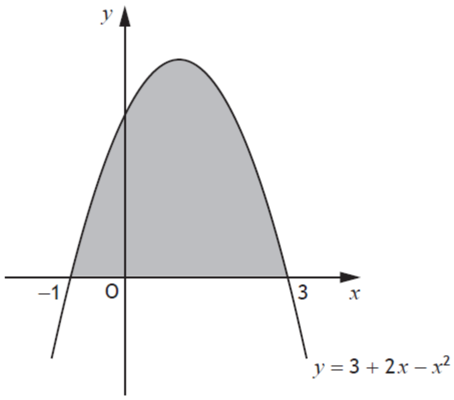
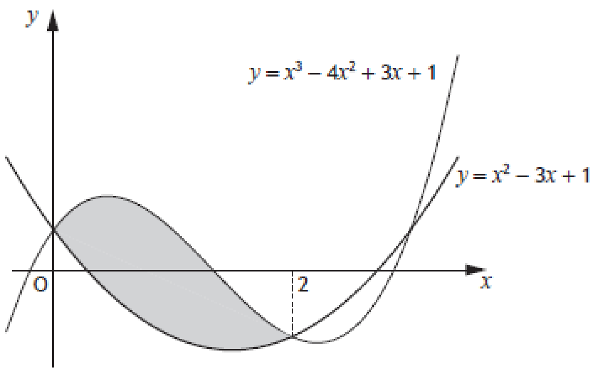
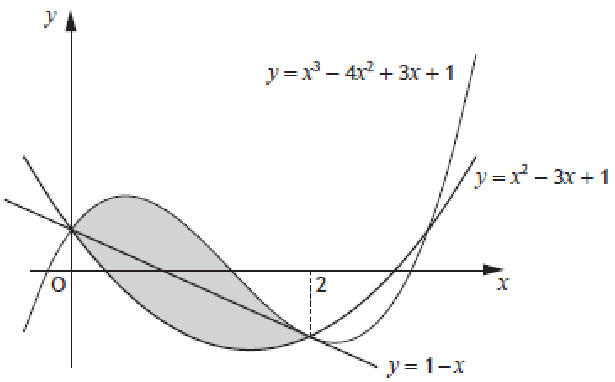
8. Integration

2018 P1 Q10	Given that • $\frac{dy}{dx} = 6x^2 - 3x + 4$, and • $y = 14$ when $x = 2$, express y in terms of x.	4
2018 P2 Q1	The diagram shows the curve with equation $y = 3 + 2x - x^2$.  Calculate the shaded area.	4
2017 P1 Q10	Two curves with equations $y = x^3 - 4x^2 + 3x + 1$ and $y = x^2 - 3x + 1$ intersect as shown in the diagram.  (a) Calculate the shaded area.	5
	The line passing through the points of intersection of the curves has equation $y = 1 - x$.  (b) Determine the fraction of the shaded area which lies below the line $y = 1 - x$.	4

2017 P1 Q15

A quadratic function, f, is defined on $\mathbb{R}$, the set of real numbers.

Diagram 1 shows part of the graph with equation $y = f(x)$.
The turning point is $(2, 3)$.

Diagram 2 shows part of the graph with equation $y = h(x)$.
The turning point is $(7, 6)$.

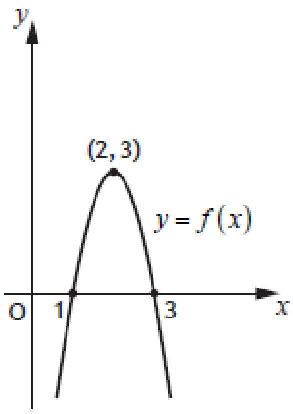


Diagram 1

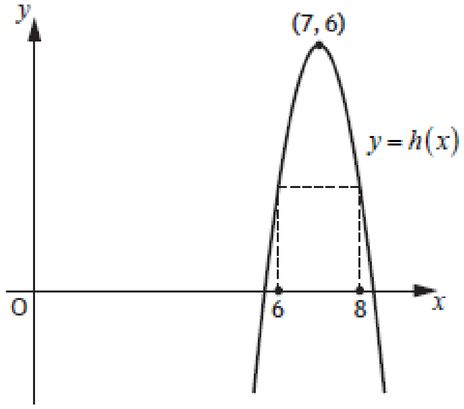


Diagram 2

(a) Given that $h(x) = f(x+a)+b$.

Write down the values of a and b. 2

(b) It is known that $\int_1^3 f(x)\,dx = 4$.

Determine the value of $\int_6^8 h(x)\,dx$. 1

(c) Given $f'(1) = 6$, state the value of $h'(8)$. 1

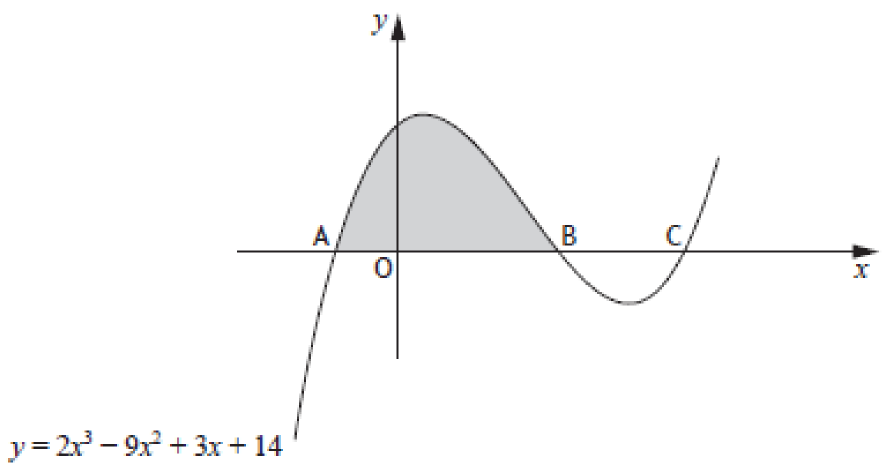
2016 P2 Q3

(a) (i) Show that $(x+1)$ is a factor of $2x^3 - 9x^2 + 3x + 14$. 2

(ii) Hence solve the equation $2x^3 - 9x^2 + 3x + 14 = 0$. 3

(b) The diagram below shows the graph with equation $y = 2x^3 - 9x^2 + 3x + 14$.

The curve cuts the x-axis at A, B and C.



(i) Write down the coordinates of the points A and B. 1

(ii) Hence calculate the shaded area in the diagram. 4

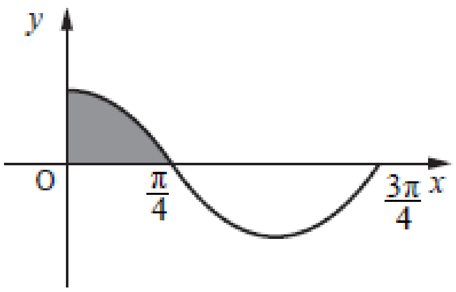
<table>
<tr>
<td>2016 P2 Q9</td>
<td>For a function f, defined on a suitable domain, it is known that:

• $f'(x) = \dfrac{2x+1}{\sqrt{x}}$

• $f(9) = 40$

Express $f(x)$ in terms of x.</td>
<td>4</td>
</tr>
<tr>
<td>2015 P1 Q12</td>
<td>The diagram shows part of the graph of $y = a\cos bx$.

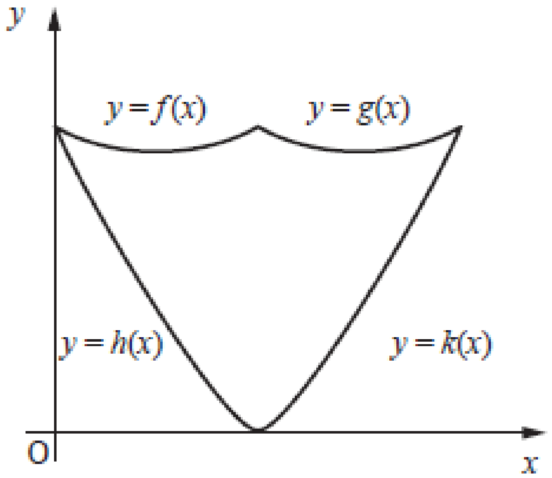
The shaded area is $\frac{1}{2}$ unit^2.

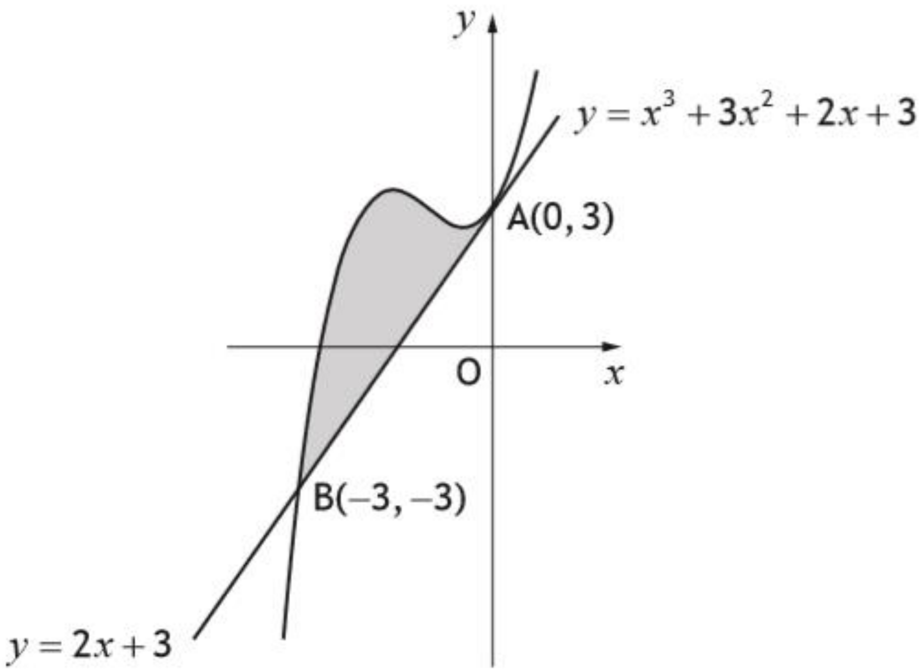


What is the value of $\int_0^{\frac{3\pi}{4}} (a\cos bx)\,dx$?</td>
<td>2</td>
</tr>
<tr>
<td>2015 P1 Q15</td>
<td>The rate of change of the temperature, T °C of a mug of coffee is given by

$$\frac{dT}{dt} = \frac{1}{25}t - k\ ,\ 0 \leq t \leq 50$$
• t is the elapsed time, in minutes, after the coffee is poured into the mug
• k is a constant
• initially, the temperature of the coffee is 100 °C
• 10 minutes later the temperature has fallen to 82 °C.

Express T in terms of t.</td>
<td>6</td>
</tr>
</table>

2015 P2 Q4	A wall plaque is to be made to commemorate the 150th anniversary of the publication of "*Alice's Adventures in Wonderland*". The edges of the wall plaque can be modelled by parts of the graphs of four quadratic functions as shown in the sketch.  • $f(x)=\frac{1}{4}x^2-\frac{1}{2}x+3$ • $g(x)=\frac{1}{4}x^2-\frac{3}{2}x+5$ • $h(x)=\frac{3}{8}x^2-\frac{9}{4}x+3$ • $k(x)=\frac{3}{8}x^2-\frac{3}{4}x$ (a) Find the x-coordinate of the point of intersection of the graphs with equations $y=f(x)$ and $y=g(x)$. The graphs of the functions $f(x)$ and $h(x)$ intersect on the y-axis. The plaque has a vertical line of symmetry. (b) Calculate the area of the wall plaque.	2 7
Spec, P1, Q1	Find $\int \frac{3x^3+1}{2x^2}\,dx,\ x \neq 0$.	
Spec, P1, Q13	The curve $y=f(x)$ is such that $\frac{dy}{dx}=4x-6x^2$. The curve passes through the point $(-1,9)$. Express y in terms of x.	4
Exp, P1, Q3	Evaluate $\int_1^2 \frac{1}{6}x^{-2}\,dx$.	3

Spec, P2, Q2	Find $\int \frac{4x^3 + 1}{x^2} dx,\ x \neq 0$.	4
Exp, P2, Q4	The line with equation $y = 2x + 3$ is a tangent to the curve with equation $y = x^3 + 3x^2 + 2x + 3$ at A(0, 3), as shown.  The line meets the curve again at B(–3, –3). Find the area enclosed by the line and the curve.	5

Marks

Land enclosed between a path and a railway line is being developed for housing.

This land is represented by the shaded area shown in Diagram 1.

- The path is represented by a parabola with equation $y = 6x - x^2$.
- The railway is represented by a line with equation $y = 2x$.
- One square unit in the diagram represents $300\,\text{m}^2$ of land.

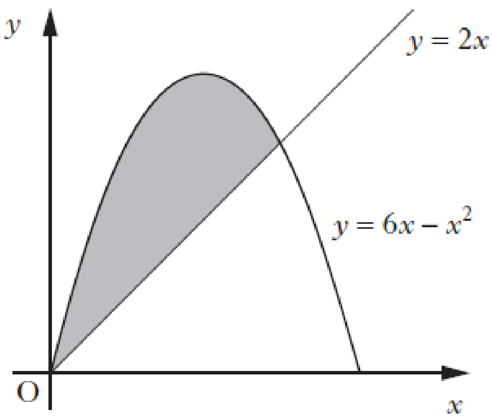


Diagram 1

2014 P2 Q7

(*a*) Calculate the area of land being developed. **5**

(*b*) A road is built parallel to the railway line and is a tangent to the path as shown in Diagram 2.

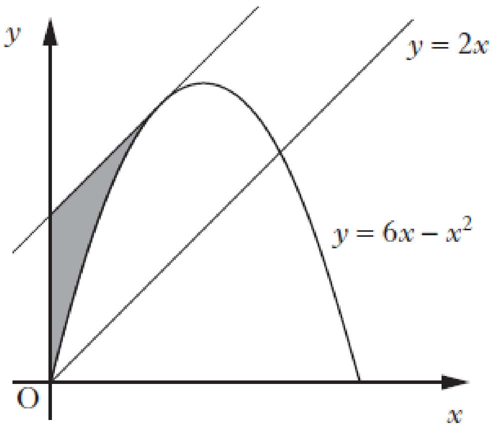
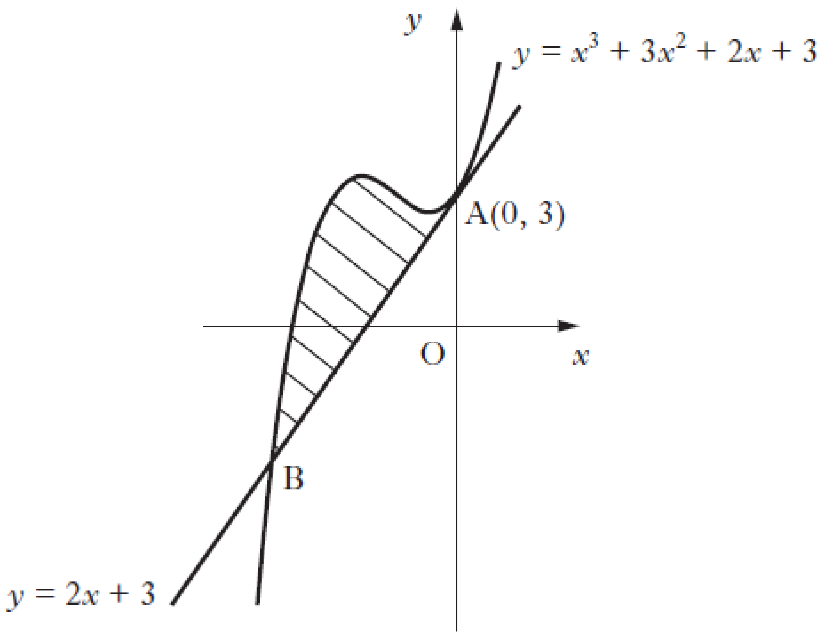
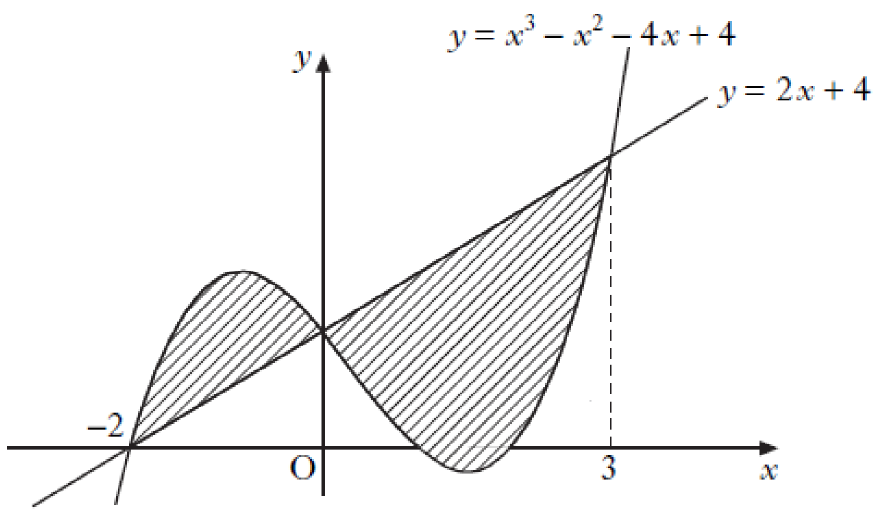


Diagram 2

It is decided that the land, represented by the shaded area in Diagram 2, will become a car park.

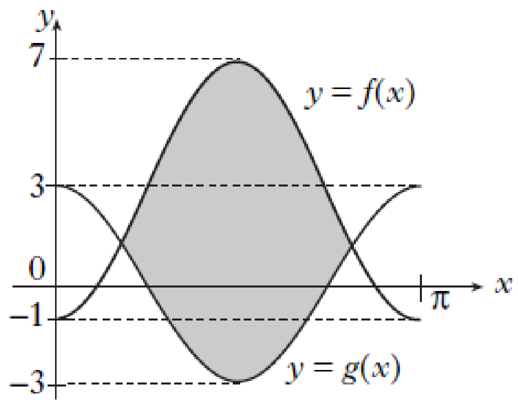
Calculate the area of the car park. **5**

2013 P2 Q4	The line with equation $y = 2x + 3$ is a tangent to the curve with equation $y = x^3 + 3x^2 + 2x + 3$ at A(0, 3), as shown in the diagram.  The line meets the curve again at B. Show that B is the point (–3, –3) and find the area enclosed by the line and the curve.	6
2011 P2 Q4	The diagram shows the curve with equation $y = x^3 - x^2 - 4x + 4$ and the line with equation $y = 2x + 4$. The curve and the line intersect at the points (–2, 0), (0, 4) and (3, 10).  Calculate the total shaded area.	10

2009 P2 Q5

The graphs of $y = f(x)$ and $y = g(x)$ are shown in the diagram.

$f(x) = -4\cos(2x) + 3$ and $g(x)$ is of the form $g(x) = m\cos(nx)$.



(*a*) Write down the values of m and n. 1

(*b*) Find, correct to one decimal place, the coordinates of the points of intersection of the two graphs in the interval $0 \leq x \leq \pi$. 5

(*c*) Calculate the shaded area. 6

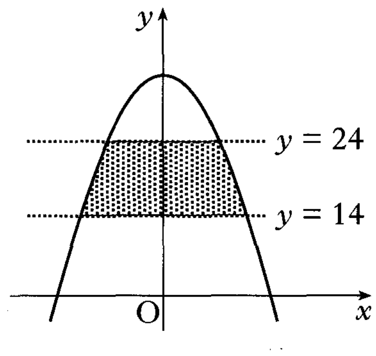
2008 P2

7. The parabola shown in the diagram has equation

$$y = 32 - 2x^2.$$

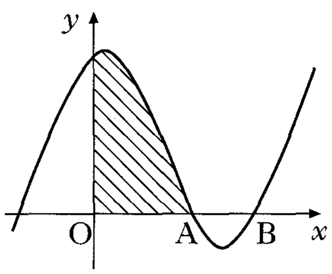
The shaded area lies between the lines $y = 14$ and $y = 24$.

Calculate the shaded area. 8



2007 P1

8. The diagram shows a sketch of the graph of $y = x^3 - 4x^2 + x + 6$.



(*a*) Show that the graph cuts the x-axis at (3, 0). 1

(*b*) Hence or otherwise find the coordinates of A. 3

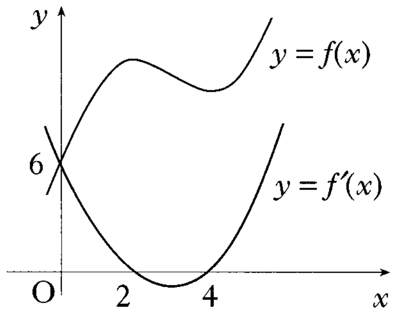
(*c*) Find the shaded area. 5

2007 P2

10. The diagram shows the graphs of a cubic function $y = f(x)$ and its derived function $y = f'(x)$.

Both graphs pass through the point (0, 6).

The graph of $y = f'(x)$ also passes through the points (2, 0) and (4, 0).



(*a*) Given that $f'(x)$ is of the form $k(x - a)(x - b)$:

(i) write down the values of a and b;

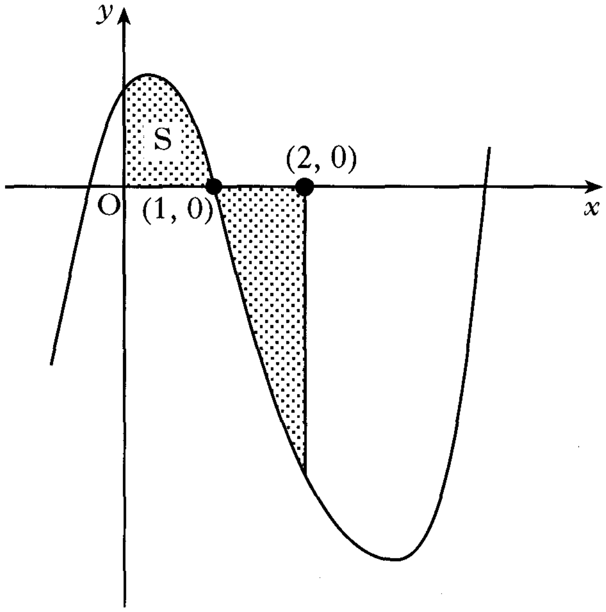
(ii) find the value of k. 3

(*b*) Find the equation of the graph of the cubic function $y = f(x)$. 4

2006 P1

6. The graph shown has equation $y = x^3 - 6x^2 + 4x + 1$.

The total shaded area is bounded by the curve, the x-axis, the y-axis and the line $x = 2$.



(a) Calculate the shaded area labelled S. **4**

(b) Hence find the total shaded area. **3**

2006 P2

5. The curve $y = f(x)$ is such that $\frac{dy}{dx} = 4x - 6x^2$. The curve passes through the point $(-1, 9)$. Express y in terms of x. **4**

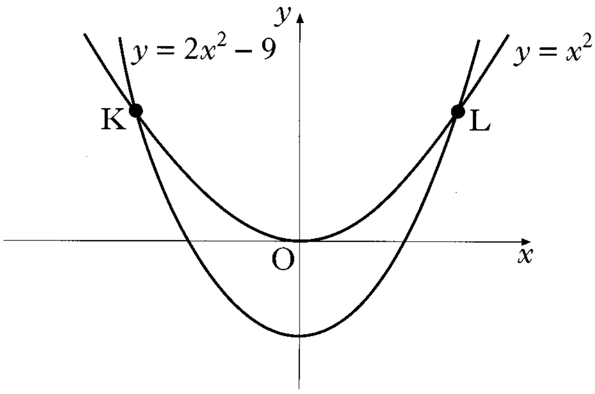
2005 P2

1. Find $\int \frac{4x^3 - 1}{x^2}\,dx,\quad x \neq 0$. **4**

2005 P2

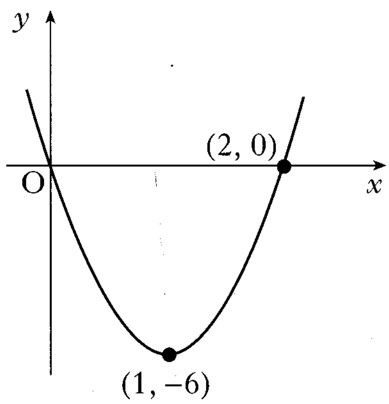
5. The curves with equations $y = x^2$ and $y = 2x^2 - 9$ intersect at K and L as shown.

Calculate the area enclosed between the curves. **8**



2004 P1

11. The diagram shows a parabola passing through the points $(0, 0)$, $(1, -6)$ and $(2, 0)$.

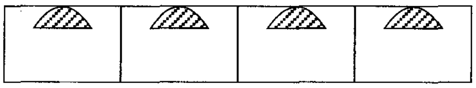
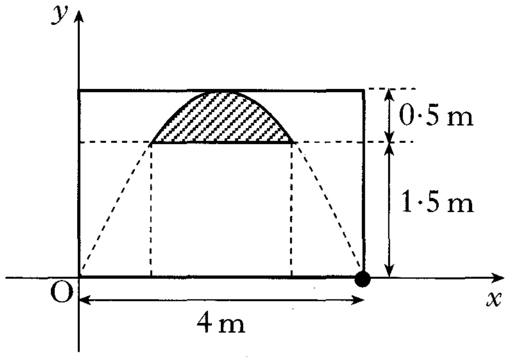
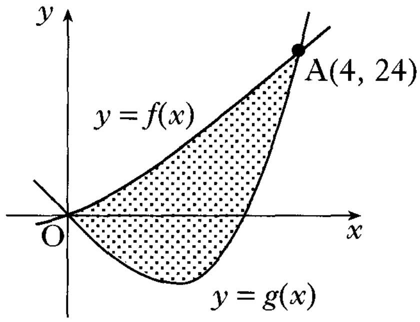


(a) The equation of the parabola is of the form $y = ax(x - b)$.

Find the values of a and b. **3**

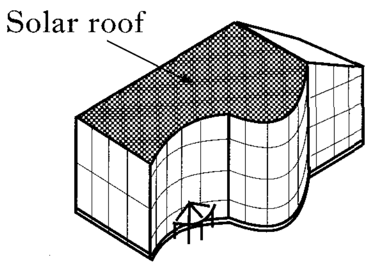
(b) This parabola is the graph of $y = f'(x)$.

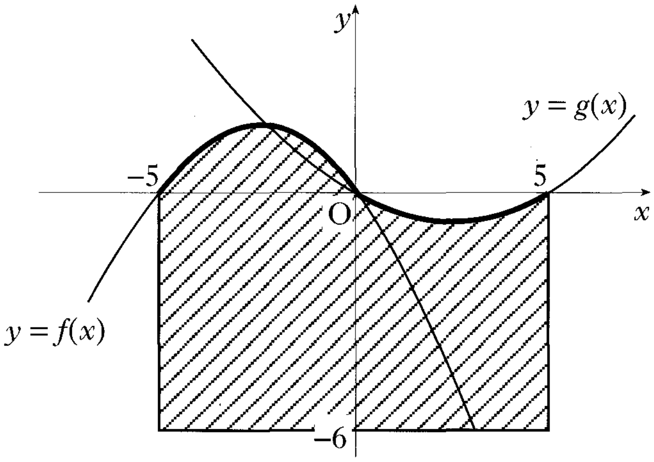
Given that $f(1) = 4$, find the formula for $f(x)$. **5**

2004 P2	**11.** An architectural feature of a building is a wall with arched windows. The curved edge of each window is parabolic.  The second diagram shows one such window. The shaded part represents the glass. The top edge of the window is part of the parabola with equation $y = 2x - \frac{1}{2}x^2$. Find the area in square metres of the glass in one window. 	8
2003 P2	**3.** The incomplete graphs of $f(x) = x^2 + 2x$ and $g(x) = x^3 - x^2 - 6x$ are shown in the diagram. The graphs intersect at A(4, 24) and the origin. Find the shaded area enclosed between the curves. 	5
2002W P1	**7.** Find $\int\left(\sqrt[3]{x} - \frac{1}{\sqrt{x}}\right)dx$.	4

2002W P2

11. An energy efficient building is designed with solar cells covering the whole of its south facing roof. The energy generated by the solar cells is directly proportional to the area, in square units, of the solar roof.





The shape of the solar roof can be represented on the coordinate plane as the shaded area bounded by the functions $f(x) = \frac{1}{4}\left(-x^2 - 5x\right)$, $g(x) = \frac{1}{12}\left(x^2 - 5x\right)$ and the lines $x = -5$, $x = 5$ and $y = -6$.

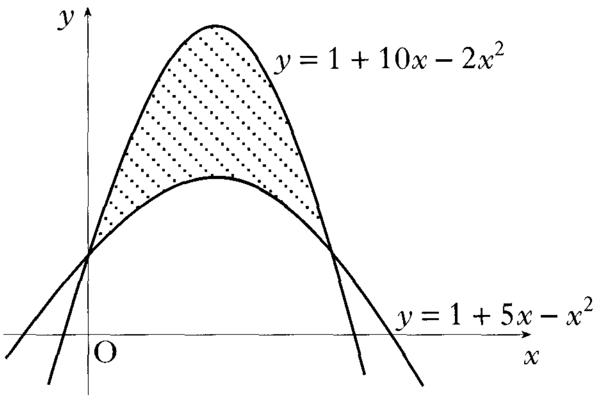
(*a*) Find the area of the solar roof. 7

(*b*) Ten square units of solar cells generate a maximum of 1 kilowatt.

What is the maximum energy the solar roof can generate in kilowatts (to the nearest kilowatt)? 1

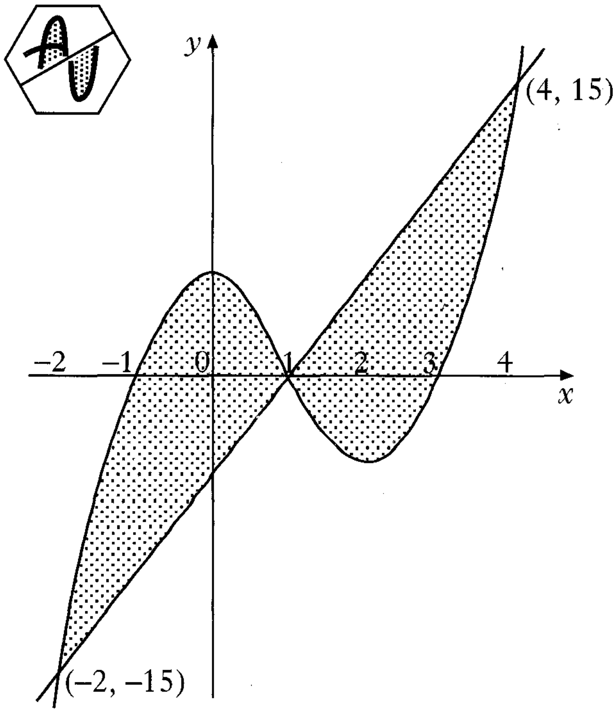
2002 P2

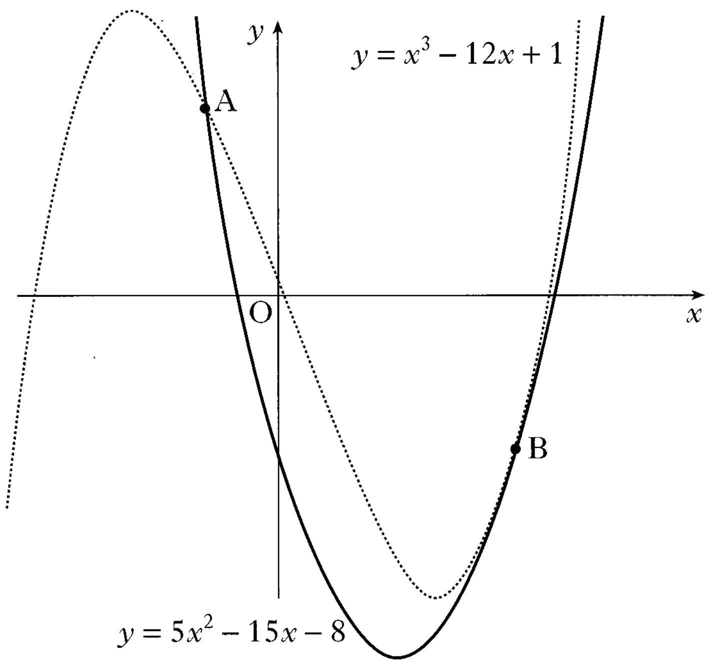
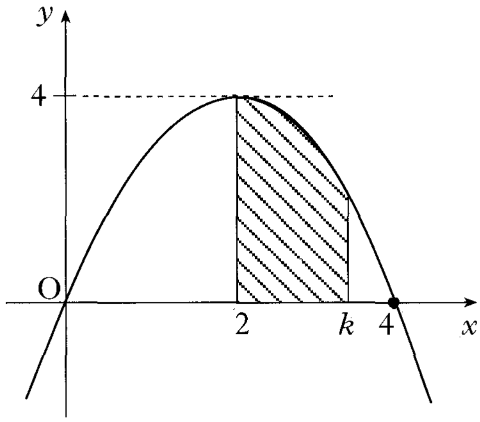
5. Calculate the shaded area enclosed between the parabolas with equations $y = 1 + 10x - 2x^2$ and $y = 1 + 5x - x^2$. 6

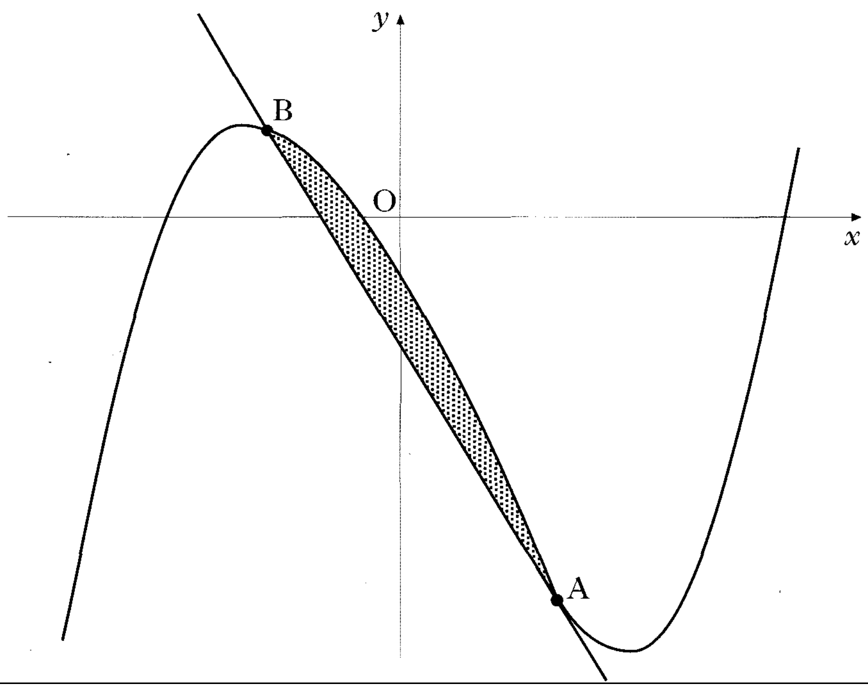


2002 P2

8. A point moves in a straight line such that its acceleration a is given by $a = 2(4 - t)^{\frac{1}{2}}$, $0 \leq t \leq 4$. If it starts at rest, find an expression for the velocity v where $a = \frac{dv}{dt}$. 4

2001 P2	**6.** Find $\int \frac{(x^2-2)(x^2+2)}{x^2}dx, \quad x \neq 0$	4
2001 P2	**8.** A firm asked for a logo to be designed involving the letters A and U. Their initial sketch is shown in the hexagon. A mathematical representation of the final logo is shown in the coordinate diagram.  The curve has equation $y = (x+1)(x-1)(x-3)$ and the straight line has equation $y = 5x - 5$. The point (1, 0) is the centre of half-turn symmetry. Calculate the total shaded area.	7

2000 P1	**4.** The diagram shows a sketch of the graphs of $y = 5x^2 - 15x - 8$ and $y = x^3 - 12x + 1$. The two curves intersect at A and touch at B, ie at B the curves have a common tangent.  (*a*) (i) Find the x-coordinates of the points on the curves where the gradients are equal. (ii) By considering the corresponding y-coordinates, or otherwise, distinguish geometrically between the two cases found in part (i). (*b*) The point A is $(-1, 12)$ and B is $(3, -8)$. Find the area enclosed between the two curves.	4 1 5
2000 P2	**4.** The parabola shown crosses the x-axis at (0, 0) and (4, 0), and has a maximum at (2, 4). The shaded area is bounded by the parabola, the x-axis and the lines $x = 2$ and $x = k$.  (*a*) Find the equation of the parabola. (*b*) Hence show that the shaded area, A, is given by $A = -\frac{1}{3}k^3 + 2k^2 - \frac{16}{3}$	2 3

Specimen 2 P1	**8.** Functions f and g are defined on the set of real numbers by $$f(x) = x - 1$$ $$g(x) = x^2.$$ (*a*) Find formulae for (i) $f(g(x))$ (ii) $g(f(x))$. (*b*) The function h is defined by $h(x) = f(g(x)) + g(f(x))$. Show that $h(x) = 2x^2 - 2x$ and sketch the graph of h. (*c*) Find the area enclosed between this graph and the x-axis.	3 3 4
Specimen 2 P1	**9.** Find $\int \frac{x^2 - 5}{x\sqrt{x}}dx$.	4
Specimen 1 P1	**7.** Find the value of $\int_1^2 \frac{u^2 + 2}{2u^2} du$.	5
Specimen 1 P2	**4.** In the diagram below, a winding river has been modelled by the curve $y = x^3 - x^2 - 6x - 2$ and a road has been modelled by the straight line AB. The road is a tangent to the river at the point A(1, −8). (*a*) Find the equation of the tangent at A. (*b*) Hence find the coordinates of B. (*c*) Find the area of the shaded part which represents the land bounded by the river and the road. 	3 4 3