Higher Mathematics Christmas Special

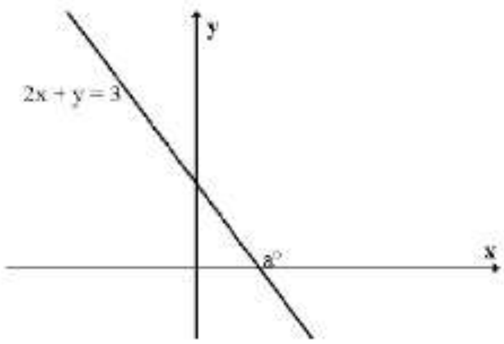
Straight Line

1. Find the equation of the line parallel to the line $3x + 2y - 10 = 0$ which passes through the point (-1 , 4).

2. In the diagram below find a°, the angle the line $2x + y = 3$ makes with the positive direction of the x-axis.

3. Find the equation of the line through the point (2,-5) perpendicular to the line AB where A is (4 , 1) & B is (6 , -3).

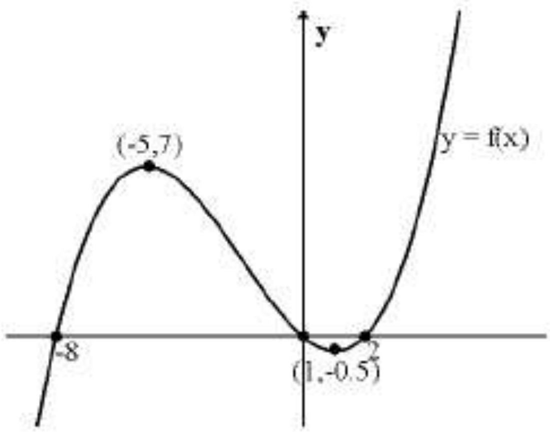
4. A is the point (2 , -1), B is (10 , -5) and C is (6 , 2). Find the:
   a) equation of the perpendicular bisector of AB.
   b) equation of the altitude from B to AC.
   c) point of intersection of these lines.



5. The triangle ABC has vertices A(2 , -5), B(8 , 1) and C(7 , 2). Find the equation of the median from C.

Functions and Graphs

6. The diagram opposite shows the graph of $y = f(x)$. Sketch the graph of:
   a) $y = -f(x) + 3$
   b) $y = -3f(x - 2)$



7. The functions f(x) & g(x) are defined on suitable domains with: $f(x) = \frac{3x - 4}{x}$ and $g(x) = \frac{4}{3 - x}$

   a) Find a formula for g(f(x)).

   b) State the connection between f(x) and g(x).

8. $f(x) = x^2 - x - 12$ and $g(x) = 3x + 1$

   a) Find a formula for f(g(x)).

   b) Solve f(g(x)) = 0.

   c) State a suitable domain for the function h(x) where $h(x) = \frac{1}{f(g(x))}$

Recurrence Relations

9. $U_{n+1} = 0.6\,U_n + 20 \quad U_0 = 40$

   a) Find n such that $U_n > 49$

   b) Explain why U_n has a limit and find the exact value of this limit.

10. A recurrence relation is defined as $U_n = aU_{n-1} + b$. The first three terms of this relation are: 160, 200 and 230. Find the values of a and b.

11. A recurrence relation is $U_{n+1} = 0.5\,U_n + 10$. Given $U_3 = 30$, find the value of U_1.

12. Two sequences are defined by the recurrence relations $U_{n+1} = 0.4\,U_n + p$ & $V_{n+1} = 0.6\,V_n + q$ If both sequences have the same limit, express p in terms of q.

13. A patient is injected with 60ml of an antibiotic drug. Every 4 hours 30% of the drug passes out of her bloodstream. To compensate for this an extra 20ml of antibiotic is given every 4 hours.

   a) Find a recurrence relation for the amount of drug in the patient's bloodstream.

   b) Calculate the amount of antibiotic remaining in the bloodstream after one day.

   c) In the long term, more than 70ml of Antibiatoc present in the bloodstream can be dangerous. Can the patient remain on this treatment course?

Differentiation

14. $f(x) = \dfrac{x^2 - 1}{\sqrt{x}}$ Find $f'(4)$.

15. $s = 3u(u^2 + 1)$. Find the rate of change of s when $u = {}^4/_3$

16. Find the equation of the tangent to the curve $y = \dfrac{x^2(x^2 - 2)}{x}$ at the point where $x = 2$.

17. A tangent to the curve $y = x^4 - 2x$ has gradient -6. Find the equation of this tangent.

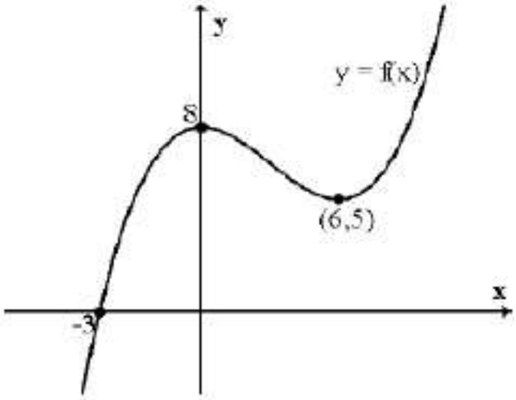
18. Show that the curve $y = x^3 - 6x^2 + 12x + 3$ is never decreasing.

19. Find the values of x for which the curve $f(x) = 2x^3 - 6x^2 - 48x + 5$ is strictly increasing.

20. $f(x) = x^4 - 4x^3 + 5$. Find the stationary points of f(x) and determine their nature.

21. Find the maximum and minimum values of $f(x) = 2x^3 - 3x^2 - 12x$ in the range $-3 \le x \le 3$.

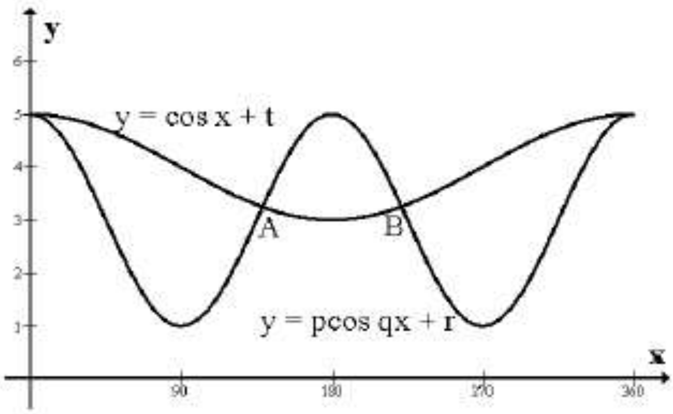
22. Shown opposite is the graph of $y = f(x)$. Sketch the graph of $y = f'(x)$.



Trigonometry

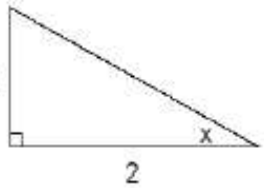
23. Solve the equations
    a) $3\tan 2x - 1 = 0$, $0 \le x \le 2\pi$
    b) $4\cos(2x - 30) + 4 = 2$, $0 \le x \le 360$
    c) $3\sin 2x = 2\cos x$, $0 \le x \le 360$

24. The diagram opposite shows the graphs of $y = p\cos qx + r$ and $y = \cos x + t$.
    a) Write down the values of p, q, r and t.
    b) Find the coordinates of A and B.



25. If $\tan x = \frac{1}{2}$, find the exact value of:

a) $\sin 2x$ (b) $\cos 2x$ (c) $\tan 2x$



26. If $\cos x = \frac{3}{5}$ and $\sin y = \frac{5}{13}$, find the exact value of $\cos(x + y)$.

Polynomials

27. $f(x) = 2x^3 - 3x^2 - 2x + 3$.

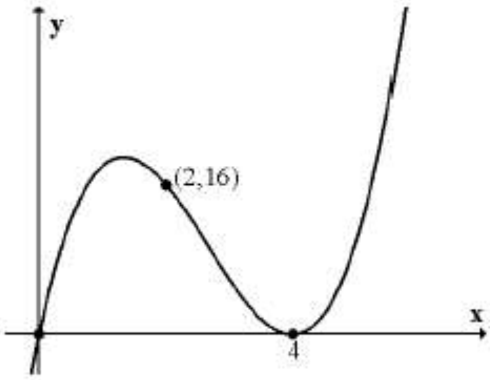
a) Show that $(x - 1)$ is a factor of $f(x)$.

b) Find the other factors of $f(x)$.

28. A function is defined as $f(x) = x^3 + 2x^2 - 5x - 6$. Given -1 is a root of $f(x)$, find the other roots.

29. The function shown in the graph opposite crosses the x-axis at 0 and 4 and the point $(2, 16)$ lies on the graph.

Find the equation of this function.



30. -3 is a root of $2x^3 - 3x^2 + px + 30 = 0$. Find p and hence find the other roots of $2x^3 - 3x^2 + px + 30 = 0$.

31. $(x - 2)$ and $(x + 4)$ are both factors of $x^3 - 2x^2 - px + q$. Find the values of p and q.

Quadratic Theory

32. a) Express $f(x) = 3x^2 + 12x - 2$ in the form $f(x) = a(x + b)^2 + c$.

b) Hence, or otherwise, write down the turning point of $f(x)$ stating whether this turning point is a maximum or minimum.

33. State the nature of the roots of: a) $3x^2 - 2x - 5 = 0$ (b) $x^2 + 3x + 7 = 0$

34. Find k if the roots of the equation: a) $(x + 1)(x + k) = -4$ are <u>equal</u>, when $k > 0$.
b) $x^2 + kx - 3k = 4x - 7$ are <u>real</u>.

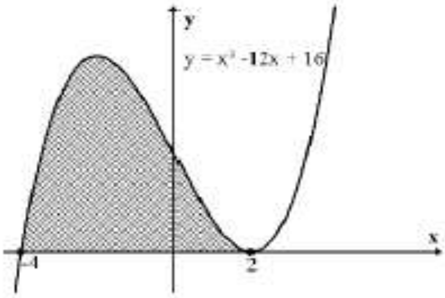
35. Show that $y = 2x^3 + x^2 + 9x + 1$ has no stationary points.

Integration

36. Find: a) $\displaystyle\int \frac{x^3 - 1}{x^2}\ dx$ (b) $\displaystyle\int_1^4 \sqrt{x}\left(\sqrt{x} - x\right)\ dx$

37. $^{dy}/_{dx} = 3x^2 - 4x + 1$. Find a formula for y given x = -1 when y = 2.

38. Calculate the shaded area in the diagram shown opposite, for the curve $y = x^3 - 12x + 16$.



39. a) Figure 1 below shows the line y = 2x + 5 and the curve $y = x^2 - x + 1$. Calculate the shaded area.

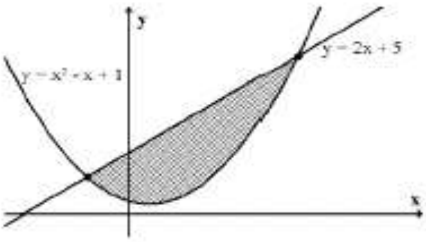


Figure 1

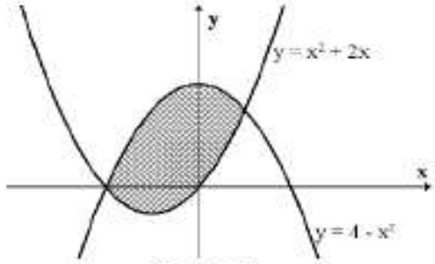
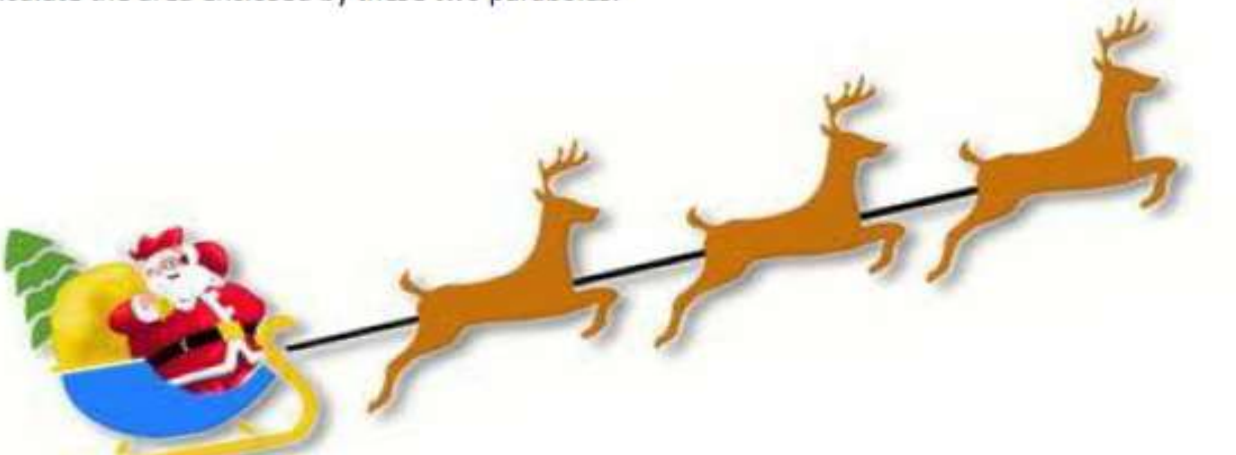


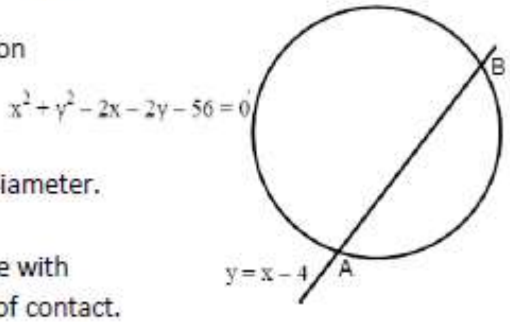
Figure 2

b) Figure 2 above shows the parabolas $y = x^2 + 2x$ and $y = 4 - x^2$.
Calculate the area enclosed by these two parabolas.



Circles

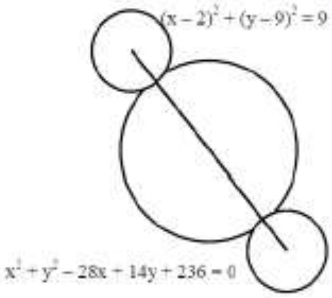
40. A circle has equation $x^2 + y^2 - 6x + 2y - 35 = 0$.
Find the equation of the tangent to this circle at the point (-3,2).



41. a) The line $y = x - 4$ intersects the circle with equation $x^2 + y^2 - 2x - 2y - 56 = 0$ at two points A and B.
Find the coordinates of A and B.

b) Find the equation of the circle which has AB as diameter.

42. Prove that the line $y = 2x + 6$ is a tangent to the circle with equation $x^2 + y^2 - 8x + 2y - 28 = 0$ and find the point of contact.



43. Three circles touch externally as shown.
The centres of the circles are collinear
and the equations of the two smaller circles are:

$(x - 2)^2 + (y - 9)^2 = 9$ and $x^2 + y^2 - 28x + 14y + 236 = 0$

Find the equation of the larger circle.