14. EXPONENTIALS AND LOGARITHMS

14.1 SIMPLIFYING LOG EXPRESSIONS

(a) $2\log_6 3 + \log_6 4$

$= \log_6 3^2 + \log_6 4$

$= \log_6 9 + \log_6 4$

$= \log_6 36$

$= \underline{\underline{2}}$

(b) $\log_4 48 - \frac{1}{2}\log_4 9$

$= \log_4 48 - \log_4 9^{\frac{1}{2}}$

$= \log_4 48 - \log_4 3$

$= \log_4 16$

$= \underline{\underline{2}}$

(c) $5\log_8 2 + \log_8 4 - \log_8 16$

$= \log_8 2^5 + \log_8 4 - \log_8 16$

$= \log_8 32 + \log_8 4 - \log_8 16$

$= \log_8 128 - \log_8 16$

$= \log_8 8$

$= \underline{\underline{1}}$

14.2 SOLVING LOG EQUATIONS

(a) $\log_x 27 = 3$

$27 = x^3$

$\underline{x = 3}$

(b) $\log_5 3x - \log_5 (x-1) = \log_5 4$

$\log_5 \frac{3x}{x-1} = \log_5 4$

$\frac{3x}{x-1} = 4$

$3x = 4(x-1)$

$3x = 4x - 4$

$-x = -4$

$\underline{x = 4}$

(c) $\log_2 x + \log_2 (x-7) = 3$

$\log_2 x(x-7) = 3$

$x(x-7) = 2^3$

$x^2 - 7x = 8$

$x^2 - 7x - 8 = 0$

$(x-8)(x+1) = 0$

$\downarrow \qquad \downarrow$

$\underline{x = 8} \qquad \cancel{x = -1}$

OUTWITH DOMAIN

14.3 SOLVING EXPONENTIAL EQUATIONS

(a) $e^{x+4} = 5$

$\log_e e^{x+4} = \log_e 5$

$(x+4)\log_e e = \ln(5)$

$x + 4 = \ln(5)$

$x = \ln(5) - 4$

$= -2.39$

(b) $2^{x-3} = 4^{2x+1}$

$\log_2 2^{x-3} = \log_2 4^{2x+1}$

$(x-3)\log_2 2 = (2x+1)\log_2 4$

$(x-3) \cdot 1 = (2x+1) \cdot 2$

$x - 3 = 4x + 2$

$-3x = 5$

$x = -\frac{5}{3}$

14.4 GROWTH AND DECAY

(a) (i) $M = M_0 e^{-kt}$

$72.5 = 80\, e^{-k(50)}$

$\frac{72.5}{80} = e^{-50k}$

$\log_e \frac{72.5}{80} = \log_e e^{-50k}$

$\ln \frac{72.5}{80} = -50k \log_e e$

$\frac{\ln \frac{72.5}{80}}{-50} = k$

$k = 0.00197$

(ii) $M = M_0 e^{-0.00197t}$

$50 = 100 e^{-0.00197t}$

$\frac{1}{2} = e^{-0.00197t}$

$\log_e \frac{1}{2} = \log_e e^{-0.00197t}$

$\ln(\frac{1}{2}) = -0.00197t \log_e e$

$\frac{\ln(\frac{1}{2})}{-0.00197} = t$

$351.85 = t$ 351.85 years

(b) (i) $B = 350e^{0.45t}$

$= 350e^{0.45(0)}$

$= 350$ bacteria

(ii) $175 = 100e^{0.45t}$

$\frac{175}{100} = e^{0.45t}$

$\log_e \frac{175}{100} = \log_e e^{0.45t}$

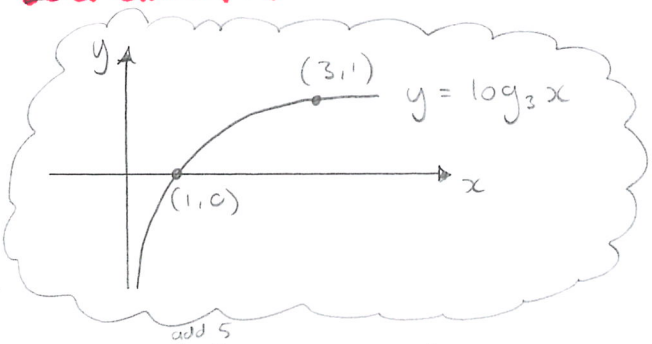
$\ln(\frac{175}{100}) = 0.45t \log_e e$

$\frac{\ln(\frac{175}{100})}{0.45} = t$

$t = 1.24$ 1.24 hours

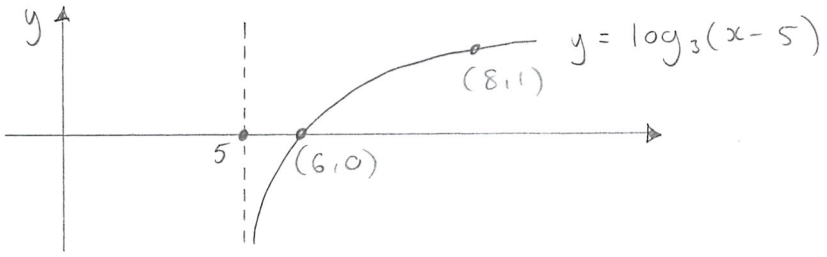
14.5 LOG GRAPHS

(a)

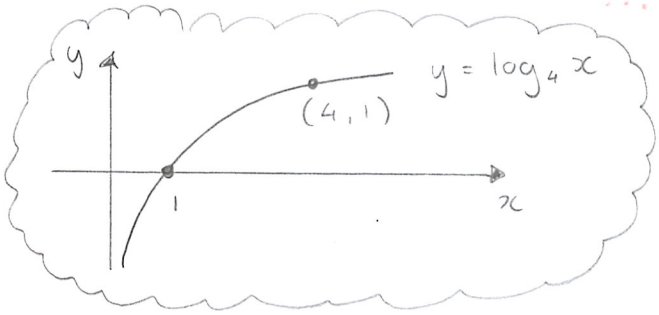


$(1,0) \xrightarrow{\text{add 5 to } x} (6,0)$

$(3,1) \longrightarrow (8,1)$



(b)

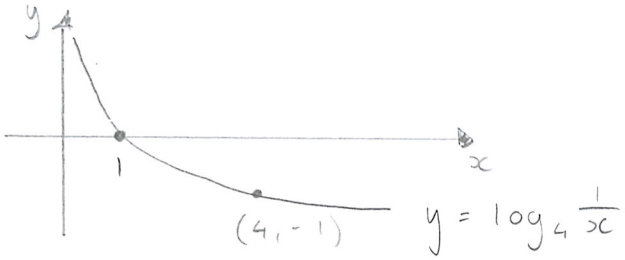


$y = \log_4 x^{-1}$

$= -\log_4 x$

$(1,0) \xrightarrow{\text{multiply } y \text{ by } -1} (1,0)$

$(4,1) \longrightarrow (4,-1)$



(c) Shifted 4 places to right $\Rightarrow$ $a = 4$

$$y = \log_b (x - 4)$$

$$2 = \log_b (40 - 4)$$

$$2 = \log_b 36$$

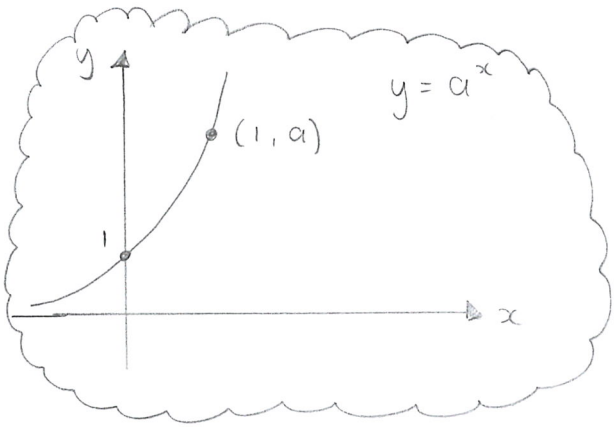
$$\log_b 36 = 2$$

$$36 = b^2$$

$$b = 6$$

14.6 EXPONENTIAL GRAPHS

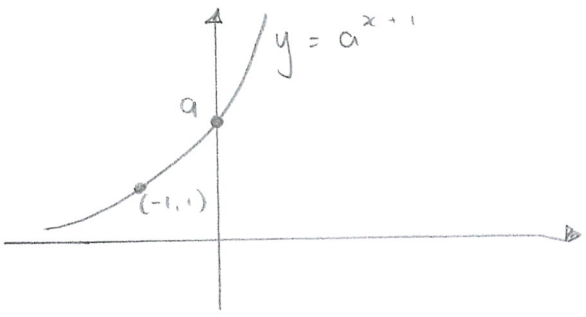
(a)



Subtract 1 From x

$(0, 1) \rightarrow (-1, 1)$

$(1, a) \rightarrow (0, a)$



(b) $y = 3^x + 1$

$p = 3^0 + 1$

$p = 1 + 1$

$\underline{p = 2}$

$y = 3^x + 1$

$10 = 3^q + 1$

$9 = 3^q$

$\underline{q = 2}$

14.7 EXPERIMENTAL DATA

(a) Let $\begin{cases} Y = \log_3 y \\ X = \log_3 x \end{cases}$

Grad: (0, 3) (3, 2)

Point: (3, 2)

$m = \frac{2-3}{3-0}$

$= -\frac{1}{3}$

$Y - 2 = -\frac{1}{3}(X - 3)$

$3Y - 6 = -X + 3$

$3Y = -X + 9$

$Y = -\frac{1}{3}X + 3$

$\log_3 y = -\frac{1}{3}\log_3 x + 3$

$\log_3 y = \log_3 x^{-\frac{1}{3}} + 3$

$\log_3 y - \log_3 x^{-\frac{1}{3}} = 3$

$\log_3 \frac{y}{x^{-\frac{1}{3}}} = 3$

$\frac{y}{x^{-\frac{1}{3}}} = 3^3$

$\frac{y}{x^{-\frac{1}{3}}} = 27$

$y = 27x^{-\frac{1}{3}}$

$\underline{k = 27}$

$\underline{n = -\frac{1}{3}}$

(b) let $Y = \log_2 y$

Grad

$m = \frac{6-2}{4-2}$

$= \frac{4}{2}$

$= 2$

Point

$(2, 2)$

$Y - 2 = 2(x - 2)$

$Y - 2 = 2x - 4$

$Y = 2x - 2$

$\log_2 y = 2x - 2$

$y = 2^{2x-2}$

$y = 2^{2x} \cdot 2^{-2}$

$y = (2^2)^x \cdot \frac{1}{2^2}$

$y = 4^x \cdot \frac{1}{4}$

$y = \frac{1}{4} \cdot 4^x$

$a = \frac{1}{4}$

$b = 4$