

## 6. POLYNOMIALS

### 6.1 SYNTHETIC DIVISION

#### 6.1.1 FULLY FACTORISING

$$\begin{array}{r|rrrr} (a) & 3 & 2 & -15 & 22 & 15 \\ & & & 6 & -27 & -15 \\ \hline & & 2 & -9 & -5 & \boxed{0} \end{array}$$

$\therefore \text{rem} = 0$   
 $(x-3)$  is a factor.

$$\begin{aligned} & (x-3)(2x^2 - 9x - 5) \\ & = \underline{\underline{(x-3)(2x+1)(x-5)}} \end{aligned}$$

$$\begin{array}{r|rrrr} (b) & -2 & 3 & -22 & -47 & 18 \\ & & & -6 & 56 & -18 \\ \hline & & 3 & -28 & 9 & \boxed{0} \end{array}$$

$\therefore \text{rem} = 0$   
 $(x+2)$  is a factor

$$\begin{aligned} & (x+2)(3x^2 - 28x + 9) \\ & = \underline{\underline{(x+2)(3x+1)(x+9)}} \end{aligned}$$

$$\begin{array}{r|rrrr} (c) & 1 & 3 & 0 & -9 & 6 \\ & & & 3 & 3 & -6 \\ \hline & & 3 & 3 & -6 & \boxed{0} \end{array}$$

$\therefore \text{rem} = 0$   
 $(x-1)$  is a factor

$$\begin{aligned} & (x-1)(3x^2 + 3x - 6) \\ & 3(x-1)(x^2 + x - 2) \\ & = \underline{\underline{3(x-1)(x+2)(x-1)}} \end{aligned}$$

## 6.1.2 UNKNOWN COEFFICIENTS

$$(a) \quad 2 \left| \begin{array}{cccc} 1 & -5 & p & -6 \\ & 2 & -6 & 2p-12 \\ \hline & -3 & p-6 & 2p-18 \end{array} \right.$$

$\therefore (x-2)$  is a factor

$$\text{rem} = 0$$

$$\therefore 2p-18 = 0$$

$$\therefore \underline{\underline{p = 9}}$$

$$(b) \quad -1 \left| \begin{array}{cccc} 1 & k & 1 & 6 \\ & -1 & -k+1 & k-2 \\ \hline & 1 & k-1 & -k+2 \end{array} \right. \quad \boxed{k+4}$$

$\therefore (x+1)$  is a factor

$$\text{rem} = 0$$

$$\therefore k+4 = 0$$

$$\underline{\underline{k = -4}}$$

$$(x+1)(x^2 - 5x + 6)$$

$$(x+1)(x-2)(x-3)$$

other two factors.

$$(c) \quad -3 \left| \begin{array}{cccc} 5 & p & q & -24 \\ & -15 & -3p+45 & 9p-3q-135 \\ \hline & 5 & p-15 & -3p+q+45 \end{array} \right. \quad \boxed{9p-3q-159}$$

$\therefore (x+3)$  is a factor

$$9p-3q-159 = 0$$

$$\boxed{9p-3q = 159}$$

$$2 \left| \begin{array}{cccc} 5 & p & q & -24 \\ & 10 & 2p+2q & 4p+2q+40 \\ \hline & 5 & p+10 & 2p+q+20 \end{array} \right. \quad \boxed{4p+2q+16}$$

$\therefore \text{rem} = -120$

$$4p+2q+16 = -120$$

$$\boxed{4p+2q = -136}$$

$$\begin{cases} 9p - 3q = 159 & (\times 2) \\ 4p + 2q = -136 & (\times 3) \end{cases}$$

$$\begin{cases} 18p - 6q = 318 \\ 12p + 6q = -408 \end{cases}$$


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$$30p = -90$$

$$p = -3$$

$$p = 3$$

$$4(3) + 2q = -136$$

$$12 + 2q = -136$$

$$2q = -148$$

$$q = -74$$

## 6.2 SOLVING CUBIC EQUATIONS

### 6.2.1 X-INTERCEPTS

(a) Crosses x-axis when  $y = 0$

$$0 = 2x^3 - 15x^2 + 22x + 15$$

$$0 = (x-3)(2x^2 - 9x - 5)$$

$$0 = (x-3)(2x+1)(x-5)$$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ x=3 & x=-\frac{1}{2} & x=5 \end{array}$$

$$\underline{(3, 0)} \quad \underline{\left(-\frac{1}{2}, 0\right)} \quad \underline{(5, 0)}$$

"try  $x=1$ "

$$\begin{array}{r|rrrr} 1 & 2 & -15 & 22 & 15 \\ & & 2 & -13 & 9 \\ \hline & 2 & -13 & 9 & 24 \end{array} \times$$

"try  $x=3$ "

$$\begin{array}{r|rrrr} 3 & 2 & -15 & 22 & 15 \\ & & 6 & -27 & -15 \\ \hline & 2 & -9 & -5 & 0 \end{array} \checkmark$$

$\therefore \text{rem} = 0$   
 $(x-3)$  is factor

(b) Crosses x-axis when  $y = 0$

$$0 = 3x^3 - 22x^2 - 47x + 18$$

$$0 = (x+2)(3x^2 - 28x + 9)$$

$$0 = (x+2)(3x-1)(x-9)$$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ x=-2 & x=\frac{1}{3} & x=9 \end{array}$$

$$\underline{(-2, 0)} \quad \underline{\left(\frac{1}{3}, 0\right)} \quad \underline{(9, 0)}$$

"try  $x=1$ "

$$\begin{array}{r|rrrr} 1 & 3 & -22 & -47 & 18 \\ & & 3 & -19 & -66 \\ \hline & 3 & -19 & -66 & 0 \end{array} \times$$

"try  $x=-2$ "

$$\begin{array}{r|rrrr} -2 & 3 & -22 & -47 & 18 \\ & & -6 & 56 & -18 \\ \hline & 3 & -28 & 9 & 0 \end{array} \checkmark$$

$\therefore \text{rem} = 0$   
 $(x+2)$  is factor

## 6.2.2 INTERSECTIONS

(a) For intersection  $y = y$

$$x^3 + 2x^2 - 4x - 3 = 3x - 7$$

$$x^3 + 2x^2 - 7x + 4 = 0$$

$$(x-1)(x^2 + 3x - 4) = 0$$

$$(x-1)(x+4)(x-1) = 0$$

$$x = 1 \quad \underline{x = -4} \quad x = 1$$

$$\begin{array}{r|rrrr} 1 & 1 & 2 & -7 & 4 \\ & & 1 & 3 & -4 \\ \hline & 1 & 3 & -4 & 0 \end{array}$$

$\therefore \text{rem} = 0$

$(x-1)$  is a factor

(Other) POC  $(-4, -19)$

(b) For intersection  $y = y$

$$x^3 - 6x^2 - 3x - 1 = x^2 - 10x - 16$$

$$x^3 - 7x^2 + 7x + 15 = 0$$

$$(x+1)(x^2 - 8x + 15) = 0$$

$$(x+1)(x-5)(x-3) = 0$$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ x = -1 & x = 5 & x = 3 \end{array}$$

$$x = -1 \quad x = 5 \quad x = 3$$

$$\underline{\underline{(-1, -5)}} \quad \underline{\underline{(5, -41)}} \quad \underline{\underline{(3, -37)}}$$

"try  $x = 1$ "

$$\begin{array}{r|rrrrr} 1 & 1 & -7 & 7 & 15 \\ & & 1 & -6 & 1 \\ \hline & 1 & -6 & 1 & 16 \end{array} \times$$

"try  $x = -1$ "

$$\begin{array}{r|rrrr} -1 & 1 & -7 & 7 & 15 \\ & & -1 & 8 & -15 \\ \hline & 1 & -8 & 15 & 0 \end{array}$$

$\therefore \text{rem} = 0$

$(x+1)$  is a factor

### 6.3 FINDING THE EQUATION OF A GRAPH

$$(a) \quad y = k(x+2)(x-5)(x-6)$$

$$30 = k(1+2)(1-5)(1-6)$$

$$30 = k(3)(-4)(-5)$$

$$30 = k(60)$$

$$\frac{1}{2} = k$$

$$\rightarrow \underline{\underline{y = \frac{1}{2}(x+2)(x-5)(x-6)}}$$

$$(b) \quad y = k(x-2)(x-2)(x-8)$$

$$64 = k(6-2)(6-2)(6-8)$$

$$64 = k(4)(4)(-2)$$

$$64 = k(-32)$$

$$-2 = k$$

$$\rightarrow \underline{\underline{y = -2(x-2)^2(x-8)}}$$

