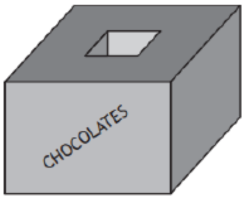
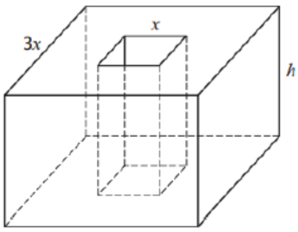
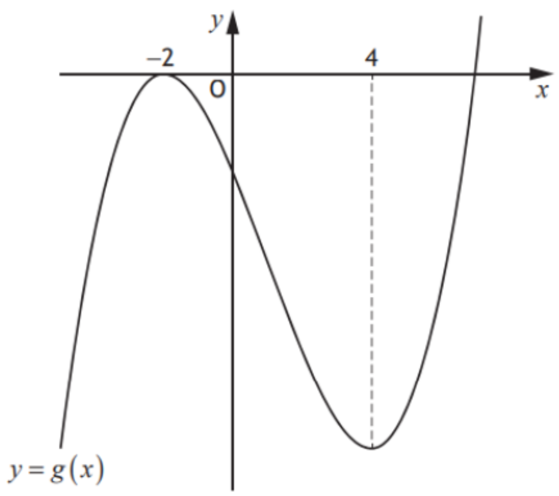
Differentiation

2019 P2 Q11	A manufacturer of chocolates is launching a new product in novelty shaped cardboard boxes.  The box is a cuboid with a cuboid shaped tunnel through it. • The height of the box is h centimetres • The top of the box is a square of side $3x$ centimetres • The end of the tunnel is a square of side x centimetres • The volume of the box is 2000 cm^3  (a) Show that the total surface area, A cm^2, of the box is given by $A = 16x^2 + \frac{4000}{x}$. (b) To minimise the cost of production, the surface area, A, of the box should be as small as possible. Find the minimum value of A.	3 6
2019 P1 Q1	Find the x-coordinates of the stationary points on the curve with equation $y = \frac{1}{2}x^4 - 2x^3 + 6$.	4
2019 P2 Q5	The diagram below shows the graph of a cubic function $y = g(x)$, with stationary points at $x = -2$ and $x = 4$.  On the diagram in your answer booklet, sketch the graph of $y = g'(x)$.	2

2019 P2 Q7	(a) Express $-6x^2 + 24x - 25$ in the form $p(x+q)^2 + r$. (b) Given that $f(x) = -2x^3 + 12x^2 - 25x + 9$, show that $f(x)$ is strictly decreasing for all $x \in \mathbb{R}$.	3 3
2018 P2 Q3	A function, f, is defined on the set of real numbers by $f(x) = x^3 - 7x - 6$. Determine whether f is increasing or decreasing when $x = 2$.	
2018 P2 Q9	A sector with a particular fixed area has radius x cm. The perimeter, P cm, of the sector is given by $$P = 2x + \frac{128}{x}.$$ Find the minimum value of P.	
2017 P1 Q8	Calculate the rate of change of $d(t) = \frac{1}{2t}$, $t \neq 0$, when $t = 5$.	
2017 P2 Q4	(a) Express $3x^2 + 24x + 50$ in the form $a(x+b)^2 + c$. (b) Given that $f(x) = x^3 + 12x^2 + 50x - 11$, find $f'(x)$. (c) Hence, or otherwise, explain why the curve with equation $y = f(x)$ is strictly increasing for all values of x.	
2017 P2 Q7	(a) Find the x-coordinate of the stationary point on the curve with equation $y = 6x - 2\sqrt{x^3}$. (b) Hence, determine the greatest and least values of y in the interval $1 \leq x \leq 9$.	
2016 P 1Q2	Given that $y = 12x^3 + 8\sqrt{x}$, where $x > 0$, find $\frac{dy}{dx}$.	

2016 P1 Q9

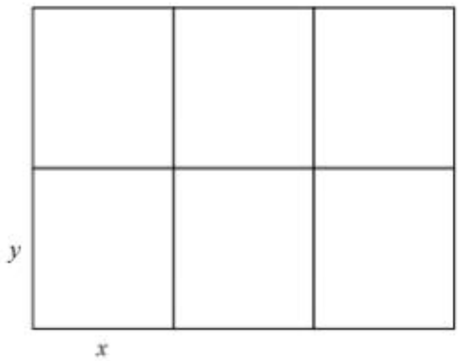
(a) Find the x-coordinates of the stationary points on the graph with equation $y = f(x)$, where $f(x) = x^3 + 3x^2 - 24x$.

(b) Hence determine the range of values of x for which the function f is strictly increasing.

2016 P2 Q7

A council is setting aside an area of land to create six fenced plots where local residents can grow their own food.

Each plot will be a rectangle measuring x metres by y metres as shown in the diagram.



(a) The area of land being set aside is $108\ \text{m}^2$.

Show that the total length of fencing, L metres, is given by

$$L(x) = 9x + \frac{144}{x}.$$

(b) Find the value of x that minimises the length of fencing required.

2015 P1 Q2

Find the equation of the tangent to the curve $y = 2x^3 + 3$ at the point where $x = -2$.

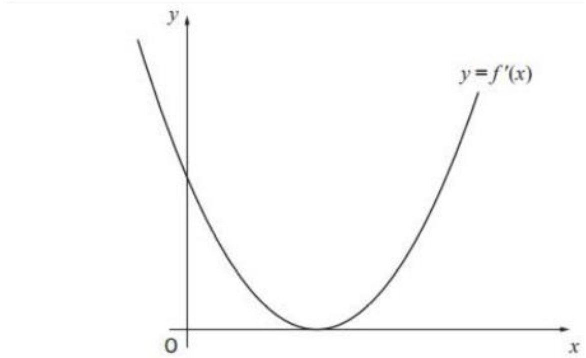
2015 P1 Q7

A function f is defined on a suitable domain by $f(x) = \sqrt{x}\left(3x - \frac{2}{x\sqrt{x}}\right)$.

Find $f'(4)$.

SPEC P1 Q11

The diagram shows the graph of $y = f'(x)$. The x-axis is a tangent to this graph.



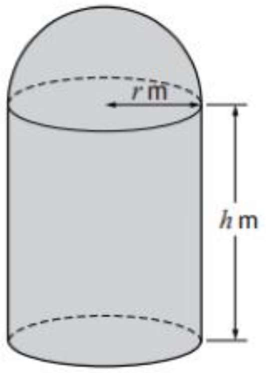
(a) Explain why the function $f(x)$ is never decreasing.

(b) On a graph of $y = f(x)$, the y-coordinate of the stationary point is negative. Sketch a possible graph for $y = f(x)$.

SPEC P2 Q8

A design for a new grain container is in the shape of a cylinder with a hemispherical roof and a flat circular base. The radius of the cylinder is r metres, and the height is h metres.

The volume of the **cylindrical** part of the container needs to be 100 cubic metres.



(a) Given that the curved surface area of a hemisphere of radius r is $2\pi r^2$ show that the surface area of metal needed to build the grain container is given by:

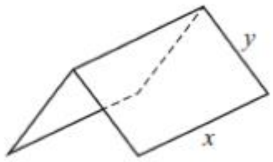
$$A = \frac{200}{r} + 3\pi r^2 \text{ square metres}$$

(b) Determine the value of r which minimises the amount of metal needed to build the container.

EXP P1 Q1

The point P (5,12) lies on the curve with equation $y = x^2 - 4x + 7$.

Find the equation of the tangent to this curve at P.

EXP P2 Q 9	A manufacturer is asked to design an open-ended shelter, as shown:  The frame of the shelter is to be made of rods of two different lengths: • x metres for top and bottom edges; • y metres for each sloping edge. The total length, L metres, of the rods used in a shelter is given by: $$L = 3x + \frac{48}{x}$$ To minimise production costs, the total length of rods used for a frame should be as small as possible. (a) Find the value of x for which L is a minimum. The rods used for the frame cost £8·25 per metre. The manufacturer claims that the minimum cost of a frame is less than £195. (b) Is this claim correct? Justify your answer.	
2014 P1 Q21	A curve has equation $y = 3x^2 - x^3$. (*a*) Find the coordinates of the stationary points on this curve and determine their nature. **6** (*b*) State the coordinates of the points where the curve meets the coordinate axes and sketch the curve. **2**	
2014 P2 Q2	A curve has equation $y = x^4 - 2x^3 + 5$. Find the equation of the tangent to this curve at the point where $x = 2$. **4**	
2013 P2 Q3b	Show that the curve with equation $$y = x^4 + 4x^3 + 2x^2 - 20x + 3$$ has only one stationary point. Find the x-coordinate and determine the nature of this point.	5

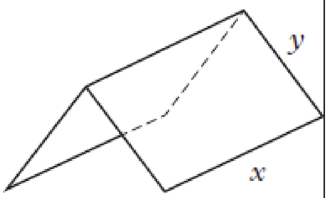
<table>
<tr>
<td>2013 P2 Q7</td>
<td>

A manufacturer is asked to design an open-ended shelter, as shown, subject to the following conditions.

Condition 1

The frame of a shelter is to be made of rods of two different lengths:

- x metres for top and bottom edges;
- y metres for each sloping edge.



Condition 2

The frame is to be covered by a rectangular sheet of material.

The total area of the sheet is $24\,\text{m}^2$.

(*a*) Show that the total length, L metres, of the rods used in a shelter is given by

$$L = 3x + \frac{48}{x}.$$

(*b*) These rods cost £8·25 per metre.

To minimise production costs, the total length of rods used for a frame should be as small as possible.

(i) Find the value of x for which L is a minimum.

(ii) Calculate the minimum cost of a frame.

</td>
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<tr>
<td>2012 P2 Q3</td>
<td>

A function f is defined on the domain $0 \le x \le 3$ by $f(x) = x^3 - 2x^2 - 4x + 6$.

Determine the maximum and minimum values of f.

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<tr>
<td>2011 P1 Q22</td>
<td>

A function f is defined on the set of real numbers by $f(x) = (x - 2)(x^2 + 1)$.

(*a*) Find where the graph of $y = f(x)$ cuts:

(i) the x-axis;

(ii) the y-axis.

(*b*) Find the coordinates of the stationary points on the curve with equation $y = f(x)$ and determine their nature.

(*c*) On separate diagrams sketch the graphs of:

(i) $y = f(x)$;

(ii) $y = -f(x)$.

</td>
<td>2

8

3</td>
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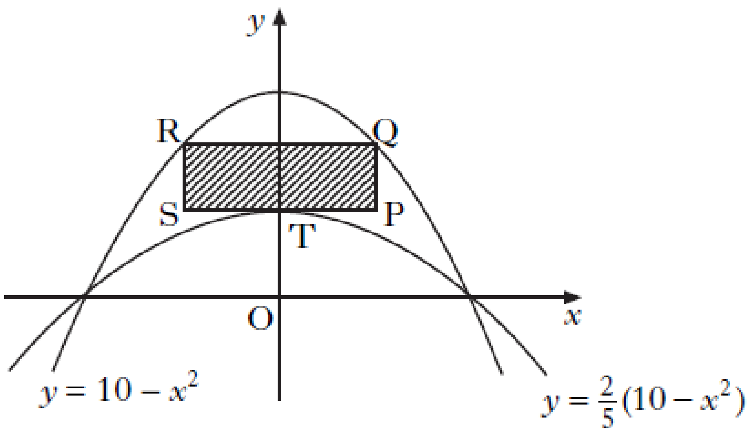
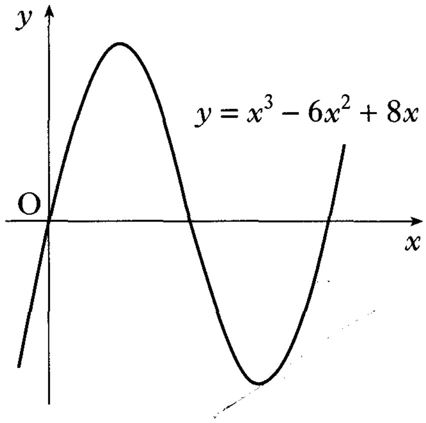
2010 P 2 Q5	The parabolas with equations $y = 10 - x^2$ and $y = \frac{2}{5}(10 - x^2)$ are shown in the diagram below.  A rectangle PQRS is placed between the two parabolas as shown, so that: • Q and R lie on the upper parabola; • RQ and SP are parallel to the x-axis; • T, the turning point of the lower parabola, lies on SP. (a) (i) If TP = x units, find an expression for the length of PQ. (ii) Hence show that the area, A, of rectangle PQRS is given by $A(x) = 12x - 2x^3$. (b) Find the maximum area of this rectangle.	3 6
2009 P2 Q1	Find the coordinates of the turning points of the curve with equation $y = x^3 - 3x^2 - 9x + 12$ and determine their nature.	8
2008 P1	**21.** A function f is defined on the set of real numbers by $f(x) = x^3 - 3x + 2$. (a) Find the coordinates of the stationary points on the curve $y = f(x)$ and determine their nature. (b) (i) Show that $(x - 1)$ is a factor of $x^3 - 3x + 2$. (ii) Hence or otherwise factorise $x^3 - 3x + 2$ fully. (c) State the coordinates of the points where the curve with equation $y = f(x)$ meets both the axes and hence sketch the curve.	6 5 4

2008 P1

22. The diagram shows a sketch of the curve with equation $y = x^3 - 6x^2 + 8x$.



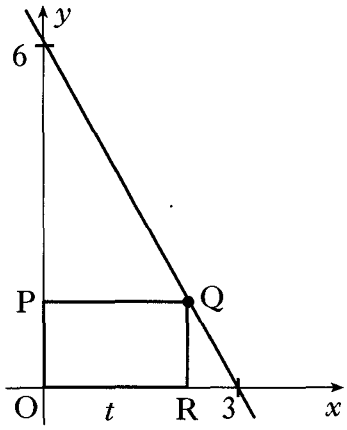
(*a*) Find the coordinates of the points on the curve where the gradient of the tangent is -1. 5

(*b*) The line $y = 4 - x$ is a tangent to this curve at a point A. Find the coordinates of A. 2

2008 P2

6. In the diagram, Q lies on the line joining (0, 6) and (3, 0).

OPQR is a rectangle, where P and R lie on the axes and OR = t.



(*a*) Show that QR = $6 - 2t$. 3

(*b*) Find the coordinates of Q for which the rectangle has a maximum area. 6

2007 P1

9. A function f is defined by the formula $f(x) = 3x - x^3$.

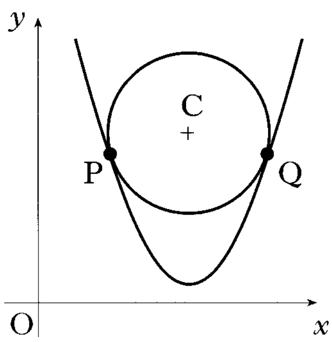
(*a*) Find the exact values where the graph of $y = f(x)$ meets the x- and y-axes. 2

(*b*) Find the coordinates of the stationary points of the function and determine their nature. 7

(*c*) Sketch the graph of $y = f(x)$. 1

2007 P2

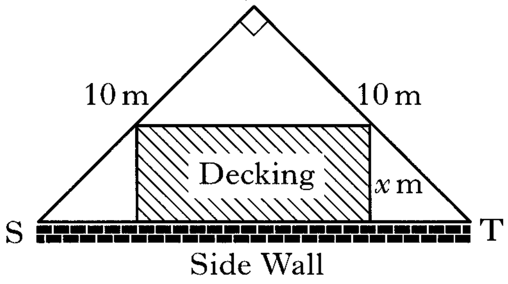
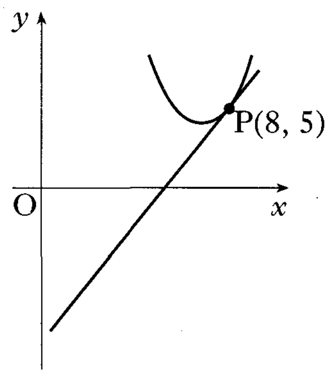
5. A circle centre C is situated so that it touches the parabola with equation $y = \frac{1}{2}x^2 - 8x + 34$ at P and Q.



(*a*) The gradient of the tangent to the parabola at Q is 4. Find the coordinates of Q. 5

(*b*) Find the coordinates of P. 2

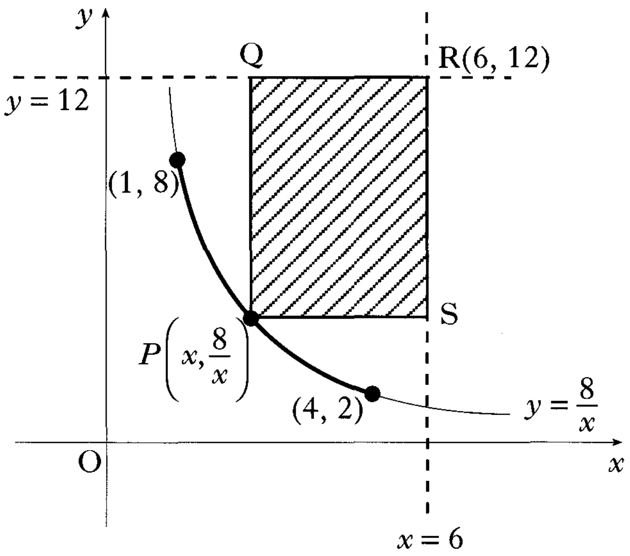
(*c*) Find the coordinates of C, the centre of the circle. 2

2007 P2	**6.** A householder has a garden in the shape of a right-angled isosceles triangle. It is intended to put down a section of rectangular wooden decking at the side of the house, as shown in the diagram.  (*a*) (i) Find the exact value of ST. (ii) Given that the breadth of the decking is x metres, show that the area of the decking, A square metres, is given by $A = (10\sqrt{2})x - 2x^2.$ (*b*) Find the dimensions of the decking which maximises its area.	3 5
2006 P2	**3.** The parabola with equation $y = x^2 - 14x + 53$ has a tangent at the point P(8, 5). (*a*) Find the equation of this tangent.  	4

2006 P2

12. PQRS is a rectangle formed according to the following conditions:

- it is bounded by the lines $x = 6$ and $y = 12$
- P lies on the curve with equation $y = \frac{8}{x}$ between (1, 8) and (4, 2)
- R is the point (6, 12).



(*a*) (i) Express the lengths of PS and RS in terms of x, the x-coordinate of P.

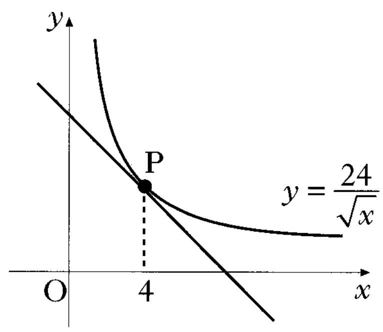
(ii) Hence show that the area, A square units, of PQRS is given by $A = 80 - 12x - \frac{48}{x}$. **3**

(*b*) Find the greatest and least possible values of A and the corresponding values of x for which they occur. **8**

2005 P2

6. The diagram shows the graph of $y = \frac{24}{\sqrt{x}}$, $x > 0$.

Find the equation of the tangent at P, where $x = 4$. **6**

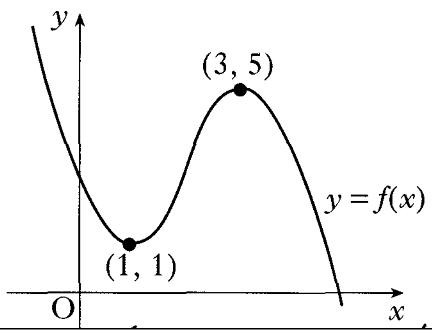
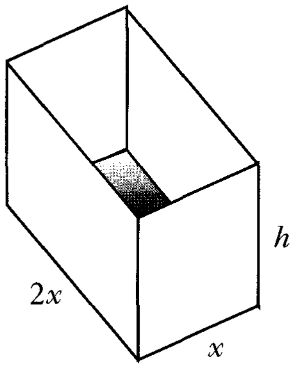
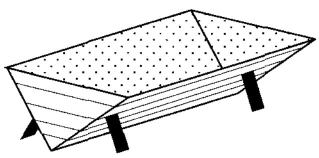
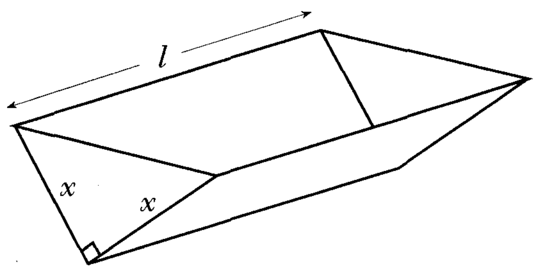


2004 P2

5. The point P(x, y) lies on the curve with equation $y = 6x^2 - x^3$.

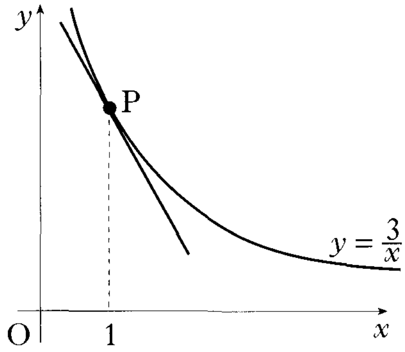
(*a*) Find the value of x for which the gradient of the tangent at P is 12. **5**

(*b*) Hence find the equation of the tangent at P. **2**

2004 P2	**7.** The graph of the cubic function $y = f(x)$ is shown in the diagram. There are turning points at (1, 1) and (3, 5). Sketch the graph of $y = f'(x)$. 	3
2004 P2	**9.** An open cuboid measures internally x units by $2x$ units by h units and has an inner surface area of 12 units^2.  (*a*) Show that the volume, V units^3, of the cuboid is given by $V(x) = \frac{2}{3}x\left(6 - x^2\right)$. (*b*) Find the exact value of x for which this volume is a maximum.	3 5
2003 P1	**5.** Given that $f(x) = \sqrt{x} + \frac{2}{x^2}$, find $f'(4)$.	5
2003 P2	**4.** (*a*) Find the equation of the tangent to the curve with equation $y = x^3 + 2x^2 - 3x + 2$ at the point where $x = 1$.	5
2003 P2	**8.** An open water tank, in the shape of a triangular prism, has a capacity of 108 litres. The tank is to be lined on the inside in order to make it watertight.  The triangular cross-section of the tank is right-angled and isosceles, with equal sides of length x cm. The tank has a length of l cm.  (*a*) Show that the surface area to be lined, A cm^2, is given by $A(x) = x^2 + \frac{432000}{x}$. (*b*) Find the value of x which minimises this surface area.	3 5

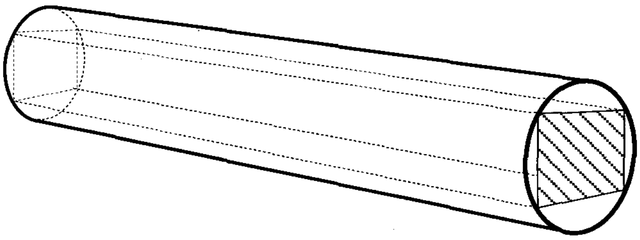
2002W P1

3. Find the equation of the tangent to the curve with equation $y = \frac{3}{x}$ at the point P where $x = 1$. **5**

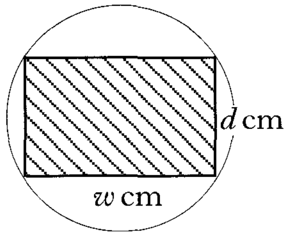


2002W P2

7. A rectangular beam is to be cut from a cylindrical log of diameter 20 cm.



The diagram shows a cross-section of the log and beam where the beam has a breadth of w cm and a depth of d cm.



The strength S of the beam is given by

$S = 1{\cdot}7\, w\ (400 - w^2)$.

Find the dimensions of the beam for maximum strength. **5**

2002 P1

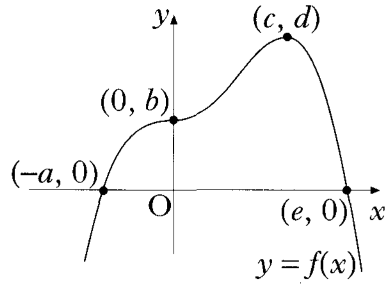
4. Find the coordinates of the point on the curve $y = 2x^2 - 7x + 10$ where the tangent to the curve makes an angle of 45° with the positive direction of the x-axis. **4**

2002 P1

6. The graph of a function f intersects the x-axis at $(-a, 0)$ and $(e, 0)$ as shown.

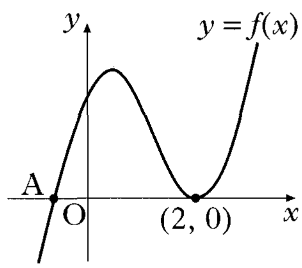
There is a point of inflexion at $(0, b)$ and a maximum turning point at (c, d).

Sketch the graph of the derived function f'. **3**



2002 P2

3. The diagram shows part of the graph of the curve with equation $y = 2x^3 - 7x^2 + 4x + 4$.



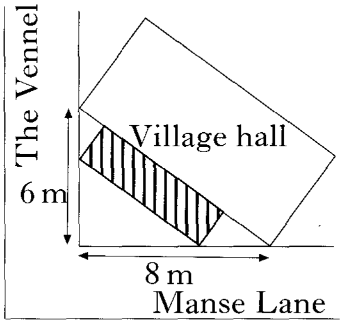
(*a*) Find the x-coordinate of the maximum turning point. **5**

(*b*) Factorise $2x^3 - 7x^2 + 4x + 4$. **3**

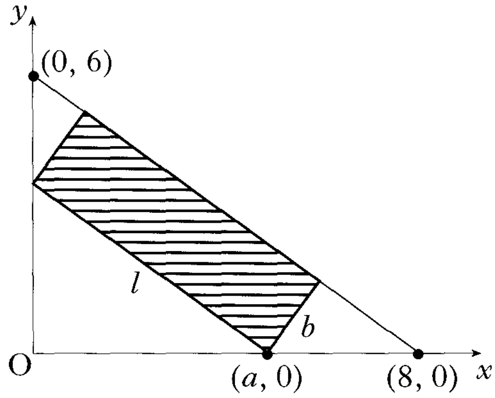
(*c*) State the coordinates of the point A and hence find the values of x for which $2x^3 - 7x^2 + 4x + 4 < 0$. **2**

2002 P2

10. The shaded rectangle on this map represents the planned extension to the village hall. It is hoped to provide the largest possible area for the extension.



The coordinate diagram represents the right angled triangle of ground behind the hall. The extension has length l metres and breadth b metres, as shown. One corner of the extension is at the point $(a, 0)$.



(*a*) (i) Show that $l = \frac{5}{4}a$.

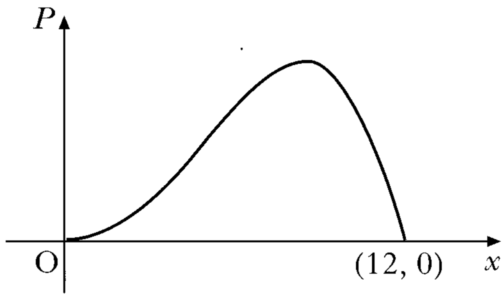
(ii) Express b in terms of a and hence deduce that the area, $A\,\text{m}^2$, of the extension is given by $A = \frac{3}{4}a(8 - a)$. 3

(*b*) Find the value of a which produces the largest area of the extension. 4

2001 P1

6. A company spends x thousand pounds a year on advertising and this results in a profit of P thousand pounds. A mathematical model, illustrated in the diagram, suggests that P and x are related by $P = 12x^3 - x^4$ for $0 \leq x \leq 12$.

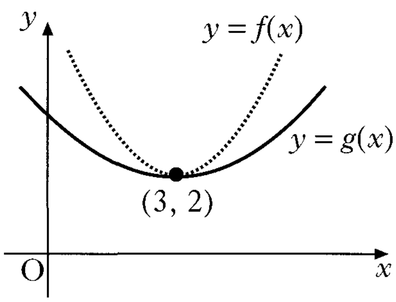
Find the value of x which gives the maximum profit. 5

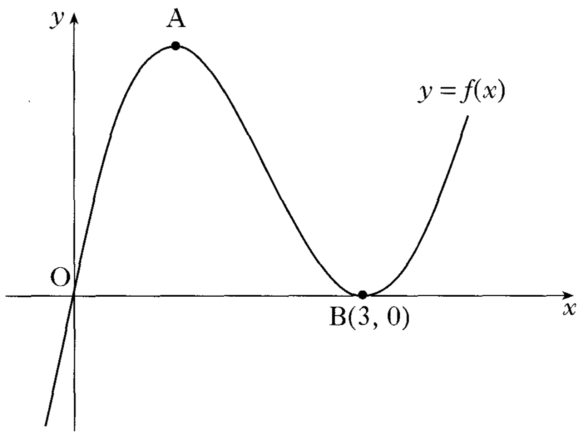
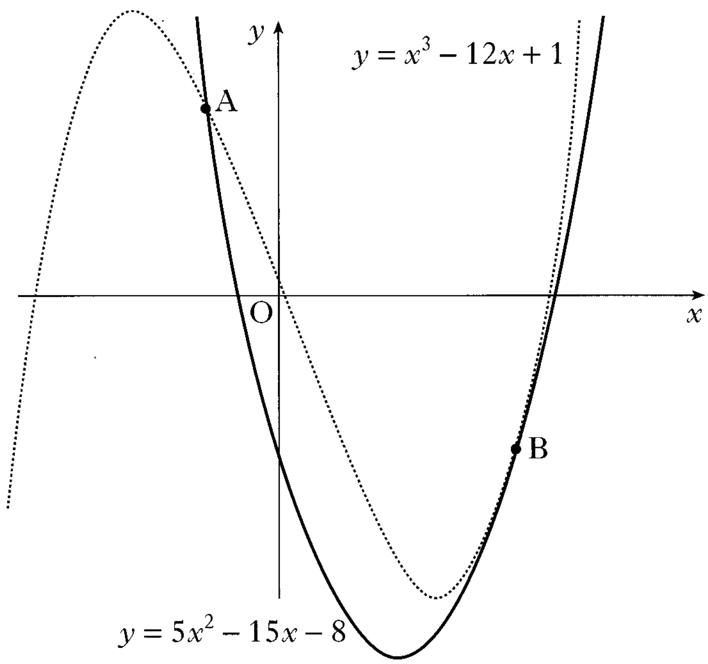


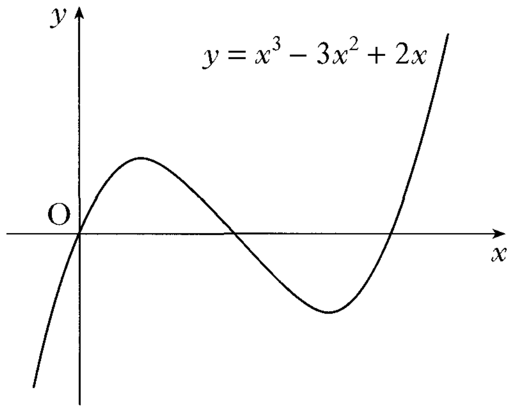
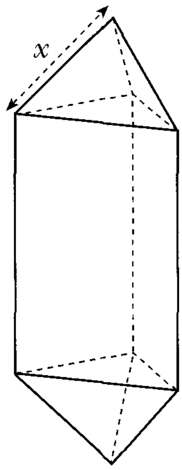
2001 P1

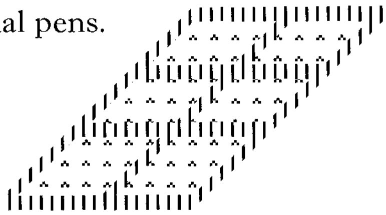
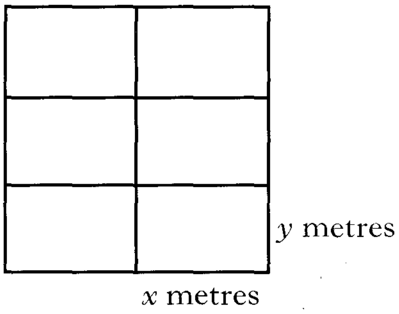
9. The diagram shows the graphs of two quadratic functions $y = f(x)$ and $y = g(x)$. Both graphs have a minimum turning point at (3, 2).

Sketch the graph of $y = f'(x)$ and on the same diagram sketch the graph of $y = g'(x)$. 2

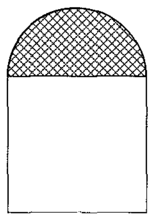


2001 P2	**2.** A curve has equation $y = x - \dfrac{16}{\sqrt{x}}$, $x > 0$. Find the equation of the tangent at the point where $x = 4$.	6
2000 P1	**2.** A sketch of the graph of $y = f(x)$ where $f(x) = x^3 - 6x^2 + 9x$ is shown below. The graph has a maximum at A and a minimum at B(3, 0).  (a) Find the coordinates of the turning point at A.	4
2000 P1	**4.** The diagram shows a sketch of the graphs of $y = 5x^2 - 15x - 8$ and $y = x^3 - 12x + 1$. The two curves intersect at A and touch at B, ie at B the curves have a common tangent.  (a) (i) Find the x-coordinates of the points on the curves where the gradients are equal. (ii) By considering the corresponding y-coordinates, or otherwise, distinguish geometrically between the two cases found in part (i).	4 1

<table>
<tr><td>2000 P2</td><td>1. The diagram shows a sketch of the graph of $y = x^3 - 3x^2 + 2x$.
(a) Find the equation of the tangent to this curve at the point where $x = 1$.
(b) The tangent at the point (2, 0) has equation $y = 2x - 4$. Find the coordinates of the point where this tangent meets the curve again.

</td><td>5
5</td></tr>
<tr><td>2000 P2</td><td>6. A goldsmith has built up a solid which consists of a triangular prism of fixed volume with a regular tetrahedron at each end.
The surface area, A, of the solid is given by
$$A(x) = \frac{3\sqrt{3}}{2}\left(x^2 + \frac{16}{x}\right)$$
where x is the length of each edge of the tetrahedron.
Find the value of x which the goldsmith should use to minimise the amount of gold plating required to cover the solid.

</td><td>6</td></tr>
<tr><td>Specimen 2 P1</td><td>3. The point P(−1, 7) lies on the curve with equation $y = 5x^2 + 2$. Find the equation of the tangent to the curve at P.</td><td>3</td></tr>
<tr><td>Specimen 2 P2</td><td>9. A curve has equation $y = 2x^3 + 3x^2 + 4x - 5$.
Prove that this curve has no stationary points.</td><td>5</td></tr>
</table>

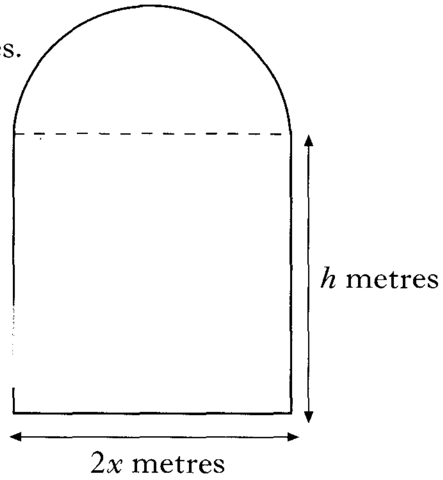
Specimen 2 P1	**10.** A zookeeper wants to fence off six individual animal pens.   Each pen is a rectangle measuring x metres by y metres, as shown in the diagram. (*a*) (i) Express the total length of fencing in terms of x and y. (ii) Given that the total length of fencing is 360 m, show that the total area, A m^2, of the six pens is given by $A(x) = 240x - \frac{16}{3}x^2$. (*b*) Find the values of x and y which give the maximum area and write down this maximum area.	4 6
Specimen 1 P1	**3.** (*a*) Show that $(x - 1)$ is a factor of $f(x) = x^3 - 6x^2 + 9x - 4$ and find the other factors. (*b*) Write down the coordinates of the points at which the graph of $y = f(x)$ meets the axes. (*c*) Find the stationary points of $y = f(x)$ and determine the nature of each. (*d*) Sketch the graph of $y = f(x)$.	3 1 5 1
Specimen 1 P2	**4.** In the diagram below, a winding river has been modelled by the curve $y = x^3 - x^2 - 6x - 2$ and a road has been modelled by the straight line AB. The road is a tangent to the river at the point A(1, −8). (*a*) Find the equation of the tangent at A.	3

9. A window in the shape of a rectangle surmounted by a semicircle is being designed to let in the maximum amount of light.



The glass to be used for the semicircular part is stained glass which lets in one unit of light per square metre; the rectangular part uses clear glass which lets in 2 units of light per square metre.

The rectangle measures $2x$ metres by h metres.



(*a*) (i) If the perimeter of the whole window is 10 metres, express h in terms of x. 2

(ii) Hence show that the amount of light, L, let in by the window is given by $L = 20x - 4x^2 - \frac{3}{2}\pi x^2$. 2

(*b*) Find the values of x and h that must be used to allow this design to let in the maximum amount of light. 5