = 3m + 6 + 4m + 4 \\ = 7m + 10$$

$$7. \quad 2(8t-2) + 5(2t-4) \\ = 16t - 4 + 10t - 20 \\ = 26t - 24$$

$$8. \quad 5(x+2) - 2(x+3) \\ = 5x + 10 - 2x - 6 \\ = 3x + 4$$

$$9. \quad x(8x-2) - 2(3x-8) \\ = 8x^2 - 2x - 6x + 16 \\ = 8x^2 - 8x + 16$$

$$10. \quad 4w - 2(1-5w) \\ = 4w - 2 + 10w \\ = 14w - 2$$

$$\text{Area 1} = x(2x+3) \\ (\text{rectangle}) \\ = 2x^2 + 3x$$

$$\text{Area 2} = 3x \times 3x \\ (\text{square}) \\ = 9x^2$$

$$\text{Total Area} = 2x^2 + 3x + 9x^2 \\ = 11x^2 + 3x$$

Expanding Double Brackets FOIL

$$1. \quad (x+3)(x+4) \\ = x^2 + 4x + 3x + 12 \\ = x^2 + 7x + 12$$

$$2. \quad (x-3)(x+4) \\ = x^2 + 4x - 3x - 12 \\ = x^2 + x - 12$$

$$3. \quad (x+3)(x-4) \\ = x^2 - 4x + 3x - 12 \\ = x^2 - x - 12$$

$$4. \quad (x-3)(x-4) \\ = x^2 - 4x - 3x + 12 \\ = x^2 - 7x + 12$$

$$5. \quad (x+3)(x-3) \\ = x^2 - 3x + 3x - 9 \\ = x^2 - 9$$

$$6. \quad (2x+3)(x-7) \\ = 2x^2 - 14x + 3x - 21 \\ = 2x^2 - 11x - 21$$

$$7. \quad (2x-3)(2x-2) \\ = 4x^2 - 4x - 6x + 6 \\ = 4x^2 - 10x + 6$$

$$8. \quad (3-2x)(3x+2) \\ = 9x + 6 - 6x^2 - 4x \\ = 6 + 5x - 6x^2$$

$$9. \quad (3y-2x)(3x+2y) \\ = 9xy + 6y^2 - 6x^2 - 4xy \\ = 5xy + 6y^2 - 6x^2$$

$$10. (-3x-2)(2x+4)$$

$$= -6x^2 - 12x - 4x - 8$$

$$= -6x^2 - 16x - 8$$

Squaring Brackets

$$1. (x+5)^2$$

$$= (x+5)(x+5)$$

$$= x^2 + 5x + 5x + 25$$

$$= x^2 + 10x + 25$$

$$2. (x+12)^2$$

$$= x^2 + 24x + 144$$

$$3. (x-5)^2$$

$$= (x-5)(x-5)$$

$$= x^2 - 5x - 5x + 25$$

$$= x^2 - 10x + 25$$

$$4. (x-11)^2$$

$$= x^2 - 22x + 121$$

$$5. (2x+1)^2$$

$$= (2x+1)(2x+1)$$

$$= 4x^2 + 2x + 2x + 1$$

$$= 4x^2 + 4x + 1$$

$$6. (3x+2)^2$$

$$= 9x^2 + 12x + 4$$

$$7. (4x-2)^2$$

$$= (4x-2)(4x-2)$$

$$= 16x^2 - 8x - 8x + 4$$

$$= 16x^2 - 16x + 4$$

$$8. (3x+2y)^2$$

$$= (3x+2y)(3x+2y)$$

$$= 9x^2 + 6xy + 6xy + 4y^2$$

$$= 9x^2 + 12xy + 4y^2$$

$$9. (p+1)^2$$

$$= p^2 + 2p + 1$$

Expanding Brackets with Trinomials

$$1. (x+3)(x^2+2x+4)$$

$$= x(x^2+2x+4) + 3(x^2+2x+4)$$

$$= x^3 + 2x^2 + 4x + 3x^2 + 6x + 12$$

$$= x^3 + 5x^2 + 10x + 12$$

$$2. (x-2)(2x^2+x-3)$$

$$= 2x^3 + x^2 - 3x - 4x^2 - 2x + 6$$

$$= 2x^3 - 3x^2 - 5x + 6$$

$$3. (2x-1)(3x^2-x+4)$$

$$= 6x^3 - 2x^2 + 8x - 3x^2 + x - 4$$

$$= 6x^3 - 5x^2 + 9x - 4$$

$$\begin{aligned}
 4. & (2-x^2)(3x^2-2x+5) \\
 & = 6x^2 - 4x + 10 - 3x^4 + 2x^2 - 5x^2 \\
 & = x^2 - 4x + 10 - 3x^4 + 2x^2 \\
 & = 10 - 4x + x^2 + 2x^2 - 3x^4
 \end{aligned}$$

$$\begin{aligned}
 5. & (x+1)(x+2)(x+5) \quad \rightarrow (x+1)(x+2) \\
 & = (x^2+3x+2)(x+5) \quad = x^2+3x+2 \\
 & = x^3+5x^2+3x^2+15x+2x+10 \\
 & = x^3+8x^2+17x+10
 \end{aligned}$$

$$\begin{aligned}
 6. & (2x+3)(x-1)(x+2) \quad \rightarrow (x-1)(x+2) \\
 & = (2x+3)(x^2+x-2) \quad = x^2+x-2 \\
 & = 2x^3+2x^2-4x+3x^2+3x-6 \\
 & = 2x^3+5x^2-x-6
 \end{aligned}$$

Mixed Expanding Brackets

$$\begin{array}{lll}
 1. -2(x-3) & 2. 5(2x+3)-4 & 3. 13-2(x+1) \\
 = -2x+6 & = 10x+15-4 & = 13-2x-2 \\
 & = 10x+11 & = 11-2x
 \end{array}$$

$$\begin{array}{lll}
 4. -2(x-10)+21 & 5. 5x-3(2x+12) & 6. 6(3x-1)+2(2x+7) \\
 = -2x+20+21 & = 5x-6x-36 & = 18x-6+4x+6 \\
 = -2x+41 & = -x-36 & = 22x
 \end{array}$$

$$\begin{array}{lll}
 7. 3(x-2)-7(2-4x) & 8. (x+3)(x+1) & 9. (5-2x)(8-5x) \\
 = 3x-6-14+28x & = x^2-x+3x-3 & = 40-25x-16x+10x^2 \\
 = 31x-20 & = x^2+2x-3 & = 40-41x+10x^2
 \end{array}$$

$$\begin{array}{ll}
 10. (x+3)(x^2+4x+5) & 11. (4-x)(2-x+3x^2) \\
 = x^3+4x^2+5x+3x^2+12x+15 & = 8-4x-12x^2-2x+x^2-3x^3 \\
 = x^3+7x^2+17x+15 & = 8-6x-11x^2-3x^3
 \end{array}$$

$$\begin{aligned}
 12. & (x+2)(x+3)(x-5) \quad \rightarrow (x+2)(x+3) \\
 & = (x^2+5x+6)(x-5) \quad = x^2+5x+6 \\
 & = x^3-5x^2+5x^2-25x+6x-30 = x^3-19x-30
 \end{aligned}$$

Factorising by Highest Common Factor (HCF)

$$1. 2x + 6 \\ = 2(x + 3)$$

$$\begin{array}{l} \frac{2}{1 \times 2} \\ \frac{6}{1 \times 6} \\ \textcircled{2} \times 3 \end{array}$$

$$2. 4x + 12 \\ = 4(x + 3)$$

$$\begin{array}{l} \frac{4}{1 \times 4} \\ \frac{12}{2 \times 2} \\ \frac{12}{1 \times 12} \\ \frac{6}{2 \times 6} \\ \frac{3}{3 \times 4} \\ \textcircled{4} \end{array}$$

$$3. 3t + 9 \\ = 3(t + 3)$$

$$4. 5a - 20 \\ = 5(a - 4)$$

$$5. 6y + 9 \\ = 3(2y + 3)$$

$$6. 4f - 10 \\ = 2(2f - 5)$$

$$7. 9g + 15 \\ = 3(3g + 5)$$

$$8. 8x + 12 \\ = 4(2x + 3)$$

$$9. 14r - 21 \\ = 7(2r - 3)$$

$$10. 12e - 15 \\ = 3(4e - 5)$$

$$11. xy + 3x \\ = x(y + 3)$$

$$12. 2ab + ad \\ = a(2b + d)$$

$$13. 5t + rt \\ = t(5 + r)$$

$$14. 5ry - rf \\ = r(5y - f)$$

$$15. 3gh - 2g \\ = g(3h - 2)$$

$$1. 2x^3 - 4x^2 \\ = 2x^2(x - 2)$$

$$2. x^2y^2 - 6xy \\ = xy(xy - 6)$$

$$3. x(y - 4x^2) \\ = x(y - 4x^2)$$

$$4. 2x^2y^2 + 6x^2y \\ = 2x^2y(y + 3)$$

$$5. 15p^2q - 3pq^3 \\ = 3pq(5p - q^2)$$

$$6. 16v^2 + 40uv \\ = 8v(2v + 5u)$$

$$7. 27y^2 - 18xy \\ = 9y(3y - 2x)$$

$$8. 30t^4 - 6t^3 \\ = 6t^3(5t - 1)$$

$$9. 30m^3 - 12m^4 \\ = 6m^3(5 - 2m)$$

$$10. 16p^2q - 15p^2q \\ = p^2q(16p - 15)$$

$$11. 15t^3 - 20t^2 \\ = 5t^2(3t - 4)$$

$$12. 28y^2 - 35y^3 \\ = 7y^2(4 - 5y)$$

Factorising by Difference of Two Squares

$$a) x^2 - 4 \\ = (x - 2)(x + 2)$$

$$x^2 = x \times x$$

$$4 = 2 \times 2 \rightarrow -2 \times 2 = -4$$

$$b) a^2 - 16 \\ = (a - 4)(a + 4)$$

$$c) b^2 - 25 \\ = (b - 5)(b + 5)$$

$$d) x^2 - 1 \\ = (x - 1)(x + 1)$$

$$e) 1 - k^2 \\ = (1 - k)(1 + k)$$

$$f) 81 - w^2 \\ = (9 - w)(9 + w)$$

$$g) 64 - h^2 \\ = (8 - h)(8 + h)$$

$$h) 100 - x^2 \\ = (10 - x)(10 + x)$$

$$i) x^2 - b^2 \\ = (x - b)(x + b)$$

$$j) w^2 - v^2 \\ = (w - v)(w + v)$$

$$k) 4a^2 - 1$$

$$= (2a - 1)(2a + 1)$$

$$2a \times 2a = 4a^2$$

$$-1 \times 1 = -1$$

$$l) x^2 - 25y^2$$

$$= (x - 5y)(x + 5y)$$

$$m) 36 - 49p^2$$

$$= (6 - 7p)(6 + 7p)$$

$$n) 81a^2 - 4b^2$$

$$= (9a - 2b)(9a + 2b)$$

$$o) 121v^2 - 100w^2$$

$$= (11v - 10w)(11v + 10w)$$

$$p) 64p^2 - 81q^2$$

$$= (8p - 9q)(8p + 9q)$$

$$q) 1 - 16a^2$$

$$= (1 - 4a)(1 + 4a)$$

$$r) 25 - 81x^2$$

$$= (5 - 9x)(5 + 9x)$$

$$s) 49 - 4k^2$$

$$= (7 - 2k)(7 + 2k)$$

$$t) 1 - 144y^2$$

$$= (1 - 12y)(1 + 12y)$$

$$1. 2b^2 - 32$$

* Check for HCF first!

$$= 2(b^2 - 16)$$

$$= 2(b - 4)(b + 4)$$

$$2. 27 - 3b^2$$

$$= 3(9 - b^2)$$

$$= 3(3 - b)(3 + b)$$

$$3. 5y^2 - 125$$

$$= 5(y^2 - 25)$$

$$= 5(y - 5)(y + 5)$$

$$4. 363 - 75b^2$$

$$= 3(121 - 25b^2)$$

$$= 3(11 - 5b)(11 + 5b)$$

$$a) \text{ Area of big square} = k^2$$

$$\text{Area of small square} = 5^2 = 25$$

$$\text{Area of pink} = k^2 - 25 = (k - 5)(k + 5) \text{ cm}^2$$

$$b) \text{ If } k = 8.5 :$$

$$\text{Area} = (8.5 - 5)(8.5 + 5) = 3.5 \times 13.5 = 47.25 \text{ cm}^2$$

Factorising Trinomials

$$1. a^2 + 12a + 11$$

$$= (a + 11)(a + 1)$$

$$\begin{array}{r|l} a & 1 \\ a & 11 \\ \hline & 11a \end{array}$$

$$a + 11a = 12a$$

$$1 \times 11 = 11$$

$$2. b^2 - 9b + 20$$

$$= (b - 5)(b - 4)$$

$$\begin{array}{r|l} b & -1 & -2 & -4 \\ b & -20 & -10 & -5 \\ \hline & -20b & -10b & -5b \end{array}$$

-ve to multiply to give a positive and add to negative

$$3. c^2 - 11c + 28$$

$$= (c - 7)(c - 4)$$

$$\begin{array}{r|l} c & -1 & -2 & -4 \\ c & -28 & -14 & -7 \\ \hline & -28c & -14c & -7c \end{array}$$

$$4. d^2 - 10d + 24 = (d - 6)(d - 4)$$

$$7. 10 - 12g + g^2 = (5 - g)(2 - g)$$

$$a) a^2 + a - 12 = (a + 4)(a - 3)$$

$$b) b^2 - 3b - 18 = (b - 6)(b + 3)$$

$$e) e^2 - e - 6 = (e - 6)(e + 5)$$

$$h) 20 - 8h + h^2 = (10 + h)(2 - h)$$

$$j) 4j^2 - 32j + 60 = (4j - 20)(j - 3)$$

$$\begin{array}{r|l} 2j & -6j \\ \hline 2j & -20 \\ \hline & -12j \\ & -60j \end{array}$$

$$\begin{array}{r|l} 4j & -3j \\ \hline 4j & -20 \\ \hline & -12j \\ & -60j \end{array}$$

$$5. e^2 + 24e + 63 = (e + 21)(e + 3)$$

$$8. 34 + 19h + h^2 = (17 + h)(2 + h)$$

$$\begin{array}{r|l} a & 1 & -1 & 2 & -2 & 3 & 4 \\ \hline a & -12 & 12 & -6 & 6 & -4 & -3 \\ \hline & -12a & 12a & -6a & 6a & -4a & -3a \end{array}$$

$$4x - 3 = -12$$

$$4a + (3a) = a$$

$$c) c^2 - 2c - 63 = (c - 9)(c + 7)$$

$$f) f^2 + 2f - 3 = (f + 3)(f - 1)$$

$$i) 2i^2 + 4i - 30 = (2i + 10)(i - 3)$$

$$d) d^2 - 5d - 36 = (d - 9)(d + 4)$$

$$g) 10 + 3g - g^2 = (5 - g)(2 + g)$$

$$\begin{array}{r|l} 2i & 1 & -1 & -30 & 30 \\ \hline 2i & -30 & 30 & 1 & -1 \\ \hline & -30i & 30i & i & -i \end{array}$$

$$10i + (-6i) = 4i$$

$$-3 \times 10 = -30$$

$$1. 2x^2 + 5x + 3 = (2x + 3)(x + 1)$$

$$\begin{array}{r|l} 2x & 3 & 1 \\ \hline x & 1 & 3 \\ \hline & 3x & 3x \end{array}$$

$$2x + 3x = 5x$$

$$1 \times 3 = 3$$

$$2. 12m^2 - 8m + 1 = (6m - 1)(2m - 1)$$

$$3. 8u^2 + 10u - 3 = (4u - 1)(2u + 3)$$

Mixed Factorising

$$a) x^2 + 3x + 2 \\ = (x + 2)(x + 1)$$

$$d) x^2 + 7x + 10 \\ = (x + 5)(x + 2)$$

$$g) a^2 + 5a \\ = a(a + 5)$$

$$j) v^2 - 10v + 16 \\ = (v - 8)(v - 2)$$

$$m) a^2 - 6a - 7 \\ = (a - 7)(a + 1)$$

$$p) x^2 - 2x - 24 \\ = (x - 6)(x + 4)$$

$$s) x^2 + x - 2 \\ = (x + 2)(x - 1)$$

$$b) m^2 - 36 \\ = (m - 6)(m + 6)$$

$$e) y^2 + 6y \\ = y(y + 6)$$

$$h) x^2 - 4 \\ = (x + 2)(x - 2)$$

$$k) 7ab + 21b \\ = 7b(a + 3)$$

$$n) 4x^2 - 9 \\ = (2x - 3)(2x + 3)$$

$$q) 9b^2 - 16 \\ = (3b + 4)(3b - 4)$$

$$t) c^2 - 13c + 12 \\ = (c - 12)(c - 1)$$

$$c) x^2 + 6x + 5 \\ = (x + 5)(x + 1)$$

$$f) t^2 + 9t + 8 \\ = (t + 8)(t + 1)$$

$$i) 2x + 3xy \\ = x(2 + 3y)$$

$$l) 1 - a^2 \\ = (1 - a)(1 + a)$$

$$o) 6st + 3s \\ = 3s(2t + 1)$$

$$r) 3x^2 - x^2 \\ = x^2(3x - 1)$$

$$u) 64y^2 - 25 \\ = (8y - 5)(8y + 5)$$

Completing the Square

$$1. x^2 + 4x \\ = (x + 2)^2 - 4$$

$4 \div 2 = 2$

$2^2 = 4$

$$5. x^2 + 2x + 7 \\ = (x + 1)^2 - 1 + 7$$

$$= (x + 1)^2 + 6$$

$$9. x^2 + 4x - 8 \\ = (x + 2)^2 - 4 - 8$$

$$= (x + 2)^2 - 12$$

$$13. x^2 - 20x - 6 \\ = (x - 10)^2 - 100 - 6$$

$$= (x - 10)^2 - 106$$

$$2. x^2 - 8x \\ = (x - 4)^2 - 16$$

$$6. x^2 + 10x + 27 \\ = (x + 5)^2 - 25 + 27$$

$$= (x + 5)^2 + 2$$

$$10. x^2 + 16x - 3 \\ = (x + 8)^2 - 64 - 3$$

$$= (x + 8)^2 - 67$$

$$14. x^2 - 2x + 5 \\ = (x - 1)^2 - 1 + 5$$

$$= (x - 1)^2 + 4$$

$$3. x^2 - 6x \\ = (x - 3)^2 - 9$$

$$7. x^2 + 6x + 2 \\ = (x + 3)^2 - 9 + 2$$

$$= (x + 3)^2 - 7$$

$$11. x^2 - 14x - 15 \\ = (x - 7)^2 - 49 + 15$$

$$= (x - 7)^2 - 34$$

$$15. x^2 - 6x + 11 \\ = (x - 3)^2 - 9 + 11$$

$$= (x - 3)^2 + 2$$

$$4. x^2 + 14x \\ = (x + 7)^2 - 49$$

$$8. x^2 + 8x + 9 \\ = (x + 4)^2 - 16 + 9$$

$$= (x + 4)^2 - 7$$

$$12. x^2 - 8x + 8 \\ = (x - 4)^2 - 16 + 8$$

$$= (x - 4)^2 - 8$$

$$16. x^2 - 12x + 21 \\ = (x - 6)^2 - 36 + 21$$

$$= (x - 6)^2 - 15$$