## Specimen Paper E

## MATHEMATICS National Qualifications - National 5 <br> Paper 1 (non-calculator)

Covering Units 1, 2 and 3

Time allowed - 1 hour

Fill in these boxes and read carefully what is printed below


1. You may NOT use a calculator.
2. Use blue or black ink. Pencil may be used for graphs and diagrams only.
3. Write your working and answers in the spaces provided. Additional space for answers is provided at the end of the booklet. If you use this space, write clearly the number of the question you are attempting.
4. Square ruled paper is provided.
5. Full credit will be given only where the solution contains appropriate working.
6. State the units for your answer where appropriate.
7. Before leaving the examination room you must give up this booklet to the invigilator. If you do not, you may lose all the marks for this paper.

## FORMULAE LIST

The roots of $a x^{2}+b x+c=0$ are $x=\frac{-b \pm \sqrt{\left(b^{2}-4 a c\right)}}{2 a}$

Sine rule:

$$
\frac{a}{\sin \mathrm{~A}}=\frac{b}{\sin \mathrm{~B}}=\frac{c}{\sin \mathrm{C}}
$$

Cosine rule:
$a^{2}=b^{2}+c^{2}-2 b c \cos \mathrm{~A}$ or $\cos \mathrm{A}=\frac{b^{2}+c^{2}-a^{2}}{2 b c}$

Area of a triangle: $\quad$ Area $=1 / 2 a b \sin C$

Volume of a sphere: $\quad$ Volume $=\frac{4}{3} \pi r^{3}$

Volume of a cone: $\quad$ Volume $=\frac{1}{3} \pi r^{2} h$

Volume of a Pyramid: $\quad$ Volume $=\frac{1}{3} \mathrm{Ah}$

Standard deviation: $\quad s=\sqrt{\frac{\sum(x-\bar{x})^{2}}{n-1}}=\sqrt{\frac{\sum x^{2}-\left(\sum x\right)^{2} / n}{n-1}}$, where n is the sample size.

## All questions should be attempted

1. Multiply and simplify

$$
(3 x-4)(2 x-7)
$$

2. (a) The line $A B$ passes through the points $(0,8)$ and $(12,0)$. On a coordinate diagram, plot $A$ and $B$ and find the equation of the line AB .
(b) The equation of the line PQ is $2 x+y=12$.

Draw this line onto the same diagram as the line AB .
(c) Write down the coordinates of T, the point of intersection of these two lines.
3. Ricky buys a pre-owned car for $£ 4000$. The value of the car depreciates at the rate of $10 \%$ per annum.

How much will the car be worth at the end of 3 years?
4. The data shows the length of films on TV during one weekend in July.

| 145 | 120 | 110 | 105 | 130 |
| :--- | :--- | :--- | :--- | :--- |
| 105 | 100 | 95 | 100 | 105 |
| 100 | 115 | 90 | 115 | 100 |

(a) Calculate the interquartile range for this data set.
(b) The length of films during one weekend in December gave a interquartile range of 17 .

Make a comment about the length of films in December compared to July.
5.


The diagram shows the graph of $y=a \cos b x^{0}, 0 \leq x \leq 360$.
Find the values of $a$ and $b$.
6. Triangles PQR and RST, with some of their measurements, are shown in the diagram opposite.

PQ is parallel to TS .
Calculate the length of TQ.

7. (a) Factorise $2 x^{2}+5 x-12$.
(b) Hence, simplify the fraction $\frac{x^{2}-16}{2 x^{2}+5 x-12}$.
8. Gareth wants to know the volume of the roof space in his house which is in the shape of a triangular prism.

He makes a sketch and takes these measurements.

Calculate the volume.

9. Two adults and three children pay $£ 17.40$ for admission to their local school concert.

One adult and two children pay $£ 10.20$ for admission to the same concert.
How much would 3 adults and 1 child have to pay to be admitted to the concert?
10. Two functions are defined as follows :

$$
\begin{aligned}
& f(x)=x^{2}+2 x-6 \\
& g(x)=7 x+8
\end{aligned}
$$

Find the value(s) of $x$ for which $3(f(x))+g(x)=0$.
11. Simplify $\cos x^{\circ} \tan x^{\circ}$ 2

| Qu | Give one mark for each • | Illustrations for awarding mark |
| :---: | :---: | :---: |
| 1. | ans : $6 x^{2}-29 x+28$ <br> 2 marks <br> - multiplies brackets <br> - ${ }^{2}$ simplifies | - ${ }^{1} 6 x^{2}-21 x-8 x+28$ <br> - $26 x^{2}-29 x+28$ |
| 2(a) <br> (b) <br> (c) | ans: $y=-2 / 3 x+8$ <br> 3 marks <br> - ${ }^{1}$ points plotted <br> - ${ }^{2}$ calculates gradient <br> - ${ }^{3}$ writes equation <br> ans: line drawn $y=-2 x+12 \quad 2$ marks <br> - ${ }^{1}$ line passes thro' $(0,12)$ <br> $\bullet^{2}$ line passes thro' $(6,0)$ <br> ans: $T(3,6)$ <br> 1 mark <br> - ${ }^{1}$ point stated | - ${ }^{1}$ diagram showing $(0,8)$ and (12.0) <br> - ${ }^{2} \mathrm{~m}=-8 / 12=-2 / 3$ <br> - ${ }^{3} y=-2 / 3 x+8$ <br> - ${ }^{1}(0,12)$ plotted or suitable alternative <br> - $2(6,0)$ plotted or suitable alternative <br> - ${ }^{1} \mathrm{~T}(3,6)$ |
| 3. | ans: £2916 <br> - ${ }^{1}$ knows how to calculate a percentage <br> $\bullet{ }^{2}$ calculates further percentages <br> - ${ }^{3}$ knows to subtract for depreciation <br> - ${ }^{4}$ calculates end of year values | - ${ }^{1} 10 \%$ of $£ 4000=£ 400$ <br> - ${ }^{2}$ 360, 324 <br> - ${ }^{3} 4000-400,3600-360,3240-324$ <br> ${ }^{4}$ 3600, 3240, 2916 |
| 4(a) <br> (b) | ans: $7 \cdot 5$ <br> 4 marks <br> - ${ }^{1}$ orders data <br> ${ }^{-2}$ find $\mathrm{Q}_{2}$ <br> ${ }^{-3}$ find $\mathrm{Q}_{1}$ and $\mathrm{Q}_{3}$ <br> - ${ }^{4}$ finds IQR <br> ans: less consistent <br> 1 mark <br> - ${ }^{1}$ suitable comment | - ${ }^{1} 90,95,100,100, \ldots \ldots .130,145$ <br> - ${ }^{2} 105$ <br> -3 100; 115 <br> -4 $(115-100)=15$ <br> - ${ }^{1}$ December films times less consistent |
| 5. | ans: $a=0 \cdot 5, b=2$ <br> 2 marks <br> - ${ }^{1}$ recognizing max $/ \min$ <br> - ${ }^{2}$ recognizing period | $\begin{aligned} & \bullet \quad a=0 \cdot 5 \\ & \bullet^{2} \quad b=2 \end{aligned}$ |

\begin{tabular}{|c|c|c|}
\hline Qu \& Give one mark for each - \& Illustrations for awarding mark \\
\hline 6. \& \begin{tabular}{l}
ans: 24 cm \\
3 marks \\
- \({ }^{1}\) recognising similar triangles \\
- \({ }^{2}\) calculating scale factor \\
-3 calculating RQ then TQ
\end{tabular} \& \begin{tabular}{l}
- \(\frac{P Q}{T S}=\frac{P R}{R S}=\frac{Q R}{R T}\) \\
\(\bullet^{2}\) S.F. \(=\frac{3}{5}\) \\
- \({ }^{3} \mathrm{RQ}=\frac{3}{5} \times 15=9 ; \mathrm{TQ}=24 \mathrm{~cm}\)
\end{tabular} \\
\hline 7(a)

(b) \& \begin{tabular}{l}
ans: $(2 x-3)(x+4)$ <br>
- ${ }^{1}$ first factor correct <br>
- ${ }^{2}$ second factor correct <br>
ans: $\frac{x-4}{2 x-3}$ <br>
2 marks <br>
- ${ }^{1}$ factorising numerator <br>
$\bullet^{2}$ simplifying fraction

 \& 

- ${ }^{1}(2 x-3) \ldots \ldots$ <br>
$\bullet^{2} \quad \ldots \ldots(x+4)$ <br>
- ${ }^{1}(x+4)(x-4)$ <br>
- ${ }^{2}$ answer
\end{tabular} <br>

\hline 8. \& | ans: $\mathbf{7 2} \mathbf{m}^{3}$ |
| :--- |
| 3 marks |
| - knows how to find volume of prism |
| $\bullet^{2}$ calculates area of cross section |
| - ${ }^{3}$ calculates volume | \& | - $1 \mathrm{~V}=$ area of X -section $\times$ length |
| :--- |
| - ${ }^{2} A=1 / 2 \mathrm{~b} \times \mathrm{h}=1 / 2 \times 6 \times 3=9 \mathrm{~m}^{2}$ |
| - ${ }^{3} \mathrm{~V}=\mathrm{Al}=9 \times 8=72 \mathrm{~m}^{3}$ | <br>


\hline 9. \& | ans: £15.60 |
| :--- |
| 5 marks |
| - ${ }^{1}$ creating two equations |
| - ${ }^{2}$ knowing to solve system of equations |
| - ${ }^{3}$ evaluating one variable |
| ${ }^{-4}$ evaluating second variable |
| - ${ }^{5}$ calculating cost | \& | - $2 A+3 C=17.40$ $A+2 C=10.20$ |
| :--- |
| - ${ }^{2}$ scales equations |
| - ${ }^{3} A=4.20$ |
| - ${ }^{4} C=3.00$ |
| ${ }^{-5} 3(£ 4.20)+£ 3.00=£ 15.60$ | <br>


\hline 10. \& | ans: $x=-5, \frac{2}{3} \quad 4$ marks |
| :--- |
| - ${ }^{1}$ substituting correctly |
| $\bullet^{2}$ creating standard quadratic equation |
| - ${ }^{3}$ factorising |
| - ${ }^{4}$ solving equation | \& | - $13\left(x^{2}+2 x-6\right)+7 x+8=0$ |
| :--- |
| - ${ }^{2} 3 x^{2}+13 x-10=0$ |
| - ${ }^{3}(3 x-2)(x+5)=0$ |
| - ${ }^{4}$ answer | <br>


\hline 11. \& | ans: $\sin x^{0}$ |
| :--- |
| 2 marks |
| - replaces $\tan x^{0}$ |
| - ${ }^{1}$ simplifies | \& | - $\cos ^{0}\left(\sin x^{0} / \cos x^{0}\right)$ |
| :--- |
| - ${ }^{1} \sin x^{0}$ | <br>

\hline \& \& Total 40 marks <br>
\hline
\end{tabular}

## Practice Paper E

## MATHEMATICS National Qualifications - National 5 <br> Paper 2 (Calculator) <br> Covering Units 1, 2 and 3

Time allowed - 1 hour and 30 minutes

Fill in these boxes and read carefully what is printed below
Full name of centre
$\square$
Town $\square$

Forename(s)
$\square$

Surname
$\square$


Candidate number


## Seat number

$\square$

Total marks - 50

1. You may use a calculator.
2. Use blue or black ink. Pencil may be used for graphs and diagrams only.
3. Write your working and answers in the spaces provided. Additional space for answers If you use this space, write clearly the number of the question you are attempting. is provided at the end of the booklet.
4. Square ruled paper is provided.
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6. State the units for your answer where appropriate.
7. Before leaving the examination room you must give up this booklet to the invigilator. If you do not, you may lose all the marks for this paper.

The roots of $a x^{2}+b x+c=0$ are $x=\frac{-b \pm \sqrt{\left(b^{2}-4 a c\right)}}{2 a}$

Sine rule:

$$
\frac{a}{\sin \mathrm{~A}}=\frac{b}{\sin \mathrm{~B}}=\frac{c}{\sin \mathrm{C}}
$$

Cosine rule:

$$
a^{2}=b^{2}+c^{2}-2 b c \cos \mathrm{~A} \text { or } \cos \mathrm{A}=\frac{b^{2}+c^{2}-a^{2}}{2 b c}
$$

Area of a triangle: $\quad$ Area $=1 / 2 a b \sin C$

Volume of a sphere: $\quad$ Volume $=\frac{4}{3} \pi r^{3}$

Volume of a cone: $\quad$ Volume $=\frac{1}{3} \pi r^{2} h$

Volume of a Pyramid: $\quad$ Volume $=\frac{1}{3} A h$

Standard deviation: $\quad s=\sqrt{\frac{\sum(x-\bar{x})^{2}}{n-1}}=\sqrt{\frac{\sum x^{2}-\left(\sum x\right)^{2} / n}{n-1}}$, where n is the sample size.

## All questions should be attempted

1. The circumference of the earth is approximately $4 \cdot 01 \times 10^{4} \mathrm{~km}$.

Calculate, correct to three significant figures, the radius of the earth, expressing your answer in standard form.

2. House prices are predicted to rise approximately $2 \cdot 5 \%$ per year, for the next few years.
A cottage bought in January 2014 cost $£ 87000$.
How much, to the nearest $£$, would the same cottage be worth in January 2017 ?
3. In the triangular shaped swimming pool shown below a swimmer dives in at A and swims directly to the opposite side BC. Angle $\mathrm{ABC}=37^{\circ}$ and angle $\mathrm{BCA}=66^{\circ}$. The length of BC is $36 \cdot 1$ metres.


Calculate, correct to three significant figures, the shortest possible distance the swimmer has to cover.
4. Solve, algebraically, the equation

$$
5 \cos x^{0}+3=0, \text { for } 0 \leq x<360 .
$$

5. Express $\frac{6 x}{y(x-2 y)}-\frac{6}{y}$ as a single fraction in its simplest form.
6. The volume of a square based pyramid, of base side $e$ and height $h$, as shown, is given by the formula $V=\frac{1}{3} e^{2} h$.

The base length is doubled and the height is halved.
What happens to the volume of the cone?

7. Find the value of $k$ so that

$$
x^{2}-3 k x+36=0 \quad \text { has equal roots. }
$$

8. The arms of a rotary clothes drier are 1.35 m long and the clothes line between them is 2.1 m long.


Calculate the angle, $x^{0}$, between the arms.
9. Triangles PQT and PRS are shown opposite.
$\mathrm{QT}=8 \mathrm{~cm}, \mathrm{RS}=10 \mathrm{~cm}$ and $\mathrm{TS}=4 \mathrm{~cm}$.
Triangle PQR is similar to triangle PST.
Calculate the length of PT.

10. The table and graph below show the relationship between the number of doctors per 10000 of population (D) and life expectancy (E) in eleven countries.

| doctors, D | 9 | 28 | 28 | 47 | 53 | 113 | 128 | 179 | 182 | 191 | 198 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| life <br> expectancy, E | 48 | 53 | 59 | 64 | 59 | 68 | 77 | 75 | 78 | 79 | 82 |


(a) Draw the line of best fit onto the diagram.
(b) Find the equation of the line of best fit.
(c) Use your answer to part (b) to predict the life expectancy in a country which has 80 doctors per 10000 of population.
11. The diagram shows a vertical flagpole $A B$ with two support wires $A C$ and $A D$.
$\angle \mathrm{ADC}=28^{\circ} \quad$ and $\angle \mathrm{ACB}=52^{\circ}$

(a) Calculate the length of the support wire AC .
(b) Calculate the height of the flagpole AB .
12. A bowling trophy is a plaque with arcs of concentric circles engraved in gold with a gold triangle at the bottom.


B
The largest circle has a radius of 7 centimetres and the largest arc, AB , is 35.4 cm in length.
(a) Calculate the angle $x^{0}$ at the centre.
(b) Calculate the area of the gold triangle.

| Qu | Give one mark for each - | Illustrations for awarding mark |
| :---: | :---: | :---: |
| 1 | ans : $\mathbf{6 . 3 8} \times \mathbf{1 0}^{\mathbf{3}} \mathbf{~ k m}$ <br> - 1 using $d=\frac{C}{\pi}$ <br> -2 calculating radius <br> - ${ }^{3}$ answer in standard form | - ${ }^{1} d=\frac{4 \cdot 01 \times 10^{4}}{\pi}=12764 \cdot 23$ <br> - $2 \quad r=\frac{12764 \cdot 23}{2}=6382 \cdot 11$ <br> ${ }^{3} \quad 6.38 \times 10^{3} \mathrm{~km}$ |
| 2 | ans: £93689 3 marks <br> - ${ }^{1}$ correct multiplier <br> - 2 knowing how to increase for3 years <br> - ${ }^{3}$ calculating answer | $\begin{array}{ll} \bullet^{1} & 1 \cdot 025 \\ \bullet^{2} & 1 \cdot 025^{3} \times £ 87000 \\ \bullet^{3} & £ 93689 \\ \hline \end{array}$ |
| 3 | ans: $\quad 20.4 \mathrm{~m} \quad 5$ marks <br> - ${ }^{1}$ attempting to calculate side AC or AB <br> - 2 calculating AC or AB using Sine Rule <br> - ${ }^{3}$ knowing shortest dist is at $90^{\circ}$ to BC <br> - ${ }^{4}$ using SOH to calculate shortest dist <br> -5 calculating correctly | - $\frac{36 \cdot 1}{\sin 77^{\circ}}=\frac{A C}{\sin 37^{\circ}}=\frac{A B}{\sin 66^{\circ}}$ <br> - ${ }^{2} \quad \mathrm{AC}=22.3 \mathrm{~m} ; \mathrm{AB}=33.8 \mathrm{~m}$ <br> - ${ }^{3}$ evidence of SOH CAH TOA <br> - $4 \sin 66^{\circ}=\frac{\text { dist }}{22 \cdot 3} ; \sin 37^{\circ}=\frac{\text { dist }}{33 \cdot 8}$ <br> - $50 \cdot 4 \mathrm{~m}$ |
| 4 | ans: $\mathbf{1 2 6 \cdot 9} \mathbf{9}^{\mathbf{0}}, \mathbf{2 3 3 \cdot 1 ^ { 0 }} \quad 3$ marks <br> - ${ }^{1}$ rearranging to find $\cos x^{\circ}$ <br> - ${ }^{2}$ finds one solution <br> - ${ }^{3}$ finds second solution | - ${ }^{1} \cos x^{\circ}=-\frac{3}{5}$ <br> - ${ }^{2} 126.9^{\circ}$ <br> - $^{3} 233.1^{\circ}$ |
| 5 | ans: $\frac{12}{x-2 y} \quad 3$ marks <br> - ${ }^{1}$ correct numerator <br> - ${ }^{2}$ correct denominator <br> $\bullet^{3}$ simplifying | -1 $6 x-6(x-2 y)=12 y$ <br> -2 $y(x-2 y)$ <br> -3 answer |
| 6 | ans: Volume is doubled 3 marks <br> - replacing $e$ with $2 e$, and $h$ with $\frac{1}{2} h$ <br> -2 simplifying expression <br> $\bullet^{3}$ conclusion | - ${ }^{1} \quad V=\frac{1}{3} \times(2 e)^{2} \times\left(\frac{1}{2} h\right)$ <br> - $2 . V=\frac{2}{3} e^{2} h$ <br> - ${ }^{3}$ answer |


| Qu | Give one mark for each - | Illustrations for awarding mark |
| :---: | :---: | :---: |
| 7 | ans: $k= \pm 4$ <br> - ${ }^{1}$ states condition for equal root <br> - ${ }^{2}$ substitutes values <br> - ${ }^{3}$ starts to solve <br> - ${ }^{4}$ solves | - $b^{2}-4 a c=0$ <br> - ${ }^{2}(3 k)^{2}-4 \times 1 \times 36=0$ <br> - ${ }^{3} \quad 9 k^{2}=144$ <br> - ${ }^{4} k= \pm 4$ |
| 8 | ans : $102^{\circ} \quad 4$ marks <br> - ${ }^{1}$ uses cosine rule <br> - ${ }^{2}$ substitutes values <br> - ${ }^{3}$ evaluates expression <br> -4 calculates required angle | - $1 \quad \cos \mathrm{~A}=\frac{b^{2}+c^{2}-a^{2}}{2 b c}$ <br> $\bullet^{2} \cos x^{0}=\frac{1 \cdot 35^{2}+1 \cdot 35^{2}-2 \cdot 1^{2}}{2 \times 1 \cdot 35 \times 1.35}$ <br> - ${ }^{3} \cos x^{0}=-0.210$ <br> - ${ }^{4} \quad x=102^{0}$ |
| 9 | ans: $\quad \mathbf{1 6 c m}$ $\mathbf{3}$ marks  <br> $\bullet^{1}$ sets up equal ratios  <br> $\bullet^{2}$ begins to solve for $x$  <br> $\bullet^{3}$ solves for $x$  | - $\frac{10}{8}=\frac{x+4}{x}$ <br> -2 $8(x+4)=10 x$ <br> $\bullet^{3} \quad x=16$ |
| 10 <br> (a) <br> (b) | ans : line of best fit $\mathbf{1}$ mark <br> - ${ }^{1}$ draws line of best fit  <br> ans : $\mathbf{E}=\mathbf{0 . 1 5 D}+\mathbf{5 2}$ $\mathbf{3}$ marks <br> - ${ }^{1}$ finds gradient  <br> $\bullet^{2}$ finds E-intercept  <br> $\bullet^{3}$ writes equation  <br> ans : $\mathbf{6 4}$ years $\mathbf{2}$ marks <br> - ${ }^{1}$ substitutes value  <br> $\bullet^{2}$ evaluates equation  | - ${ }^{1}$ line drawn <br> - ${ }^{1} m=0 \cdot 15$ (or suitable alternative) <br> - ${ }^{2} \mathrm{c}=52$ (or suitable alternative) <br> - ${ }^{3} \mathrm{E}=0 \cdot 15 \mathrm{D}+52$ <br> - ${ }^{1} \quad \mathrm{E}=0.15 \times 80+52$ <br> $\bullet^{2}=64$ |
| 11 <br> (a) <br> (b) | ans: $\mathbf{1 1 . 5} \mathrm{m}$ <br> - ${ }^{1}$ calculates supplementary angle <br> - ${ }^{2}$ calculates third angle of triangle <br> -3 knows to use sine rule <br> - ${ }^{4}$ substitutes correctly <br> ${ }^{-5}$ evaluates length <br> ans: 9.1 m <br> 2 marks <br> - ${ }^{1}$ uses SOHCAHTOA <br> $\bullet^{2}$ calculates length | - ${ }^{1} \angle \mathrm{ACD}=180^{\circ}-52^{\circ}=128^{\circ}$ <br> - $2 \angle \mathrm{DAC}=180^{\circ}-\left(128^{\circ}+28^{\circ}\right)=24^{\circ}$ <br> - ${ }^{3} \mathrm{a} / \sin \mathrm{A}=\mathrm{d} / \sin \mathrm{D}$ <br> - ${ }^{4} 10 / \sin 24=\mathrm{AC} / \sin 28$ <br> - ${ }^{5} \mathrm{AC}=11.5 \mathrm{~m}$ <br> - $\quad \sin 52^{\circ}=\mathrm{AB} / 11.5$ <br> - ${ }^{2} \mathrm{AB}=11.5 \times \sin 52^{\circ}=9.1$ |


| Qu | Give one mark for each - |  | Illustrations for awarding mark |
| :---: | :---: | :---: | :---: |
| $\begin{aligned} & \mathbf{1 2} \\ & \text { (a) } \end{aligned}$ | ans: $\mathbf{2 9 0}^{\mathbf{o}}$ <br> - ${ }^{1}$ sets up ratio <br> - ${ }^{2}$ substitutes values <br> - ${ }^{3}$ evaluates $x^{0}$ | $3 \text { marks }$ | - $\frac{\text { angle at centre }}{360}=\frac{\text { arc }}{\text { circumferemce }}$ <br> - $2 \frac{x}{360}=\frac{35 \cdot 4}{14 \pi}$ <br> - $^{3} x=290^{\circ}$ |
| (b) | ans: $23 \mathrm{~cm}^{2}$ <br> - ${ }^{1}$ calculates angle in triangle <br> - ${ }^{2}$ uses triangle formula <br> -3 evaluates area | 3 marks | - ${ }^{1} 360^{\circ}-290^{\circ}=70^{\circ}$ <br> - $2 \mathrm{~A}=1 / 2 \mathrm{ab} \sin \mathrm{C}=1 / 2 \times 7 \times 7 \times \sin 70^{\circ}$ <br> - $33 \mathrm{~cm}^{2}$ |
|  |  |  | Total 50 marks |

