Functions

2019 P2 Q8	A function, f, is given by $f(x)=\sqrt[3]{x}+8$. The domain of f is $1\leq x\leq 1000,\ x\in\mathbb{R}$. The inverse function, f^{-1}, exists. (a) Find $f^{-1}(x)$. (b) State the domain of f^{-1}.	 3 1
2019 P1 Q12	Functions f and g are defined by • $f(x)=\frac{1}{\sqrt{x}}$, where $x>0$ • $g(x)=5-x$, where $x\in\mathbb{R}$. (a) Determine an expression for $f(g(x))$. (b) State the range of values of x for which $f(g(x))$ is undefined.	 2 1
2018 P1 Q2	A function $g(x)$ is defined on $\mathbb{R}$, the set of real numbers, by $g(x)=\frac{1}{5}x-4.$ Find the inverse function, $g^{-1}(x)$.	 3
2018 P2 Q6	Functions, f and g, are given by $f(x)=3+\cos x$ and $g(x)=2x$, $x\in\mathbb{R}$. (a) Find expressions for (i) $f(g(x))$ and (ii) $g(f(x))$.	 2 1
2017 P1 Q1	Functions f and g are defined on suitable domains by $f(x)=5x$ and $g(x)=2\cos x$. (a) Evaluate $f(g(0))$. (b) Find an expression for $g(f(x))$.	 3
2017 P1 Q6	A function, h, is defined by $h(x)=x^3+7$, where $x\in\mathbb{R}$. Determine an expression for $h^{-1}(x)$.	 1 2

2016 P1 Q6	Functions f and g are defined on $\mathbb{R}$, the set of real numbers. The inverse functions f^{-1} and g^{-1} both exist. (a) Given $f(x) = 3x + 5$, find $f^{-1}(x)$. (b) If $g(2) = 7$, write down the value of $g^{-1}(7)$.	2 3
2016 P1 Q12	The functions f and g are defined on $\mathbb{R}$, the set of real numbers by $f(x) = 2x^2 - 4x + 5$ and $g(x) = 3 - x$. (a) Given $h(x) = f(g(x))$, show that $h(x) = 2x^2 - 8x + 11$. (b) Express $h(x)$ in the form $p(x+q)^2 + r$.	3 1
2015 P1Q5	A function g is defined on $\mathbb{R}$, the set of real numbers, by $g(x) = 6 - 2x$. (a) Determine an expression for $g^{-1}(x)$. **2** (b) Write down an expression for $g(g^{-1}(x))$. **1**	
2015 P2 Q2	Functions f and g are defined on suitable domains by $f(x) = 10 + x$ and $g(x) = (1 + x)(3 - x) + 2$. (a) Find an expression for $f(g(x))$. **2** (b) Express $f(g(x))$ in the form $p(x + q)^2 + r$. **3** (c) Another function h is given by $h(x) = \frac{1}{f(g(x))}$. What values of x cannot be in the domain of h? **2**	
2014 P2 Q3	Functions f and g are defined on suitable domains by $f(x) = x(x - 1) + q$ and $g(x) = x + 3$. (a) Find an expression for $f(g(x))$. **2** (b) Hence, find the value of q such that the equation $f(g(x)) = 0$ has equal roots. **4**	

2012 P2 Q1	Functions f and g are defined on the set of real numbers by • $f(x) = x^2 + 3$ • $g(x) = x + 4$. (*a*) Find expressions for: (i) $f(g(x))$; (ii) $g(f(x))$. (*b*) Show that $f(g(x)) + g(f(x)) = 0$ has no real roots.	3 3
2011 P2 Q2	Functions f, g and h are defined on the set of real numbers by • $f(x) = x^3 - 1$ • $g(x) = 3x + 1$ • $h(x) = 4x - 5$. (*a*) Find $g(f(x))$. (*b*) Show that $g(f(x)) + xh(x) = 3x^3 + 4x^2 - 5x - 2$. (*c*) (i) Show that $(x - 1)$ is a factor of $3x^3 + 4x^2 - 5x - 2$. (ii) Factorise $3x^3 + 4x^2 - 5x - 2$ fully. (*d*) Hence solve $g(f(x)) + xh(x) = 0$.	2 1 5 1
2009 P2 Q2	Functions f and g are given by $f(x) = 3x + 1$ and $g(x) = x^2 - 2$. (*a*) (i) Find $p(x)$ where $p(x) = f(g(x))$. (ii) Find $q(x)$ where $q(x) = g(f(x))$. (*b*) Solve $p'(x) = q'(x)$.	
2008 P1	**23.** Functions f, g and h are defined on suitable domains by $f(x) = x^2 - x + 10$, $g(x) = 5 - x$ and $h(x) = \log_2 x$. (*a*) Find expressions for $h(f(x))$ and $h(g(x))$.	3
2007 P1	**3.** Functions f and g, defined on suitable domains, are given by $f(x) = x^2 + 1$ and $g(x) = 1 - 2x$. Find: (*a*) $g(f(x))$; (*b*) $g(g(x))$.	2 2

2006 P1	**3.** Two functions f and g are defined by $f(x) = 2x + 3$ and $g(x) = 2x - 3$, where x is a real number. (a) Find expressions for: (i) $f(g(x))$; (ii) $g(f(x))$. (b) Determine the least possible value of the product $f(g(x)) \times g(f(x))$.	 3 2
2005 P1	**4.** Functions $f(x) = 3x - 1$ and $g(x) = x^2 + 7$ are defined on the set of real numbers. (a) Find $h(x)$ where $h(x) = g(f(x))$. (b) (i) Write down the coordinates of the minimum turning point of $y = h(x)$. (ii) Hence state the range of the function h.	 2 2
2003 P1	**9.** Functions $f(x) = \frac{1}{x-4}$ and $g(x) = 2x + 3$ are defined on suitable domains. (a) Find an expression for $h(x)$ where $h(x) = f(g(x))$. (b) Write down any restriction on the domain of h.	 2 1
2002W P1	**9.** The function f, defined on a suitable domain, is given by $f(x) = \frac{3}{x+1}$. (a) Find an expression for $h(x)$ where $h(x) = f(f(x))$, giving your answer as a fraction in its simplest form. (b) Describe any restriction on the domain of h.	 3 1
2002 P1	**3.** Functions f and g are defined on suitable domains by $f(x) = \sin(x°)$ and $g(x) = 2x$. (a) Find expressions for: (i) $f(g(x))$; (ii) $g(f(x))$.	 2
2001 P1	**7.** Functions $f(x) = \sin x$, $g(x) = \cos x$ and $h(x) = x + \frac{\pi}{4}$ are defined on a suitable set of real numbers. (a) Find expressions for: (i) $f(h(x))$; (ii) $g(h(x))$.	 2
2000 P2	**3.** $f(x) = 3 - x$ and $g(x) = \frac{3}{x}$, $x \neq 0$. (a) Find $p(x)$ where $p(x) = f(g(x))$. (b) If $q(x) = \frac{3}{3-x}$, $x \neq 3$, find $p(q(x))$ in its simplest form.	 2 3

Specimen 2 P1	**8.** Functions f and g are defined on the set of real numbers by $f(x) = x - 1$ $g(x) = x^2$. (a) Find formulae for (i) $f(g(x))$ (ii) $g(f(x))$.	3