7. QUADRATICS

7.1 COMPLETING THE SQUARE

(a) $2\left[x^2 - 4x\right] + 7$

$= 2\left[(x-2)^2 - 4\right] + 7$

$= 2(x-2)^2 - 8 + 7$

$= \underline{2(x-2)^2 - 1}$

(b) $3\left[x^2 + 2x\right] - 1$

$= 3\left[(x+1)^2 - 1\right] - 1$

$= 3(x+1)^2 - 3 - 1$

$= \underline{3(x+1)^2 - 4}$

(c) $5\left[x^2 - 10x\right] - 100$

$= 5\left[(x-5)^2 - 25\right] - 100$

$= 5(x-5)^2 - 125 - 100$

$= \underline{5(x-5)^2 - 225}$

7.2 QUADRATIC INEQUATIONS

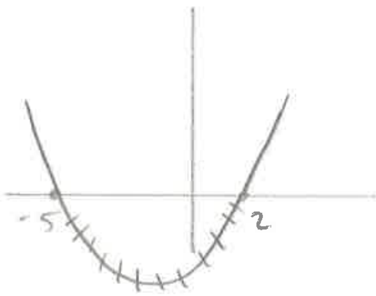
(a) $x^2 + 3x - 10 < 0$

$x^2 + 3x - 10 = 0$

$(x+5)(x-2) = 0$

$\downarrow \qquad \downarrow$

$x = -5 \qquad x = 2$



$\underline{-5 < x < 2}$

(b) $6 - x - x^2 \leq 0$

$-x^2 - x + 6 \leq 0$

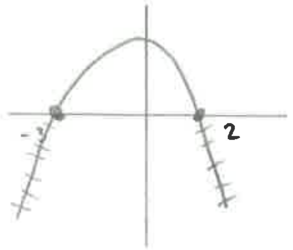
$-x^2 - x + 6 = 0$

$-(x^2 + x - 6) = 0$

$-(x + 3)(x - 2) = 0$

$\downarrow \quad \downarrow$

$x = -3 \quad x = 2$



$\underline{x \leq -3} \quad \underline{x \geq 2}$

7.3 DISCRIMINANT

7.3.1 UNKNOWN VALUES

(a) For 2 real and distinct roots $\Delta > 0$.

$$3x^2 - 2x + 2 - k = 0$$

$a = 3 \quad b = -2 \quad c = 2 - k$

$$\begin{aligned} \Delta &= (-2)^2 - 4(3)(2-k) \\ &= 4 - 4(6 - 3k) \\ &= 4 - 24 + 12k \\ &= 12k - 20 \end{aligned}$$

$\because \Delta > 0$

$12k - 20 > 0$

$12k > 20$

$\underline{k > \frac{5}{3}}$

(b) For equal roots $\Delta = 0$

$$3x^2 + px + 3 = 0$$

$a = 3 \quad b = p \quad c = 3$

$$\Delta = (p)^2 - 4(3)(3)$$
$$= p^2 - 36$$

$\because \Delta = 0$

$$p^2 - 36 = 0$$

$$(p+6)(p-6) = 0$$

$p = -6 \quad p = 6$

(c) For no real roots $\Delta < 0$

$$x^2 - mx + m + 3 = 0$$

$a = 1 \quad b = -m \quad c = m + 3$

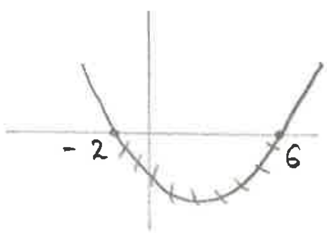
$$\Delta = (-m)^2 - 4(1)(m+3)$$
$$= m^2 - 4(m+3)$$
$$= m^2 - 4m - 12$$

$\because \Delta < 0$

$$m^2 - 4m - 12 < 0$$

$$m^2 - 4m - 12 = 0$$
$$(m-6)(m+2) = 0$$

$m = 6 \quad m = -2$



$-2 < m < 6$

7.3.2 ALWAYS / NEVER REAL ROOTS

(a) For real roots $\Delta \geq 0$

$$2x^2 + px - 3 = 0$$

$a = 2 \quad b = p \quad c = -3$

$$\Delta = (p)^2 - 4(2)(-3)$$
$$= p^2 - 4(-6)$$
$$= p^2 + 24$$

$\because p^2 \geq 0$

$p^2 + 24 > 0$

$\therefore \Delta > 0$

$\therefore$ roots are real for all values of p. ◿

(b) For real roots $\Delta \geq 0$

$$(1-2k)x^2 - 5kx - 2k = 0$$

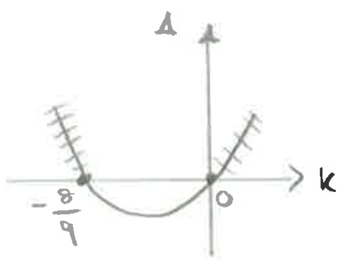
$a = 1 - 2k \quad b = -5k \quad c = -2k$

$$\Delta = (-5k)^2 - 4(1-2k)(-2k)$$
$$= 25k^2 - 4(-2k + 4k^2)$$
$$= 25k^2 + 8k - 16k^2$$
$$= 9k^2 + 8k$$

$9k^2 + 8k = 0$

$k(9k + 8) = 0$

$k = 0 \qquad k = -\frac{8}{9}$



$\Delta \geq 0$ when $k \leq -\frac{8}{9}$ and $k \geq 0$.

$\therefore$ real roots for all integer values of k. ◿