13. FURTHER CALCULUS

13·1 CHAIN RULE

(a) $$\frac{dy}{dx} = 3(4x-9)^2 \cdot 4$$

$$= 12(4x-9)^2$$

(b) $$f(x) = (5-x)^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2}(5-x)^{-\frac{1}{2}} \cdot (-1)$$

$$= -\frac{1}{2\sqrt{5-x}}$$

$$f'(-4) = -\frac{1}{2\sqrt{5+(4)}}$$

$$= -\frac{1}{6}$$

(c) $$f(x) = 2(3x+1)^{-1}$$

$$f'(x) = -2(3x+1)^{-2} \cdot 3$$

$$= -\frac{6}{(3x+1)^2}$$

$$f'(-1) = -\frac{6}{(3(-1)+1)^2}$$

$$= -\frac{6}{(-2)^2}$$

$$= -\frac{3}{2}$$

13.2 INTEGRATING $(ax+b)^n$

(a) $$\frac{(6x-5)^5}{5.6} + C$$

$$= \frac{(6x-5)^5}{30} + C$$

(b) $$\int (2x+1)^{-\frac{1}{2}}\,dx$$

$$= \frac{(2x+1)^{\frac{1}{2}}}{\frac{1}{2}\cdot 2} + C$$

$$= \sqrt{2x+1} + C$$

(c) $$\int_{-1}^{2} 5(4x+1)^{-3}\,dx$$

$$= \left[\frac{5(4x+1)^{-2}}{-2\cdot 4}\right]_{-1}^{2}$$

$$= \left[\frac{5(4x+1)^{-2}}{-8}\right]_{-1}^{2}$$

$$= \left[-\frac{5}{8(4x+1)^2}\right]_{-1}^{2}$$

$$= \left(-\frac{5}{8(4(2)+1)^2}\right) - \left(-\frac{5}{8(4(-1)+1)^2}\right)$$

$$= -\frac{5}{8\cdot 81} + \frac{5}{72}$$

$$= -\frac{5}{648} + \frac{5}{72}$$

$$= \frac{5}{81}$$

13·3 DIFFERENTIATING sin(x) AND cos(x)

(a) $\frac{dy}{dx} = 4\cos(3x) \cdot 3$

$= \underline{\underline{12\cos 3x}}$

(b) $f(x) = 3x^2 - \cos(5x)$

$f'(x) = 6x + \sin(5x) \cdot 5$

$= 6x + 5\sin 5x$

$f'\left(\frac{\pi}{6}\right) = 6\left(\frac{\pi}{6}\right) + 5\sin\left(5 \times \frac{\pi}{6}\right)$

$= \underline{\underline{5 \cdot 64}}$

(c) $f(x) = (\sin x)^2$

$f'(x) = 2(\sin x) \cdot \cos x$

$= 2\sin x\cos x$

$= \underline{\underline{\sin 2x}}$ $\square$

13·4 INTEGRATING sin(x) AND cos(x)

(a) $\underline{\underline{\frac{4\sin(5x)}{5} + c}}$

(b) $\int 5x^{\frac{1}{2}} - 3\sin(2x)\, dx$

$= \frac{5x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{3\cos(2x)}{2} + c$

$= \underline{\underline{\frac{10\sqrt{x^3}}{3} + \frac{3\cos 2x}{2} + c}}$

(c) $\displaystyle\int_0^{\frac{5\pi}{6}} 3 - \cos(5x)\,dx$

$$= \left[3x - \frac{\sin(5x)}{5}\right]_0^{\frac{5\pi}{6}}$$

$$= \left(3\left(\frac{5\pi}{6}\right) - \frac{\sin\left(5 \times \frac{5\pi}{6}\right)}{5}\right) - \left(3(0) - \frac{\sin(5 \times 0)}{5}\right)$$

$$= \underline{\underline{7 \cdot 75}}$$

13·5 APPLICATIONS

(a) $f(x) = 3(\cos x)^2$

$$f'(x) = 6(\cos x) \cdot (-\sin x)$$

$$= -6 \sin x \cos x$$

$$f'\left(\frac{5\pi}{6}\right) = -6 \sin\left(\frac{5\pi}{6}\right) \cos\left(\frac{5\pi}{6}\right)$$

$$= \underline{\underline{\frac{3\sqrt{3}}{2}}}$$

(b) $\dfrac{dy}{dx} = 4 \sin(3x)$

$$y = \frac{-4 \cos(3x)}{3} + C$$

$$\sqrt{3} = \frac{-4 \cos\left(3 \times \frac{5\pi}{6}\right)}{3} + C$$

$$\sqrt{3} = \frac{-4 \cos\left(\frac{5\pi}{2}\right)}{3} + C$$

$$\sqrt{3} = 0 + C$$

$$C = \sqrt{3}$$

$$\underline{\underline{y = \frac{-4 \cos(3x)}{3} + \sqrt{3}}}$$