N5

Specimen Paper D

MATHEMATICS
National Qualifications - National 5
Paper 1 (non-calculator)
Covering Units 1, 2 and 3

Time allowed - 1 hour

Fill in these boxes and read carefully what is printed below

Full name of centre | **Town**

Forename(s) | **Surname**

Date of birth
Day Month Year

Candidate number

Seat number

Total marks - 40

1. You may NOT use a calculator.
2. Use **blue** or **black** ink. Pencil may be used for graphs and diagrams only.
3. Write your working and answers in the spaces provided. Additional space for answers is provided at the end of the booklet. If you use this space, write clearly the number of the question you are attempting.
4. Square ruled paper is provided.
5. Full credit will be given only where the solution contains appropriate working.
6. State the units for your answer where appropriate.
7. Before leaving the examination room you must give up this booklet to the invigilator. If you do not, you may lose all the marks for this paper.

FORMULAE LIST

The roots of $ax^2 + bx + c = 0$ are $x = \dfrac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

Sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$ or $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

Area of a triangle: Area = $\frac{1}{2} ab \sin C$

Volume of a sphere: Volume = $\dfrac{4}{3}\pi r^3$

Volume of a cone: Volume = $\dfrac{1}{3}\pi r^2 h$

Volume of a Pyramid: Volume = $\dfrac{1}{3}Ah$

Standard deviation: $s = \sqrt{\dfrac{\sum(x - \bar{x})^2}{n-1}} = \sqrt{\dfrac{\sum x^2 - (\sum x)^2 / n}{n-1}}$, where n is the sample size.

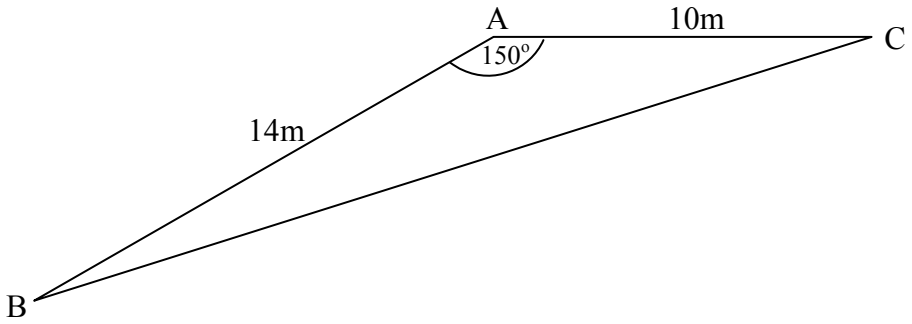
All questions should be attempted

Marks

Do not write in this margin.

1. Express $y = x^2 - 6x + 7$ in the form $y = (x - a)^2 + b$. **2**

2. In triangle ABC, AB = 14m and AC = 10m. Angle BAC = 150°.



Given that sin 150° = 0·5, calculate the area of triangle ABC. **3**

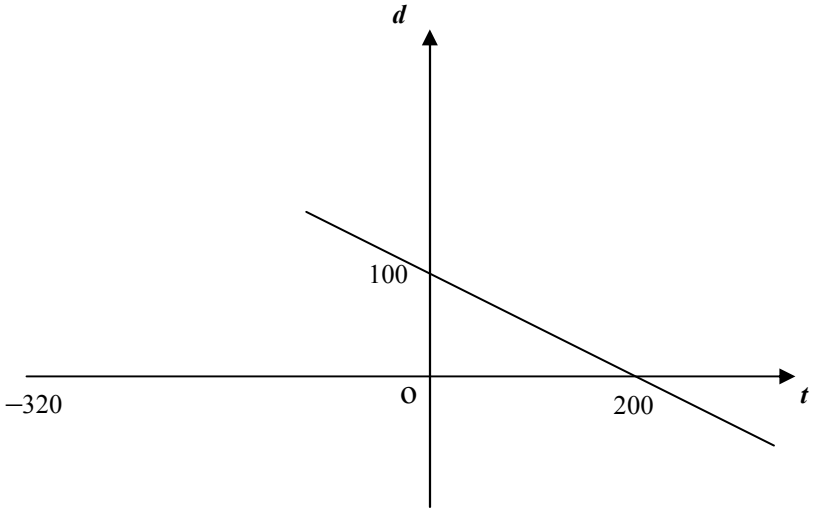
3. Evaluate $\frac{6}{7}$ of $\left(\frac{2}{3} - \frac{1}{2}\right)$ **2**

4. **(a)** If $h(x) = 5x - 2x^2$, find the value of $h(-2)$ **2**

(b) Factorise **fully** $15y^2 - 3y$ **1**

(c) Hence, or otherwise, express $\frac{15y^2 - 3y}{25y^2 - 1}$ in its simplest form. **2**

5.



(a) Find the equation of the line in terms of ***t*** and ***d***. **4**

(b) If the line were continued, would it pass through the point P(–320, 250)? **Give a reason for your answer.** **2**

6. **(a)** Remove the brackets and simplify

$$(t-5v)(3t+2v)$$ **2**

(b) Solve the inequality:

$$3-4(3x-4) \geq 3(2-3x)$$ **4**

(c) Solve **algebraically** the equation

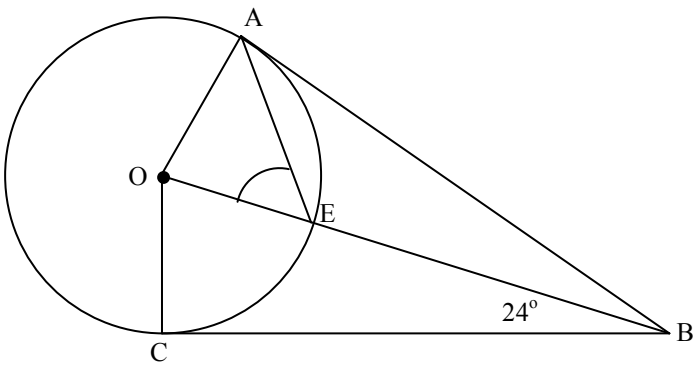
$$\frac{m}{3} - \frac{(m-3)}{2} = 1$$ **3**

7. Given the following vectors

$$\overrightarrow{AB} = \begin{pmatrix} 2 \\ 2 \\ 4 \end{pmatrix} \quad \text{and} \quad \overrightarrow{AC} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$

Find $|\mathbf{2\overrightarrow{AB} - 2\overrightarrow{AC}}|$, expressing the result as a surd in its simplest form **4**

8.



The diagram above shows a quadrilateral ABCO. BA and BC are tangents to the circle, centre O, and E is the point where OB meets the circle.

Find the size of angle OEA. **3**

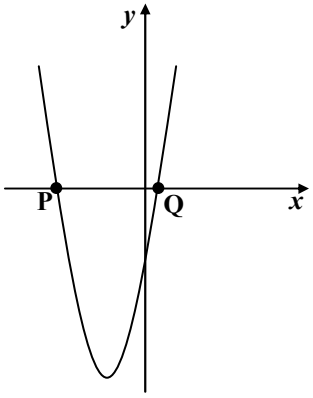
Marks

9. **(a)** Simplify $m^3 \times m^{-5}$ **1**

(b) Evaluate $81^{\frac{3}{4}}$ **2**

10. The graph in the diagram has equation

$y = 2x^2 + 5x - 3$ and cuts the x-axis at **P** and **Q**.



Find the coordinates of the points **P** and **Q**. **3**

End of Question Paper

	Give 1 mark for each •	Illustration(s) for awarding each mark
1	**ans:** $y = (x-3)^2 - 2$ **2 marks** $\bullet^1$ bracket correct $\bullet^2$ value of b correct	 $\bullet^1$ $(x-3)^2$……. $\bullet^2$ ……-2
2	**ans : 35 cm^2** **3 marks** $\bullet^1$ knows to use area of triangle formula $\bullet^2$ subs values into formula $\bullet^3$ calculates area	 $\bullet^1$ evidence $\bullet^2$ $A = \frac{1}{2} \times 14 \times 10 \times 0 \cdot 5$ $\bullet^3$ $A = 35\text{cm}^2$
3	**ans:** $\frac{1}{7}$ **2 marks** $\bullet^1$ subtract fractions $\bullet^2$ multiply fractions	 $\bullet^1$ 1/6 $\bullet^2$ 1/7 or equivalent
4(a)	**ans: –18** **2 marks** $\bullet^1$ interpret function notation $\bullet^2$ evaluate function	 $\bullet^1$ $5 \times (-2) - [\, 2 - (-2) - (-2) \,]$ $\bullet^2$ -18
(b)	**ans: $3y(5y-1)$** **1 mark** $\bullet^1$ factorises	 $\bullet^1$ $3y\,(5y-1)$
(c)	**ans:** $\frac{3y}{5y+1}$ **2 marks** $\bullet^1$ factorise denominator $\bullet^2$ simplify	 $\bullet^1$ $(5y-1)(5y+1)$ $\bullet^2$ $3y/(5y+1)$
5(a)	**ans: $d = -1/2t + 100$** **4 marks** $\bullet^1$ for starting to find m $\bullet^2$ for calculating m $\bullet^3$ for finding c $\bullet^4$ for equation with d and t	 $\bullet^1$ $m = (100-0) / (0-200)$ $\bullet^2$ …. $= -\frac{1}{2}$ or equiv. $\bullet^3$ $c = 100$ $\bullet^4$ $d = -\frac{1}{2}\, t + 100$
(b)	**ans: No - point does *not* satisfy equation** **2 marks** $\bullet^1$ substitutes values in equation $\bullet^2$ valid conclusion	 $\bullet^1$ $250 = -\frac{1}{2} \times (-320) + 100$ $\bullet^2$ $250 \neq 160 + 100$, so point *not* on line.

	Give 1 mark for each •	Illustration(s) for awarding each mark
6. (a)	**ans:** $3t^2 - 13vt - 10v^2$ **2 marks**	
	•[1] for finding $3t^2 and -10v^2$ •[2] for finding $-13vt$	•[1] $3t^2 and -10v^2$ •[2] $-13vt$
(b)	**ans:** $x \leq \frac{13}{3}$ **4 marks**	
	•[1] removing brackets •[2] collecting like terms •[3] knows to reverse inequality •[4] solving inequation	•[1] $3 - 12x + 16 \geq 6 - 9x$ •[2] $-3x \geq -13$ •[3] $x \leq$ •[4] $\frac{13}{3}$
(c)	**ans:** $m = 3$ **3 marks**	
	•[1] add the fractions •[2] multiply expressions •[3] solve equation	•[1] multiply by 6 or take common denominator •[2] ……$-m + 9$ •[3] $m = 3$
7.	**ans:** $6\sqrt{2}$ **4 marks**	
	•[1] finds $2\overrightarrow{AB} - 2\overrightarrow{AC}$ •[2] knows how to find the magnitude •[3] finds the magnitude •[4] expresses as a surd in its simplest form	•[1] $\begin{pmatrix} 0 \\ 6 \\ 6 \end{pmatrix}$ •[2] $\sqrt{(0^2 + 6^2 + 6^2)}$ •[3] $\sqrt{72}$ •[4] $6\sqrt{2}$
8.	**ans:** $57°$ **3 marks**	Steps can be shown on diagram but angle OEA must be stated explicitly
	•[1] knows angle ABO = 24^o and angle A is right •[2] finds angle AOB •[3] final answer.	•[1] evidence •[2] angle AOB = 56^o •[3] angle OEA = 57^o
9(a)	**ans:** m^{-2} **1 mark**	
	•[1] follows rule for indices	•[1] m^{-2}
(b)	**ans:** 27 **2 marks**	
	•[1] interprets fractional index •[2] simplifies	•[1] $\sqrt[4]{81^3}$ •[2] 27
10	**ans:** **P(– 3, 0) and Q(0·5, 0)** **3 marks**	
	•[1] equates to 0 and factorises •[2] solves equation •[3] states coordinates of P and Q	•[1] $(2x - 1)(x + 3) = 0$ •[2] $x = 0{\cdot}5$ or $x = -3$
		Total: **40 marks**

N5

Practice Paper D

MATHEMATICS
National Qualifications - National 5
Paper 2 (Calculator)
Covering Units 1, 2 and 3

Time allowed - 1 hour and 30 minutes

Fill in these boxes and read carefully what is printed below

Full name of centre | **Town**

Forename(s) | **Surname**

Date of birth
Day Month Year

Candidate number

Seat number

Total marks - 50

1. **You may use a calculator.**
2. Use **blue** or **black** ink. Pencil may be used for graphs and diagrams only.
3. Write your working and answers in the spaces provided. Additional space for answers is provided at the end of the booklet. If you use this space, write clearly the number of the question you are attempting.
4. Square ruled paper is provided.
5. Full credit will be given only where the solution contains appropriate working.
6. State the units for your answer where appropriate.
7. Before leaving the examination room you must give up this booklet to the invigilator. If you do not, you may lose all the marks for this paper.

FORMULAE LIST

The roots of $ax^2 + bx + c = 0$ are $x = \dfrac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

Sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$ or $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

Area of a triangle: Area = $\frac{1}{2} ab \sin C$

Volume of a sphere: Volume = $\dfrac{4}{3}\pi r^3$

Volume of a cone: Volume = $\dfrac{1}{3}\pi r^2 h$

Volume of a Pyramid: Volume = $\dfrac{1}{3}Ah$

Standard deviation: $s = \sqrt{\dfrac{\sum(x - \bar{x})^2}{n-1}} = \sqrt{\dfrac{\sum x^2 - (\sum x)^2 / n}{n-1}}$, where n is the sample size.

Do not write in this margin.

Marks

All questions should be attempted

1. Multiply out the brackets and collect like terms

$(x - 1)(x^2 + 5x - 2)$ **3**

2. **(a)** A quality control examiner on a production line measures the weight in grams of cakes coming off the line. In a sample of eight cakes the weights were

150 147 148 153 149 143 145 151

Use appropriate formulae to calculate the mean and standard deviation. Show all your working clearly**.** **4**

(b) On a second production line, a sample of 8 cakes gives a mean of 148·25 and a standard deviation of 6·1.

Compare the two production lines by referring to the consistency of the weight. **1**

3. **(a)** Express $\frac{3}{x} - \frac{3}{x+1}$ as a single fraction in its simplest form $(x \neq 0, x \neq -1)$. **3**

(b) Change the subject of the formula to v in $b = \frac{v-u}{c}$ **2**

4. Solve the following trigonometric equation:

$3\sin x° + 2 = 1, \quad 0 \leq x \leq 360$ **3**

5. A charter aeroplane, when full, can carry 96 passengers. Some of these passengers will be travelling business class while others will be travelling economy class.

Let B be the number of business class passengers and E be the number of economy passengers.

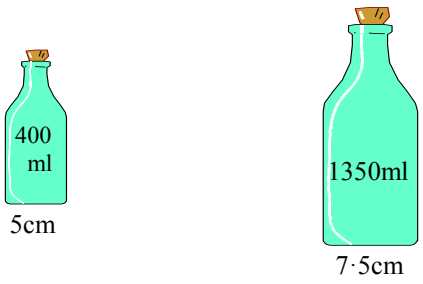
(a) Given that the plane is full, use the information above to write down a simple equation involving B and E. **1**

Each business class passenger is allowed to have 65kg of luggage but an economy passenger is allowed only 35kg. The total weight of luggage on board is 4140kg for one flight.

(b) Assuming that each passenger has taken their maximum amount of luggage, write down another equation involving B and E. **2**

(c) Find the number of business and the number of economy class passengers on board. **3**

6. The national soft drink of Spain is called " Elaborado del Hierro" and it is sold in two main bottle sizes.



The smaller bottle has a base diameter of 5cm and holds 400ml.

The larger bottle has a base diameter of 7·5cm and it holds 1350ml.

The bottles look alike but could they actually be **mathematically** similar?
(Show calculations to justify your answer.) **4**

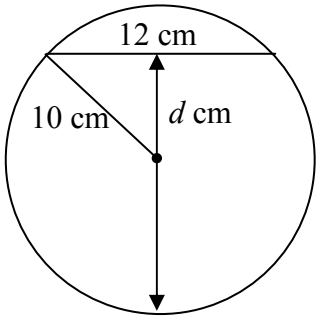
7. Solve the equation $2x^2 - x - 7 = 0$.

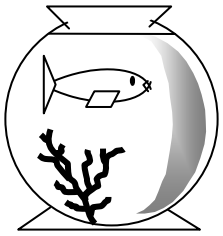
Give your answers **correct to 1 decimal place.** **4**

Marks

8. A goldfish bowl is filled with water to a certain level.

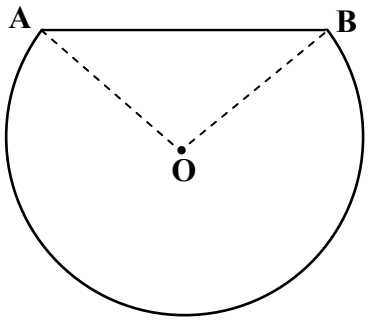
A cross section through the centre of the bowl is circular.

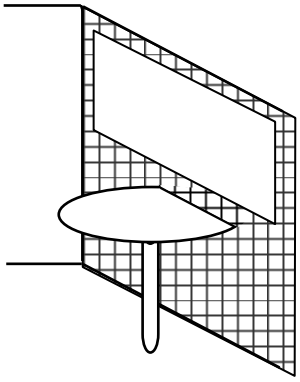




If the width of the water surface is 12 cm and the radius is 10 cm, find the depth of the water, d cm, in the bowl. **5**

9. The tables in Carlo's Coffee shop are circular with a segment removed so that they will fit against a wall.



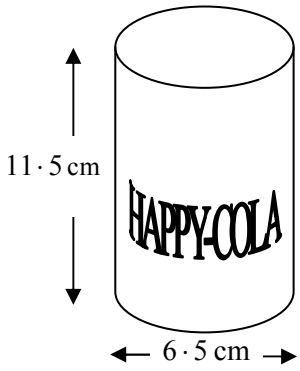


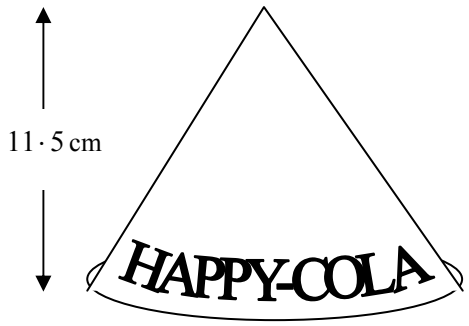
Angle AOB is 90^{o}, where O is the centre of the circle, and the diameter of the tables is 120 cm.

The tables have to be covered in a heat resistant material. What area of material will be needed to exactly cover the table? **6**

Marks

10. "HAPPY-COLA" have decided to issue a "limited edition" cone-shaped can to celebrate their 50th anniversary.
Their normal can is a cylinder whose height is 11·5 cm and whose **diameter** is 6·5 cm.

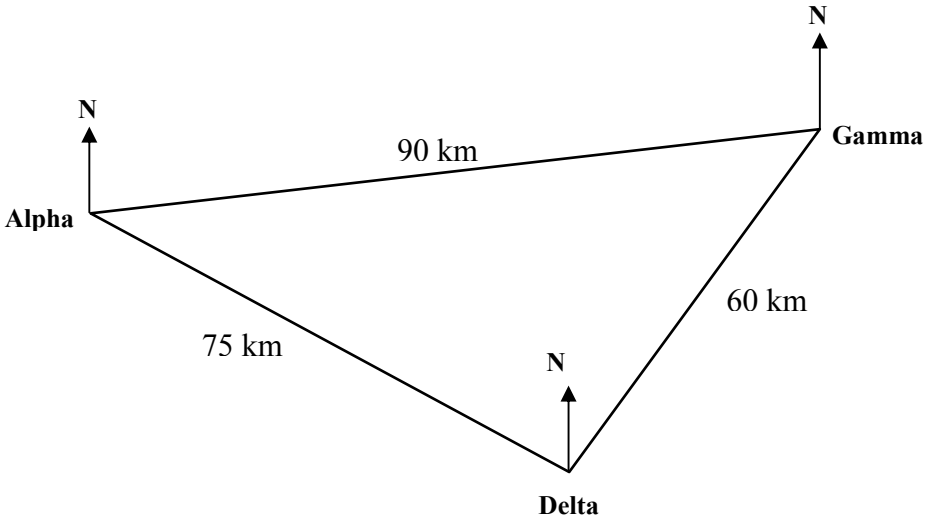




If the height of the cone is to be the same as the height of the cylinder, i.e. 11·5 cm, and the volume of the cone is to be the same as the volume of the cylinder, calculate the diameter of the cone.
(Answer in centimetres giving your answer correct **to 1 decimal place**) **4**

11. Three oil platforms, Alpha, Gamma and Delta are situated in the North Sea as shown in the diagram below.

The distances between the oil platforms are shown in the diagram.

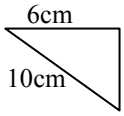


If the bearing of Delta from Alpha is 125°, what is the bearing of Gamma from Alpha?

5

End of Question Paper

Qu	Give one mark for each •	Illustrations for awarding mark
1	**ans : $x^3 + 4x^2 - 7x + 2$ 3 marks** •1 multiplies bracket by 1st term •2 multiplies bracket by 2nd term •3 simplifies	 •1 $x^3 + 5x^2 - 2x$ . . . •2 . . . $-x^2 - 5x + 2$ •3 $x^3 + 4x^2 - 7x + 2$
2(a)	**ans : 148·25, 3·24 4 marks** •1 calculates mean •2 calculates $(x - \bar{x})^2$ and totals •3 substitutes into formula •4 calculates standard deviation	 •1 mean = 1186 ÷ 8 = 148·25 •2 3·0625, 1·5625, 0·0625, 22·5625, 0·5625, 27·5625, 10·5625, 7·5625; 73·5 •3 $s = \sqrt{\frac{73 \cdot 5}{7}}$ Alternative solution at end of marking scheme •4 s = 3·24
(b)	**ans : any suitable comment 1 mark** •1 compares samples	 •1 eg 2nd line has a larger spread of values
3(a)	**ans: $\frac{3}{x(x+1)}$ 3 marks** •1 correct numerator •2 correct denominator •3 for simplifying numerator	 •1 $3(x + 1) - 3x$ •2 $x(x + 1)$ •3 $= 3$
(b)	**ans: $v = bc + u$ 2 marks** •1 multiplies through by b •2 adds u to both sides	 •1 $v - u = bc$ •2 $v = bc + u$
4	**ans: 199·5°, 340·5° 3 marks** •1 manipulation to $\sin x^o = \ldots$ •2 finds one solution •3 finds second solution	 •1 $\sin x^o = -1/3$ •2 $x = 199 \cdot 5°$ •3 $x = 340 \cdot 5°$ N.B. $x = -19 \cdot 5°$ is not acceptable for •2
5(a)	**ans: B + E = 96 1 mark** •1 equation	 •1 B + E = 96
(b)	**ans: 65B + 35E = 4140 2 marks** •1 part of equation •2 further part of equation	 •1 65B + 35E… •2 65B + 35E = 4140
(c)	**ans: B = 26, E = 70 3 marks** •1 scales sim. equations •2 solves for both variables •3 states number of each passenger	 •1 65B + 35E = 4140, 35B + 35E = 3360 •2 B = 26; E = 70 •3 26 business class and 70 economy class

Qu	Give one mark for each •	Illustrations for awarding mark
6	**ans: Yes, bottles could be similar 4 marks** •[1] calculating the linear scale factor •[2] knowing to **cube** the S.F. •[3] calculating the new volume •[4] consistent conclusion	 •[1] $7{\cdot}5 / 5 =$ S.F. •[2] $1{\cdot}5^3 = 3{\cdot}375$ •[3] $V = 400 \times 3{\cdot}375 = 1350$ •[4] bottles **could** be similar since volumes are consistent with similar shapes
7	**ans: 2·1 or –1·6 4 marks** •[1] knows to use quadratic formula •[2] evaluates discriminant •[3] substitutes values •[4] finds values of x correctly rounded	 •[1] evidence •[2] $b^2 - 4ac = 57$ •[3] $x = \dfrac{1 \pm \sqrt{57}}{4}$ •[4] $x = 2{\cdot}1, -1{\cdot}6$
8	**ans : 18 cm 5 marks** •[1] knows to use Pythagoras •[2] assembles facts in right triangle •[3] uses Pythagoras •[4] calculates x •[5] calculates depth	 •[1] evidence •[2] see diagram •[3] $x^2 + 6^2 = 10^2$ •[4] $x = 8$ •[5] $d = 8 + 10 = 18$ cm 
9	**ans : 10282·3cm^2 6 marks** •[1] knows to find angle at centre •[2] knows area of sector is ¾ circle •[3] substitutes radius •[4] calculates area •[5] knows to calculate area of triangle •[6] calculates total area	 •[1] angle at centre is 270° •[2] area of sector = ¾ πr^2 •[3] area of sector = ¾ $\pi \times 60^2$ •[4] area = 8482·3 cm^2 •[5] Δ area = ½ × bh = ½ × 60 × 60 = 1800 cm^2 •[6] 10282·3 cm^2
10	**ans: Diameter ≈ 11·2 cm 4 marks** •[1] volume of cylinder •[2] vol. of cyl. = vol. of cone (strategy) •[3] calculating r^2 •[4] final answer	 •[1] volume of cyl. = 381·4 cm^3 [$\pi = 3{\cdot}14$] •[2] $381{\cdot}4 = 1/3\pi r^2 h$ •[3] $r^2 = 31{\cdot}7$cm •[4] D = 11·2cm
11	**ans : 084° 5 marks** •[1] knows to use cosine rule •[2] substitutes correctly •[3] calculates angle •[4] subtracts to find angle •[5] writes bearing	 •[1] $\cos x^\circ = \ldots$ •[2] $= (90^2 + 75^2 - 60^2)/(2 \times 90 \times 75)$ •[3] $x = 41{\cdot}4^\circ$ •[4] $125^\circ - 41^\circ = 84^\circ$ •[5] 084°
		Total 50 marks

ALTERNATIVE SOLUTION TO QUESTION 2

2(a)	**ans : 148·25, 3·24** **4 marks** •[1] calculates mean •[2] finds $\sum x^2$ and $(\sum x)^2$ •[3] substitutes into formula •[4] calculates standard deviation	 •[1] mean = $1186 \div 8 = 148{\cdot}25$ •[2] $3\sum x^2 = 175898$; $(\sum x)^2 = 1406596$ •[3] $s = \sqrt{\frac{175898 - (1406596/8)}{7}}$ •[4] $s = 3{\cdot}24$
(b)	**ans : any suitable comment** **1 mark** •[1] compares samples	 •[1] eg 2nd line has a larger spread of values