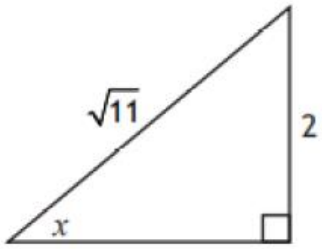
Higher : Addition Formulae — Revision

2018 P1 Q13

The right-angled triangle in the diagram is such that $\sin x = \frac{2}{\sqrt{11}}$ and $0 < x < \frac{\pi}{4}$.



(a) Find the exact value of:

(i) $\sin 2x$

(ii) $\cos 2x$.

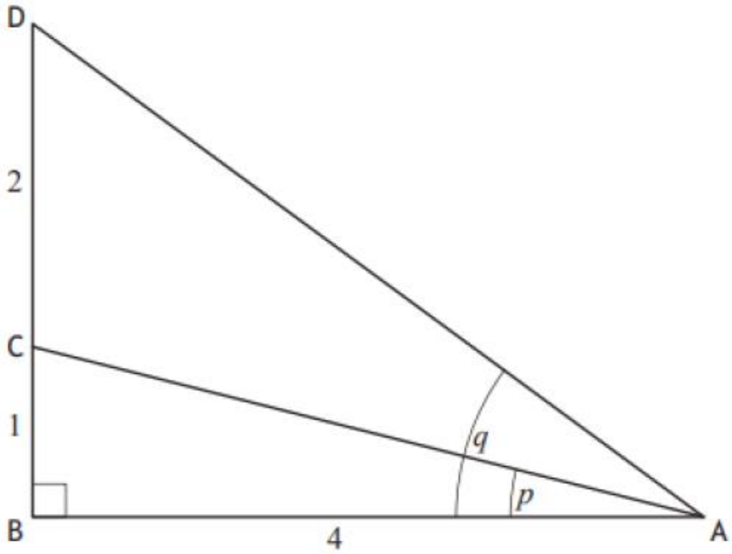
(b) By expressing $\sin 3x$ as $\sin(2x + x)$, find the exact value of $\sin 3x$.

2017 P2 Q6

Solve $5\sin x - 4 = 2\cos 2x$ for $0 \leq x < 2\pi$.

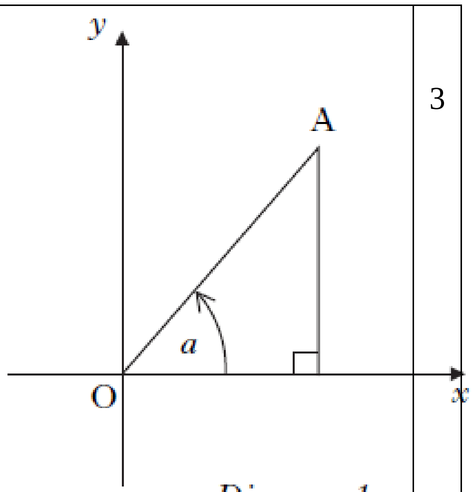
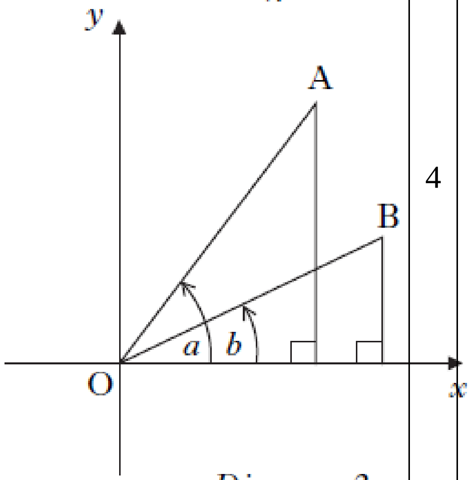
2016 P1 Q13

Triangle ABD is right-angled at B with angles BAC = p and BAD = q and lengths as shown in the diagram below.



Show that the exact value of $\cos(q - p)$ is $\frac{19\sqrt{17}}{85}$.

2015 P1 Q10	Given that $\tan 2x = \frac{3}{4}$, $0 < x < \frac{\pi}{4}$, find the exact value of (a) $\cos 2x$ (b) $\cos x$.	
2014 P2Q6	Solve the equation $\sin x - 2\cos 2x = 1$ for $0 \leq x < 2\pi$.	5
2013 P2 Q8	Solve algebraically the equation $\sin 2x = 2\cos^2 x$ for $0 \leq x < 2\pi$	6
2011 P1 Q23	(*a*) Solve $\cos 2x° - 3\cos x° + 2 = 0$ for $0 \leq x < 360$. (*b*) Hence solve $\cos 4x° - 3\cos 2x° + 2 = 0$ for $0 \leq x < 360$.	5 2
2010 P2 Q4	Solve $2\cos 2x - 5\cos x - 4 = 0$ for $0 \leq x < 2\pi$.	5

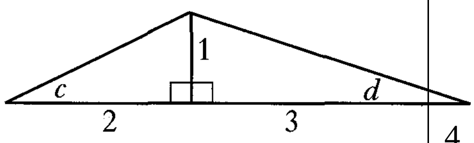
2010 P1 Q23	(*a*) Diagram 1 shows a right angled triangle, where the line OA has equation $3x - 2y = 0$. (i) Show that $\tan a = \frac{3}{2}$. (ii) Find the value of $\sin a$.  *Diagram 1*	3
	(*b*) A second right angled triangle is added as shown in Diagram 2. The line OB has equation $3x - 4y = 0$. Find the values of $\sin b$ and $\cos b$.  *Diagram 2*	4
	(*c*) (i) Find the value of $\sin(a - b)$. (ii) State the value of $\sin(b - a)$.	4
2009 P1 Q24	(*a*) Using the fact that $\frac{7\pi}{12} = \frac{\pi}{3} + \frac{\pi}{4}$, find the exact value of $\sin\left(\frac{7\pi}{12}\right)$. (*b*) Show that $\sin(A + B) + \sin(A - B) = 2\sin A \cos B$. (*c*) (i) Express $\frac{\pi}{12}$ in terms of $\frac{\pi}{3}$ and $\frac{\pi}{4}$. (ii) Hence or otherwise find the exact value of $\sin\left(\frac{7\pi}{12}\right) + \sin\left(\frac{\pi}{12}\right)$.	
2008 P2	**5.** Solve the equation $\cos 2x° + 2\sin x° = \sin^2 x°$ in the interval $0 \le x < 360$.	5

2007 P1 Q6

6. Solve the equation $\sin 2x° = 6\cos x°$ for $0 \leq x \leq 360$. 4

2007 P2 Q2

2. The diagram shows two right-angled triangles with angles c and d marked as shown.



(a) Find the exact value of $\sin(c + d)$. 4

(b) (i) Find the exact value of $\sin 2c$.

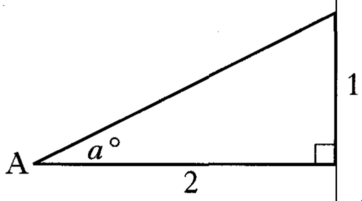
(ii) Show that $\cos 2d$ has the same exact value. 4

2006 P1 Q7

7. Solve the equation $\sin x° - \sin 2x° = 0$ in the interval $0 \leq x \leq 360$. 4

2006 P2 Q8

8. The diagram shows a right-angled triangle with height 1 unit, base 2 units and an angle of $a°$ at A.



(a) Find the exact values of:

(i) $\sin a°$;

(ii) $\sin 2a°$. 4

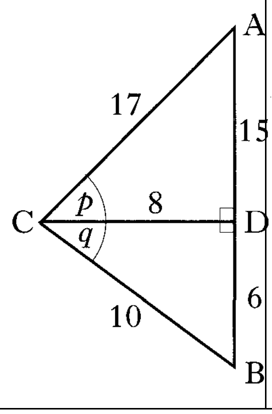
(b) By expressing $\sin 3a°$ as $\sin(2a + a)°$, find the exact value of $\sin 3a°$. 4

2005 P1 Q9

9. If $\cos 2x = \frac{7}{25}$ and $0 < x < \frac{\pi}{2}$, find the exact values of $\cos x$ and $\sin x$. 4

2005 P2

2. Triangles ACD and BCD are right-angled at D with angles p and q and lengths as shown in the diagram.



(a) Show that the exact value of $\sin(p + q)$ is $\frac{84}{85}$. 4

(b) Calculate the exact values of:

(i) $\cos(p + q)$;

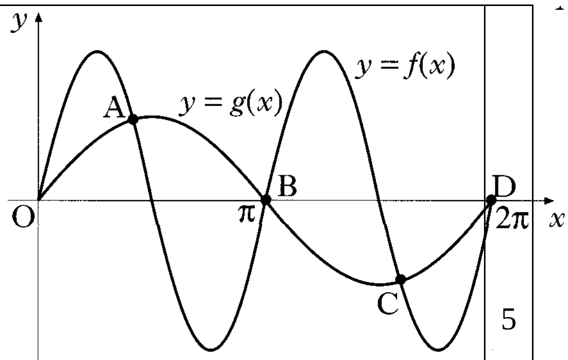
(ii) $\tan(p + q)$. 3

2005 P2 Q8

8. Two functions, f and g, are defined by $f(x) = k\sin 2x$ and $g(x) = \sin x$ where $k > 1$.

The diagram shows the graphs of $y = f(x)$ and $y = g(x)$ intersecting at O, A, B, C and D.

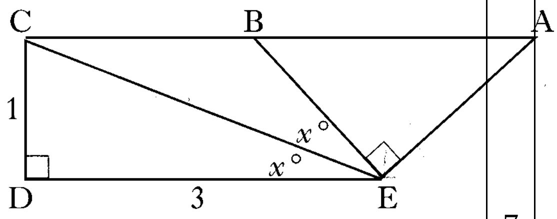
Show that, at A and C, $\cos x = \frac{1}{2k}$. — 5



2004 P1 Q10

10. In the diagram angle DEC = angle CEB = $x°$ and angle CDE = angle BEA = $90°$.
CD = 1 unit; DE = 3 units.

By writing angle DEA in terms of $x°$, find the exact value of $\cos(\hat{DEA})$. — 7



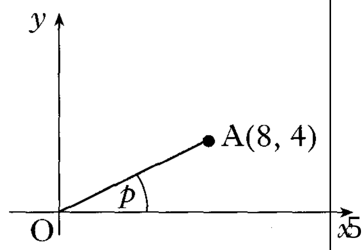
2003 P1 Q10

10. A is the point (8, 4). The line OA is inclined at an angle p radians to the x-axis.

(*a*) Find the exact values of:

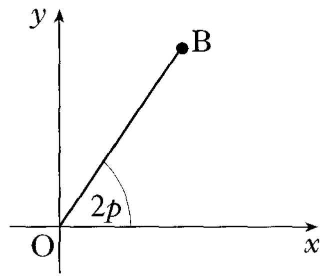
(i) $\sin(2p)$;

(ii) $\cos(2p)$. — 5



The line OB is inclined at an angle $2p$ radians to the x-axis.

(*b*) Write down the exact value of the gradient of OB. — 1



2003 P2 Q10

10. Solve the equation $3\cos(2x) + 10\cos(x) - 1 = 0$ for $0 \le x \le \pi$, correct to 2 decimal places. — 5

2002W P2 Q5

5. Solve the equation $\cos 2x - 2\sin^2 x = 0$ in the interval $0 \le x < 2\pi$. — 4

2002 P1 Q3

3. Functions f and g are defined on suitable domains by $f(x) = \sin(x°)$ and $g(x) = 2x$.

(*a*) Find expressions for:

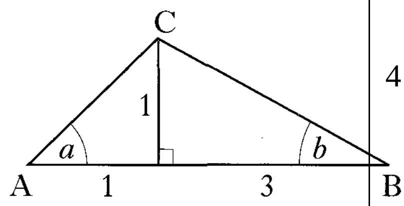
(i) $f(g(x))$;

(ii) $g(f(x))$. — 2

(*b*) Solve $2f(g(x)) = g(f(x))$ for $0 \le x \le 360$. — 5

2002 P1 Q5

5. In triangle ABC, show that the exact value of $\sin(a+b)$ is $\frac{2}{\sqrt{5}}$. **4**

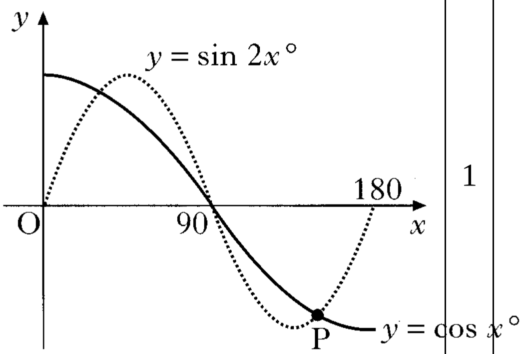


2001 P1 Q5

5. (*a*) Solve the equation $\sin 2x° - \cos x° = 0$ in the interval $0 \le x \le 180$. **4**

(*b*) The diagram shows parts of two trigonometric graphs, $y = \sin 2x°$ and $y = \cos x°$.

Use your solutions in (*a*) to write down the coordinates of the point P. **1**



2001 P1 Q7

7. Functions $f(x) = \sin x$, $g(x) = \cos x$ and $h(x) = x + \frac{\pi}{4}$ are defined on a suitable set of real numbers.

(*a*) Find expressions for:

(i) $f(h(x))$;

(ii) $g(h(x))$. **2**

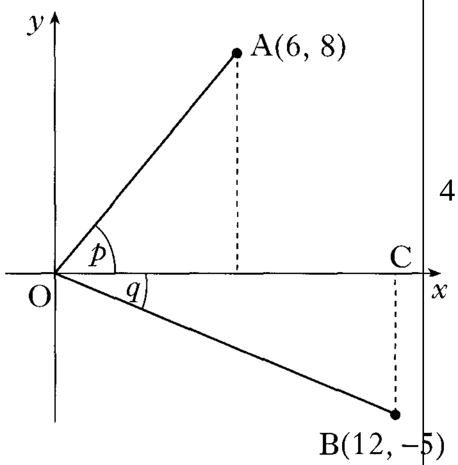
(*b*) (i) Show that $f(h(x)) = \frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x$.

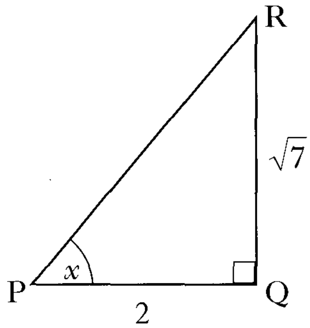
(ii) Find a similar expression for $g(h(x))$ and hence solve the equation $f(h(x)) - g(h(x)) = 1$ for $0 \le x \le 2\pi$. **5**

2000 P1 Q1

1. On the coordinate diagram shown, A is the point (6, 8) and B is the point (12, –5). Angle AOC = p and angle COB = q.

Find the exact value of $\sin(p + q)$. **4**



Spec 2 P1 Q7	**7.** Using triangle PQR, as shown, find the exact value of $\cos 2x$. 	3
Spec 1 P1 Q4	**4.** If $x°$ is an acute angle such that $\tan x° = \frac{4}{3}$, show that the exact value of $\sin(x+30)°$ is $\frac{4\sqrt{3}+3}{10}$.	3
Spec 1 P2 Q7	**7.** (*a*) Show that $2\cos 2x° - \cos^2 x° = 1 - 3\sin^2 x°$.	2
	(*b*) **Hence** (i) write the equation $2\cos 2x° - \cos^2 x° = 2\sin x°$ in terms of $\sin x°$ (ii) solve this equation in the interval $0 \leq x < 90$.	3