S4 - October Revision Solutions

Fractions

1a) $2\frac{1}{3} + 3\frac{2}{5}$

$= \frac{7}{3} + \frac{17}{5}$

$= \frac{35+51}{15}$

$= \frac{86}{15}$

b.) $\frac{\cancel{9}^{3}}{\cancel{4}_{4\cancel{16}}} \times \frac{\cancel{4}^{1}}{\cancel{15}_{5}}$

$= \frac{3}{20}$

c.) $\frac{4}{5} \div \frac{16}{25}$

$= \frac{\cancel{4}^{1}}{\cancel{5}_{1}} \times \frac{\cancel{25}^{5}}{\cancel{16}_{4}}$

$= \frac{5}{4}$

d.) $\frac{1}{2} + \frac{\cancel{3}^{1}}{\cancel{4}_{2}} \times \frac{\cancel{2}^{1}}{\cancel{3}_{1}}$

$= \frac{1}{2} + \frac{1}{2}$

$= 1$

e.) $\frac{2}{3}\left(\frac{2}{3} + \frac{1}{4}\right)$

$= \frac{2}{3}\left(\frac{8+3}{12}\right)$

$= \frac{\cancel{2}^{1}}{3} \times \frac{11}{\cancel{12}_{6}}$

$= \frac{11}{18}$

f.) $\frac{4}{5} - \frac{\cancel{3}^{1}}{\cancel{4}_{1}} \times \frac{\cancel{8}^{2}}{\cancel{9}_{3}}$

$= \frac{4}{5} - \frac{2}{3}$

$= \frac{12-10}{15}$

$= \frac{2}{15}$

Expanding Brackets.

2a.) $(3a-2)(4a+5)$

$= 12a^2 + 15a - 8a - 10$

$= 12a^2 + 7a - 10$

b.) $3 - 2(x+3)(x-4)$

$= 3 - 2(x^2 + x - 12)$

$= 3 - 2x^2 - 2x + 24$

$= -2x^2 - 2x + 27$

c.) $3(x+2)^2$

$= 3(x+2)(x+2)$

$= 3(x^2 + 4x + 4)$

$= 3x^2 + 12x + 12$

d.) $(x+4)(x^2 + 4x - 1)$

$= x^3 + 4x^2 - x + 4x^2 + 16x - 4$

$= x^3 + 8x^2 + 15x - 4$

e.) $(2x-3)(3x^2 + 5x - 2)$

$= 6x^3 + 10x^2 - 4x - 9x^2 - 15x + 6$

$= 6x^3 + x^2 - 19x + 6$

Equations / Inequations

3a) $3(2x+1) = 2(2x+5)$

$6x + 3 = 4x + 10$

$2x = 7$

$x = \frac{7}{2}$

b) $\frac{3x(x+4)}{3} + 3^{x}1 = 3^{x}5$

$x + 4 + 3 = 15$

$x + 7 = 15$

$x = 8$.

c.) $\frac{{}^{4}12x\,x}{3} + \frac{{}^{3}12x\,3x}{4} = 12 \times 8$

$4x + 9x = 96$

$13x = 96$

$x = \frac{96}{13}$

d.) $3x + 2 > 2x + 8$

$x > 6$

e.) $2(2x-3) < 9x$

$4x - 6 < 9x$

$4x - 9x < 6$

$-5x < 6$

$x > -\frac{6}{5}$

f.) $\frac{10}{x} > 5$

$10 > 5x$

$5x < 10$

$x < 2$.

Percentages

4.) $1.0555^{4} \times 188500$

$= 233961.45$

$= €234\,000$

5.) $1.0205^{3} \times 1200$

$= 1275.32$

$CI = €75.32$

$= €75$

6.) $\% = \frac{151-138}{138} \times 100$

$\% = 9.4\%$

7.) $115\% = €32500$

$1\% = €32500/115$

$100\% = €28260.87$

8.) $80\% = 4800$

$1\% = 4800/80$

$100\% = 6000$ people.

Linear Relationships

9a) $2y = 4x + 5$

$y = 2x + \frac{5}{2}$

$m = 2 \quad c = \frac{5}{2}$

b.) $3y - 2x - 4 = 0$

$3y = 2x + 4$

$y = \frac{2}{3}x + \frac{4}{3}$

$m = \frac{2}{3} \quad c = \frac{4}{3}$

c.) $3x + 5y - 3 = 0$

$5y = -3x + 3$

$y = -\frac{3}{5}x + \frac{3}{5}$

$m = -\frac{3}{5} \quad c = \frac{3}{5}$

10a) P: $(3,5)$ Gr: $m = \frac{4}{2} = 2$ E: $y - 5 = 2(x - 3)$, $y - 5 = 2x - 6$, $y - 2x = -1$

b) P: $(1,4)$ Gr: $m = \frac{2}{3}$ E: $y - 4 = \frac{2}{3}(x - 1)$, $3y - 12 = 2x - 2$, $3y - 2x = 10$

c.) P: $(3,5)$ Gr: $m = \frac{9}{5}$ E: $y - 5 = \frac{9}{5}(x - 3)$, $5y - 25 = 9x - 27$, $5y - 9x = -2$

11a) when $x = 0$, $y = 4$, $(0,4)$ — when $y = 0$, $x = -\frac{4}{3}$, $(-\frac{4}{3}, 0)$

b.) when $x = 0$, $y = -1$, $(0,-1)$ — when $y = 0$, $x = \frac{2}{3}$, $(\frac{2}{3}, 0)$

c.) when $x = 0$, $y = \frac{5}{2}$, $(0, 5/2)$ — when $y = 0$, $x = 5/3$, $(5/3, 0)$

12.a.) P: $(20,25)$

Gr: $m = \frac{85-25}{75-20}$, $m = \frac{60}{55}$, $m = \frac{12}{11}$

E: $y - 25 = \frac{12}{11}(x - 20)$

$11y - 275 = .12x - 240$

$11y - 12x = 35$

$11M - 12P = 35$

b.) $P = 66 \rightarrow 11M - 12 \times 66 = 35$

$11M = 827$

$M = 75.2\%$

Factorising

13a) $2x^2 - 8x$

$2x(x-4)$

b.) $12a^2 - 15ab$

$3a(4a-5b)$

c.) $x^2 - 36$

$= (x-6)(x+6)$

d.) $1 - a^2$

$= (1-a)(1+a)$

e.) $9x^2 - 16y^2$

$= (3x-4y)(3x+4y)$

f.) $3x^2 - 27$

$= 3(x^2-9)$

$= 3(x-3)(x+3)$

g.) $x^2 + 2x - 15$

$= (x+5)(x-2)$

h) $x^2 - x - 12$

$= (x-4)(x+3)$

i) $x^2 + 10x - 24$

$= (x+12)(x-2)$

j.) $2x^2 - 7x + 3$

$= (2x-1)(x-3)$

k.) $7x^2 - 29x + 4$

$= (7x-1)(x-4)$

l.) $5x^2 - 17x + 6$

$= (5x-2)(x-3)$

Circle Geometry

14a) $\frac{AL}{\pi \times 10} = \frac{135}{360}$

$AL = \frac{135 \times \pi \times 10}{360}$

$AL = 11.8\text{cm}$

b) $\frac{AL}{\pi \times 16.4} = \frac{159}{360}$

$AL = \frac{159 \times \pi \times 16.4}{360}$

$= 22.8\text{cm}$

15a) $\frac{\text{Area}}{\pi \times 5^2} = \frac{135}{360}$

$\text{Area} = \frac{135 \times \pi \times 5^2}{360}$

$\text{Area} = 29.5\text{cm}^2$

b) $\frac{\text{Area}}{\pi \times 8.2^2} = \frac{159}{360}$

$\text{Area} = \frac{159 \times \pi \times 8.2^2}{360}$

$\text{Area} = 93.3\text{cm}^2$

16) $\frac{\text{Area}_1}{\pi \times 15^2} = \frac{105}{360}$ $\quad \frac{\text{Area}_2}{\pi \times 6^2} = \frac{105}{360}$

$\text{Area}_1 = \frac{105 \times \pi \times 15^2}{360}$ $\quad \text{Area}_2 = \frac{105 \times \pi \times 6^2}{360}$

$\text{Area}_1 = 206.2\text{cm}^2$ $\quad = 32.99\text{cm}^2$

$\therefore \text{Area} = 206.2 - 32.99$

$= 173.2\text{cm}^2$

17a) $p = 55°$

$q = 90°$

$r = 45°$

b) $m = 65°$

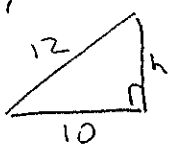
$n = 90°$

$h = 25°$

$k = 90°$

$g = 25°$

18)



$h^2 = 12^2 - 10^2$

$h^2 = 144 - 100$

$h = \sqrt{44}$

$h = 6.6\text{ cm}$

$\therefore x = 6.6 + 12$

$= 18.6\text{cm}$

Complete the Square

19a) $(x+2)^2 - 2^2 + 5$

$= (x+2)^2 + 1$

b) $(x-3)^2 - 9 - 4$

$= (x-3)^2 - 13$

c) $(x+\frac{5}{2})^2 - \frac{25}{4} - \frac{8}{4}$

$= (x+\frac{5}{2})^2 - \frac{33}{4}$

Algebraic Fractions

a) $\frac{3x+4}{x^2}$

b) $\frac{12b-4a}{6ab}$

$= \frac{6b-2a}{3ab}$

c) $\frac{4(2x-2)+3(x+1)}{(x+1)(2x-2)}$

$= \frac{11x-5}{(x+1)(2x-2)}$

d) $\frac{2(x+2)+3(x+5)}{6}$

$= \frac{5x+19}{6}$

e) $\frac{3(2a-1)-2(a+4)}{6}$

$= \frac{4a-11}{6}$

f) $\frac{5(x^2-2)+4(2x^2+3)}{20}$

$= \frac{13x^2+2}{20}$

g) $\frac{(x-5)(x+5)}{(x+5)(x+2)}$

$= \frac{x-5}{x+2}$

h) $\frac{(x+4)(x+2)}{(x+4)(x+3)}$

$= \frac{x+2}{x+3}$

i) $\frac{(x+8)(x-4)}{(x+8)(x+2)}$

$= \frac{x-4}{x+2}$

Surds

a) $\sqrt{9}\sqrt{2} + \sqrt{2} + \sqrt{25}\sqrt{2}$

$= 3\sqrt{2} + \sqrt{2} + 5\sqrt{2}$

$= 9\sqrt{2}$

b) $6\sqrt{3} - \sqrt{9}\sqrt{3} + \sqrt{100}\sqrt{3}$

$= 6\sqrt{3} - 3\sqrt{3} + 10\sqrt{3}$

$= 13\sqrt{3}$

c) $\sqrt{49}\sqrt{2} + \sqrt{16}\sqrt{2} - \sqrt{2}$

$= 7\sqrt{2} + 4\sqrt{2} - \sqrt{2}$

$= 10\sqrt{2}$

d) $\sqrt{16}\sqrt{3} + 2\sqrt{3} + \sqrt{25}\sqrt{3}$

$= 4\sqrt{3} + 2\sqrt{3} + 5\sqrt{3}$

$= 11\sqrt{3}$

22a) $\frac{4}{\sqrt{4}}$

$= \frac{4}{2}$

$= 2$

b) $\frac{2\sqrt{5}}{5}$

c) $\frac{3\sqrt{6}}{6_2}$

$= \frac{\sqrt{6}}{2}$

Indices

23a) $12a^6$

b) $8b^3 \times 4b^{-6}$

$= 32b^{-3}$

c) $125m^6 \div 10m^2$

$= 12.5m^4$ or $\frac{25}{2}m^4$

d) $\frac{8c^8}{6c^{10}}$

$= \frac{4}{3c^2}$

e) $\frac{6x^4}{4x^7}$

$= \frac{3}{2x^3}$

f) $15a^{\frac{23}{21}}$

24a) $\sqrt[3]{8^2}$

$= 2^2$

$= 4$

b) $\sqrt{9^3}$

$= 3^3$

$= 27$

c) $\frac{1}{\sqrt{8}}$

$= \frac{1}{2\sqrt{2}}$

d) 1

e) $\sqrt{4} + 3^\circ$

$= 2 + 1$

$= 3$

f) $(2x)^\circ$

$= 1$

Change the subject

25a) $x = \frac{10 - y}{2}$

b) $x = \sqrt{\frac{y - 4}{3}}$

c) $x = \sqrt{\frac{1 + z}{4}}$

d) $x = \frac{3y}{2}$

e) $x = \frac{5z}{y^2}$

f) $\sqrt{2xy} = 12$

$2xy = 144$

$x = \frac{72}{y}$

Pythagoras

26a)

LS	SS
5^2	3^2+4^2
$=25$	$=25$

By Converse of Pythagoras this is a right angled triangle as $5^2 = 3^2 + 4^2$.

b)

LS	SS
11^2	7^2+8^2
$=121$	$=113$

By Converse of Pythagoras this is not a right angled triangle as $11^2 \neq 7^2 + 8^2$.

c)

LS	SS
13^2	12^2+5^2
$=169$	$=169$

By Converse of Pythagoras this is a right angled triangle as $13^2 = 12^2 + 5^2$

Statistics

27a) $L = 12$
$Q_1 = 14$
$Q_2 = 15$
$Q_3 = 19$
$H = 20$

$SIQR = \frac{19-14}{2}$

$= \frac{5}{2}$

$= 2.5$

b) 5, 7 | 8, 10 | 12, 13 | 14, 21

$L = 5$
$Q_1 = 7.5$
$Q_2 = 11$
$Q_3 = 13.5$
$H = 21$

$SIQR = \frac{13.5 - 7.5}{2}$

$= 3$

28a) $\bar{x} = 20$

x	$(x-\bar{x})$	$(x-\bar{x})^2$
14	-6	36
25	5	25
14	-6	36
28	8	64
19	-1	1
		$\Sigma = 162$

$s.d. = \sqrt{\frac{162}{5-1}} = \underline{\underline{6.4}}$

b) $\bar{x} = 25.5$

x	$(x-\bar{x})$	$(x-\bar{x})^2$
21	-4.5	20.25
24	-1.5	2.25
26	0.5	0.25
24	-1.5	2.25
23	-2.5	6.25
35	9.5	90.25
		$\Sigma = 121.5$

$s.d. = \sqrt{\frac{121.5}{6-1}}$

$= \underline{\underline{4.9}}$

Simultaneous equations

29a) $3a + 2b = 10 \quad \times 3$

$5a - 3b = 4 \quad \times 2$

$9a + 6b = 30$

$+ \quad 10a - 6b = 8$

$19a = 38$

$a = 2$

$3(2) + 2b = 10$

$2b = 4$

$b = 2$

b) $3c - 4d = -9$

$8c + 4d = 20$

$11c = 11$

$c = 1$

$3(1) - 4d = -9$

$-4d = -12$

$d = 3$

c) $4f + 2g = 6 \quad (\times 5)$

$3f + 5g = 1 \quad (\times -2)$

$20f + 10g = 30$

$+ \quad -6f - 10g = -2$

$14f = 28$

$f = 2$

$4(2) + 2g = 6$

$2g = -2$

$g = -1$

30a) $2a + 3c = 41 \quad (\times 4)$

b) $3a + 4c = 58 \quad (\times -3)$

c) $8a + 12c = 164$

$-9a - 12c = -174$

$-a = -10$

$a = 10$

$2(10) + 3c = 41$

$3c = 21$

$c = 7$

1 adult costs £10

1 child costs £7

VOLUME

31) $V_{HS} = \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \pi \times 7^3$

$= 718.4\text{cm}^3$

32) $V_1 = \frac{1}{3} \times \pi \times 6^2 \times 5.6$

$= 211.1\text{cm}^3$

$V_2 = \frac{1}{3} \times \pi \times 1^2 \times 1.9$

$= 1.99\text{cm}^3$

$V_{SHADED} = 211.1 - 1.99$

$= 209.1$

$= 209\text{cm}^3$

Function Notation

33a) $f(2) = 3(2)^2 + 9(2)$

$= 12 + 10$

$= \underline{22}$

b) $g(-3) = 2(-3)^2 + 6(-3)$

$= 18 - 18$

$= \underline{0}$

c) $f(p) = 3p - 4$

$\therefore \quad 3p - 4 = 14$

$3p = 18$

$p = \underline{6}$