Higher Mathematics October Revision

1. A sequence is defined by the recurrence relation $U_n = 0.4U_{n-1} + 10$, where $U_1 = 34$

        a. Calculate the value of U_0 and U_2.
        b. State why this sequence has a limit.
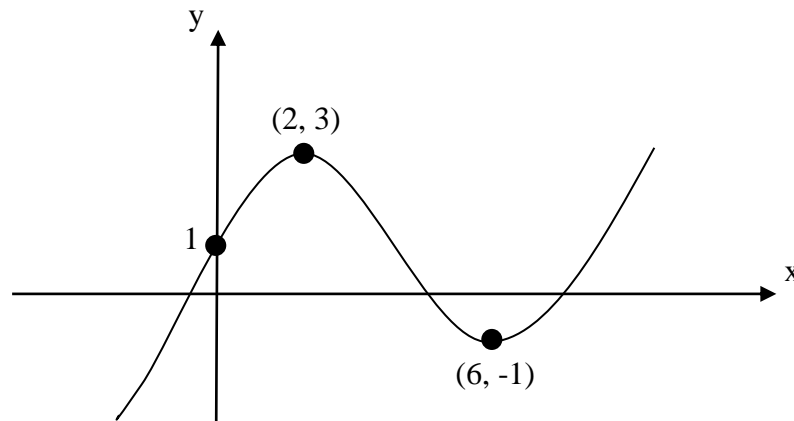   c. Hence, calculate the limit of this sequence when $n \to \infty$.

2. A straight line, which passes through the point (3, -2), is parallel to the straight line with the equation $5y - 3x + 1 = 0$.

Find the equation of this straight line.

3. A sequence is defined by the recurrence relation $U_{n+1} = kU_n + 9$.

If the limit of the recurrence relation is 24, find the value of k.

4. The diagram below shows the graph of $y = f(x)$.

On separate diagrams, sketch and annotate the following graphs:

        a. $y = f(x) + 4$ b. $y = f(x - 1) - 2$

        c. $y = -f(x)$ d. $y = f(2x) + 2$

5. The vertices of the triangle PQR are P(2, 6), Q(-4, -4) and R(-3, 7).

    a. Find the equation of the median from R.
    b. Find the equation of the altitude from Q.
    c. The median from R and altitude from Q intersect at point T. Find the coordinates of the point of intersection.

6. A curve has the equation $y = 4x^2 - 5x + 1$.

A tangent to this curve has a gradient of 3. Find the equation of this tangent.

7. The distance, d metres, travelled by a rocket is calculated using the formula $d(t) = 10t^3 - 7t$, where t is the time in seconds after the rocket takes off.

Calculate the speed of the rocket after 8 seconds.

8. A function f is defined by $f(x) = x^4 + 8x^3 - 6$.

    a. Find the coordinates of the points where the graph of $y = f(x)$ crosses the y-axis.
    b. Find the stationary points and determine their nature.

9. For what values of x is the function $h(x) = 2x^3 + 3x^2 - 12x + 1$ decreasing?

10. Find the exact value of:

    a. $\tan 60^\circ$ b. $\sin 225^\circ$ c. $\cos \dfrac{5\pi}{6}$

11. A curve has the equation $y = (x - 2)(x^2 + 6x - 1)$.

Find the equation of the tangent to this curve at the point where x = -1

12. A function is given by $f(x) = x^3 + ax^2 + bx + 2$.

Given that $(x - 1)$ and $(x + 2)$ are factors of $x^3 + ax^2 + bx + 2$, find the values of a and b.

13. Find the inverse of the following functions:

    a. $f(x) = 2x - 1$ b. $g(x) = 5x^2 - 3$ c. $p(x) = 7 + 4\sqrt{x}$

14. A function is given by $2x^3 - x^2 - 5x - 2$.

    a. Show that $(x + 1)$ is a factor of the function.
    b. Hence, fully factorise the function.

15. A sequence is defined by the recurrence relation $U_n = 0.7U_{n-1} + 18$, where $U_1 = 3$

    a. Calculate the value of U_0 and U_2.
    b. State why this sequence has a limit.
    c. Hence, calculate the limit of this sequence when $n \rightarrow \infty$.

16. A function has the equation $f(x) = x^3 - 6x^2 + 9x$. A tangent to the curve is drawn at the point where x = 0.

    a. Find the equation of the tangent to the curve.
    b. This tangent meets the curve at a second point P. Find the coordinates of point P.

17. A triangle has vertices A(-1, 12), B(-2, -5) and C(7, -2).

    a. Find the equation of the median BD.
    b. Find the equation of the altitude AE.
    c. Hence, find the coordinates of the point of intersection between the 2 lines.

18. Given that x = -2 and x = 1 are two roots of the equation $x^3 + px^2 - 6x + q = 0$, establish values of p and q and hence find the third root of the equation.

19. A function is defined by the formula $f(x) = 4x^2(x - 3)$ where x is a member of Real numbers.

    a. Write down the coordinates of the points where the curve with equation
       y = f(x) meets the coordinates axes.
    b. Find the stationary points of y = f(x) and determine their nature.
    c. Sketch the curve.

20. Find the interval in which $f(x) = x^3 + 3x^2 - 9x + 5$ is decreasing.

21. A function is given by $f(x) = 3x^2 - 2x^3$.

Determine the interval where the function is increasing.

22. If $f(x) = 2x - 5$ and $g(x) = 2x^2$

    a. Find f(g(x))   b. g(f(x))

23. Functions are given by:

    $f(x) = x^2 - x$ $g(x) = 3x + 1$ $h(x) = \dfrac{1}{3}(x - 7)$

Find   a. f(g(x))
           b. h(g(x))
           c. f(h(x))

24. A function is given by $y = \sqrt{x - 3}$.

Find the restriction on the domain for the function above.

25. The point A has coordinates (7,4).

The straight lines with equations x + 3y + 1 = 0 and 2x + 5y = 0 intersect at point B.

    a. Find the gradient of AB.
    b. Hence show that AB is perpendicular to only one of these 2 lines.

26. Solve the following equations for $0 \leq x \leq 360$:

    a. $4\sin 2x - 1 = 0$ b. $6\cos 4x + 1 = 0$

27. Find the exact value of:

    a. $\tan 45^{\circ}$ b. $\sin 150^{\circ}$ c. $\cos 210^{\circ}$

28. Convert the following into radians:

    a. 90° b. 300° c. 15°

29. Solve the following equations for $0 \leq x \leq 360$:

$$7\sin^2 x - 2\sin x - 5 = 0$$

30. Express in the form $p(x + q)^2 + r$:

    a. $y = 2x^2 - 8x + 2$ b. $y = 5x^2 + 10x + 1$

    c. $y = 3x^2 - 9x - 1$ d. $y = 2 - 4x - x^2$

31. Find the values of b if $x^2 + bx + 36 = 0$ has two real and equal roots.

32. Find the range of values of d if $x^2 + 6x + d^2 = 0$ has no real roots.

33. Find the value of k for which the equation $x^2 + 4(k - 1)x = -4$ has real roots.

34. Prove that $x^2 - (m + 3)x + 2m + 3 = 0$ has no real roots for $-1 < m < 3$.

35. Prove that the line $y = x - 1$ is a tangent to the curve $y = x^2 - x$ and find the point of contact.