

Mathematics Department

Higher Exercise 1

1. A sequence is defined by the recurrence relation $U_{n+1} = 3U_n - 1$, where $U_0 = 2$.

Calculate the next three consecutive terms in the sequence.

- 2. A sequence is defined by the recurrence relation $U_{n+1} = 0.5U_n + 3$ where $U_1 = 2$.
 - a. Calculate the value of U_2 and U_3 .
 - b. Explain why the sequence above has a limit.
 - c. Find the limit of this sequence when n $\rightarrow \infty$.
- 3. A straight line passing through the point (3, -1) is parallel to the line 5y 2x + 1 = 0.

Find the equation of this line.

- 4. A sequence is defined by the recurrence relation $U_n = 0.8U_{n-1} + 1000$, where $U_1 = 500$.
 - a. Calculate the value of U_0 and U_2 .
 - b. What is the smallest value of n for which $U_n > 3000$?
 - c. Find the limit of this sequence when $n \rightarrow \infty$.
- 5. Two functions, f and g, defined on suitable domains, are given by f(x) = 3x 1and g(x) = 7 - 2x.

Find f(g(x)) in its simplest form.

6. A sequence is defined by the recurrence relation $U_{n+1} = kU_n + 7$.

If the limit of the recurrence relation is 21, find the value of k.

- 7. A lake next to a waste factory currently contains approximately 30 tonnes of pollutant. Due to health regulations the factory runs a filtration process where they remove 75% of the waste each month, however an extra 1.5 tonnes is released into the lake over the same month.
 - a. Establish a recurrence relation to describe this situation.
 - b. Health inspectors inform the factory that a level of 2.1 tonnes of waste or less will be acceptable. In the long run will the factory reach an acceptable level of waste in the lake?