NATIONAL
QUALIFICATIONS

Mathematics

CFE Higher Prelim Examination 2015/2016

Paper 1

Assessing Pre CFE Units 1, 2 & Vectors

Time allowed - 1 hour 10 minutes

Read carefully

Calculators may NOT be used in this paper.

1. Full credit will be given only where the solution contains appropriate working.
2. Answers obtained by readings from scale drawings will not receive any credit.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Trigonometric formulae:

$$\begin{aligned}
\sin(A \pm B) &= \sin A\cos B \pm \cos A\sin B \\
\cos(A \pm B) &= \cos A\cos B \mp \sin A\sin B \\
\sin 2A &= 2\sin A\cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

Scalar Product: $\boldsymbol{a} \cdot \boldsymbol{b} = |\boldsymbol{a}|\,|\boldsymbol{b}|\cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$.

or

$\boldsymbol{a} \cdot \boldsymbol{b} = \boldsymbol{a}_1\boldsymbol{b}_1 + \boldsymbol{a}_2\boldsymbol{b}_2 + \boldsymbol{a}_3\boldsymbol{b}_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\ dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

All questions should be attempted

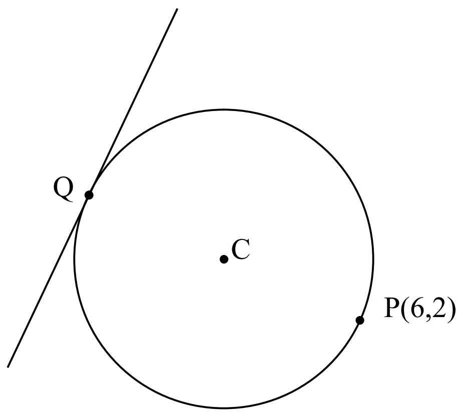
1. Given that $f(x) = 2x - \frac{1}{x}$, $x > 0$, evaluate $f'(-1)$. **4**

2. A function is given by $f(x) = x^3 + ax^2 - x + 2$.

(a) Given that $(x - 2)$ is a factor of the function, find the value of a. **3**

(b) Hence, fully factorise the function. **1**

3. The circle below, with centre C, has as its equation $x^2 + y^2 - 4x - 8y = 0$.



(a) Show that the point P(6, 2) lies on the circumference of this circle. **1**

(b) Hence find the equation of the tangent to the circle at the point Q, where PQ is a diameter of the circle. **5**

4. A certain acute angle, A , is such that $\tan A = \frac{1}{\sqrt{2}}$.

(a) Show clearly that the exact value of $\sin 2A$ is $\frac{2}{3}\sqrt{2}$. **4**

(b) Hence show that $\tan A + \sin 2A = \frac{7}{6}\sqrt{2}$. **2**

5. Two recurrence relationships are defined as follows with a and b being constants and taking the same values in each relationship.

$$U_{n+1} = aU_n + 3b \qquad\qquad V_{n+1} = (2a)V_n + b$$

(a) Given that both recurrence relationships have a limit of 60, find the values of a and b. **4**

(b) If both of the relationships have the same initial value of 20 find the difference between the terms U_2 and V_2. **3**

6. Points P, Q and R are (2, -3, 4), (6, 1, 2) (12, 7, -1) respectively.

Shows that the points are collinear. **3**

7. Find $\int x\left(10\sqrt{x} + 6x\right)\,dx$. **4**

8. An equation is given as $\dfrac{4k + 3x}{x} = \dfrac{4 - x}{k}$, where $x \neq 0$, $k \neq 0$.

(a) Show that this equation can be written in the form

$$x^2 + (3k - 4)x + 4k^2 = 0.$$ **3**

(b) Hence find the values of k which would result in the above equation having **equal roots**. **5**

9. A function is defined as $f(x) = \dfrac{40}{x^2 - 6x + 13}$, for $x \in R$.

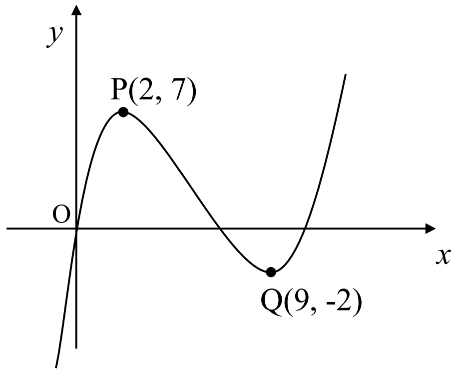
Express the function in the form $f(x) = \dfrac{40}{(x-a)^2 + b}$, and hence state the maximum value of the function *f*. **4**

10. Part of the graph of the function $y = f(x)$ is shown opposite.

Sketch the graph of the related function

$$y = f(-x) + 2$$

showing clearly the image positions of P and Q.

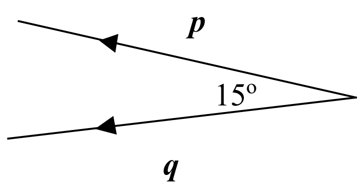


3

11. For what values of x is the function $f(x) = x^3 - 12x + 2$ decreasing? **4**

12. (a) Find cos 15° **4**

The diagram shows vectors $\boldsymbol{p}$ and $\boldsymbol{q}$, where $|\boldsymbol{p}| = 2$ and $|\boldsymbol{q}| = \sqrt{6}$.



(b) Hence, find the value of $\boldsymbol{p}.(\boldsymbol{p} + \boldsymbol{q})$. **4**

[END OF QUESTION PAPER 1]

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Mathematics
CFE Higher Prelim Examination 2015/2016
Paper 2
Assessing Pre CFE Units 1, 2 & Vectors

Time allowed - 1 hour 30 minutes

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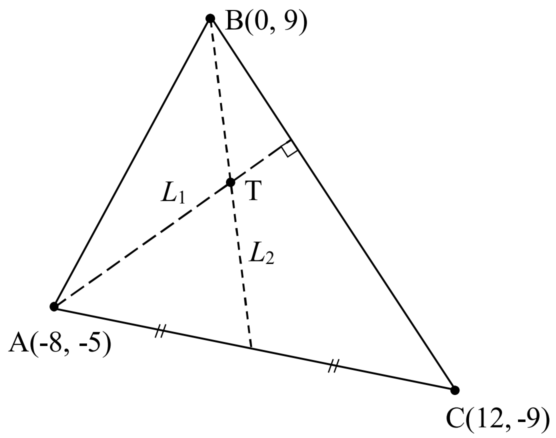
All questions should be attempted

1. Triangle ABC has vertices A(-8, -5), B(0, 9) and C(12, -9).

L_1 is the altitude from A to BC and L_2 is the median from B to AC.



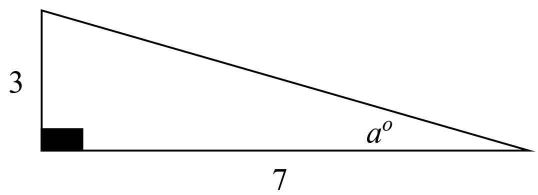
(a) Find the equation of the altitude L_1. **3**

(b) Find the equation of the median L_2. **3**

(c) Find the coordinates of T, the point of intersection of L_1 and L_2. **3**

2. Given $\mathbf{p} = -3\mathbf{i} - 5\mathbf{j} + \mathbf{k}$ and $\mathbf{q} = 4\mathbf{i} + m\mathbf{j} - \mathbf{k}$ are perpendicular, find the value of m. **3**

3. Triangle EFG is shown opposite.



(a) Find the exact values of (i) $\sin 2a$. **2**
(ii) $\cos 2a$. **2**

(b) Hence, find the exact value of $\tan 2a$. **2**

4. Two functions f and g are defined on the set of real numbers as follows :

$$f(x)=3x-4 \quad \text{and} \quad g(x)=\frac{x+16}{9} \; .$$

(a) Evaluate $f\big(g(-4)\big)$. **1**

(b) Find an expression , in its simplest form, for $g\left(f(x)\right)$. **2**

(c) Hence verify that $f^{-1}(x) \;=\; g\left(f(x)\right)$ **2**

5. Solve $2\sin 2x^\circ - \cos x^\circ = 0$ where $0 \le x \le 2\pi$. **5**

6. Two circles, which do not touch or overlap, have as their equations

$$(x-4)^2+(y-10)^2=50 \quad \text{and} \quad x^2+y^2+8x-4y+18=0 \quad .$$

(a) Show that the **exact** distance between the centres of the two circles is $8\sqrt{2}$ units. **3**

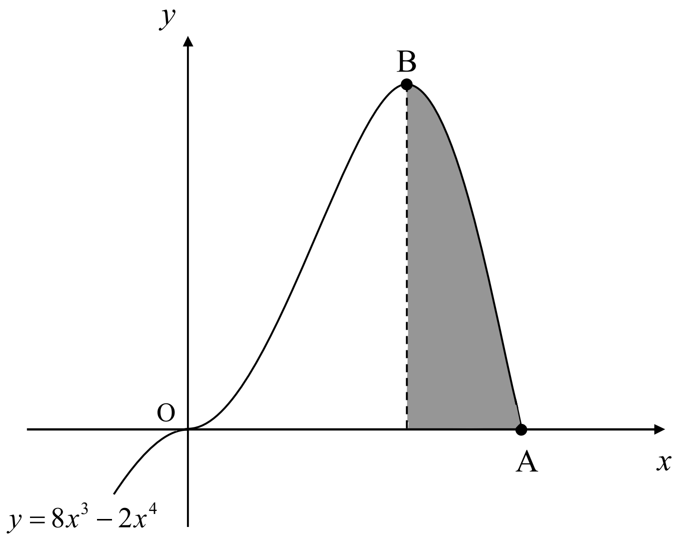
(b) Hence show that the shortest distance between the two circles is equal to the diameter of the smaller circle. **4**

7. Triangle EFV has vertices E(2, 0, -7), F(2, 2, -7) and V(1, 1, 3).

(a) Find the components of $\overrightarrow{VF}$ and $\overrightarrow{VE}$. **2**

(b) Calculate angle EVF. **5**

8. Part of the graph of the curve with equation $y = 8x^3 - 2x^4$ is shown below.



(a) Find the coordinates of the point A. **2**

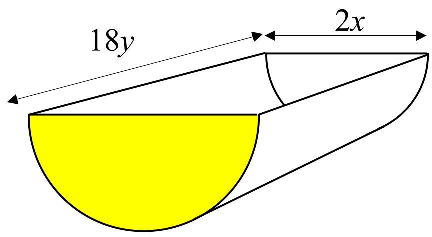
(b) Find the coordinates of the stationary point B. **4**

(c) Hence calculate the shaded area, in square units. **5**

9. Show that the line with equation $2x - y + 5 = 0$ is a tangent to the circle $x^2 + y^2 = 5$ and find the point of contact. **5**

10. An open topped animal feeding trough is in the shape of a prism with a semi-circular cross section and dimensions $18y$ by $2x$ as shown in the diagram.

All dimensions are in centimetres.



The volume of the trough can be found using the formula $V = \frac{1}{2}\pi r^2 l$, where r is the radius and l is the length of the trough.

(a) (i) Given that the trough has to hold 45,000cm^3 of feed, show clearly that y can be expressed in terms of x as

$$y = \frac{5000}{\pi x^2}.$$ **2**

(ii) Hence show that its internal surface area, A(x), is given by:

$$A(x) = \pi x^2 + \frac{90000}{x}.$$ **4**

(b) Find the value of x which will minimise this surface area and calculate the corresponding value of y when x takes this value.
Give your answers correct to one decimal place. **6**

[END OF QUESTION PAPER 2]