2019 P2 Q11

11.	(a)		
		•[1] express A in terms of x and h	•[1] $(A=)16x^2+16xh$
		•[2] express height in terms of x	•[2] $h=\frac{2000}{8x^2}$
		•[3] substitute for h and complete proof	•[3] $A=16x^2+16x\times\frac{2000}{8x^2}$ leading to $A=16x^2+\frac{4000}{x}$

Notes:

1. At •[1] accept any unsimplified form of $16x^2+16xh$.
2. The substitution for h at •[3] must be clearly shown for •[3] to be available.
3. For candidates who omit some of the surfaces of the box, only •[2] is available.

Commonly Observed Responses:

	(b)		
		•[4] express A in differentiable form	•[4] $16x^2+4000x^{-1}$
		•[5] differentiate	•[5] $32x-4000x^{-2}$
		•[6] equate expression for derivative to 0	•[6] $32x-4000x^{-2}=0$
		•[7] process for x	•[7] 5
		•[8] verify nature	•[8] table of signs for a derivative (see below) $\therefore$ minimum or $A''(x)=96>0 \Rightarrow$ minimum
		•[9] evaluate A	•[9] $A=1200$ or min value $=1200$

2019 P1 Q1

•[1] $2x^3$ ... or ... $-6x^2$

•[2] $2x^3-6x^2=0$

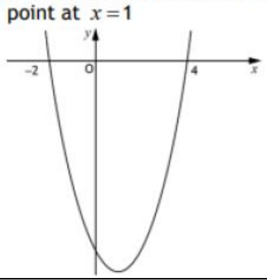
•[3] $2x^2(x-3)$

•[4] 0 and 3

2019 P2 Q5

•[1] parabola with roots at -2 and 4

•[2] parabola with a minimum turning point at $x=1$



2019 P2 Q7

$(a) - 6(x-2)^2 - 1$

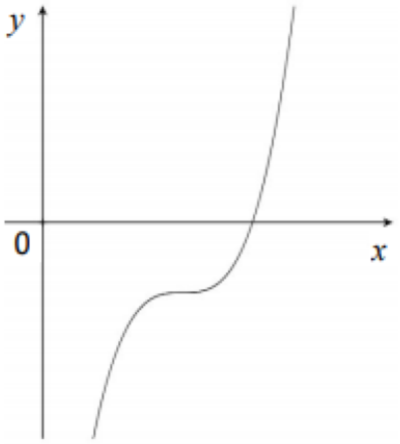
•[4] $-6x^2+24x-25$

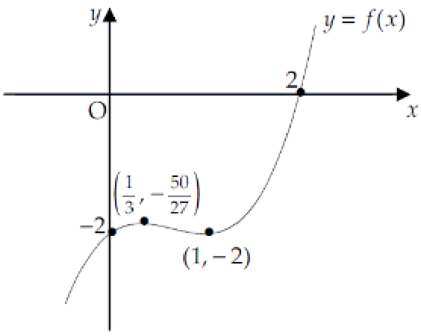
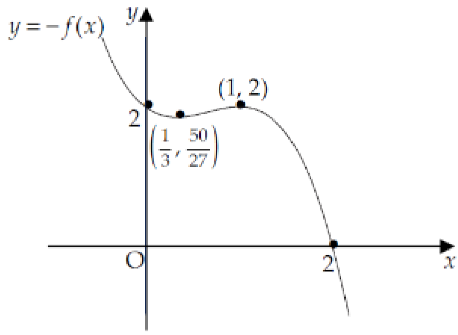
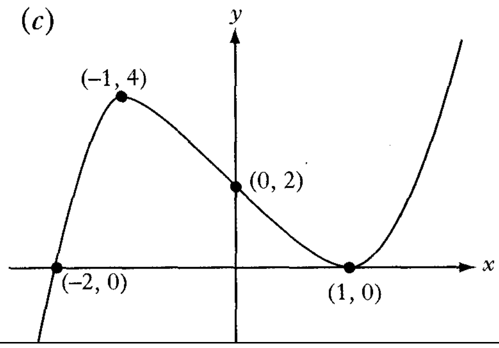
•[5] $f'(x)=-6(x-2)^2-1$ and $(x-2)^2\geq 0\ \forall x$

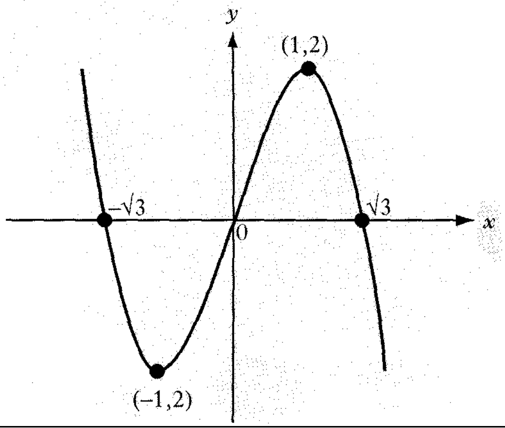
•[6] eg $\therefore -6(x-2)^2-1<0\ \forall x$

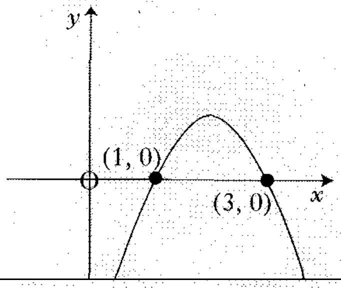
$\Rightarrow$ always strictly decreasing

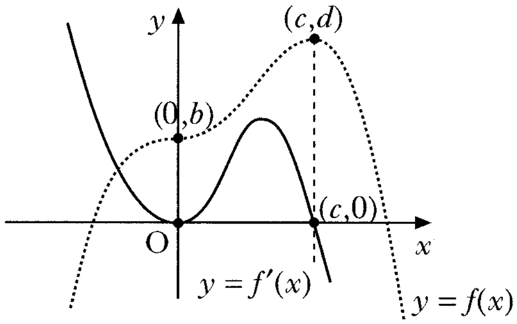
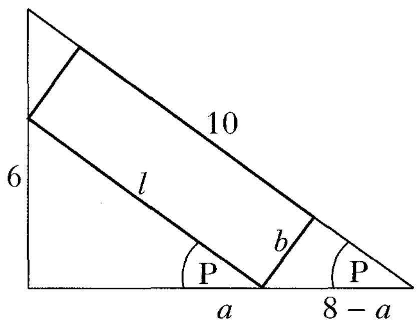
2018 P2 Q3	f *is increasing* (*differentiate, then sub* $x = 2$, *since positive,* f *is increasing*)	
2018 P2 Q9	$P = 32$ *or* minimum *value* $= 32$	
2017 P1 Q8	$x = -\frac{1}{50}$	
2017 P2 Q4	(a) $3(x+4)^2 + 2$ (b) $3x^2 + 24x + 50$ (c) $f'(x) = 3(x+4)^2 + 2$ and $(x+4)^2 \geq 0 \;\; \forall x$ $\therefore 3(x+4)^2 + 2 > 0 \Rightarrow$ always strictly increasing	
2017 P2 Q7	(a) $x = 4$ (b) greatest 8, least 0	
2016 P 1Q2	$\frac{dy}{dx} = 36x^2 + 4x^{\frac{-1}{2}}$	
2016 P1 Q9	(a) $x = -4, 2$ (b) $x < -4 \quad x > 2$	
2016 P2 Q7	(a) Proof (b) $x = 4$	
2015 P1 Q2	$y = 24x + 35$	

2015 P1 Q7	$9\frac{1}{8}$	
SPEC P1 Q11	(a) $m = f'(x) \geq 0$ (b) 	
SPEC P2 Q8	(a) Proof (b) $r = 2.20m$	
EXP P1 Q1	$y - 12 = 6(x - 5)$	
EXP P2 Q 9	(a) $a = 4cm$ (b) No (£198 > £195)	
2014 P1 Q21	min. at $(0,0)$ and max. at $(2,4)$	
2014 P2 Q2	$4x^3 - 6x^2$ $y - 5 = 8(x - 2)$	
2013 P2 Q3b	$x = 1$ nature table **and** minimum	

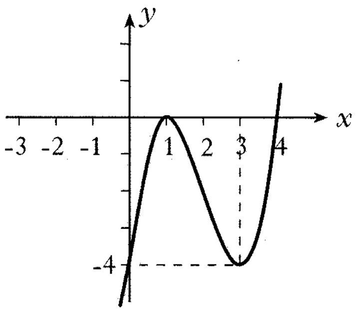
2013 P2 Q7	(a) $L = 3x + 4y$ $y = \frac{24}{2x}$ $L = 3x + 4 \times \frac{24}{2x}$ and complete (b) $x = 4$ $L = 24$ cost $24 \times £8{\cdot}25 = £198$
2012 P2 Q3	differentiate x^3 or $-2x^2$ correctly max. 6 **and** min. -2
2011 P1 Q22	(a) $(2, 0)$ $(0, -2)$ (b) Max @ $(^1/_3, ^{-50}/_{27})$ Min @ $(1, -2)$ (c) (i)  (ii) 
2010 P 2 Q5	(a) •1 4 or $(0,4)$ •2 $10 - x^2 - 4$ •3 $2x \times (6 - x^2) = 12x - 2x^3$ (b) Max **and** $8\sqrt{2}$ or decimal equivalent
2009 P2 Q1	Max TP (-1, 17) Min TP (3, -15)
2008 P1 Q21	(*a*) (–1, 4) maximum (1, 0) minimum (*b*) (i) $x = 1$, $f(x) = 0$ so $(x - 1)$ is a factor (ii) $(x - 1)(x - 1)(x + 2)$ (*c*) 
2008 P1 Q22	(a) (1,3), (3,-3) (b) (1,3)

2008 P2 Q6	(a) proof (b) (1.5,3)
2007 P1 Q9	(*a*) $(-\sqrt{3}, 0)$, (0,0), $(\sqrt{3},0)$ (*b*) (1,2): maximum (−1,−2): minimum (*c*) 
2007 P2 Q5	(*a*) Q = (12,10) (*b*) P = (4,10) (*c*) C = (8,11)
2007 P2 Q6	(*a*) (i) ST = $\sqrt{200}$ (ii)Length of decking = $\sqrt{200} - 2x$ So A = $x(\sqrt{200} - 2x)$ $= (10\sqrt{2}) - 2x^2$ (*b*) $x = \frac{10\sqrt{2}}{4}$ length = $5\sqrt{2}$
2006 P2 Q3	(*a*) $y - 5 = 2(x - 8)$
2006 P2 Q12	(*a*) (i) PS = $6 - x$ RS = $12 - \frac{8}{x}$ (ii) Area = $(6 - x)\left(12 - \frac{8}{x}\right)$ and complete (*b*) max.A = 32 at $x = 2$ **and** min.A = 20 at $x = 1$ **or** $x = 4$
2005 P2 Q6	$y - 12 = -\frac{3}{2}(x - 4)$

2004 P2 Q5	(*a*) $x = 2$ (*b*) $y = 12x - 8$	
2004 P2 Q7		
2004 P2 Q9	(*a*) $A = 2x^2 + 2xh + 4xh = 12$ $V = 2x \times x \times h$ $V = 2x \times \frac{12 - 2x^2}{6}$ $V = \frac{2}{3}x(6 - x^2)$ (*b*) $x = \sqrt{2}$	
2003 P1 Q5	$\frac{3}{16}$	
2003 P2 Q4	(*a*) $y = 4x - 2$	
2003 P2 Q8	(*a*) length = $\frac{108000}{\frac{1}{2}x^2}$ $SA = 2 \times \frac{1}{2}x^2 + 2x \times \text{length}$ $SA = x^2 + \frac{432000}{x}$ $\frac{dA}{dx} = 2x - \frac{432000}{x^2}$ $\frac{dA}{dx} = 0$ $x = 60$ Justify minimum using, e.g. nature table (*b*) 60	
2002W P1 Q3	$y + 3x = 6$	

2002W P2 Q7	Solve $\frac{dS}{dw} = 0$ and test for max/min $d = 20\sqrt{\frac{2}{3}}$	
2002 P1 Q4	(2, 4)	
2002 P1 Q6		
2002 P2 Q3	(a) $f'(x) = 6x^2 - 14x + 4 = 0$ $x = \frac{1}{3}$ (b) $(x-2)(2x+1)(x-2)$ (c) $A(-\frac{1}{2},0)$, $x < -\frac{1}{2}$	
2002 P2 Q10	(a) proof  $\cos P = \frac{8}{10} = \frac{a}{l} \Rightarrow l = \frac{10}{8}a = \frac{5}{4}a$ $\sin P = \frac{6}{10} = \frac{b}{8-a} \Rightarrow b = \frac{6}{10}(8-a)$ $\text{Area} = lb = \frac{5}{4}a \times \frac{6}{10}(8-a) = \frac{3}{4}a(8-a)$ (b) Solve $\frac{dA}{da} = 0$ and test for max/min $a = 4$	
2001 P1 Q6	$x = 9$	
2001 P1 Q9	straight line for f' through (3,0), $m_{f'} > 0$ straight line for g' through (3,0), $m_{f'} > m_{g'} > 0$	

2001 P2 Q2	$y = 2x - 12$	
2000 P1 Q2	(a) $A = (1,4)$	
2000 P1 Q4	(a) (i) $x = \frac{1}{3}$ and $x = 3$ (ii) parallel and coincident	
2000 P2 Q1	(a) $x + y = 1$ (b) $(-1, -6)$	
2000 P2 Q6	$x = 2$	
Specimen 2 P1 Q3	$y - 7 = -10(x - (-1))$	
Specimen 2 P2 Q9	$\frac{dy}{dx} = 6x^2 + 6x + 4$ $b^2 - 4ac = -60$ $6x^2 + 6x + 4$ has no roots $\frac{dy}{dx} = 0$ has no solutions so curve has no stationary points	
Specimen 2 P1 Q10	**10.** (a) length $= 9y + 8x = 360$ $A = 3y \times 2x = 2x.3.\frac{1}{9}(360 - 8x)240x - \frac{16}{3}x^2$ (b) $A'(x) = 240 - \frac{32}{3}x$ $A'(x) = 0 \quad \Rightarrow x = 22\frac{1}{2}, \quad y = 20$ x: $22\frac{1}{2}^-$, $22\frac{1}{2}$, $22\frac{1}{2}^+$ $A'(x)$: $+$, 0, $-$ maximum $A_{max} = 2700$	

Specimen 1 P1 Q3	(*a*) $f(1) = 0$, $(x-4)$, $(x-1)$ (*b*) (1,0), (4,0), (0, −4) (*c*) max at (1,0), min at (3, −4) (*d*) 	
Specimen 1 P2	(*a*) $y = -5x - 3$	
Specimen 1 P2 Q9	(*a*) (i) $h = \frac{1}{2}(10 - \pi x - 2x)$ (ii) $L = 2 \times 2xh + \frac{1}{2}\pi x^2$ $= 4x \times \frac{1}{2}(10 - \pi x - 2x) + \frac{1}{2}\pi x^2$ $= 20x - 2\pi^2 - 4x^2 + \frac{1}{2}\pi x^2$ (*b*) $x = \frac{20}{3\pi + 8}$, $h = \frac{5(\pi + 4)}{3\pi + 8}$	