N5

Specimen Paper F

MATHEMATICS
National Qualifications - National 5
Paper 1 (non-calculator)
Covering Units 1, 2 and 3

Time allowed - 1 hour

Fill in these boxes and read carefully what is printed below

Full name of centre

Town

Forename(s)

Surname

Date of birth
Day   Month   Year

Candidate number

Seat number

Total marks - 40

1.   You may <u>NOT</u> use a calculator.
2.   Use **blue** or **black** ink. Pencil may be used for graphs and diagrams only.
3.   Write your working and answers in the spaces provided. Additional space for answers is provided at the end of the booklet. If you use this space, write clearly the number of the question you are attempting.
4.   Square ruled paper is provided.
5.   Full credit will be given only where the solution contains appropriate working.
6.   State the units for your answer where appropriate.
7.   Before leaving the examination room you must give up this booklet to the invigilator. If you do not, you may lose all the marks for this paper.

FORMULAE LIST

The roots of $ax^2 + bx + c = 0$ are $x = \dfrac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

Sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$ or $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

Area of a triangle: Area $= \frac{1}{2}\, ab \sin C$

Volume of a sphere: Volume $= \dfrac{4}{3} \pi r^3$

Volume of a cone: Volume $= \dfrac{1}{3} \pi r^2 h$

Volume of a Pyramid: Volume $= \dfrac{1}{3} Ah$

Standard deviation: $s = \sqrt{\dfrac{\sum (x - \bar{x})^2}{n - 1}} = \sqrt{\dfrac{\sum x^2 - (\sum x)^2 / n}{n - 1}}$, where n is the sample size.

All questions should be attempted

1. Solve algebraically the system of equations

$$y = 3x + 2$$
$$2x + 3y = 50$$

3

2. Simplify $\dfrac{\sqrt{72} - \sqrt{8}}{16}$ expressing your answer as a surd in its simplest form.

2

3. The function $f(x)$ is given by the formula $f(x) = 2x^2 - 5$, where x is a real number.
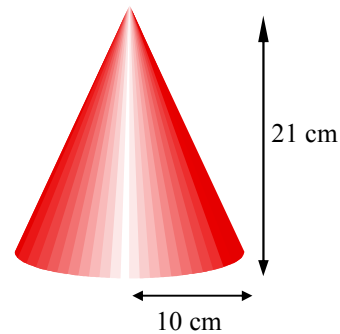
   (a) Find the value of $f(-3)$.

   2

   (b) Find the **values** of a for which $f(a) = 45$.

   3

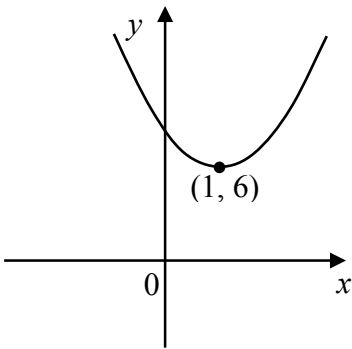4. The diagram shows a cone with radius 10 centimetres and height 21 centimetres.

   Taking $\pi = 3\cdot14$, calculate the volume of the cone.

   21 cm

   10 cm

   3

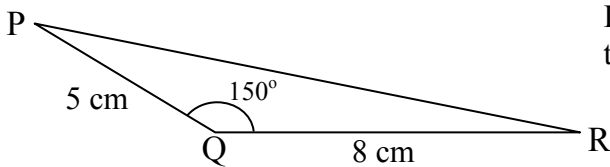5. Simplify $\dfrac{(x+2)^2}{x^2 - 2x - 8}$

   2

6.

(1, 6)

The equation of the parabola is of the form

$$y = (x + p)^2 + q.$$

Write down the equation of the parabola and state the equation of the axis of symmetry

2

7.

5 cm

150°

8 cm

If $\sin 150^\circ = \frac{1}{2}$, calculate the area of triangle PQR.

2

8. **(a)** Simplify $\dfrac{6x^3 y^{-\frac{2}{3}}}{3xy^{-\frac{1}{3}}}$

2

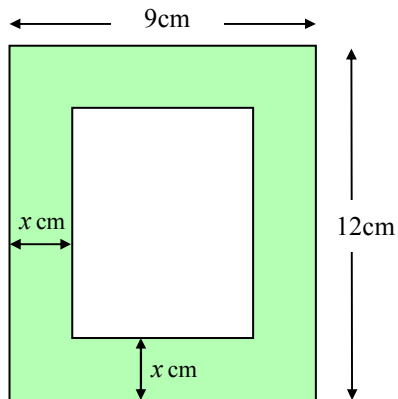**(b)** Evaluate the expression if $x = -2$ and $y = 27$

3

9. Given that $P = \dfrac{3b - c}{b}$, express b in terms of A and c.

3

10. Sketch the graph of $f(x) = \sin(x - 60)^\circ$, $0 \le x \le 360$

3

11. Sandy found a small photo-frame and decided to put one of her favourite photographs in it. The diagram below shows the dimensions of the frame.

9cm

x cm

12cm

x cm

The width of the wooden surround is x cm.

Unfortunately the glass in the centre of the frame was cracked and had to be replaced.

(a) Show that the area of glass needed for the centre of the frame can be given by the formula

$$A = \left(4x^2 - 42x + 108\right) \text{ cm}^2$$

3

(b) If the area of glass needed was 54cm², find a possible value for x.

4

12. Simplify $\dfrac{6 - 6\sin^2 x}{3\cos x}$

3

End of question paper

National 5 *Practice Paper F* Paper 1 Marking Scheme

Qu	Give one mark for each ●	Illustrations for awarding mark
1	**ans :** $x = 4, y = 14$ **3 marks** ●¹ substitutes for y in second equation ●² solves for x ●³ solves for y	 ●¹ $2x + 3(3x + 2) = 50$ ●² $11x = 44 \Rightarrow x = 4$ ●³ $y = (3 \times 4) + 2 = 14$
2	**ans :** $\sqrt{2}/4$ **2 marks** ●¹ simplifies surds ●² simplifies fraction	 ●¹ $6\sqrt{2} - 2\sqrt{2} = 4\sqrt{2}$ ●² $\sqrt{2}/4$
3a	**ans:** 13 **2 marks** ●¹ interpret function notation ●² evaluate function	 ●¹ $2(-3)^2 - 5$ ●² 13
b	**ans:** $-5, 5$ **3 marks** ●¹ substitute correctly ●² attempts to solve equation ●³ correctly solves equation	 ●¹ $2a^2 - 5 = 45$ ●² $a = \sqrt{25}$ ●³ $a = \pm 5$
4	**ans :** 2198cm³ **3 marks** ●¹ knows how to calculate volume ●² starts to evaluate ●³ calculates volume	 ●¹ $V = \frac{1}{3} \times \pi \times 10^2 \times 21$ ●² $V = 314 \times 7$ ●³ 2198cm³
5	**ans :** $(x+2)/(x-4)$ **2 marks** ●¹ factorises denominator ●² simplifies fraction	 ●¹ $(x-4)(x+2)$ ●² $(x+2)/(x-4)$
6	**ans :** $y = (x-1)^2 + 6$; $x = 1$ **2 marks** ●¹ states equation ●² states equation of axis of symmetry	 ●¹ $y = (x-1)^2 + 6$ ●² $x = 1$
7	**ans :** 10 cm² **2 marks** ●¹ knows to use area formula ●² calculates area	 ●¹ A = ½ ab sin C = ½ $\times 5 \times 8 \times$ ½ ●² 10
8(a)	**ans :** $2x^2 y^{-\frac{1}{3}}$ **2 marks** ●¹ simplifies numbers and terms in x ●² simplifies terms in y	 ●¹ $6x^3/3x = 2x^2$ ●² $y^{-\frac{2}{3}}/y^{-\frac{1}{3}} = y^{-\frac{1}{3}}$
(b)	**ans :** $8/3$ **3 marks** ●¹ substitutes values ●² evaluates numerator ●³ evaluates numerator	 ●¹ $2 \times (-2)^2 \times 27^{-\frac{1}{3}}$ ●² $8 \times \ldots$ ●³ $\ldots \times 1/3$

Qu	Give one mark for each ●	Illustrations for awarding mark
9	**ans:** $b = \dfrac{-c}{P-3}$ or $b = \dfrac{c}{3+P}$ **3 marks** ●[1] eliminates the fractions ●[2] collects like terms and takes c.f. ●[3] divides to state answer	●[1] $Pb = 3b - c$, ●[2] $Pb - 3b = -c$; $b(P-3) = -c$, ●[3] $b = \dfrac{-c}{P-3}$ or $b = \dfrac{c}{3+P}$
10	**ans :** **graph** **3 marks** ●[1] graph has sine shape ●[2] graph shifted 60° right ●[3] graph drawn within correct limits	
11 (a)	**ans:** **proof** **3 marks** ●[1] finding an expression for length ●[2] finding an expression for breadth ●[3] calculating area and simplifies to answer	●[1] $12 - 2x$ ●[2] $9 - 2x$ ●[3] $A = (12 - 2x)(9 - 2x)$
(b)	**ans:** **1·5 cm** **4 marks** ●[1] equating expression to 54 ●[2] attempting to solve the quadratic equation ●[3] correctly solving equation ●[4] selects appropriate solution	●[1] $4x^2 - 42x + 54 = 0$ ●[2] $2(x-9)(2x-3) = 0$ ●[3] $x = 1·5$ or 9 ●[4] $1·5$cm
12	**ans :** $2 \cos x^o$ **3 marks** ●[1] factorises numerator ●[2] substitutes ●[3] simplifies	●[1] $6(1 - \sin^2 x^o)$ ●[2] $6 \cos^2 x^o$ ●[3] $2 \cos x^o$
		Total: **40 marks**

MATHEMATICS
National Qualifications   -   National 5
Paper 2 (Calculator)
Covering Units 1, 2 and 3

Time allowed - 1 hour and 30 minutes

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Full name of centre

Town

Forename(s)

Surname

Date of birth
Day   Month   Year

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Seat number

Total marks - 50

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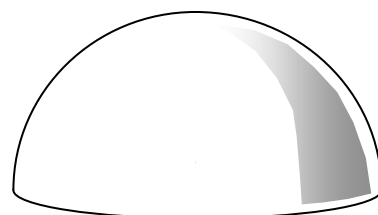**1.**   Multiply out the brackets and simplify the expression

$$3x^3 - 2x(x^2 - 5x + 4)$$ **3**

2.   A lime-flavoured jelly is in the shape of a hemisphere.
The diameter is 18 centimetres.

As the jelly sits in a warm room it begins to melt
and loses 5% of its solid volume every hour.

What would be the solid volume of the jelly left
after 3 hours?   **6**

← 18 cm →

3.   **Thermogreen Ltd** are developing a new greenhouse which claims to maintain an
average interior temperature of 18^0 C.

Over a period of time the following temperatures (in degrees
Celsius) were recorded:

16·8   20·3   17·4   18·2   19·5   17·6   19·1   17·8   16·5   18·8

(a)   Calculate the mean temperature   **1**

(b)   Calculate the standard deviation correct to 1 decimal place.   **3**

(c)   Trading Standards expect the mean temperature to be within $0 \cdot 4^0$ C of the
stated temperature and the standard deviation to be less than $1 \cdot 5^0$ C.
Will **Thermogreen** be able to market their new greenhouse?   **1**

4. Brendan's golf fees have increased by 14% from last year and he now has to pay £741.

How much did he have to pay last year? **2**

5. Solve the equation
$$3x^2 + 4x - 3 = 0.$$

Give your answers **correct to 1 decimal place** **4**

6. Two vectors are defined as $V_1 = 4\boldsymbol{i} + \boldsymbol{j} + \sqrt{8}\,\boldsymbol{k}$ and $V_2 = 8a\,\boldsymbol{i} + 6a\,\boldsymbol{k}$,

where a is a constant and <u>all</u> coefficients of $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$ are greater than zero..

Given that $\left| V_2 \right| = 2\left| V_1 \right|$, calculate the value of a. **3**

7. Solve, **algebraically**, the equation

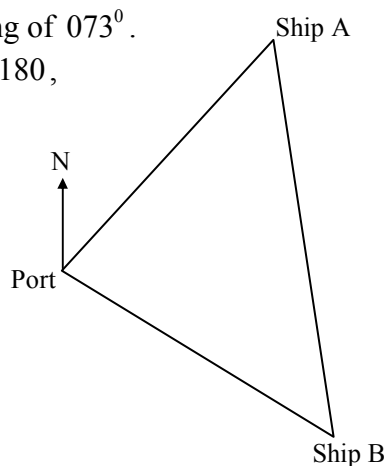$$3\tan x^0 + 4 = 3, \text{ for } 0 \le x < 360.$$ **3**

8. Two ships leave port at exactly the same time.
Ship A travels at 12 kilometres per hour on a bearing of 073^0.
Ship B sets out on a bearing of x^0, where $90 \le x \le 180$,
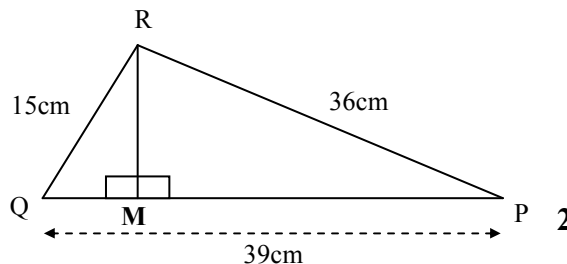at 15 kilometres per hour.

After 3 hours the two ships are 30km apart.

Calculate, correct to three significant figures,
the bearing Ship B must have followed
when leaving port. **5**

9. Triangle PQR has sides as shown.
PQ = 39cm, PR = 15cm, RQ = 36cm.

(a) Prove that angle PRQ is a right angle. **2**

(b) If the area of triangle PQR is 270cm², calculate the length of altitude RM, correct to
1 decimal place. **3**

10. The petrol tank in a car is cylindrical in shape as shown in diagram 1 below.
The tank is 85cm long and has a radius of 18cm.
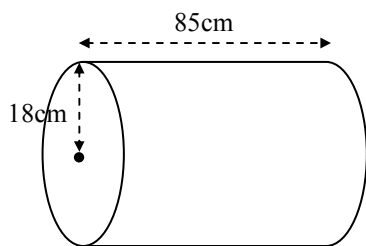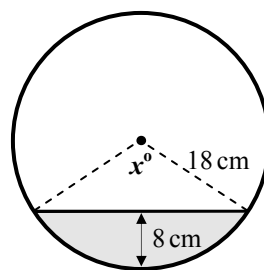

diagram 1   diagram 2

Diagram 2 shows the cross-sectional view of the tank. There is petrol in the tank
to a maximum depth of 8cm as shown.

(a) Find the size of the angle marked, x^o, in diagram 2. **2**

(b) Calculate the volume of petrol in the tank. **4**

11. Here are words which are used to describe the roots of a quadratic equation.

REAL   NOT REAL   EQUAL   UNEQUAL   RATIONAL   IRRATIONAL

Which of them describe(s) the roots of the quadratic equation $3x^2 - 4x - 5 = 0$? **4**

12. A triangular shaped poster is split into a black section and a white section as shown in the diagram. BE is parallel to CD. BE = 7cm and CD = 20cm.
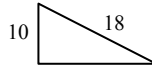
A

B 7m E

C 20m D
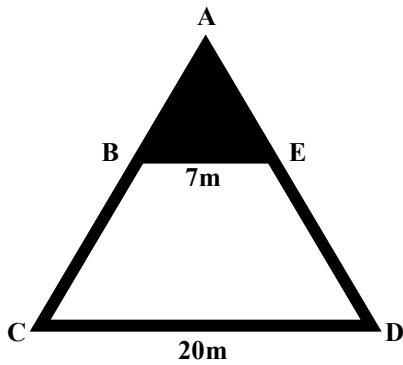
Given that the area of the black section is $147cm^2$, calculate the area of the white section.

4

End of Question Paper

Qu	Give one mark for each ●	Illustrations for awarding mark
1	**ans : $x^3 + 10x^2 - 8x$** **3 marks** ●¹ terms correctly evaluated ●² applies negative sign ●³ simplifies	●¹ $2x^3 - 10x^2 + 8x$ ●² $-2x^3 + 10x^2 - 8x$ ●³ $x^3 + 10x^2 - 8x$
2	**ans : 1309·1 cm³** **6 marks** ●¹ knows to calculate volume of hemisphere ●² substitutes radius correctly ●³ evaluates volume of hemisphere ●⁴ calculates percentages ●⁵ knows to subtract ●⁶ evaluates remaining volume	●¹ $V_{hemisphere} = \frac{1}{2} \times \frac{4}{3}\pi \times r^3$ ●² $V_{hemisphere} = \frac{1}{2} \times \frac{4}{3}\pi \times 9^3$ ●³ 1526·8 cm³ ●⁴ 76·5, 72·5, 68·9 ●⁵ 1451·5, 1378 ●⁶ 1309·1
3(a)	**ans: 18·2** **1 mark** ●¹ calculating mean	●¹ $\dfrac{182}{10} = 18 \cdot 2$
(b)	**ans : 1·2** **3 marks** ●¹ knowing how to calculate sd ●² correctly calculating sd ●³ rounding	●¹ $\sum x = 182 \quad \sum x^2 = 3325 \cdot 68$ ●² $sd = \sqrt{\dfrac{3325 \cdot 68 - \dfrac{182^2}{10}}{9}}$ ●³ answer
(c)	**ans : Yes as mean and sd < standard set** **1 mark** ●¹ makes comparison	●¹ suitable answer
4	**ans: £650** **2 marks** ●¹ suitable strategy ●² answer	●¹ £741 ÷ 1·14 ●² £650
5	**ans: 0·5 and −1·9** **4 marks** ●¹ knows to use quadratic formula ●² calculates discriminant ●³ finds first solution ●⁴ finds second solution	●¹ evidence ●² $b^2 - 4ac = 52$ ●³ $x = 0 \cdot 5$ ●⁴ $x = -1 \cdot 9$

Qu	Give one mark for each ●	Illustrations for awarding mark
6	**ans:** $a = 1$ **3 marks** ●^{1} finds magnitude of V_1 ●^{2} finds expression for magnitude of V_2 ●^{3} equates $2V_1 = V_2$ and solves for a	 ●^{1} $\|V_1\| = 5$ ●^{2} $\|V_2\| = 10a$ ●^{3} $2 \times 5 = 10 \times a$; $a = 1$
7	**ans:** $\mathbf{161 \cdot 6^0, \ 341 \cdot 6^o}$ **3 marks** ●^{1} rearranging to find $\tan x =$ ●^{2} finds first solution ●^{3} finds second solution	 ●^{1} $\tan x = -\dfrac{1}{3}$ ●^{2} $161 \cdot 6^o$ ●^{3} $341 \cdot 6^o$
8	**ans :** $\mathbf{115^o}$ **5 marks** ●^{1} interpreting information ●^{2} using suitable formula ●^{3} substituting correctly ●^{4} calculating interior angle ●^{5} stating bearing	 ●^{1} triangle with sides 36, 45 &30 ●^{2} cosine rule $\cos a = \dfrac{b^2 + c^2 - a^2}{2bc}$ ●^{3} $\cos a = \dfrac{36^2 + 45^2 - 30^2}{2 \times 36 \times 45}$ ●^{4} angle $= 42^0$ ●^{5} answer
9(a)	**ans:** **proof** **2 marks** ●^{1} knowing to use Converse of Pythagoras ●^{2} completing proof	 ●^{1} If Δ is R.A. then $a^2 = b^2 + c^2$ ●^{2} LHS = RHS = 1521
(b)	**ans :** **13·8cm** **3 marks** ●^{1} knowing to use area of Δ ●^{2} knowing QP(base) and RM(height) ●^{3} calculations	 ●^{1} $\dfrac{1}{2} b \times h = 270$ ●^{2} $\dfrac{1}{2} \times 39 \times \text{RM} = 270$ ●^{3} answer
10 (a)	**ans:** $\mathbf{112^o}$ **2 marks** ●^{1} identifying R.A. triangle ●^{2} finding angle at apex	 ●^{1} ●^{2} $\cos x° = \dfrac{10}{18} \Rightarrow x = 56°, \ 2x = 112°$
(b)	**ans:** **14195 cm^3** **4 marks** ●^{1} calculating area of sector ●^{2} calculating area of triangle ●^{3} calculating cross sectional area ●^{4} calculating volume of fuel	 ●^{1} sector $= \dfrac{112}{360} \times \pi \times 18^2 = 317 \, \text{cm}^2$ ●^{2} $\dfrac{1}{2} \times 18 \times 18 \times \sin 112 = 150 \ \text{cm}^2$ ●^{3} CSA $= 317 - 150 = 167 \, \text{cm}^2$ ●^{4} $14195 \, \text{cm}^3$

Qu	Give one mark for each ●		Illustrations for awarding mark
11	**ans: real, unequal, irrational**	**4 marks**	
	●[1] knows to calculate discriminant		●[1] evidence
	●[2] calculates discriminant		●[2] $b^2 - 4ac = 76$
	●[3] chooses any two suitable words		●[3] real, unequal
	●[4] chooses a third suitable word		●[4] irrational
12	**ans: 1053 cm^2**	**4 marks**	
	●[1] finds linear scale factor		●[1] linear scale factor = $\dfrac{20}{7}$
	●[2] calculating area scale factor		●[2] area scale factor = $\left(\dfrac{20}{7}\right)^2$
	●[3] calculating area of whole poster		●[3] area(poster) = $\left(\dfrac{20}{7}\right)^2 \times 147 = 1200$
	●[4] calculating area of larger section		●[4] area(section) = answer
			Total 50 marks