

3. RECURRENCE RELATIONS

3.1 BASIC OPERATIONS

$$(a) \quad U_2 = 0.4(5) + 8 \\ = \underline{\underline{10}}$$

$$U_3 = 0.4(10) + 8 \\ = \underline{\underline{12}}$$

$$(b) \quad U_1 = \frac{1}{3}(300) - 40 \\ = 100 - 40 \\ = \underline{\underline{60}}$$

$$U_2 = \frac{1}{3}(60) - 40 \\ = 20 - 40 \\ = \underline{\underline{-20}}$$

$$(c) \quad 456 = 5U_4 + 6 \\ 450 = 5U_4 \\ \underline{\underline{90 = U_4}}$$

3.2 LIMITS

$$(a) \quad \underline{\underline{\text{Limit exists } \because -1 < 0.4 < 1}}$$

$$L = \frac{8}{1-0.4} \\ = \frac{8}{0.6} \\ = \frac{80}{6} \\ = \underline{\underline{\frac{40}{3}}}$$

$$(b) \quad \underline{\underline{\text{Limit exists } \because -1 < \frac{1}{3} < 1}}$$

$$L = \frac{-5}{1-\frac{1}{3}} \\ = \frac{-5}{\frac{2}{3}} \\ = \underline{\underline{-\frac{15}{2}}}$$

(c) Limit exists $\because -1 < \frac{1}{5} < 1$

$$\begin{aligned}L &= \frac{9}{1 - \frac{1}{5}} \\&= \frac{9}{\frac{4}{5}} \\&= \frac{45}{4}\end{aligned}$$

3.3 CONTEXT EXAMPLES

(a) (i) $U_{n+1} = \frac{9}{10} U_n + 20$

(ii) $L = \frac{20}{1 - \frac{9}{10}}$
 $= \frac{20}{\frac{1}{10}}$

Yes \because $200 > 190$

= 200 penguins

(b) (i) $100\% - 0.5\% = 99.5\%$

$U_{n+1} = 0.995 U_n + 10500$

(ii) $L = \frac{10500}{1 - 0.995}$
 $= \frac{10500}{0.005}$

= 2,100,000 trees in the long run

(c) (i) $100\% + 6\% = 106\%$

$U_{n+1} = 1.06 U_n - 150$ ($U_0 = 7150$)

(ii) $U_1 = 1.06(7150) - 150$
 $= 7429$

$U_2 = 7724.74$

$U_3 = 8038.22$

$U_4 = 8370.52$ pupils

4 years \because $8370.52 > 8100$

3.4 SIMULTANEOUS EQUATIONS

$$(a) \begin{cases} 13 = 5a + b \rightarrow \textcircled{1} \\ 29 = 13a + b \rightarrow \textcircled{2} \end{cases}$$

$$16 = 8a \rightarrow \textcircled{2} - \textcircled{1}$$

$$\underline{\underline{2 = a}}$$

$$\underline{\underline{a = 2}}$$

$$13 = 5(2) + b$$

$$\underline{\underline{3 = b}}$$

$$(b) \begin{cases} 1 = 2p + q \rightarrow \textcircled{1} \\ -4 = 1p + q \rightarrow \textcircled{2} \end{cases}$$

$$\underline{\underline{5 = p}} \rightarrow \textcircled{1} - \textcircled{2}$$

$$\underline{\underline{p = 5}}$$

$$-4 = 5 + q$$

$$\underline{\underline{-9 = q}}$$

3.5 EXPRESSIONS WITH UNKNOWNNS

$$(a) (i) u_1 = 2m + 5$$

$$u_2 = m(2m + 5) + 5$$

$$= 2m^2 + 5m + 5$$

$$(ii) 8 = 2m^2 + 5m + 5$$

$$0 = 2m^2 + 5m - 3$$

$$0 = (2m - 1)(m + 3)$$

$$\downarrow$$
$$\underline{\underline{m = \frac{1}{2}}}$$

$$\downarrow$$
$$m = -3$$

$$\therefore -3 < -1$$

$$\therefore -1 < \frac{1}{2} < 1$$

