National 5 Maths

Course Notes

Contents

Expanding Single Brackets

Reminder: Gathering Terms

Collect the like terms in the expression $5a + 2b + 3a - 6b$

Examples:

1. $6(t + 5)$
2. $7(3a + 4b - 5c)$
3. $-(3 - y)$
4. $7t - 3(t - 5)$
5. $7(3x - 2) - 5(4x - 3)$

Expanding Double Brackets

There are a number of strategies that can be used to expand double brackets:

Expand: $(2x + 3)(x + 1)$

Method 1: First
Outside
Inside
Last

Method 2:

Method 3: Breaking up 1st bracket

Use your preferred method to expand the brackets below:

(1) $(3x + 7)(2x + 5)$

(2) $(3x+2)^2$

(3) $(2x + 1)(2x - 1)$

Brackets with Trinomials

Expanding brackets that contain a trinomial expression is a simple extension of the problems above:

(1) $(x + 5)(2x^2 + 3x + 6)$

(2) $(3 - x)(x^2 + 2x + 4)$

(3) Expand $(x + 2)(x + 4)(x - 3)$ (Extension)

Factorising Expressions

An expression can be factorised in one of the following 3 ways:

- Highest Common Factor - Common Factor is taken outside ()s
- Difference of 2 Squares - Two matching ()s with different signs
- Trinomial - An expression with 3 parts but no HCF.

Highest Common Factor

1. $2a + 2b$
2. $6a - 8b$
3. $5a - 10$
4. $m^2 + m$
5. $3x^2 + 2x$
6. $t^3 - t^2 + t$

Difference of 2 Squares

1. $x^2 - 9$
2. $x^2 - 81$
3. $y^2 - 25$
4. $100 - x^2$
5. $25 - 16x^2$
6. $9x^2 - 64y^2$

Simple Trinomials

1. $x^2 + 6x + 5$

2. $x^2 + 7x + 12$

3. $x^2 - 16x + 15$

4. $x^2 - 10y + 25$

5. $x^2 + x - 6$

6. $x^2 + 2x - 8$

7. $x^2 - 3x - 40$

8. $x^2 - x - 20$

<u>More Complex Trinomials</u>

1. $2x^2 + 12x + 18$

2. $2a^2 + 25a + 12$

3. $3x^2 - 5x + 2$

4. $4t^2 - 12t + 9$

5. $9x^2 + 3x - 2$

Completing the Square

Completing the square requires us to take an expression in the form $ax^2 + bx + c$, where a, b and c are all integers.

The aim is to rewrite the expression with a squared bracket. This will help us later when we are working with graphs of quadratic expressions.

eg. $(x + p)^2 + q$ or $(x + a)^2 + b$

Examples:

(1) Write $x^2 + 2x$ in the form $(x + p)^2 + q$

(2) Write $x^2 + 10x$ in the form $(x + a)^2 + b$

(3) Write $x^2 - 8x$ in the form $(x + p)^2 + q$

(4) Write $x^2 + 6x + 20$ in the form $(x + p)^2 + q$

(5) Write $x^2 - 8x + 4$ in the form $(x + a)^2 + b$

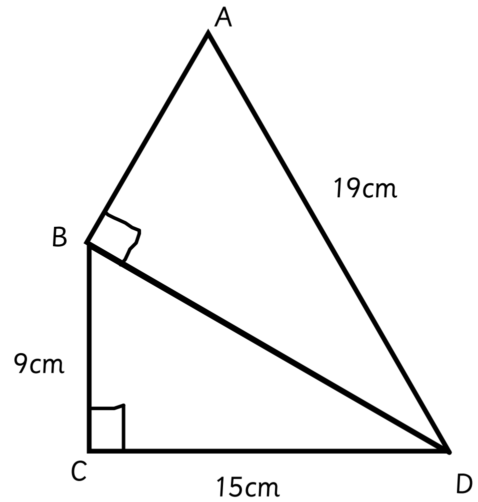
(6) Write $x^2 + 10x - 4$ in the form $(x + p)^2 + q$

Pythagoras

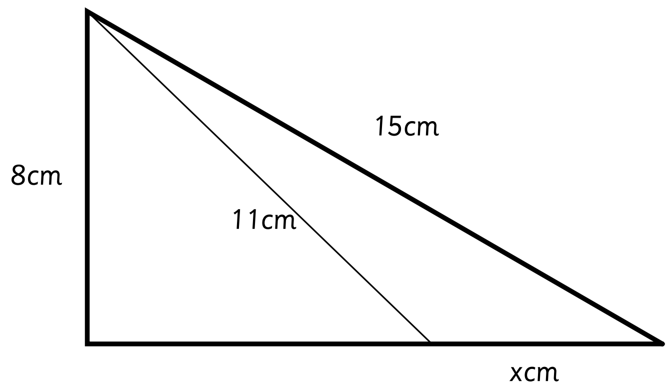
You will have already covered Pythagoras' theorem at level 4. In National 5, we extend our knowledge to more complex 2D problems as well as working in 3D.

Examples:

(1) Calculate the length of the side marked AB giving your answer to 2 decimal places.



(2) Calculate the length x giving your answer to 2 significant figures.

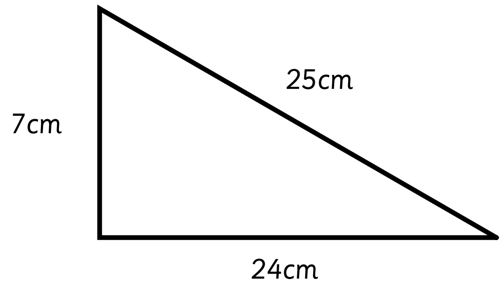


Converse of Pythagoras

We know we can use Pythagoras' Theorem to find a missing side in a triangle as long as we know it is right angled. Conversely, if we know all the sides in a triangle, we can check whether or not it is right angled by using Pythagoras. We call this "The converse of Pythagoras' Theorem".

Examples:

(1) Determine if this triangle is right-angled or not.



(2) A joiner is checking a window frame before the glazier fit a rectangular piece of glass. The window appears to be rectangular with a height of 1267mm, a breadth of 349mm and a diagonal of 1338mm.

Is the window frame ready for the glass to be fitted?

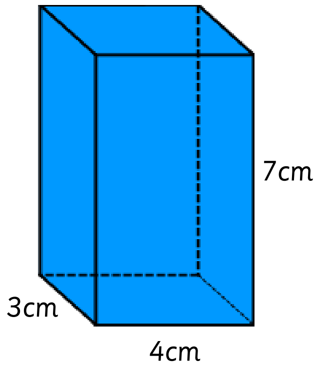
Pythagoras in 3D Shapes

We can use Pythagoras in some 3D shapes to find missing measurements using knowledge of perpendicular lines.

Examples:

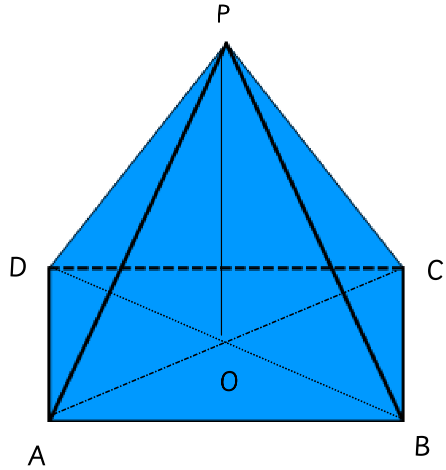
(1) Calculate the length of the space diagonal AG in the cuboid.

Give your answer to 1 decimal place.



(2) A Pyramid shaped tent has a square base with sides of length 5m. A pole OP of length 4m is used to hold up the centre of the tent. Four poles are used to connect the corners of the base to P.

How long are these poles?

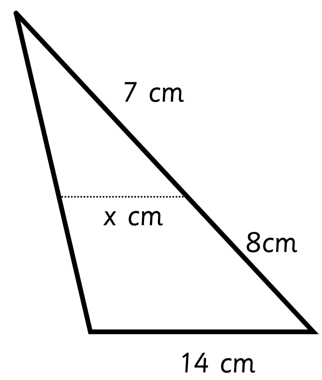


Similar Triangles

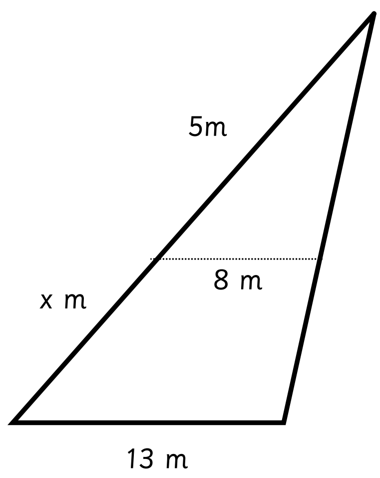
We can use properties of angles and parallel lines to calculate missing values in similar triangles.

Examples:

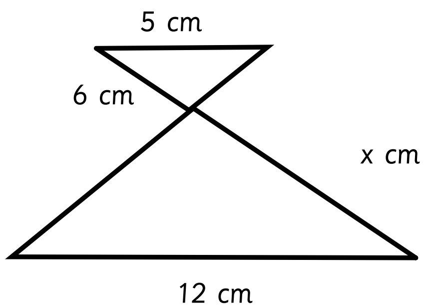
(1) Find the value of x to 2 decimal places.



(2) Find x giving your answer to 3 significant figures.



(3) Calculate x:



Similar Areas and Volumes

We already know how to find a basic scale factor. To calculate a similar area or volume, we only need to square or cube the scale factor.

Length = LSF x original

Area = LSF^2 x original

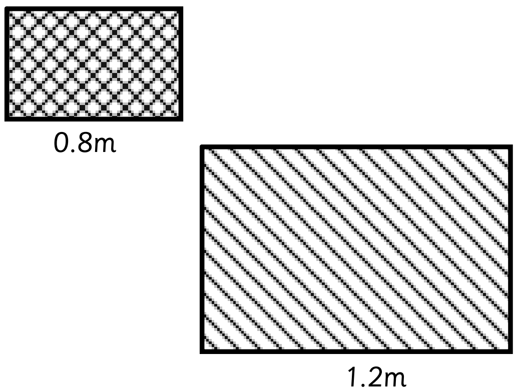
Volume = LSF^3 x original

Examples:

(1) Two rugs are mathematically similar.

The large rug has an area of $3.6m^2$

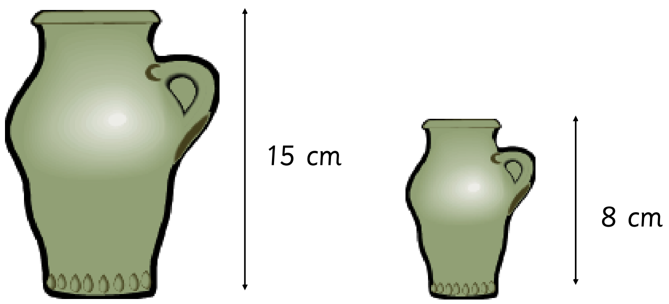
Calculate the area of the small rug correct to 1 decimal place.



(2) The following vases are mathematically similar.

The small vase has a volume of 250ml.

Calculate the volume of the larger vase to the nearest millilitre.

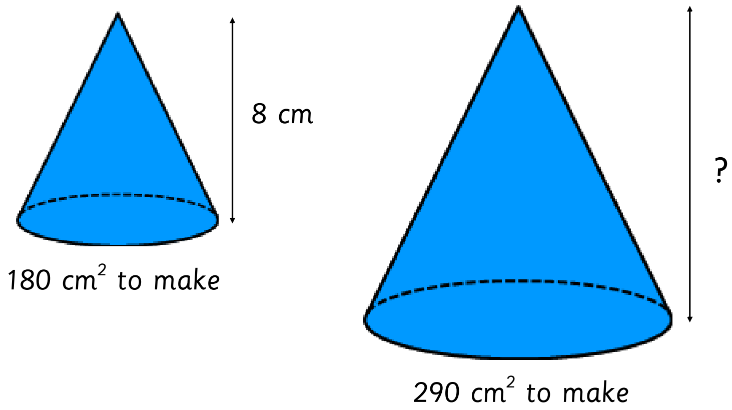


Reverse Scale Factor

Sometimes, we may be given similar areas or volumes in order to work backwards to find a missing length. If we are working in reverse, we must consider inverse operations.

Examples:

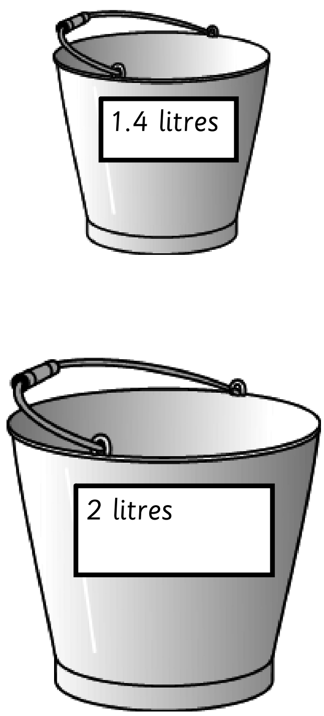
(1) The cones shown are mathematically similar. Find the height of the larger cone.



(2) Two paint tins are similar in shape as are their labels.

The area of label on the small tin is $80cm^2$.

Calculate the area of the label on the large tin.

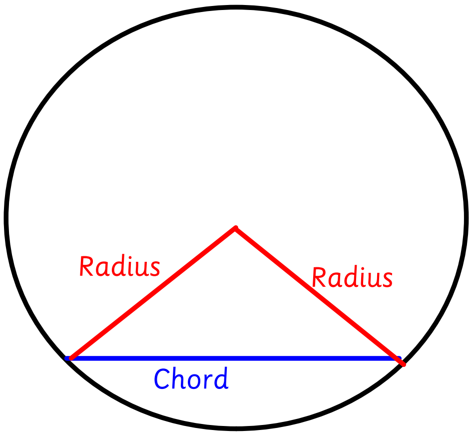


Triangles in Circles

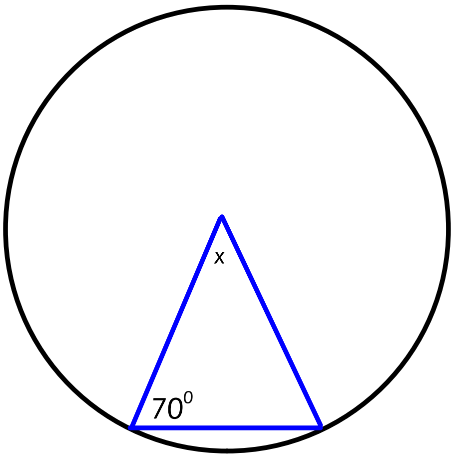
Any line joining two points on a circumference of a circle is called a chord

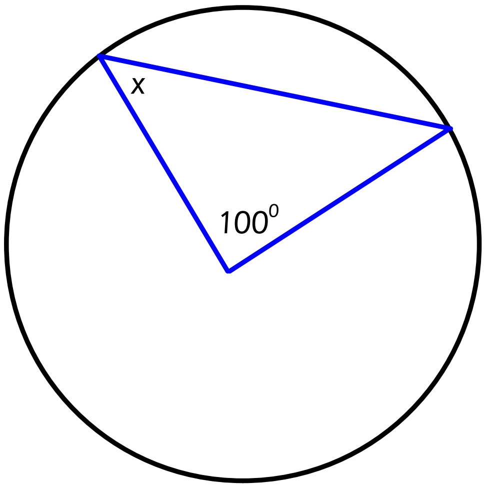
A diameter is therefore just a special chord

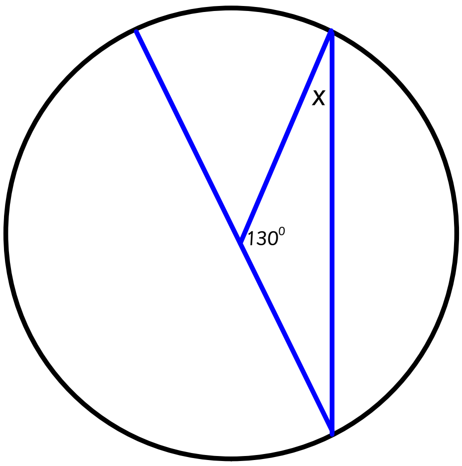
that passes through the centre of the circle
We can create an isosceles triangle by joining the ends of the chord to the centre of the circle



We can use facts about isoceles triangles to calculate unknown angles



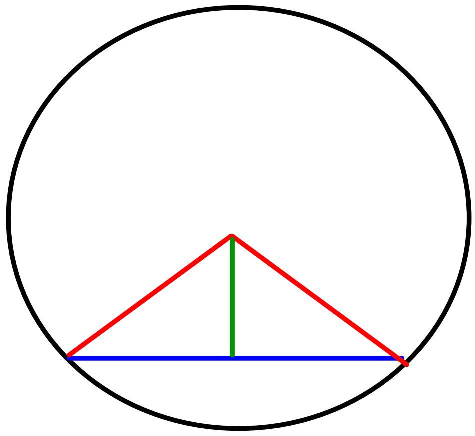




Perpendicular Bisectors

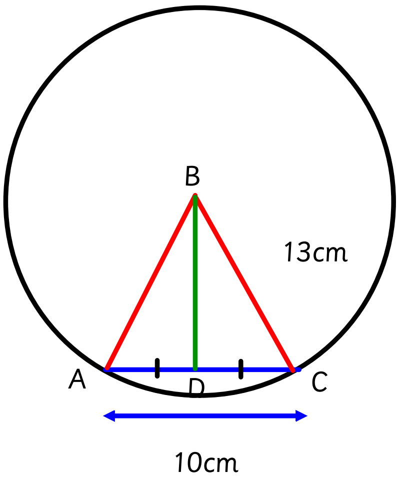
A perpendicular bisector cuts another line in half and at right angles.

This line generates 2 right angled triangles and therefore allows us to use Pythagoras and trig to calculate unknown sides and angles

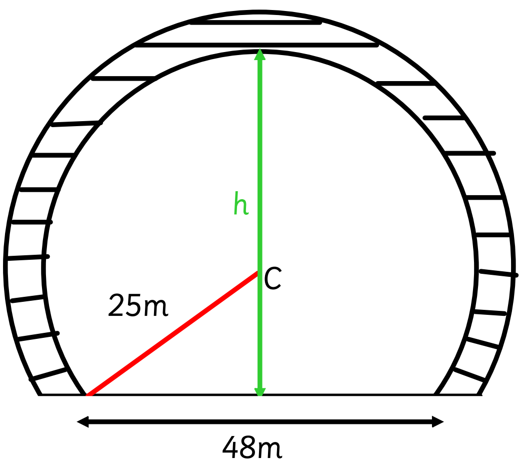


Examples:

(1) Calculate the length of the line BD.

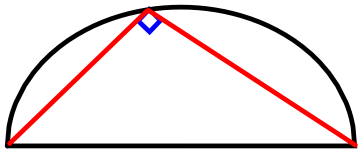


(2) Calculate the height of the tunnel.



Triangles in Semi-Circles

If you use 2 chords to create a triangle inside a semi circle, them the angle formed at the circumference will be a right angle

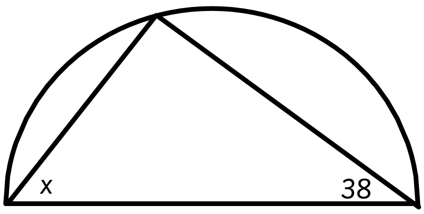


You may then be able to use angle facts,

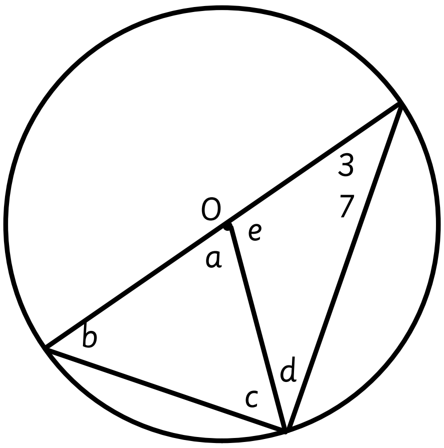
pythagoras and trig to calculate unknown sides and angles.

Examples:

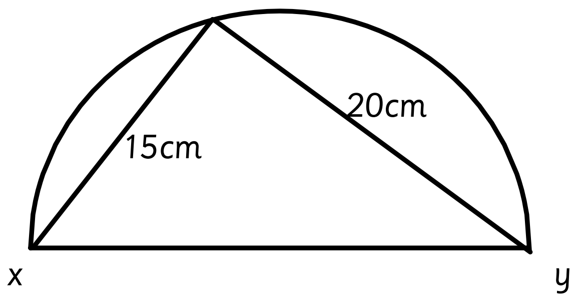
(1) Find the size of angle x.



(2) Find the size of the missing angles.



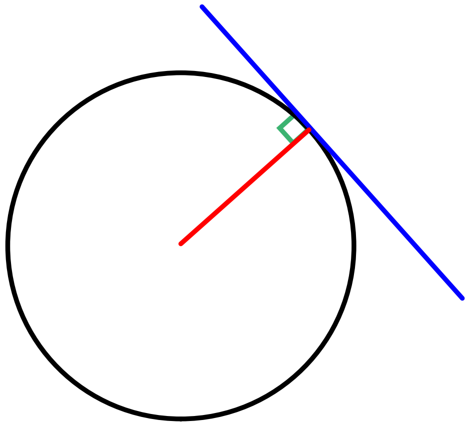
(3) Find the length of the diameter xy.



Tangents

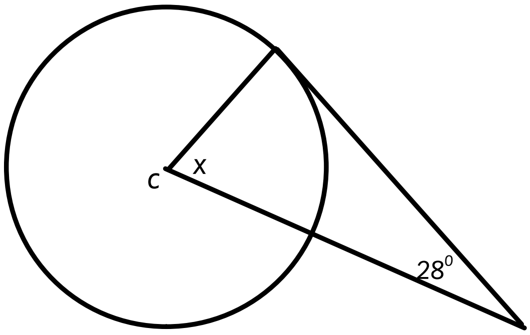
A tangent is a line that touches but does not cut a circle.

When a radius is draw to meet the tangent, where they touch forms a right angle.

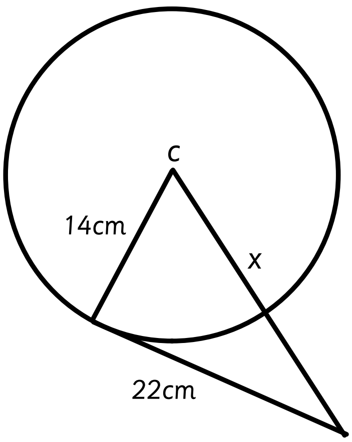


Examples:

(1) Find angle x.

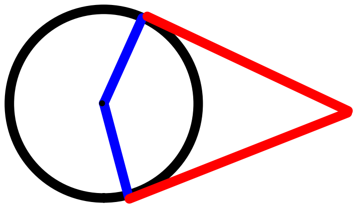


(2) Find the length of side x.



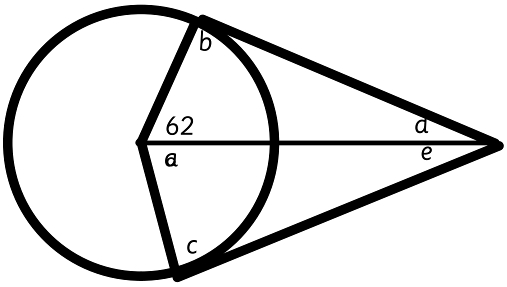
The Tangent Kite

When two tangents from the same circle meet they create a TANGENT KITE where the other two sides area made up of radii.

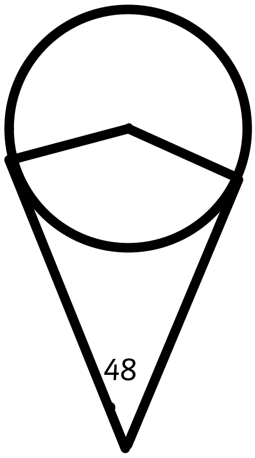


Examples:

(1) Fill in the missing angles



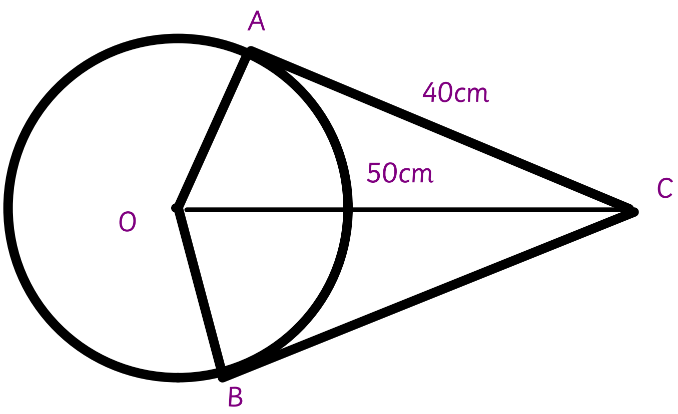
(2) Fill in the missing angles



(3) The diagram below shows two equal tangents which are 40cm long.

The line OC is 50cm long.

Calculate the length of the radius.



Gradient

Reminder:
The **gradient** of a line describes the steepness of the slope.

Positive Gradient (sloping up the page)

Negative Gradient (sloping down the page)

Zero Gradient (horizontal line)

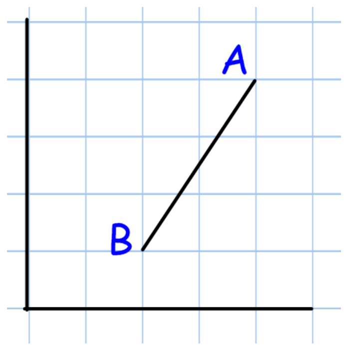
Undefined Gradient (vertical line)

Previously, we learned that the gradient is found by:

$$\textbf{gradient (m)} = \frac{\textbf{vertical distance}}{\textbf{horizontal distance}}$$

We can't however use this to help us find the gradient between two points.

We know the gradient between A and B is $^3/_2$ by looking at the vertical and horizontal distance travelled.
But what if we were only given the points A(4,4) and B(2,1)?



In order to calculate the gradient using co-ordinates, we need to adapt our formula.

$$\textbf{Gradient} = \frac{\textbf{vertical change}}{\textbf{horizontal change}} = \frac{\textbf{difference in y co-ordinates}}{\textbf{difference in x-co-ordinates}}$$

To find the gradient between two points A and B:

$$m_{AB} = \frac{Y_A - Y_B}{X_A - X_B}$$

Find the gradient of the lines joining the following pairs of points.

(1) A(4,8) and B(3,4)

(2) A(5,9) and B(6,7)

(3) R(-3,5) and S(2,-1).

Problems with Gradients

(1) A straight line with gradient zero passes through A(5,4) and B(7,*x*).
What is the value of *x*?

(2) The gradient of the line passing through C(b,5) and D(12,15) is $^5/_2$.
What is the value of b?

(3) If the gradient of the line EF is $^2/_3$, E is the point (2,-5) and F (-4,*a*).
Find the value of *a*.

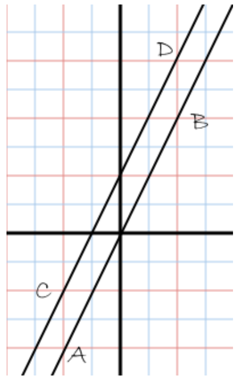
Drawing Straight Lines

Straight lines can be expressed in terms of the gradient m and the y-intercept using the equation $y = mx + c$.

We can also use this equation to help us draw straight line graphs.

Example:

(1) Find the equation of the following straight lines:



(2) Find the equation of the line shown.

We can use $y=mx + c$ when we know the gradient and have a diagram showing the y-intercept.

(3) Find the equation of the straight line passing through the point (8,5) that meets the y-axis at (0,1)

Rearranging Straight Line Equations

Sometimes, we are given the equation of a straight line in an alternative form. We can rearrange this to $y = mx + c$ to help us find key features of the graphs.

Examples:

(1) Find the gradient of the straight line having equation

$$2x + 3y + 6 = 0$$

(2) A straight line has equation $4y - 3x + 9 = 0$, find:

(a) the gradient of the line

(b) where the line crosses the y-axis

(c) where the line crosses the x-axis

(3) A straight line has equation $y = -\frac{3}{4}x + 6$. Find the coordinates of the point where this line intercepts the x-axis.

Equation of a Line

The equation $y = mx + c$ can also be found using $y - b = m(x - a)$.

To use this formula we must know the gradient and a point on the line.

Examples

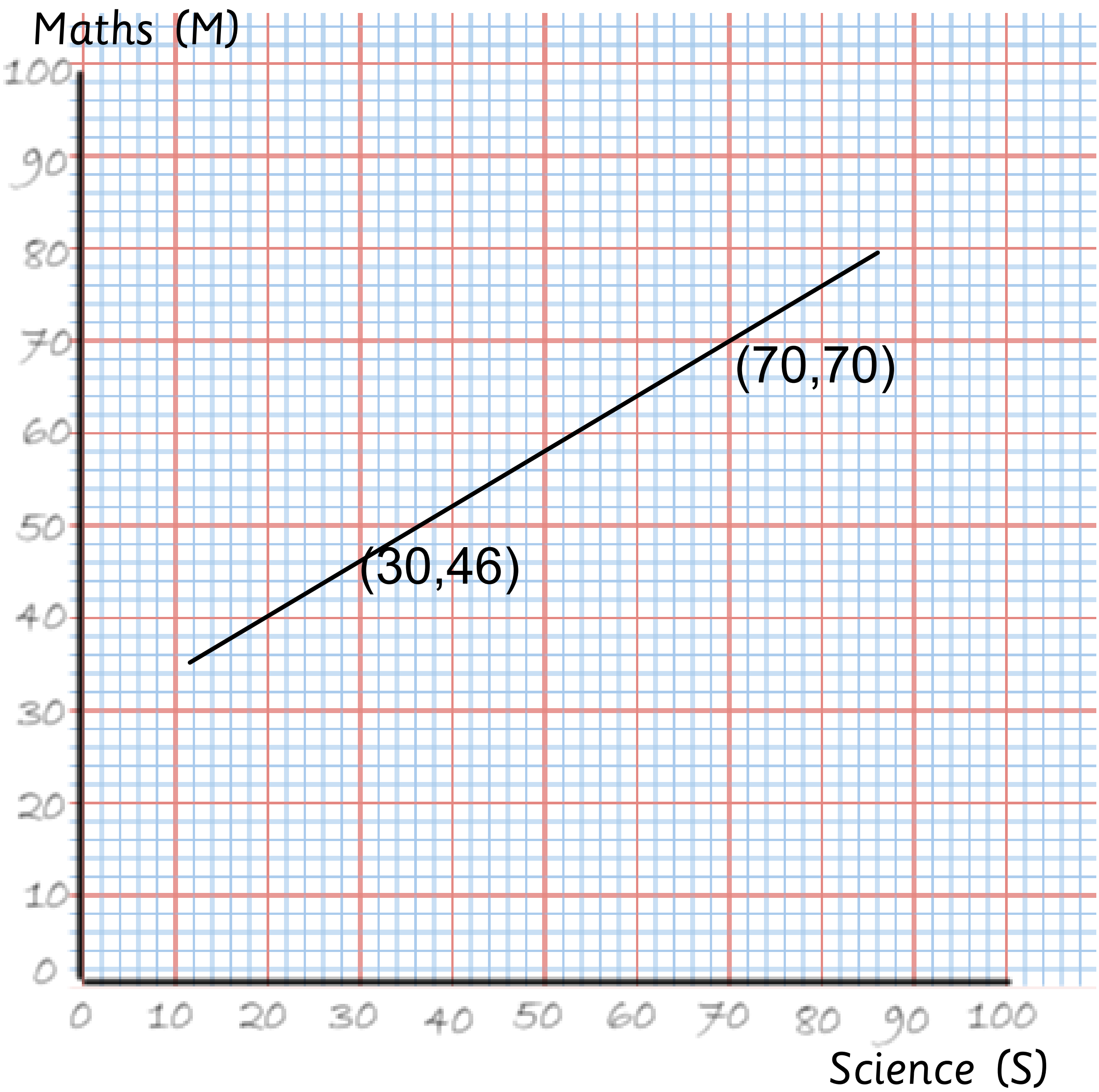
(1) A straight line has gradient 2 and passes through the point (6,3). Write down the equation of this line.

(2) A straight line has gradient $^{3}/_{4}$ and passes through the point (8, -5).
Find the equation of the line expressing your answer in the form $y = mx + c$.

(3) A straight line passes through the points (3,-5) and (-1,7).

Find the equation of the line and express it in the form $y = mx + c$.

Scatter Graphs (Best Fitting Lines)



(a) Find the equation of the line of best fit in terms of M and S

(b) Use your equation to estimate the science mark for a pupil who scored 80% in the maths test.

Function Notation

A function takes numbers in and applies a rule to them to give a new set of numbers.

Examples:

(1) For the function $f(x) = 3x - 4$ find:

a) $f(2)$

b) $f(-3)$

(2) A function, g is defined by $g(x) = x^2 - 4x$. Find the value of $g(-5)$.

(3) The function h is given by $h(x) = 7 - 4x$.

Find the value of x for which $h(x) = 13$

Equations with Brackets

(1) *Solve the equation* $2(3x - 8) - 3x = 17$

(2) *Solve for x* $5(x + 6) - 2 = 5 + 3(3x + 5)$

(3) *Solve the equation* $3(4x - 7) - 4(2x - 1) = 9$

(4) A litre carton of orange juice costs x pence.

a) Write down the cost of 5 cartons of juice.

The cost of a litre carton os apple and raspberry smoothie is 82p more than the cost of a litre of orange juice.

b) Write down an expression for the cost of a carton of smoothie

Jenny buys 5 cartons of orange juice and 3 cartons of smoothie. Altogether she pays £19.90.

c) Write down and solve the equation that represents this situation and find the value of x.

Equations with Fractions

(1) *Solve* $\frac{x}{5} = 6$

(2) *Solve* $\frac{3}{4}(2x - 1) = 9$

(3) *Solve* $\frac{5x + 2}{4} - \frac{2x - 3}{7} = -1$

Inequations

Solving inequations requires the same knowledge and skills as solving equations. with only one exception.

When dividing through by a negative, the direction of the inequality changes.

(1) Solve $2x - 5 \geq 7x - 3$

(2) Solve the inequation $20 - 2(3x + 8) \geq 8 - 5x$

(3) Solve the inequality $11 - 2(1 + 3x) < 39$ SQA 2015 paper 1

Simultaneous Equations - Graphical Solution

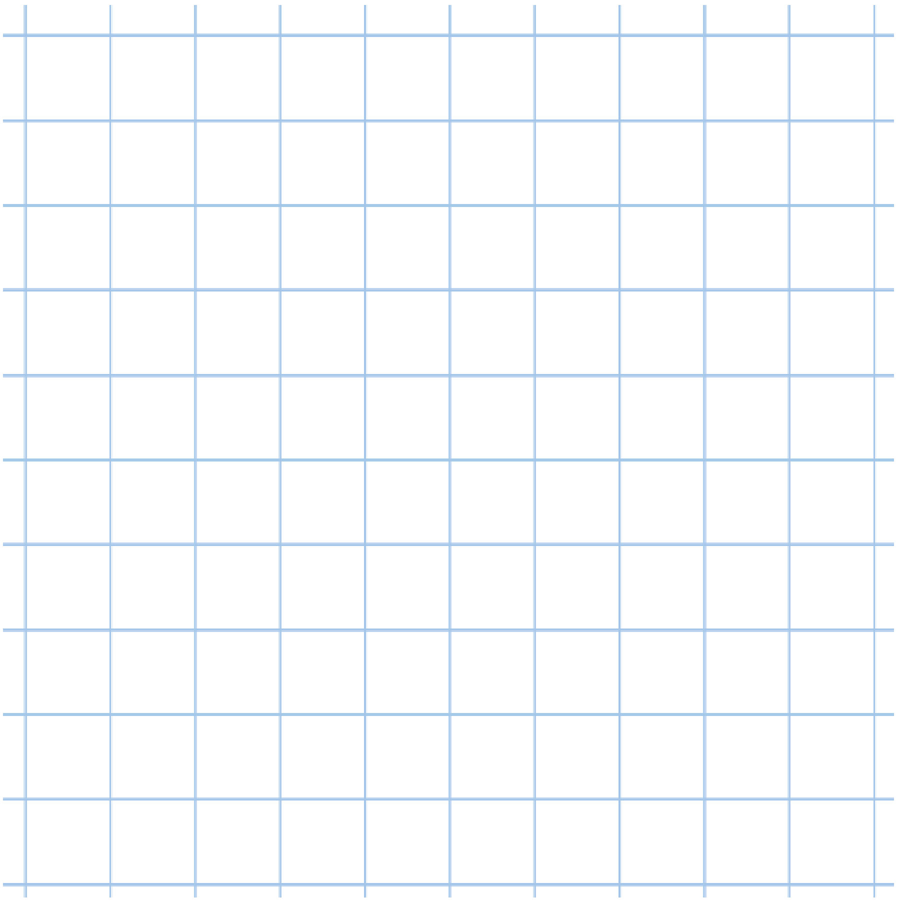
Simultaneous Equations are two equations that can be solved at the same time.

A good example of solving simultaneous equations is finding the point of intersection between two straight lines.

Example

Two straight lines have the *equations* $x + y = 6$ and $x + 2y = 8$.

Find the point of intersection of the lines using a diagram.



Simultaenous Equations - Algebraic

Examples: Solve the following systems of equations

(1) $2x + y = 11$

$x + y = 7$

(2) $5x - 2y = 16$

$x - 2y = 8$

(3) $-3x + 8y = -14$

$3x + 5y = 1$

More Simultaenous Equations

Sometimes we must prepare a systems of equations before we solve them.

(1) $3x + 2y = -12$

$x - y = 1$

(2) $2x + 3y = 2$

$3x - 4y = 20$

(3) $3x + 4y = 5$

$2y = x + 5$

Changing the Subject

Sometimes we need to rearrange a formula to make it more useful to us in the context of a question. This is a skill you will use in Science and Engineering as well.

If we "change the subject of a formula to x" we are being asked to solve a formula for x or "get the x term on its own".

(1) Make x the subject of the formula

(a) $x + b = c$ (b) $y = 5x$

(2) Change the subject to the letter in brackets:

(a) $C = 2\pi r$ (r) (b) $y = 3x - 5$ (x)

(c) $s = 2 - tx$ (t)

Change of Subject with Fractions

(1) Change the subject of $y = \frac{x}{3}$ to x.

(2) Make x the subject of the formula $y = \frac{2x}{5}$

(3) Make p the subject of the formula $f = \frac{4 - p}{g}$

(4) Make k the subject of the formula $q = \frac{k}{2} - \frac{p}{3}$

Change of Subject to a Denominator

The most complex case of changing the subject of a formula occurs when the subject we require is the denominator of a fraction.

(1) Make x the subject of the formula $y = \frac{5}{x}$

(2) Make x the subject of the formula $y = 4 + \frac{3}{x}$

(3) Make g the subject of $T = \frac{l}{g}$

(4) Make t the subject of the formula $G = \frac{m}{t - 4}$

(5) Change the subject of the formula to p: $H = \frac{\pi}{k - pr}$

Change the Subject - Powers and Brackets

(1) Make H the subject:

$$S = \frac{\sqrt{(WT)}}{60}$$

(2) Make r the subject

$$V = \frac{4}{3}\pi r^3$$

(3) Make v the subject

$$s = \frac{t(u + v)}{2}$$

(4) Make d the subject

$$T = k\sqrt{d} + e$$

(5) Make h the subject:

$$b = \sqrt{\frac{3V}{h}}$$

Fractions Revision

At 4th level, we considered addition and subtraction of both fractions and mixed numbers. The following examples should remind you of your previous learning.

Steps to Success:

1. convert to improper fractions if required
2. create a common denominator
3. use equivalent fractions
4. add/ subtract the numerators
5. 5. convert back if appropriate

Examples:

(1) $1\frac{3}{4} + \frac{5}{6}$

(2) $3\frac{3}{10} - 1\frac{5}{6}$

(3) Both Lee and Mary have a packet of the same sweets. Mary eats $^1/_3$ of her packet. Lee eats $^3/_4$ of his packet.

(a) Find the difference between the amount each person eats.

(b) Lee gives the rest of his sweets to Mary. What fraction of a packet does Mary have now?

<u>Multiplying Fractions</u>

When we work with fractions in maths, we often use the word "of" to describe a multiplication e.g. find $^1/_2$ x 20 means find $^1/_2$ of 20.

Examples:

(1) Find $^3/_4$ x 28

When we consider two simple fractions it is often much easier to consider using "of" to complete the calculation.

(2) $^1/_2$ x $^1/_2$

From the example above we can see that the rule for multiplying fractions is: "to multiply the numerators together and then multiply the denominators" or: $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$

(3) $^3/_8$ x $^1/_9$

(4) $^5/_8$ x 12

(5) $1\frac{1}{2} \times 1\frac{3}{5}$

Dividing by a Fraction

When we first learned to divide, when considering 20 ÷ 5, we often find it easier to think about "how many 5s go into 20." This logic is also helpful when we divide by a fraction:

Examples:

(1) $4 \div {}^1/_2$ is the same as… how many halves in 4 wholes?

(2) ${}^1/_2 \div {}^1/_4$

From the example above we can see that the rule for dividing by a fraction is to flip the second fraction and multiply:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

(3) ${}^2/_3 \div 5$

(4) ${}^2/_3 \div {}^1/_5$

(5) $1\frac{3}{5} \div \frac{4}{9}$

(6) $2\frac{1}{4} \div 1\frac{4}{5}$

Percentage Increase/ Decrease

Percentages are often used to measure the change in value of property, cars and retail goods in order to help customers make comparisons and search for the best value for money. In order to calculate many of these percentages we must consider decimal multipliers.

Decimal Multipliers

<u>An increase by 5%</u>

100% + 5%

= 105%

= 1.05

<u>A decrease by 2.3%</u>

100% - 2.3%

= 97.7%

= 0.977

Decimal Multiplier

To find the resultant amount of a percentage change, we multiply the original amount by the decimal multiplier.

Examples:

(1) A TV is reduced by 15% in a sale. It originally costs £600, what is the price now?

(2) The population of a small town increased by 6.5% over one year. There were 12400 people in the town at the start of the year, how many were there at the end?

Give your answer to 2SF.

Percentage Change Over Time

When we consider the long term impact of a percentage change we consider repeated calculations. For example if the value of a £20 000 car decreases by 10% per year over 3 years.

Examples:

(1) A population of bacteria increased by 15% each hour. If the population at 10am was 120, what would it be at 2pm?

Method 1: Repeated Calculation	Method 2: Using Powers

(2) A car is valued at £9600. It is expected to depreciate in value by 15% per annum. What will the car be worth after 4 years?

Method 1: Repeated Calculation	Method 2: Using Powers

Reverse Percentage Calculations

Sometimes, we will be given the value of an item after a percentage change, i.e. the new amount and will be expected to work out the original amount. We apply our knowledge of proportion to these problems to work out 100%. These questions may be calculator or non-calculator depending on the values.

Non-Calculator:

Examples:

(1) Last year, the value of a car decreased by 20%. The current value of the car is £8640, what was it worth at the start of the year?

(2) A laptop costs £583 including VAT at 10%. What was the cost of the laptop before the VAT was added?

Calculator:

(3) A cottage has appreciated in calue by 3% over the past year. If it is now worth £154 000, what was its value at the start of the year? Answer to the nearest hundred pounds.

(4) An electricity bill for the six months from May to November was £436.27 including VAT at 5%. What was the cost before VAT?

Standard Deviation

The standard deviation is a measure of spread of a set of data in comparison to the mean.

- a low standard deviation suggests that there is a low amount of variation from the mean - this gives a more consistent data set.
- a high standard deviation suggests a high amount of variation from the mean and there for represents a data set with low/ poor consistency.

Examples:

(1) The price of a bar of chocolate in five different shops is:

44p, 49p, 50p, 52p, 55p

Calculate the mean and standard deviation of this sample.

(2) The marks (out of 100) of six pupils in a maths test are:

52 60 77 88 91

Calculate the mean and standard deviation of the group.

(3) A machine is used to put drawing pins into boxes. A sample of 8 boxes is taken and the number of drawing pins in each is counted. The results are shown below:

102 102 101 98 99 101 103 102

a) Calculate the mean and standard deviation of this sample

b) A sample of 8 boxes is taken from another machine. This sample has a mean of 103 and a standard deviation of 2.1.
Write down two valid comparisons between the samples.

Quartiles and Quartile Range

The Interquartile range is a measure of spread of a set of data in comparison to the median.

- a low IQR or SIQR suggests that there is a low amount of variation from the median - this gives a more consistent data set.
- a high IQR or SIQR suggests a high amount of variation from the median and there for represents a data set with low/ poor consistency.

$$IQR = Q_3 - Q_1$$

$$SIQR = \frac{Q_3 - Q_1}{2}$$

Examples:

(1) Find the median and quartiles of this list of values.

3 4 5 5 7 9 11 11 12

(2) Find the median, quartiles, IQR and SIQR for the following data set 19, 10, 12, 5, 20, 3, 13, 7, 5

(3) Mrs. Sutherland compares the price of a carton of milk in 10 different shops.

70 76 83 86 95 98 113 117 122 130

a) Create a 5 figure summary for this data.

b) Hence, calculate the semi-interquartile range.

(4) Ten couples took part in a dance competition. The couples were given a score in each round. The scores in the first round were:

16 27 12 18 26 21 27 22 18 17

a) Calculate the median and SIQR of these scores.

b) In the second round, the median was 26 and the semi-interquartile range was 2.4. Make two valid comparisons between the scores in the first and second round.

Simplify Algebraic Fractions

Simplify the following fractions:

$$\frac{30}{65} \qquad \frac{x}{x^5}$$

$$\frac{8x}{12x^2} \qquad \frac{x^2y^3}{x^2y^2}$$

$$\frac{(x+5)^2}{x+5} \qquad \frac{(x-2)^6}{(x-2)^3}$$

$$\frac{(x^2-7)^2}{(x^2-7)^4} \qquad \frac{(x-2)(x-3)}{(x-2)}$$

$$\frac{(x+5)(x-3)}{(x-2)(x+5)} \qquad \frac{x(y-2)^3}{x(y-2)}$$

Factorising to Reduce Algebraic Fractions

$$\frac{5x + 10}{20}$$

$$\frac{15}{6x - 3}$$

$$\frac{x - 3}{x^2 - 3x}$$

$$\frac{x - 4}{x^2 - 16}$$

$$\frac{9x^2 - y^2}{3x - y}$$

$$\frac{2x^2 - 5x - 12}{x - 4}$$

$$\frac{a^2 - 25}{a^2 + 13a + 40}$$

$$\frac{8x^2 - 18}{2x^2 - 11x - 21}$$

Multiplication and Division of Algebraic Fractions

Express the following as single fractions in their simplest form:

$$\frac{4}{x} \times \frac{5}{x}$$

$$\frac{4xy}{5z} \times \frac{5z^2}{7x}$$

$$\frac{x^2 - 9}{x + 4} \times \frac{x^2 + 7x + 12}{x - 3}$$

$$x^4 \div \frac{2}{x}$$

$$\frac{2}{x^2} \div \frac{4}{x}$$

$$\frac{x^2 - 4}{x^4} \div \frac{x + 2}{x^2}$$

$$\frac{3x}{2y^2} \div \frac{x^2}{y} \times \frac{xy}{6}$$

Addition and Subtraction of Algebraic Fractions

Express as a single fraction in its simplest form:

$$\frac{x + 3}{4} + \frac{x - 2}{6}$$

Express as a single fraction in its simplest form

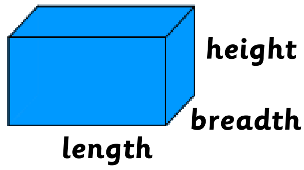
$$\frac{5}{x - 1} + \frac{2}{x + 4}$$

Express as a single fraction in its simplest form

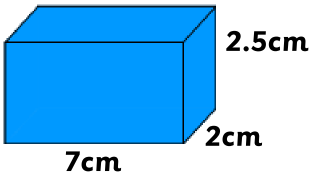
$$\frac{3}{2x + 3} - \frac{2}{4x - 2}$$

Volume of 3D Objects

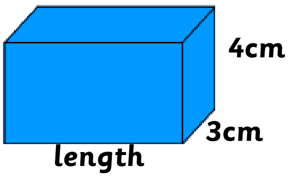
$$V_{cuboid} = L \times B \times H$$



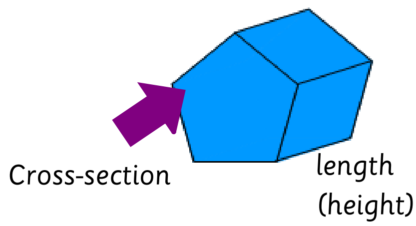
Example 1: Find the volume of the cuboid shown:



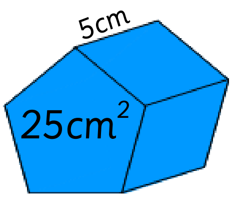
Example 2: The cuboid shown has a volume of 72cm^3. Calculate its length.



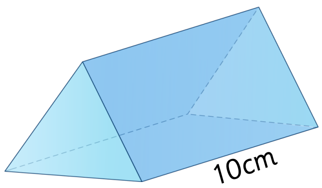
$$V_{prism} = A_{cross\ section} \times h$$



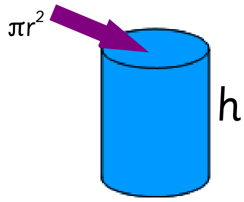
Example 1: Find the volume of the prism shown:



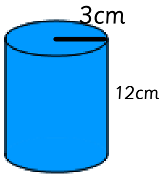
Example 2: The prism shown has a volume of 140cm^3. Calculate the area of its cross-section.



$$V_{cylinder} = \pi r^2 h$$

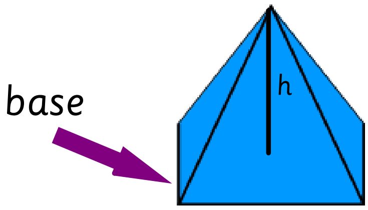


Example 1: Calculate the volume of the cylinder shown.

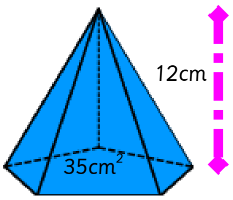


Example 2: Calculate the height of a cylinder with volume $402.1cm^3$ and radius 4.

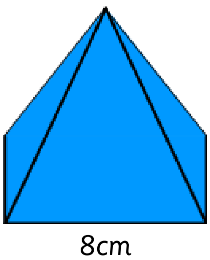
$$V_{pyramid} = \frac{1}{3} A_{base} \times h$$



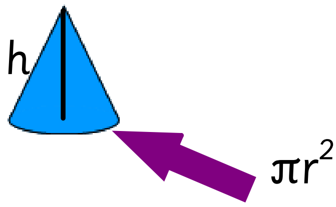
Example 1: Calculate the volume of the pyramid shown.



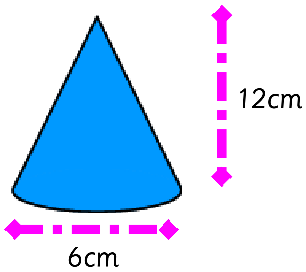
Example 2: A square based pyramid has a volume of $640cm^3$. Calculate its height.



$$V_{cone} = \frac{1}{3}\pi r^2 h$$

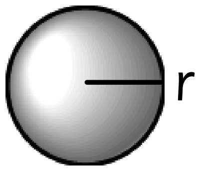


Example 1: Calculate the volume of the cone shown.



Example 2: Calculate the height of a cone with volume $42cm^3$ and radius 2cm.

$$V_{sphere} = \frac{4}{3}\pi r^3$$

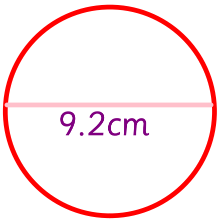


Example 1: Calculate the volume of a sphere with radius 3cm.

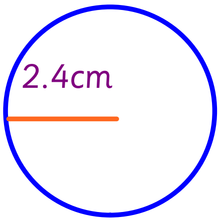
Example 2: Calculate the radius of a sphere with volume $113.1cm^3$

Arcs and Sectors

Circle Revision

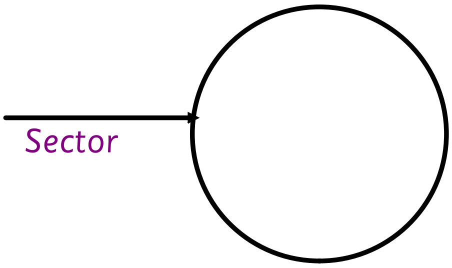


Calculate the circumference $C = \pi \times \text{diameter}$

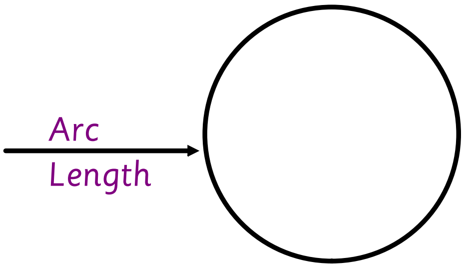


Calculate the area $A = \pi \times \text{radius}^2$

A ***sector*** of a circle is a fraction (or part) of a whole circle.



The ***arc length*** of a sector refers to a fraction of the whole circle's circumference.



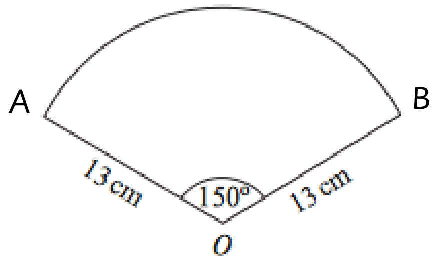
Calculating the Arc Length

The arc length formula is based on the formula for the whole circumference of a circle.

$$\textbf{Arc Length} = \frac{x^{o}}{360} \times \pi \times \textbf{diameter}$$

Example

Find the length of the arc AB.



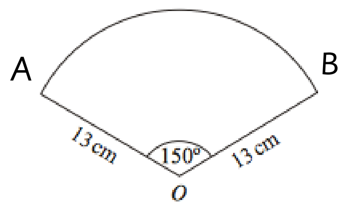
Calculating the Area of a Sector

The area of a sector formula is based on the formula for the whole area of a circle.

$$\textbf{Asector} = \frac{x^{o}}{360} \times \pi \times (\textbf{radius})^2$$

Examples

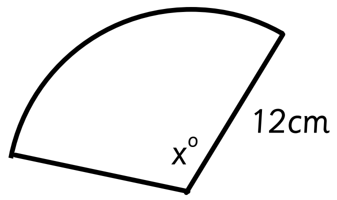
Find the area of the arc AB.



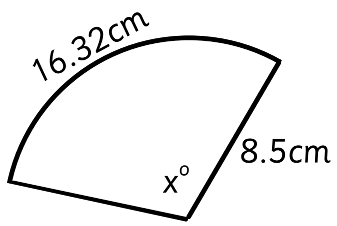
Working Backwards Arcs and Sectors

Examples

(1) Calculate the angle x^o given that the arc length is 12.56cm

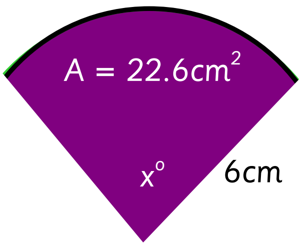


(2) Find x^o



(3) Calculate the angle x^o given that a sector has radius 4cm and an area of $16.76cm^2$.

(4) Find x^o



Simplify Surds

Simplify the following:

(1) $6\sqrt{2} + 2\sqrt{2} + 7\sqrt{5}$

(2) $11\sqrt{7} - 8\sqrt{7} + \sqrt{7}$

(3) $4\sqrt{3} - 6\sqrt{2} + 5\sqrt{3} + 2\sqrt{2}$

(4) Express each of the following in their simplest form:

$\sqrt{12}$ $\sqrt{72}$ $5\sqrt{18}$ $\sqrt{18} + 5\sqrt{2}$

Multiply/ Divide Surds

Simplify the following leaving your answer in surd form where necessary

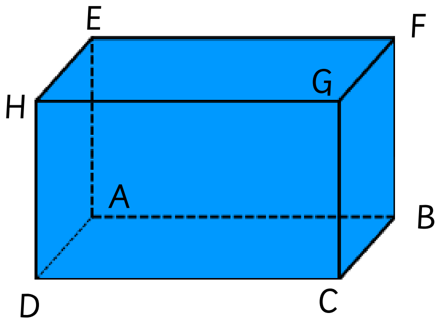
$\sqrt{3} \times \sqrt{5}$ $\sqrt{2} \times \sqrt{18}$ $\dfrac{\sqrt{21}}{\sqrt{3}}$

$\sqrt{54} \div \sqrt{6}$ $5\sqrt{2} \times 6\sqrt{3}$ $\dfrac{4\sqrt{6} \times 2\sqrt{5}}{3\sqrt{15}}$

Problems with Surds

Find the exact length of the space diagonal AG

Given that GC = 2cm, CB = 4cm AB = 5cm:



Rationalising the Denominator

Rationalise the Denominator of: $\frac{5}{\sqrt{3}}$ and $\frac{\sqrt{7}}{\sqrt{8}}$

Brackets and Surds

Multiply the bracket out and simplify $\sqrt{5}(2 + \sqrt{8})$

Indices

We know already that we can use indices (powers) to represent repeated multiplication. Index terms can be manipulated using a set of rules in order to simplify and evaluate expressions.

Examples

(1) Calculate the following values:

$$3^5 \qquad 0.2^3 \qquad \left(\frac{2}{3}\right)^4 \qquad 1^4 \qquad 5^0$$

(2) Write the following expressions using index notation:

$$2 \times 2 \times 2 \times 2 \times 2 =$$

$$f \times f \times f \times f =$$

Negative Powers

A term with a positive power represents repeated addition. Likewise, a term with a negative power (or index) represents repeated division i.e the term belongs on the denominator of a fraction (in most cases).

Examples

(1) Write the following expressions with a positive index.

$$4^{-3} \qquad x^{-2} \qquad 6n^{-3} \qquad (2a)^{-2} \qquad \frac{2a^{-4}}{3}$$

(2) Evaluate 4^{-2} leaving your answer as a fraction.

Multiply and Divide Indices

When we multiply two index terms with the same base, we add the powers. Conversely, when we are dividing two index terms with the same base we must subtract the powers.

Examples

Simplify the following expressions writing your answers in positive index form:

$3^4 \times 3^5$ $\qquad$ $7^2 \times 7^{-5}$

$5x^6 \times 6x^4$ $\qquad$ $10y^3 \times 4y^{-5}$

$8^7 \div 8^4$ $\qquad$ $2^3 \div 2^9$

$5^3 \div 5^{-7}$ $\qquad$ $18a^9 \div 6a^2$

Raising Powers

When raising a power to another power, we simply multiply the powers, eg. $(x^2)^2$.

Examples

$(x^7)^4$ $\qquad$ $(7^6)^{-2}$ $\qquad$ $(y^{-3})^{-4}$ $\qquad$ $(4a)^3$

Fractional Indices

When a term has a fractional power, we must consider a root and a power.
Examples:

(1) Write in index form:

$$\sqrt[4]{a^3} \qquad\qquad \sqrt{m^5}$$

(2) Evaluate:

$$49^{\frac{1}{2}} \qquad\qquad 32^{\frac{3}{5}} \qquad\qquad 32^{-\frac{3}{5}}$$

Expanding Brackets with Indices

Multiply the bracket and simplify: $x^{\frac{1}{2}}(2x^2 + x^{-\frac{1}{2}})$

Scientific Notation

(1) Write the following numbers in scientific notation:

34

568

35 807

0.8

0.345

0.0078

(2) Write the following numbers in expanded (full) form:

2.8×10^1

1.342×10^4

9.11×10^2

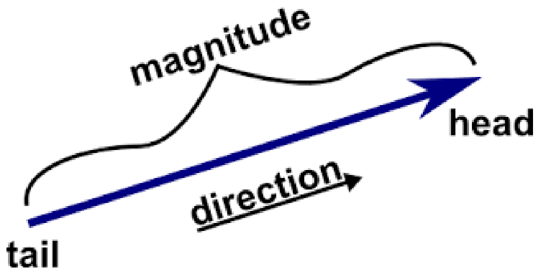
3.6×10^{-1}

1.78×10^{-2}

5.54×10^{-2}

What is a Vector?

Pupils studying physics will already be familiar with Vectors.



A vector is a quantity that has both size (magnitude) and direction.

An example of a vector is wind with speed 9mph travelling in a direction of 272^{o}.

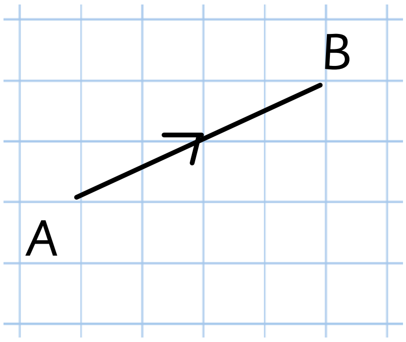
Wind is an example of a vector, but vectors are used by the following people:

- Pilots/Captains of ships - travelling at a certain speed in a certain direction
- GPS - this works around the use of vectors
- Engineers use vectors to make some calculations easier
- Sports - Vectors are applied in many sports. Eg. When passing in football, the ball needs to be passed in a certain direction at a certain speed, in order for it to go to the correct person.
- Mountain Rescue - Vectors will be used to find the location of people missing or injured.

Now that you know what a vector is, we are now going to look at how these are used in Mathematics.

Naming Vectors and their Components

Vectors can be named in 2 ways:



$\overrightarrow{AB}$ or **a** - electronically

$\underline{a}$ - handwritten

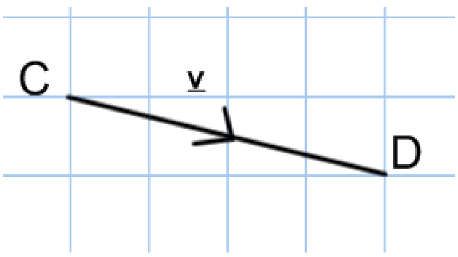
We might also like to describe them in terms of their components:

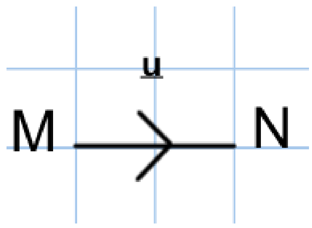
$$\begin{pmatrix} \text{left/right} \\ \text{up/down} \end{pmatrix} \qquad \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

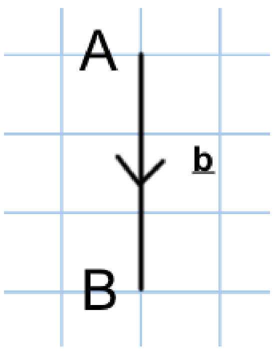
NOTE: these are NOT fractions and should not include a line!

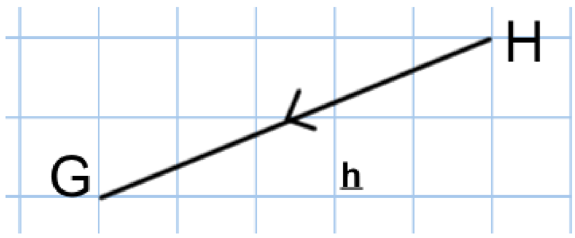
Practice

Name the following vectors in 2 ways and write down their components.









Sketching Vectors

Sketch the following vectors.

$\overrightarrow{AB} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ $\quad \underline{\mathbf{c}} = \begin{pmatrix} -2 \\ 0 \end{pmatrix}$ $\quad \overrightarrow{MN} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$ $\quad \underline{\mathbf{v}} = \begin{pmatrix} -3 \\ -2 \end{pmatrix}$

Sketching Vector Addition & Subtraction

Sketch the following vectors of $\underline{\mathbf{a}}$, $\underline{\mathbf{b}}$ and $\underline{\mathbf{c}}$:

$\underline{\mathbf{a}} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ $\quad \underline{\mathbf{b}} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$ $\quad \underline{\mathbf{c}} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$

(a) $\underline{\mathbf{a}} + \underline{\mathbf{b}}$ (b) $\underline{\mathbf{b}} - \underline{\mathbf{c}}$ (c) $3\underline{\mathbf{b}}$

(d) $-2\underline{\mathbf{c}}$ (e) $2\underline{\mathbf{a}} + \underline{\mathbf{b}}$ (f) $2\underline{\mathbf{a}} - \underline{\mathbf{c}}$

Operations with Vectors

Given that $\underline{r} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$ $\underline{s} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$ calculate:

$\underline{r} + \underline{s}$

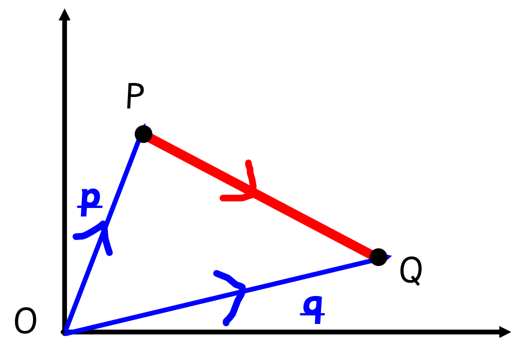
$\underline{r} - \underline{s}$

$4\underline{r}$

$2\underline{r} + 3\underline{s}$

Vector Journeys

Describe $\overrightarrow{PQ}$ in terms of vectors $\underline{p}$ and $\underline{q}$



Magnitude of Vectors

The magnitude of a vector describes its length or size and is a direct application of Pythagoras' Theorem.

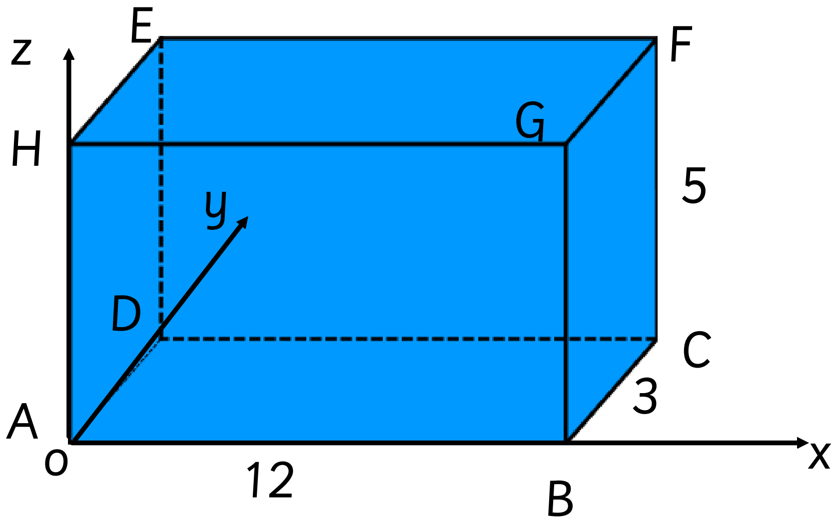
$\underline{a} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ calculate $|\underline{a}|$

Calculate |b - a|

$a = \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix}$ $b = \begin{pmatrix} 7 \\ -2 \\ 4 \end{pmatrix}$

3D Coordinates

State the coordinates of each vertex of this cuboid.



Quadratic Equations

A quadratic equation is an equation of the form

$$ax^2 + bx + c = 0$$

We can solve quadratic equations in three ways:

- algebraically (factorising) when equal to zero
- using the quadratic formula (decimal places and sig figs).
- drawing a diagram.

Factorising to Solve

Examples

Solve the following quadratic equations algebraically:

(a) $x(x - 3) = 0$

(b) $4x^2 - 9 = 0$

(c) $x^2 + 4x + 3 = 0$

(d) $2x^2 + 3x - 5 = 0$

More Quadratic Equations

Quadratic equations can only be solved using factorising when the equation is equal to zero.

Examples

(1) Solve $x^2 = 3x + 10$

(2) Solve for x: $x(x - 4) = 2x - 5$

(3) $(x + 3)^2 = 9$

(4) Find the roots (solutions) of the equation $x + 5 = \frac{14}{x}$

(5) Find the value(s) of x: $x = \frac{15}{x + 2}$

Intersection of a Curve and a Line

Just as we can find where a quadratic (parabola) crosses the x-axis and y-axis, we can also find the co-ordinates where a parabola meets any straight line graph.

Examples

(1) Find the points of intersection of the curve $y = x^2 + 4x - 7$ and the line $y = 5$.

(2) Find the co-ordinates of the points where the curve $y = x^2 - 4x + 5$ and the line $y = 3x - 1$.

(3) Find the points of intersection of the curve $y = 2x^2 - 4x + 5$ and straight line with equation $2x + y = 9$.

The Quadratic Formula

The quadratic formula can be used to solve quadratic equations that cannot be factorised easily.

These questions often read:

"Find the solutions of the equation to 1dp"

"Solve the quadratic equation to 2 significant figures"

"Find the roots of the equation giving your answer to 2 decimal places"

The Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Examples:

(1) Solve the equation $2x^2 + 4x + 1 = 0$ giving your answer to 2 decimal places.

(2) Solve the equation $3x^2 - 2x = 7$ giving your answers to 2 significant figures.

The Discriminant

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The discriminant $b^2 - 4ac$ of a quadratic equation helps us determine information about the roots (solutions) of an equation.

There are three possible outcomes when using the discriminant:

Result	Outcome	What does it mean?
$b^2 - 4ac > 0$	2 real unequal roots	the graph crosses the x-axis in two places
$b^2 - 4ac = 0$	2 repeated roots	the graph turns on the x-axis
$b^2 - 4ac < 0$	no real roots	the graph does not cross the x-axis

Examples: determine the nature of the roots of the following.

(a) $3x^2 + 2x + 1 = 0$

(b) $4x^2 - 12x + 9 = 0$

(c) $x^2 + 7x - 2 = 0$

(d) Show that the equation $5x^2 - 2x + 1 = 0$ has no real roots.

Solving Discriminant Problems

At times, we are given information about the discriminant and are asked to find a range of possible values that satisfy the equation.

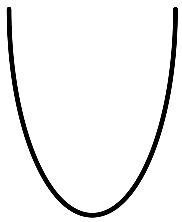
(a) Find the range of values of p such that $2x^2 + 4x + p = 0$ has no real roots.

(b) For what values of q does the equation $x^2 + qx = 3q$ have equal (repeated) roots?

Quadratic Graphs - TP Origin

We can sketch a quadratic graph by evaluating the function for a range of values just as we do for straight line graphs.

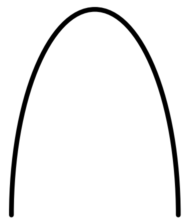
Quadratic graphs are called parabolas and have two general forms:



"U-shaped parabolas"

When x^2 G0

eg $2x^2 + 5x + 3 = 0$



"N-shaped parabolas"

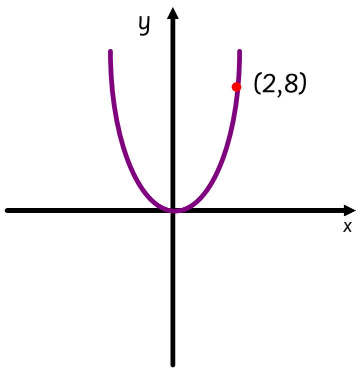
When x^2 q 0

eg $4x - x^2 = 0$

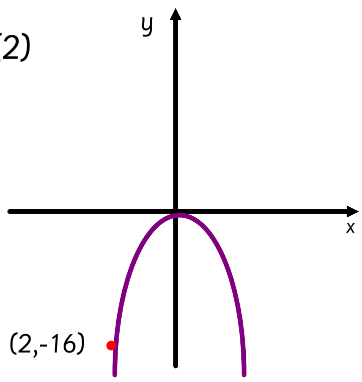
We can determine the equation of a parabola in a range of ways, each of these strategies requires us to pick out key information from a diagram and then substitute in.

(1) The diagram shows the parabola with equation $y = kx^2$.

What is the value of k?



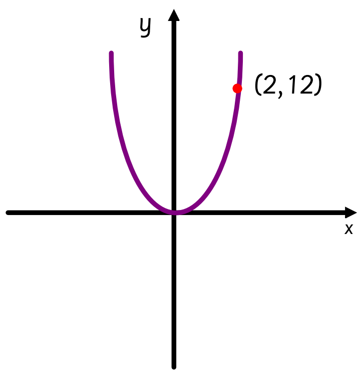
(2)



The diagram shows the parabola with equation $y = kx^2$. What is the value of k?

(3) The diagram shows the parabola with equation $y = kx^2$.

What is the value of k?



Graphs and Turning Points

Reminder:

Earlier in the course, we learned to complete the square for an expression:

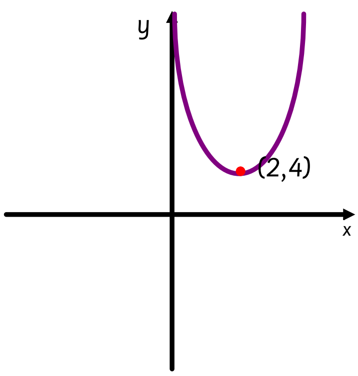
eg. Write $x^2 + 6x + 13$ in the form $(x + p)^2 + q$.

If given a parabola with an equation in completed square form, we can determine the turning point of the graph.

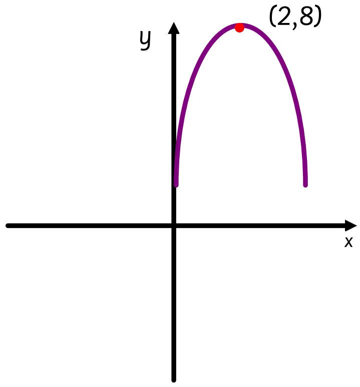
Examples:

Work out the values of p and q for each of the examples below:

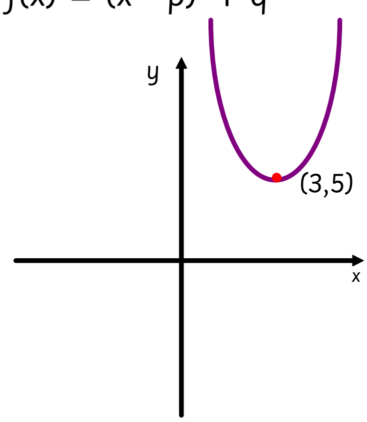
(1) $y = (x - p)^2 + q$



(2) $y = -(x + q)^2 + q$



(3) $f(x) = (x - p)^2 + q$



Sketching Parabolas (1)

(1) Sketch the graph of $f(x) = (x - 1)^2 + 5$

(2) Sketch the graph of $y = (x + 3)^2 - 4$

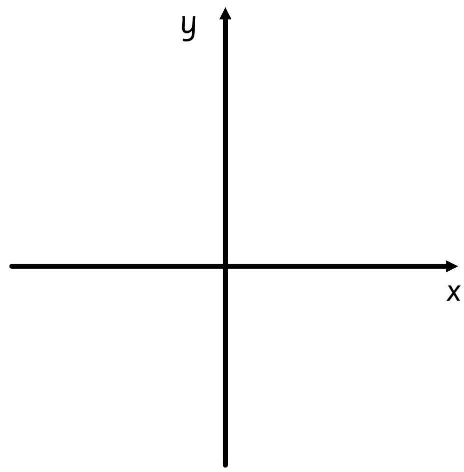
(3) Sketch the graph of $g(x) = -(x - 2)^2 + 6$

Roots of Quadratic Graphs

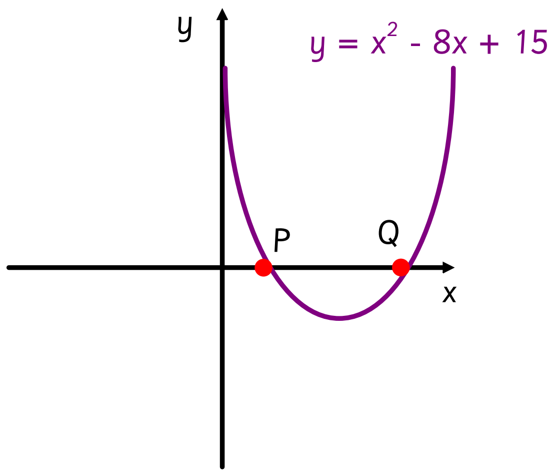
We can find the values at which a graph crosses the x-axis by solving a quadratic equation.

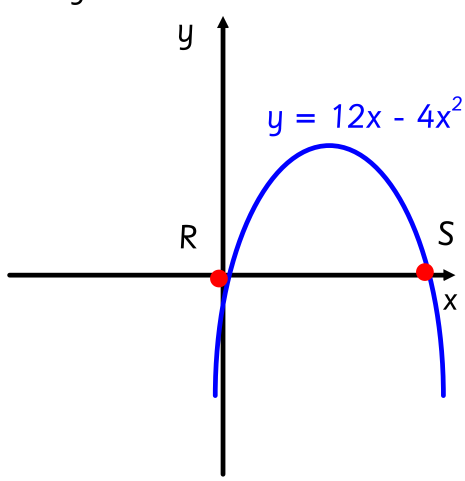
Examples

1. Determine where the graph $y = x^2 - 3x - 10$ crosses the x-axis.



2. Find the co-ordinates P,Q,R and S on the diagrams below:





Sketching Parabolas (1)

When sketching a quadratic we much consider key features:

- on x-axis y=o use this to find the roots
- find the centre or complete the square to find the TP
- on y-axis x=0, use this to find the y-intercept
- sketch and mark the axis of symmetry.

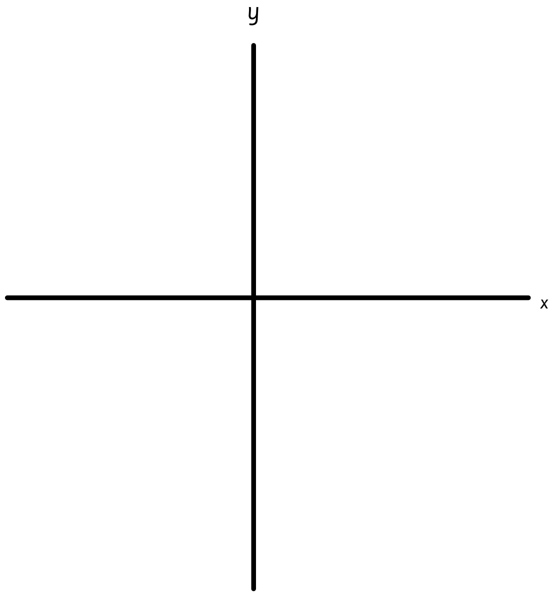
(1) Sketch the graph of $y = (x - 2)(x - 4)$

(2) Sketch the graph of $y = (x + 1)(x - 2)$

Sketching Parabolas (3)

Sketch $y = x^2 + 4x + 5$

roots (x-axis)



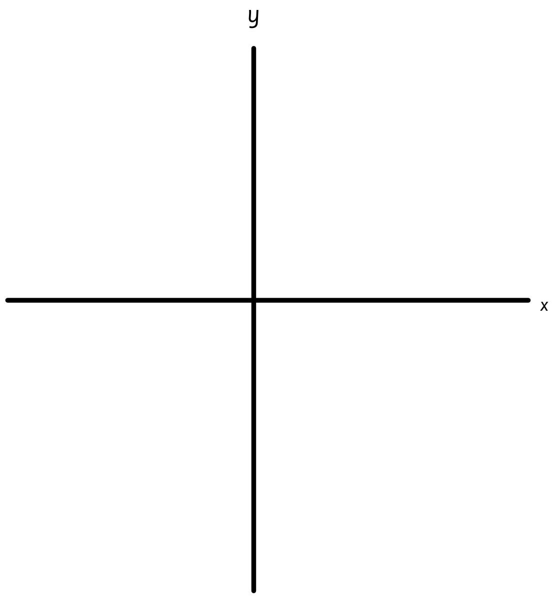
turning point

(y-axis)

axis of symmetry

Sketch $x^2 - 2x - 8$

roots (x-axis)



turning point

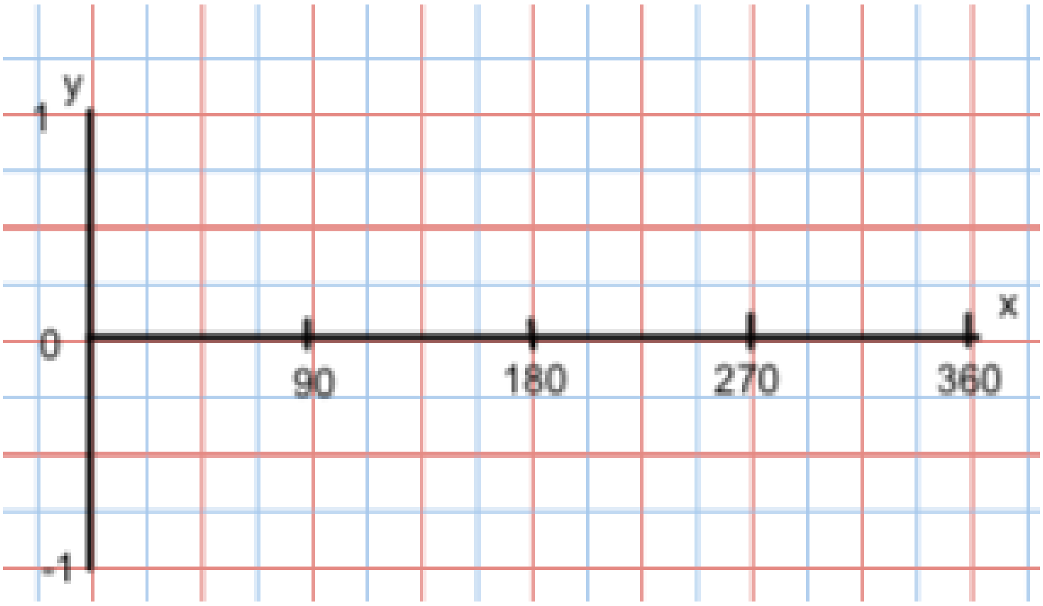
(y-axis)

axis of symmetry

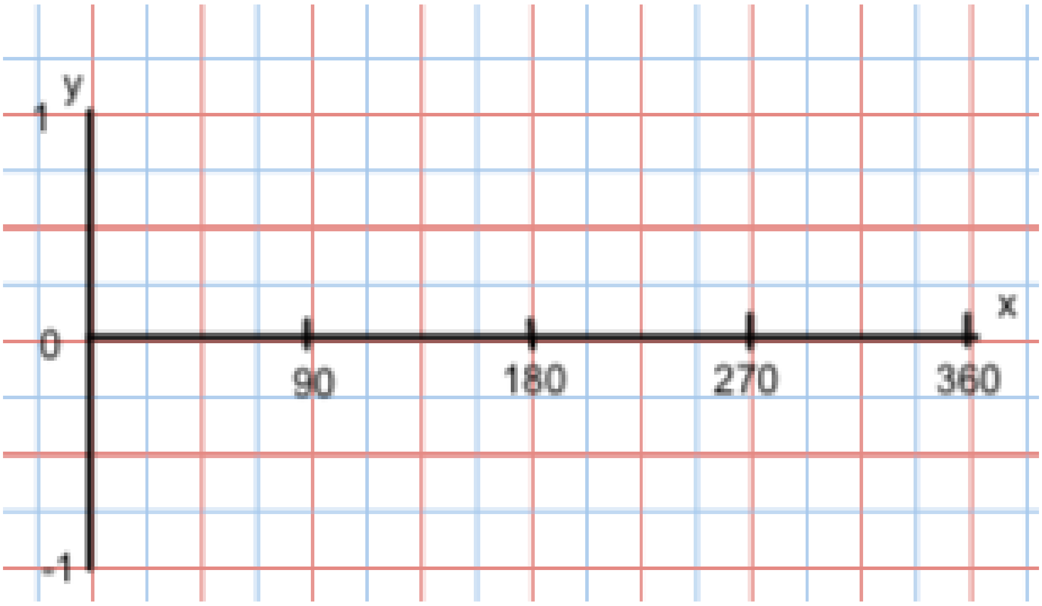
Intro to Trig Graphs

By completing the following tables of values, make neat diagrams of the graphs of y=sinx° and y=cosx°.

x°	0	45	90	135	180	225	270	315	360
y=sinx°									

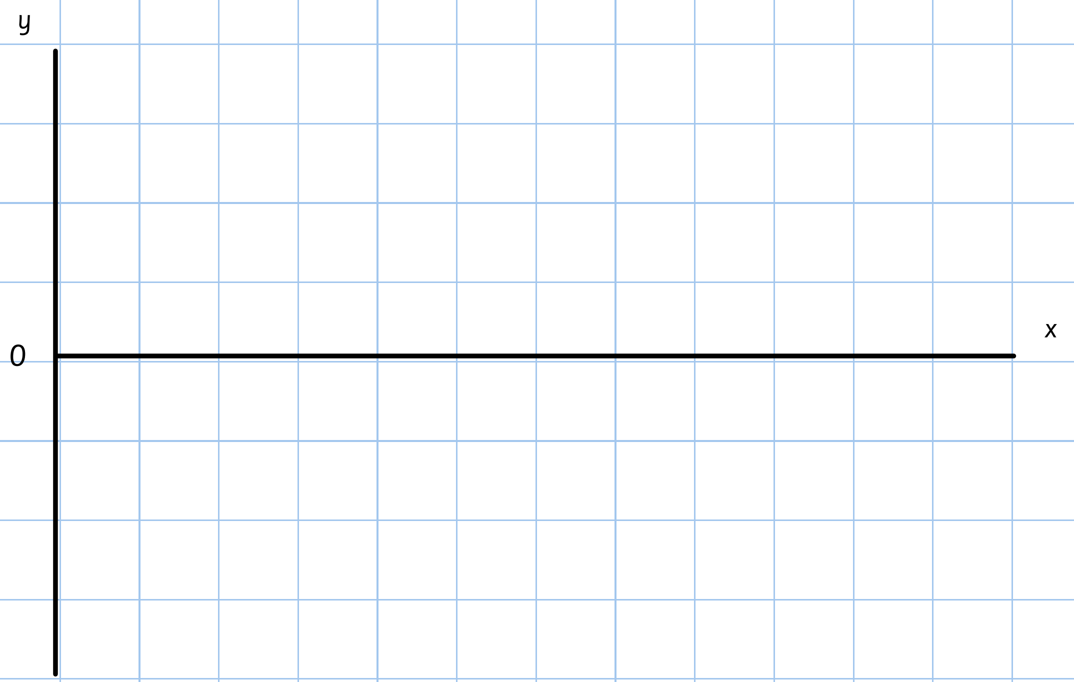


x°	0	45	90	135	180	225	270	315	360
y=cosx°									

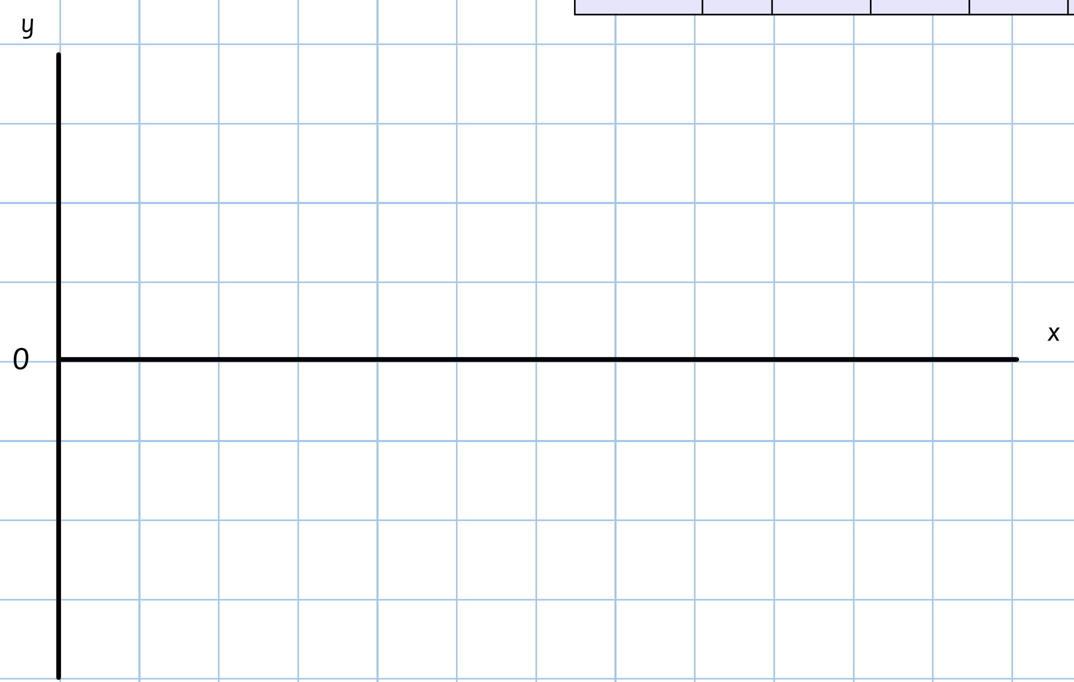


Task 1: By completing the following tables of values, make neat diagrams of the graphs of $y=2\sin x°$ and $y=3\cos x°$.

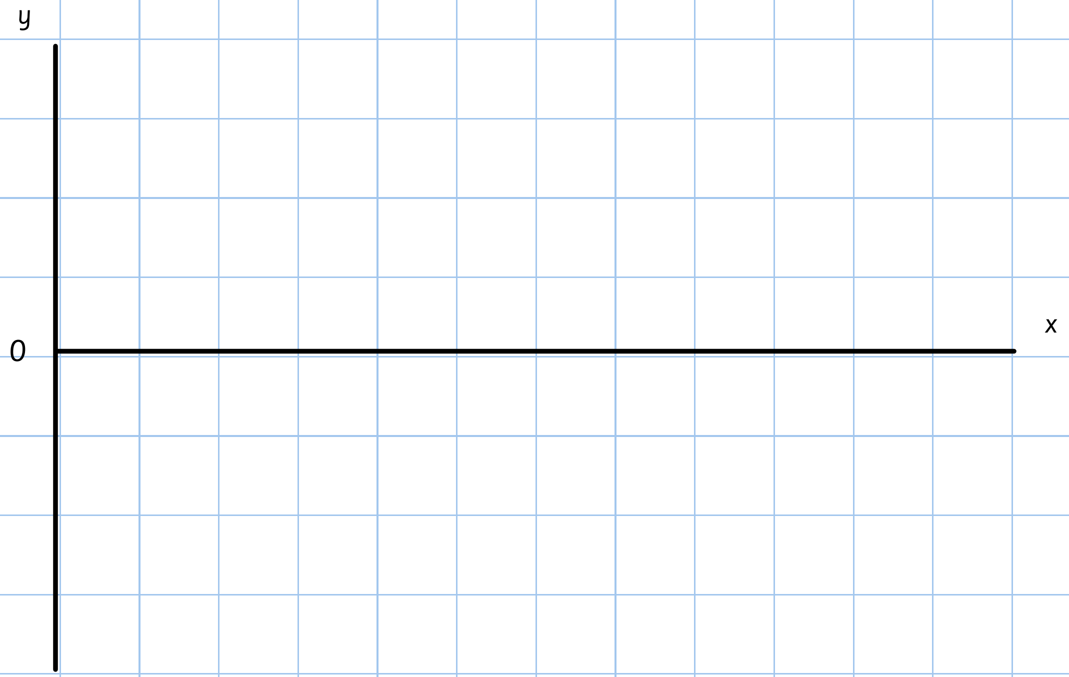
x°					
y=2sinx°					

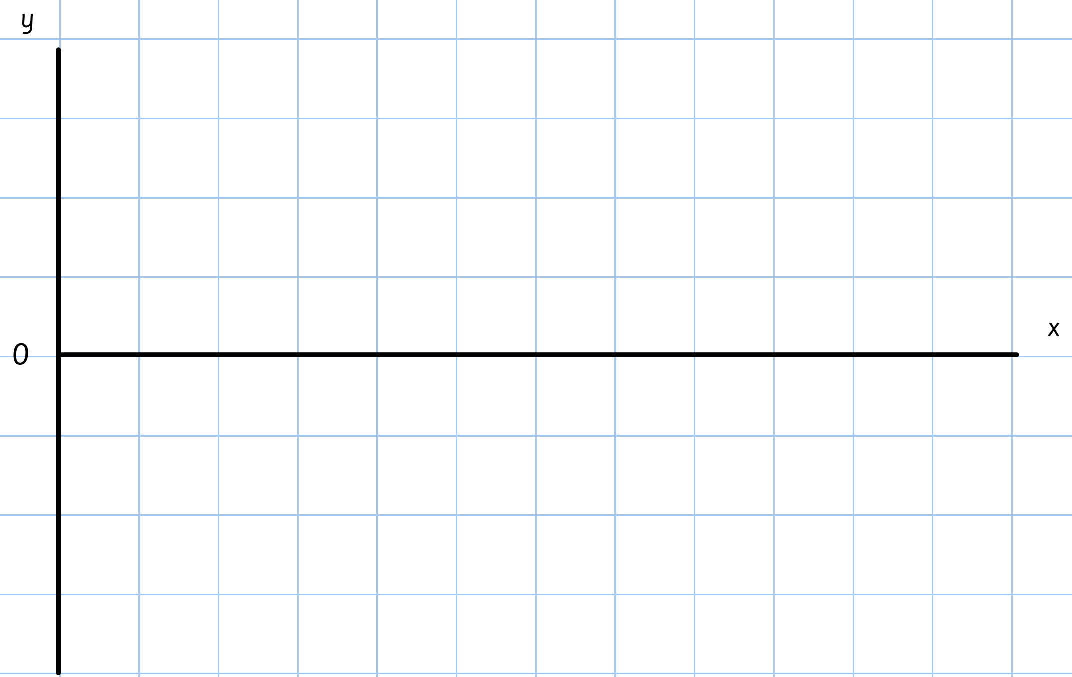


x°	0	90	180	270	360
y=3cosx°					



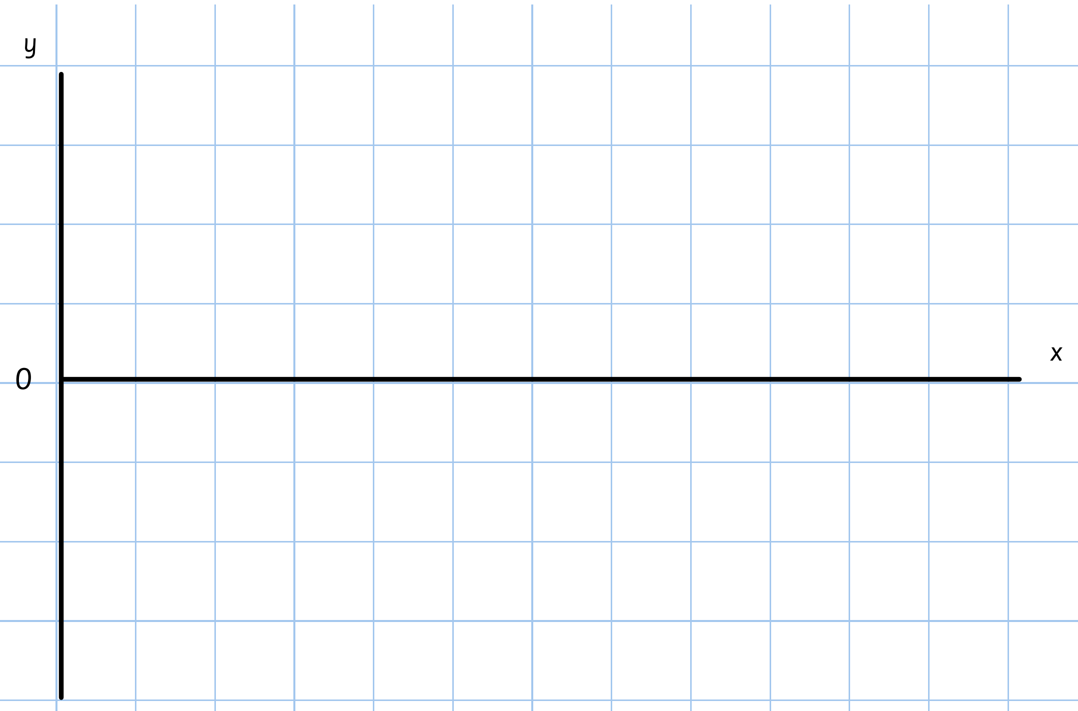
Task 2: By completing the following tables of values, make neat diagrams of the graphs of $y=\sin 4x°$ and $y=\cos 3x°$



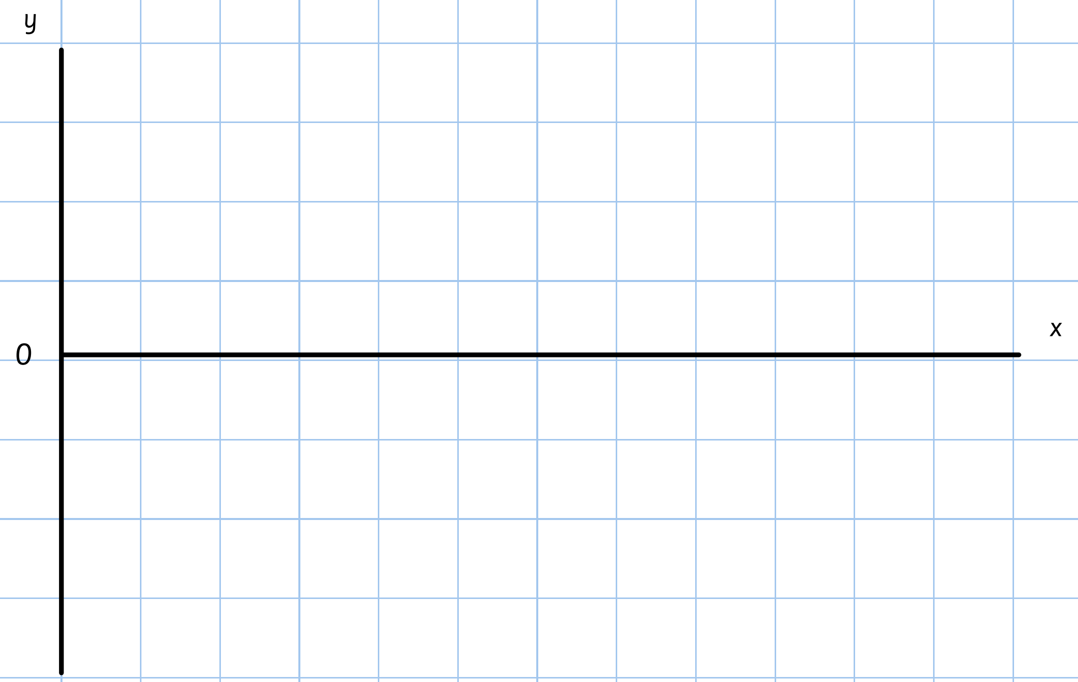


Task 3:

1. Sketch y = sinx
2. now sketch y = sinx + 1



3. Sketch y = 3cosx
4. Now sketch 3cosx - 2



Intro to Trig Equations

1. $\sin x^\circ = -0.342$

2. $\tan x^\circ = 1.732$

3. $\sin x^\circ = 0.259$

Further Equations

Solve the following equations for $0 \leq x \leq 320$ giving your answers to the nearest degree.

(1) $4\sin x - 1 = 0$

(2) $3\cos x + 4 = 2$

(3) $4\cos x + 3 = -1$

(4) $4 + \tan x = 0$

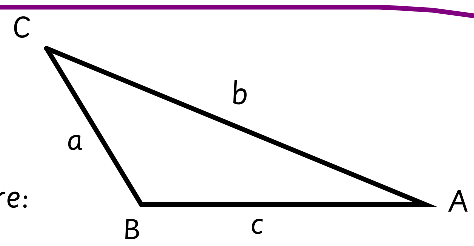
<u>Area of a Triangle</u>

We already know how to find the area of a triangle if we know the base and perpendicular height i.e. base x height ÷ 2.

We now consider a formula for a triangle's area when we do not know the perpendicular height.

$$\text{Area}\Delta = \frac{1}{2}\,ab\sin C$$

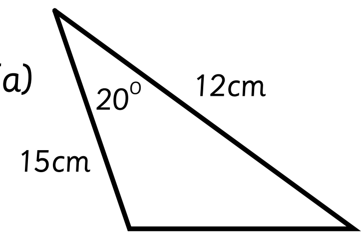
Where:



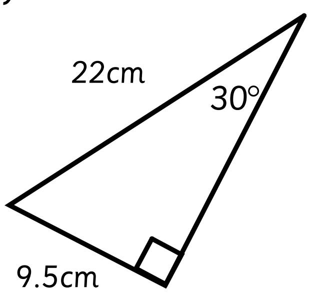
Examples:

Calculate the area of the triangles below:

(a)



(b)

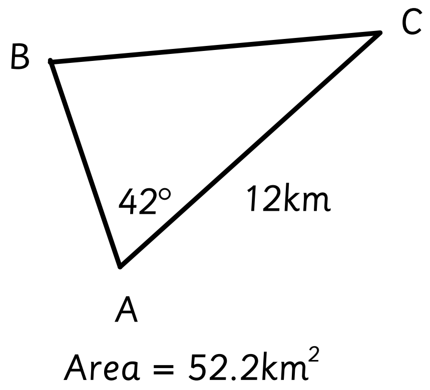


Reverse Area

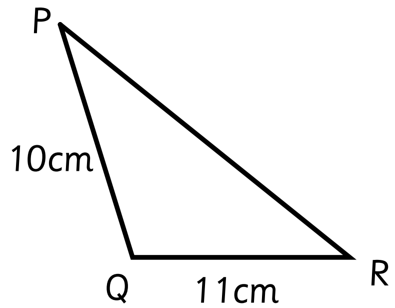
Sometimes, just as with volume or arcs and circles, we are required to work backwards to find a missing value in a triangle.

Examples:

(1) Calculate the length of the missing side BA.



(2) given that the following triangle has an area of $45cm^2$, calculate the size of angle PQR.



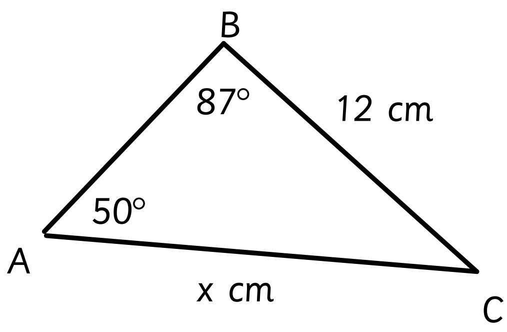
The Sine Rule (Sides)

The sine rule describes the relationship between pairs of sides and angles in triangles.

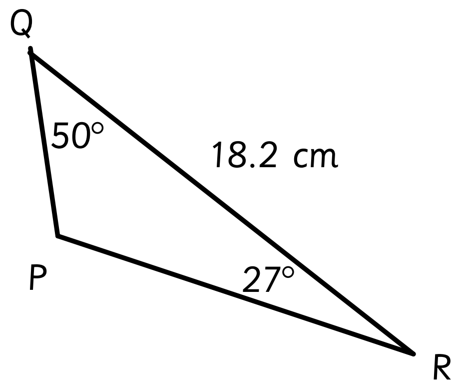
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Examples:

(1) find the value of x:



(2) find the length of side PQ

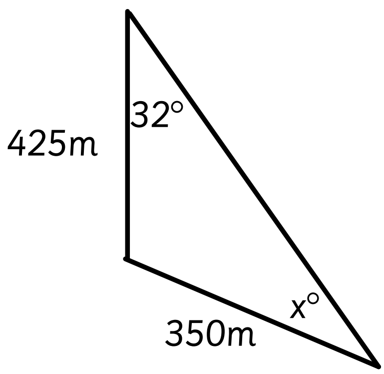


Sine Rule (Angle)

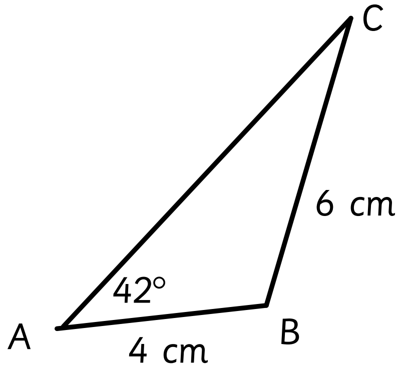
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Examples:

(1) Find the size of angle x° giving your answer to 1dp.



(2) Calculate angle ACB giving your answer correct to 1 decimal place.



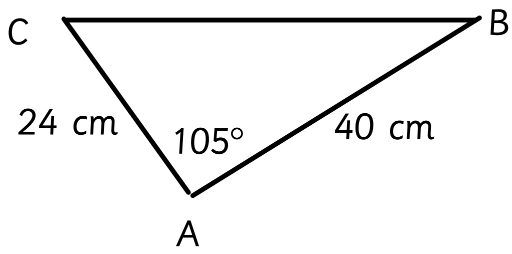
Cosine Rule (Side)

We use the cosine rule to help us find a side when we know two sides and the included angle.

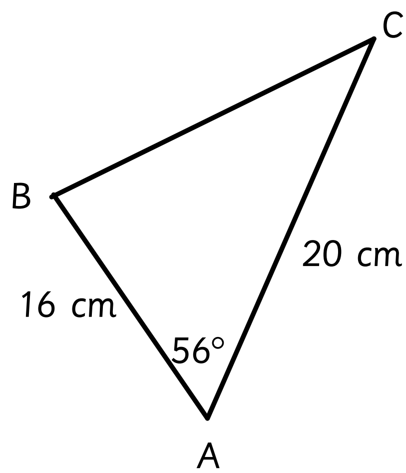
$$a^2 = b^2 + c^2 - 2bc\cos A$$

Examples:

(1) Calculate the size of the missing side in the triangle to 1 decimal place.



(2) find the length of side BC:



Cosine Rule (Angles)

$$a^2 = b^2 + c^2 - 2bc\cos A$$

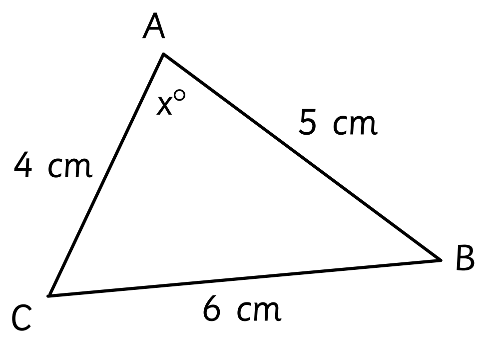
$$2bc\cos A + a^2 = b^2 + c^2$$

$$2bc\cos A = b^2 + c^2 - a^2$$

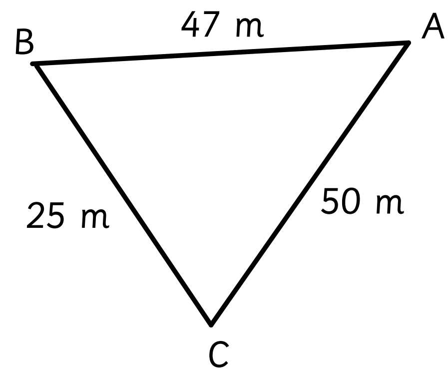
$$\boxed{\cos A = \frac{b^2 + c^2 - a^2}{2bc}}$$

Examples:

(1) Find the value of x°



(2) Find the size of angle BAC



Trig Identities

$$\sin^2 x + \cos^2 x = 1 \qquad \tan x = \frac{\sin x}{\cos x}$$

$$\sin^2 x = 1 - \cos^2 x \qquad \sin x = \tan x \cos x$$

$$\cos^2 x = 1 - \sin^2 x \qquad \cos x = \frac{\sin x}{\tan x}$$

Examples:

(1) Prove that: $2\sin^2 x + 3 = 5 - 2\cos^2 x$

(2) Show that: $\dfrac{1 - \cos^2 x}{\tan^2 x} = \cos^2 x$

$$\sin^2 x + \cos^2 x = 1 \qquad \tan x = \frac{\sin x}{\cos x}$$

(3) Show that: $(\cos x - \sin x)(\cos x + \sin x) = 1 - 2\sin^2 x$

(4) Show clearly that: $(1 - \cos^2 x^\circ) + (1 - \sin^2 x^\circ) = 1$