

10. CIRCLE

10-1 EQUATION OF A CIRCLE

10-1.1 FINDING CENTRE AND RADIUS

(a) $C(5, -7) \quad r = 6$

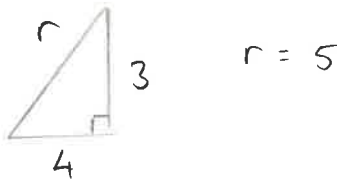
(b) $C(-10, 0) \quad r = \sqrt{45} \quad (3\sqrt{5})$

10-1.2 FINDING EQUATION

(a) $(x+3)^2 + (y-5)^2 = 17$

(b)

<u>Centre</u>	<u>Radius</u>
$(-2, 5)$	$(-2, 5) \quad (2, 2)$

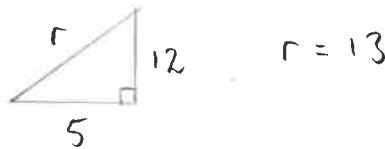


$(x+2)^2 + (y-5)^2 = 25$

(c)

<u>Centre</u>	<u>Radius</u>
$(9, -10) \quad (-1, 14)$	$(4, 2) \quad (9, -10)$

$C(4, 2)$



$(x-4)^2 + (y-2)^2 = 169$

10-2 GENERAL EQUATION OF A CIRCLE

(a) $g = -2 \quad f = 4 \quad c = 16$

$C(2, -4)$

$r = \sqrt{(-2)^2 + (4)^2 - 16}$

$r = 2$

(b) $g = 5$ $f = -3$ $c = -16$

$C(-5, 3)$ $r = \sqrt{(5)^2 + (-3)^2 - (-16)}$

$r = \sqrt{50}$ ($5\sqrt{2}$)

$(x+5)^2 + (y-3)^2 = 50$

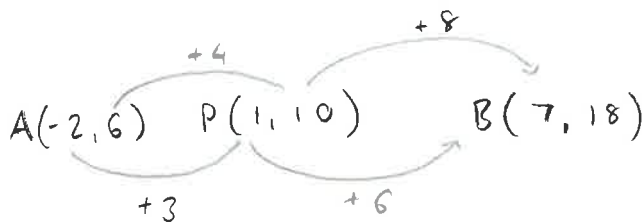
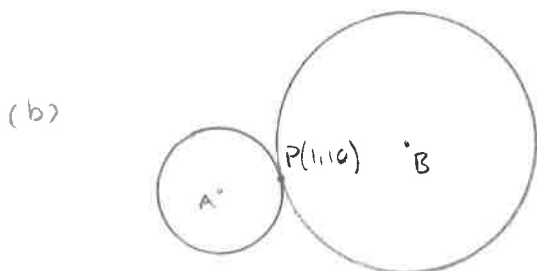
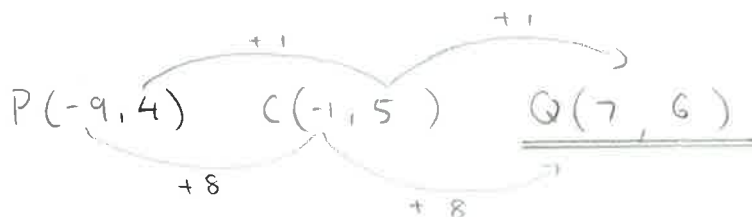
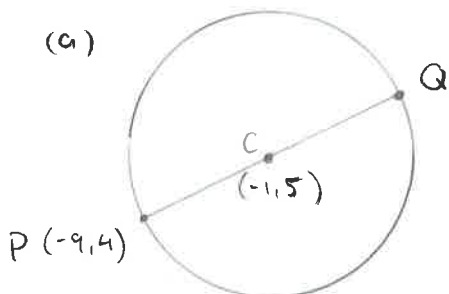
(c) $(x+5)^2 + (y-6)^2 = 4$

$x^2 + 10x + 25 + y^2 - 12y + 36 = 4$

$x^2 + y^2 + 10x - 12y + 61 = 4$

$x^2 + y^2 + 10x - 12y + 57 = 0$ \square

10.3 STEPPING OUT



$g = 2$ $f = -6$ $c = -24$

$A(-2, 6)$ $r_A = \sqrt{(2)^2 + (-6)^2 - (-24)}$

$r_A = 8$

$\therefore r_B = 16$

$(x-7)^2 + (y-18)^2 = 256$

10.4 TOUCHING CIRCLES

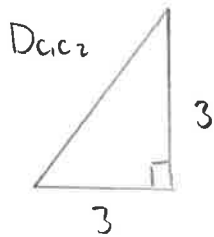
(a) $C_1(3, 2)$ $r_1 = \sqrt{2}$

$$g = -6 \quad f = -5 \quad c = 53$$

$$C_2(6, 5) \quad r_2 = \sqrt{(-6)^2 + (-5)^2 - 53}$$

$$r_2 = \sqrt{8}$$

$$r_2 = 2\sqrt{2}$$



$$D_{C1C2} = \sqrt{18}$$

$$= 3\sqrt{2}$$

$$r_1 + r_2 = \sqrt{2} + 2\sqrt{2}$$

$$= 3\sqrt{2}$$

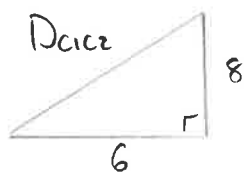
$\therefore D_{C1C2} = r_1 + r_2$, circles touch (externally) at a single point. \square

(b) $C_1(4, 1)$ $r_1 = 2$

$$g = -10 \quad f = -9 \quad c = 37$$

$$C_2(10, 9) \quad r_2 = \sqrt{(-10)^2 + (-9)^2 - 37}$$

$$r_2 = 12$$



$$D_{C1C2} = 10$$

$$r_2 - r_1 = 10$$

$\therefore D_{C1C2} = r_2 - r_1$, circles touch (internally) at a single point. \square

10.5 TANGENTS TO CIRCLES

10.5-1 FINDING THE EQUATION OF A TANGENT

(a) Grad Pt

$(1, -3)$ $(-3, 0)$ $(-3, 0)$

$$m = \frac{3}{-4}$$

$$m_{\perp} = \frac{4}{3}$$

$$y - 0 = \frac{4}{3}(x + 3)$$

$$\underline{3y - 4x = 12}$$

(b) Grad PE
 $g=5$ $f=-3$ $c=-16$ $(-10, 8)$

$(-5, 3)$ $(-10, 8)$

$$m = \frac{5}{-5}$$

$$= -1$$

$$m_{\perp} = 1$$

$$y - 8 = 1(x + 10)$$

$$\underline{\underline{y - x = 18}}$$

10.5.2 PROVING THAT A LINE IS A TANGENT

(a) $y - x = 13$

$$y = x + 13$$

$$x^2 + y^2 - 6y - 41 = 0$$

$$x^2 + (x+13)^2 - 6(x+13) - 41 = 0$$

$$x^2 + x^2 + 26x + 169 - 6x - 78 - 41 = 0$$

$$2x^2 + 20x + 50 = 0$$

$$2(x^2 + 10x + 25) = 0$$

$$2(x + 5)(x + 5) = 0$$

$$x = -5 \text{ twice}$$

repeated root \Rightarrow tangency. \square

POC $(-5, 8)$

(b) $y = 7 - x$

$$x^2 + y^2 - 12x - 10y + 53 = 0$$

$$x^2 + (7-x)^2 - 12x - 10(7-x) + 53 = 0$$

$$x^2 + 49 - 14x + x^2 - 12x - 70 + 10x + 53 = 0$$

$$2x^2 - 16x + 32 = 0$$

$$2(x^2 - 8x + 16) = 0$$

$$2(x - 4)(x - 4) = 0$$

$$x = 4 \text{ twice}$$

repeated root \Rightarrow tangency. \square

POC $(4, 3)$