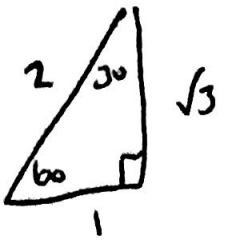
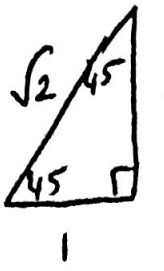
Exact Values





1/ $\sin 60$

$= \frac{\sqrt{3}}{2}$

2/ $\tan 45$

$= 1$

3/ $\cos 30$

$= \frac{\sqrt{3}}{2}$

4/ $\tan 135$

$= -\tan 45$

$= -1$

5/ $\sin 120$

$= \sin 60$

$= \frac{\sqrt{3}}{2}$

6/ $\cos 300$

$= \cos 60$

$= \frac{1}{2}$

7/ $\tan 225$

$= \tan 45$

$= 1$

8/ $\sin 210$

$= -\sin 30$

$= -\frac{1}{2}$

9/ $\cos 315$

$= \cos 45$

$= \frac{1}{\sqrt{2}}$

10/ $\sin 135$

$= \sin 45$

$= \frac{1}{\sqrt{2}}$

Rate of Change

1 $d'(t) = 3t^2$ $\quad 3(2)^2 = \underline{\underline{12}}$

2 $v'(t) = 2t + 1$ $\quad 2(3) + 1 = \underline{\underline{7}}$

3 $s'(t) = 6t^2$ $\quad 6(4)^2 = \underline{\underline{96}}$

4 $a'(t) = -3t^{-4} = \dfrac{-3}{t^4}$ $\quad \dfrac{-3}{2^4} = \underline{\underline{\dfrac{-3}{16}}}$

5 $b'(t) = 3t^2 - 2t + 1$

$3(-1)^2 - 2(-1) + 1$

$= 3 + 2 + 1$

$= \underline{\underline{6}}$

5 At S.P's $\frac{dy}{dx} = 0$ $\quad \frac{dy}{dx} = 6x^2 + 6x - 36 = 0$

$6(x^2 + x - 6) = 0$

$6(x - 2)(x + 3) = 0$

$x = -3, 2$

$x = -3, \quad y = 2(-3)^3 + 3(-3)^2 - 36(-3) + 4$

$= -54 + 27 + 108 + 4$

$= 85$

$x = 2, \quad y = 2(2)^3 + 3(2)^2 - 36(2) + 4$

$= 16 + 12 - 72 + 4$

$= -40$

x	$\to$	-3	$\to$		$\to$	2	$\to$
$\frac{dy}{dx}$	$+$	0	$-$		$-$	0	$+$
	/		−	\	\	—	/

$(-3, 85)$ Max T.P

$(2, -40)$ Min T.P

3 (cont'd)

x	$\to$	-1	$\to$	$\to$	3	$\to$
$\frac{dy}{dx}$	$+$	0	$-$	$-$	0	$+$
	/	$-$	\	\	$-$	/

$(-1, 13)$ Max T.P

$(3, -19)$ Min T.P

4 At S.P's $\frac{dy}{dx} = 0$

$\frac{dy}{dx} = 6x^2 - 18x + 12 = 0$

$6(x^2 - 3x + 2) = 0$

$6(x-1)(x-2) = 0$

$x = 1, 2$

$x = 1, \; y = 2(1)^3 - 9(1)^2 + 12(1) + 1$

$= 2 - 9 + 12 + 1$

$= 6$

$x = 2, \; y = 2(2)^3 - 9(2)^2 + 12(2) + 1$

$= 16 - 36 + 24 + 1$

$= 5$

x	$\to$	1	$\to$	$\to$	2	$\to$
$\frac{dy}{dx}$	$+$	0	$-$	$-$	0	$+$
	/	$-$	\	\	$-$	/

$(1, 6)$ Max T.P

$(2, 5)$ Min T.P

Stationary Points

1 At S.P's $\frac{dy}{dx} = 0$ $\quad \frac{dy}{dx} = 2x - 4 = 0$

$2(x-2) = 0$

$x = 2$

$x = 2, \; y = (2)^2 - 4(2) + 6$

$= 4 - 8 + 6$

$= 2 \quad (2, 2)$

x	$\rightarrow 2 \rightarrow$
$\frac{dy}{dx}$	$- \quad 0 \quad +$
	$\backslash \; \_ \; /$

$(2, 2)$ Min T.P

2 At S.P's $\frac{dy}{dx} = 0$ $\quad \frac{dy}{dx} = 2x + 6 = 0$

$2(x+3) = 0$

$x = -3$

$x = -3, \; y = (-3)^2 + 6(-3) + 3$

$= 9 - 18 + 3$

$= -6$

x	$\rightarrow -3 \rightarrow$
$\frac{dy}{dx}$	$- \quad 0 \quad +$
	$\backslash \; \_ \; /$

$(-3, -6)$ Min. T.P

3 At S.P's $\frac{dy}{dx} = 0$ $\quad \frac{dy}{dx} = 3x^2 - 6x - 9 = 0$

$3(x^2 - 2x - 3) = 0$

$3(x+1)(x-3) = 0$

$x = -1, 3$

$x = -1, \; y = (-1)^3 - 3(-1)^2 - 9(-1) + 8$

$= -1 - 3 + 9 + 8$

$= 13$

$x = 3, \; y = (3)^3 - 3(3)^2 - 9(3) + 8$

$= 27 - 27 - 27 + 8$

$= -19$

Equations of Tangents

1

$\frac{dy}{dx} = 3x^2$

$m = 3(2)^2$

$= 12$

$(2, 3) \quad m = 12$

$y - 3 = 12(x - 2)$

$y - 3 = 12x - 24$

$y = 12x - 21$

2

$\frac{dy}{dx} = 2x - 2$

$m = 2(4) - 2$

$= 6$

$(4, 5) \quad m = 6$

$y - 5 = 6(x - 4)$

$y - 5 = 6x - 24$

$y = 6x - 19$

3

$\frac{dy}{dx} = 3x^2 + 2$

$m = 3(3)^2 + 2$

$= 29$

$(3, 1) \quad m = 29$

$y - 1 = 29(x - 3)$

$y - 1 = 29x - 87$

$y = 29x - 86$

4

$\frac{dy}{dx} = 2x + 3$

$m = 2(-2) + 3$

$= -1$

$(-2, 3) \quad m = -1$

$y - 3 = -(x + 2)$

$y - 3 = -x - 2$

$y = -x + 1$

5

$\frac{dy}{dx} = 3x^2 + 4x + 3$

$m = 3(-1)^2 + 4(-1) + 3$

$= 3 - 4 + 3$

$= 2$

$(-1, 5) \quad m = 2$

$y - 5 = 2(x + 1)$

$y - 5 = 2x + 2$

$y = 2x + 7$

9/ $f(x) = \frac{x}{\sqrt{x}} - \frac{1}{\sqrt{x}}$

$= x^{1/2} - x^{-1/2}$

$f'(x) = \frac{1}{2}x^{-1/2} + \frac{1}{2}x^{-\frac{3}{2}}$

$= \frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{x^3}}$

10/ $y = \frac{x^2}{x^{1/2}} + \frac{3x}{x^{1/2}}$

$= x^{\frac{3}{2}} + 3x^{1/2}$

$y' = \frac{3}{2}x^{1/2} + \frac{3}{2}x^{-1/2}$

$= \frac{3}{2}\sqrt{x} + \frac{3}{2\sqrt{x}}$

Derivatives - Products & Quotients

1/ $y = 3x^2 - 12x$

$\frac{dy}{dx} = 6x - 12$

2/ $f(x) = 8x^2 + 10x - 3$

$f'(x) = 16x + 10$

3/ $y = 25x^2 - 20x + 4$

$\frac{dy}{dx} = 50x - 20$

5/ $f(x) = x^{\frac{5}{2}} - 5x^3$

$f'(x) = \frac{5}{2}x^{\frac{3}{2}} - 15x^2$

4/ $f(x) = x^{\frac{3}{2}} + x^{\frac{1}{2}}$

$f'(x) = \frac{3}{2}x^{\frac{1}{2}} + \frac{1}{2}x^{-\frac{1}{2}}$

$= \frac{3}{2}\sqrt{x} + \frac{1}{2\sqrt{x}}$

6/ $f(x) = \frac{2}{x} - \frac{x^3}{x}$

$= 2x^{-1} - x^2$

$f'(x) = -2x^{-2} - 2x$

$= \frac{-2}{x^2} - 2x$

7/ $f(x) = \frac{4x}{x} - \frac{3x^5}{x}$

$= 4 - 3x^4$

$f'(x) = -12x^3$

8/ $y = \frac{7x^3}{x^3} + \frac{2x}{x^3} - \frac{5}{x^3}$

$= 7 + 2x^{-2} - 5x^{-3}$

$y' = -4x^{-3} + 15x^{-4}$

$= \frac{-4}{x^3} + \frac{15}{x^4}$

Basic Derivatives

1/ $4x^3$

2/ $-2x^{-3}$

$= \frac{-2}{x^3}$

3/ $\frac{1}{2}x^{-1/2}$

$= \frac{1}{2\sqrt{x}}$

4/ $\frac{3}{4}x^{-1/4}$

$= \frac{3}{4\sqrt[4]{x}}$

5/ $-\frac{2}{3}x^{-\frac{5}{3}}$

$= \frac{-2}{3\sqrt[3]{x^5}}$

6/ $10x$

7/ $-27x^{-4}$

$= \frac{-27}{x^4}$

8/ $5x^4 + 3x^2$

9/ $21x^6 + 4x$

10/ $12x^2 - 3$

4 a) $u_{n+1} = 0.7u_n + 15$, $u_0 = 120$

b) $u_1 = 99$

$u_2 = 84.3$

$u_3 = 74.01$

$u_4 = 66.807$

66 sheep after 4 months.

c) $0.3L = 15$

$L = \frac{15 \times 10}{3}$

$L = 50$

The number of sheep will level out at 50 in the long run.

5 a) $u_{n+1} = 0.13u_n + 45$,

b) $0.87L = 45$

$L = \frac{45}{0.87}$

$L = 51.724$

The number of pests will level out at 51 pests in the long run.

Creating a Rec. Relation

1 a) $u_{n+1} = 0.82u_n + 30$, $u_0 = 180$

b) $u_1 = 177.6$
$u_2 = 175.632$
$u_3 = 174.01824$

174 litres at end of 3rd hour.

2 a) $u_{n+1} = 1.05u_n - 55$, $u_0 = 2000$

b) $u_1 = 2045$
$u_2 = 2092.25$

2,092.25 newtons at end of 2nd hour.

3 a) $u_{n+1} = 0.85u_n + 22$, $u_0 = 100$

b) $u_1 = 107$
$u_2 = 112.95$
$u_3 = 118.0075$

118 elephants after 3 years.

c) $0.15L = 22$

$L = \frac{22}{0.15}$

$L = 146.667$

Yes they were as the population stabilises at 146 elephants

Limit (Rec. Relations)

1 a) since $-1 < 0.35 < 1$

b) $0.65L = 3$

$L = \frac{3}{0.65}$

$L = \frac{60}{13}$

2 a) since $-1 < \frac{4}{5} < 1$

b) $\frac{1}{5}L = 2$

$L = 10$

3 a) since $-1 < 0.55 < 1$

b) $0.45L = 7$

$L = \frac{7}{0.45}$

$L = \frac{140}{9}$

4 a) Since $-1 < 0.14 < 1$

b) $0.86L = 8$

$L = \frac{8}{0.86}$

$L = \frac{400}{43}$

5 a) Since $-1 < \frac{1}{3} < 1$

b) $\frac{2}{3}L = 9$

$2L = 9 \times 3$

$2L = 27$

$L = \frac{27}{2}$

Finding Terms (Rec. Relations)

1 $u_1 = 8$

$u_2 = 10$

$u_3 = 11$

2 $v_1 = 10$

$v_2 = 10$

$v_3 = 10$

3 $u_6 = -1.6$

$u_7 = -7.72$

$u_8 = -10.474$

4 a) $p_2 = 32$

$p_3 = 56$

b) $20 = 2p_0 - 8$

$28 = 2p_0$

$p_0 = 14$

5 a) $k_3 = 14$

$k_4 = \frac{41}{3}$

b) $15 = \frac{1}{3}k_1 + 9$

$6 = \frac{1}{3}k_1$

$18 = k_1$

6 a) $u_6 = -8$

$u_7 = -10$

$u_8 = -12$

b) $-6 = u_4 - 2$

$-4 = u_4$

$-4 = u_3 - 2$

$-2 = u_3$

Inverse Functions

1/ $y = 4x^2 - 1$

$y + 1 = 4x^2$

$\frac{y+1}{4} = x^2$

$\sqrt{\frac{y+1}{4}} = x$

$f^{-1}(x) = \sqrt{\frac{x+1}{4}}$

2/ $y = 7x^3 + 3$

$y - 3 = 7x^3$

$\frac{y-3}{7} = x^3$

$\sqrt[3]{\frac{y-3}{7}} = x$

$f^{-1}(x) = \sqrt[3]{\frac{x-3}{7}}$

3/ $y = \frac{8}{x} + 2$

$y - 2 = \frac{8}{x}$

$x(y-2) = 8$

$x = \frac{8}{y-2}$

$f^{-1}(x) = \frac{8}{x-2}$

4/ $y = 3 - 5x^3$

$5x^3 = 3 - y$

$x^3 = \frac{3-y}{5}$

$x = \sqrt[3]{\frac{3-y}{5}}$

$f^{-1}(x) = \sqrt[3]{\frac{3-x}{5}}$

5/ $y = \frac{7 + 3\sqrt{x}}{2}$

$2y = 7 + 3\sqrt{x}$

$2y - 7 = 3\sqrt{x}$

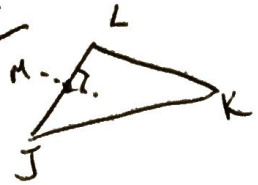
$\frac{2y-7}{3} = \sqrt{x}$

$\left(\frac{2y-7}{3}\right)^2 = x$

$f^{-1}(x) = \left(\frac{2x-7}{3}\right)^2$

Perpendicular Bisector (cont'd)

5/



$M(2, 4)$ $\quad m_{JL} = \frac{-2-10}{1-3} = \frac{-12}{-2} = 6$

$m_{\perp} = -\frac{1}{6}$

$y - 4 = -\frac{1}{6}(x-2)$

$6y - 24 = -x + 2$

$6y = -x + 26$

Composite Functions

1/ $f(x+2) = 3(x+2) - 2$

$= 3x + 6 - 2$

$= 3x + 4$

2/ $f(x-1)$

$= 5(x-1)^2 - 2$

$= 5(x^2 - 2x + 1) - 2$

$= 5x^2 - 10x + 5 - 2$

$= 5x^2 - 10x + 3$

3/ $g(7-4x)$

$= 6(7-4x)^2 + 1$

$= 6(49 - 56x + 16x^2) + 1$

$= 294 - 336x + 96x^2 + 1$

$= 295 - 336x + 96x^2$

4/ $g(x+5)$

$= 3(x+5)^2 - 2(x+5)$

$= 3(x^2 + 10x + 25) - 2x - 10$

$= 3x^2 + 30x + 75 - 2x - 10$

$= 3x^2 + 28x + 65$

5/ $f(3x+2)$

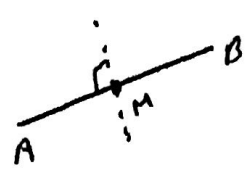
$= (3x + 2 + 1)^2$

$= (3x + 3)^2$

$= 9x^2 + 18x + 9$

Perpendicular Bisector

1



$M(4, 6)$ $\quad m_{AB} = \frac{9-3}{7-1} = \frac{6}{6} = 1$

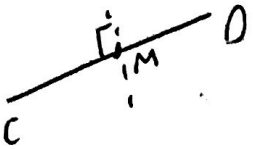
$m_{\perp} = -1$

$y - 6 = -(x - 4)$

$y - 6 = -x + 4$

$y = -x + 10$

2



$M(1, 4)$ $\quad m_{CD} = \frac{2-6}{4+2} = \frac{-4}{6} = -\frac{2}{3}$

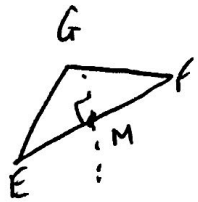
$m_{\perp} = \frac{3}{2}$

$y - 4 = \frac{3}{2}(x - 1)$

$2y - 8 = 3x - 3$

$2y = 3x + 5$

3



$M(3, 2)$ $\quad m_{EF} = \frac{7+1}{3-1} = \frac{8}{2} = 4$

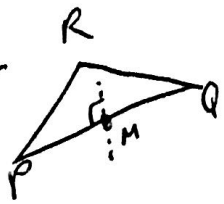
$m_{\perp} = -\frac{1}{4}$

$y - 2 = -\frac{1}{4}(x - 3)$

$4y - 8 = -x + 3$

$4y = -x + 11$

4



$M(2, -1)$ $\quad m_{PQ} = \frac{1+3}{6+2} = \frac{4}{8} = \frac{1}{2}$

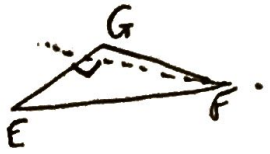
$m_{\perp} = -2$

$y + 1 = -2(x - 2)$

$y + 1 = -2x + 4$

$y = -2x + 3$

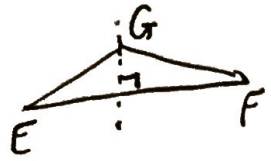
Altitude

1 

$m_{EG} = \frac{10-6}{-2-2} = \frac{6}{-4} = -1$, $m_{\perp r} = 1$

$(-6, 0)$ $y - 0 = 1(x + 6)$

$y = x + 6$

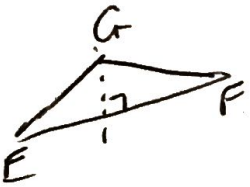
2 

$m_{EF} = \frac{0-6}{-6-2} = \frac{-6}{-8} = \frac{3}{4}$, $m_{\perp r} = -\frac{4}{3}$

$(-2, 10)$ $y - 10 = -\frac{4}{3}(x + 2)$

$3y - 30 = -4x - 8$

$3y = -4x + 22$

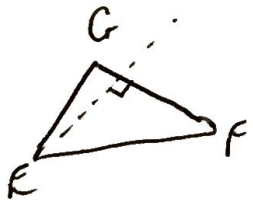
3 

$m_{EF} = \frac{7+1}{3-1} = \frac{8}{2} = 4$, $m_{\perp r} = -\frac{1}{4}$

$(-5, 2)$ $y - 2 = -\frac{1}{4}(x + 5)$

$4y - 8 = -x - 5$

$4y = -x + 3$

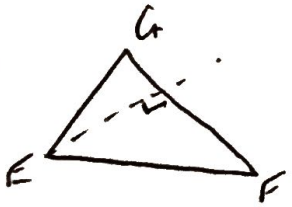
4 

$m_{FG} = \frac{2-7}{-5-3} = \frac{-5}{-8} = \frac{5}{8}$, $m_{\perp r} = -\frac{8}{5}$

$(1, -1)$ $y + 1 = -\frac{8}{5}(x - 1)$

$5y + 5 = -8x + 8$

$5y = -8x + 3$

5 

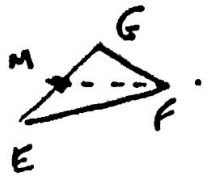
$m_{FG} = \frac{4-2}{-1+3} = \frac{2}{2} = 1$, $m_{\perp r} = -1$

$(7, -4)$ $y + 4 = -(x - 7)$

$y + 4 = -x + 7$

$y = -x + 3$

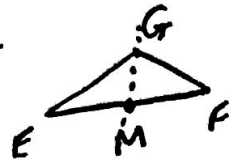
Median

1 

$M(0, 8)$ $\quad m_{MF} = \frac{0-8}{-6-0} = \frac{-8}{-6} = \frac{4}{3}$

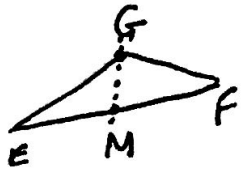
$y - 8 = \frac{4}{3}(x - 0)$

$3y - 24 = 4x$

2 

$M(-2, 3)$ $\quad m_{MG} = \frac{10-3}{-2+2} = \frac{7}{0}$ undefined.

so vertical line $\quad x = -2$

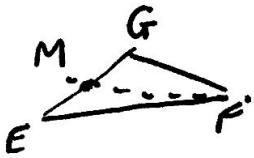
3 

$M(2, 3)$ $\quad m_{MG} = \frac{2-3}{-5-2} = \frac{-1}{-7} = \frac{1}{7}$

$y - 3 = \frac{1}{7}(x - 2)$

$7y - 21 = x - 2$

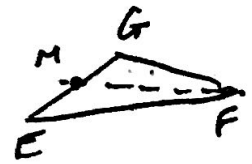
$7y = x + 19$

4 

$M(-2, 2)$ $\quad m_{MF} = \frac{7-2}{3+2} = \frac{5}{5} = 1$

$y - 2 = 1(x + 2)$

$y = x + 4$

5 

$M(-3, -2)$ $\quad m_{MF} = \frac{1+2}{8+3} = \frac{3}{11}$

$y + 2 = \frac{3}{11}(x + 3)$

$11y + 22 = 3x + 9$

$11y = 3x - 13$

Parallel & Perpendicular

1

$5y = 3x - 1$

$y = \frac{3}{5}x - \frac{1}{5}$

$m = \frac{3}{5}$ $(3, 2)$

$y - 2 = \frac{3}{5}(x - 3)$

$5y - 10 = 3x - 9$

$5y = 3x + 1$

2

$7y = -4x + 3$

$y = -\frac{4}{7}x + \frac{3}{7}$

$m = -\frac{4}{7}$, $m_{\perp} = \frac{7}{4}$

$(1, -3)$

$y + 3 = \frac{7}{4}(x - 1)$

$4y + 12 = 7x - 7$

$4y = 7x - 19$

3

$6y = 4x - 7$

$y = \frac{4}{6}x - \frac{7}{6}$

$y = \frac{2}{3}x - \frac{7}{6}$

$m = \frac{2}{3}$ $(-4, 5)$

$y - 5 = \frac{2}{3}(x + 4)$

$3y - 15 = 2x + 8$

$3y = 2x + 23$

4

$9y = -4x + 5$

$y = -\frac{4}{9}x + \frac{5}{9}$

$m = -\frac{4}{9}$, $m_{\perp} = \frac{9}{4}$

$(8, 0)$

$y - 0 = \frac{9}{4}(x - 8)$

$4y = 9x - 72$

5

$3y = 2x - 8$

$y = \frac{2}{3}x - \frac{8}{3}$

$m = \frac{2}{3}$ $(-7, -2)$

$y + 2 = \frac{2}{3}(x + 7)$

$3y + 6 = 2x + 14$

$3y = 2x + 8$