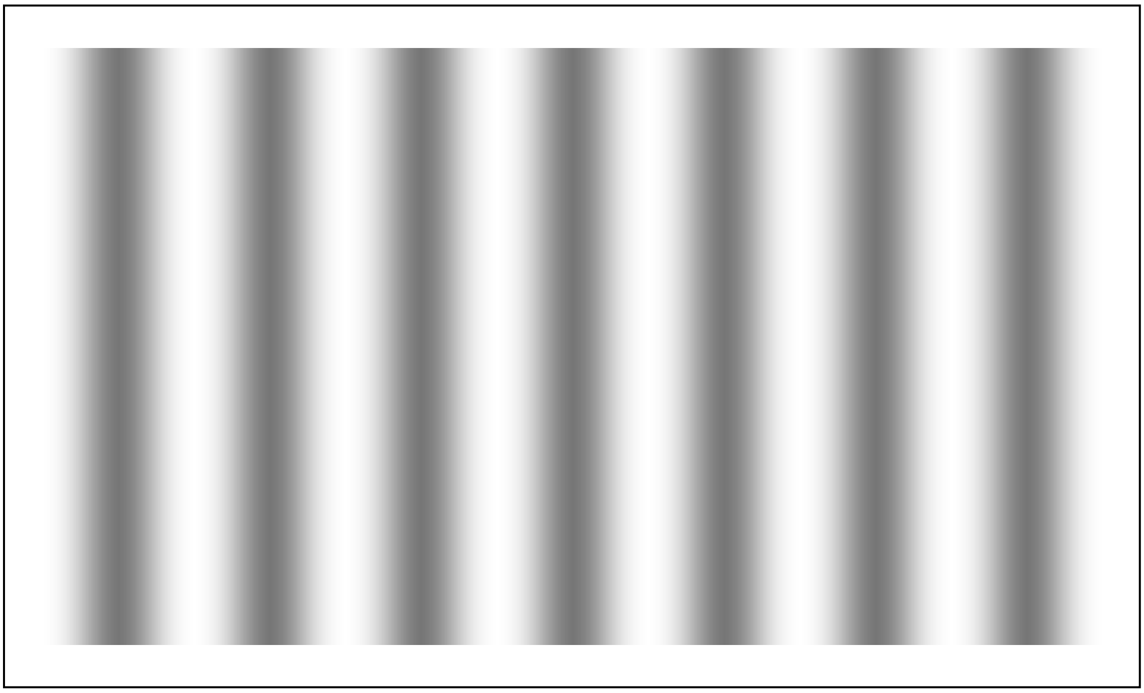
Name ______________________________________

Class ____________ Teacher ______________________

Particles and Waves



Summary Notes Part 2

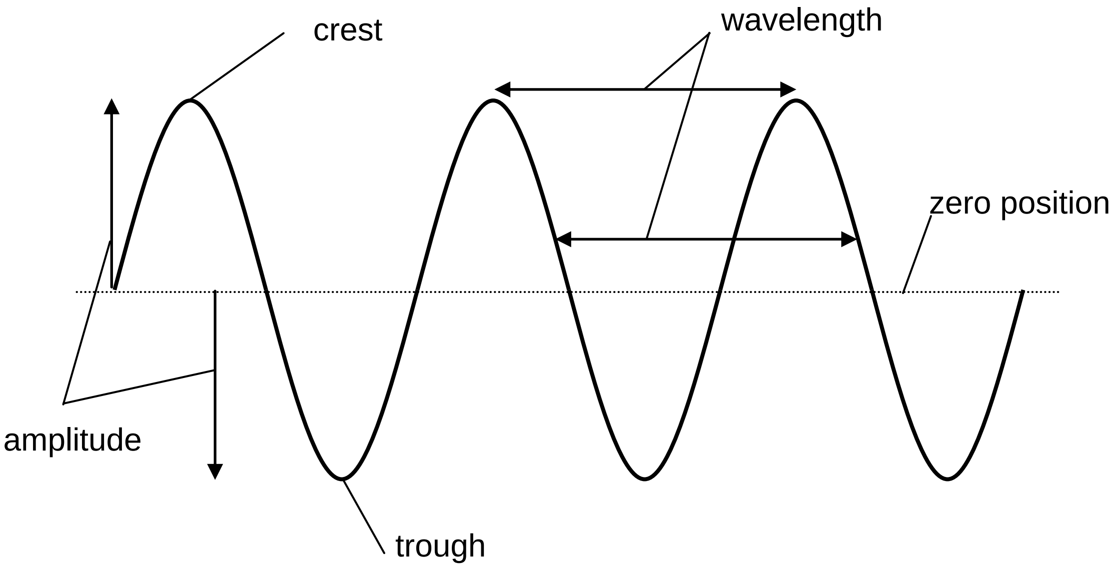
Version 2013

Contents

Waves Revision

All waves transfer energy away from their source.

Waves can be described using common terms.



The **amplitude** of a wave is the maximum displacement of a particle way from its zero position. Amplitude is measured in metres (m). The energy of a wave depends on the amplitude of a wave. The larger the amplitude the more energy the wave has.

The **wavelength** of the wave is the minimum distance in which the wave repeats itself. This equals the distance between two adjacent compressions in a longitudinal wave or the distance between two adjacent crests in a transverse wave. Wavelength is given the symbol λ (pronounced 'lambda') and is measured in metres (m).

The **frequency** of the wave is the number of wavelengths produced by its source each second or the number of wavelengths passing a point each second. It is given the symbol f and is measured in hertz (Hz).

Where N is the number of wavelengths passing a point in time t then frequency is given by:

$$f = \frac{N}{t}$$

The **period** of a wave is the time it takes for one complete wavelength to be produced by a source or the time for one complete wavelength to pass a point. It is given the symbol T and is measured in seconds (s).

Period and frequency are closely related:

$$T = \frac{1}{f} \quad \text{and} \quad f = \frac{1}{T}$$

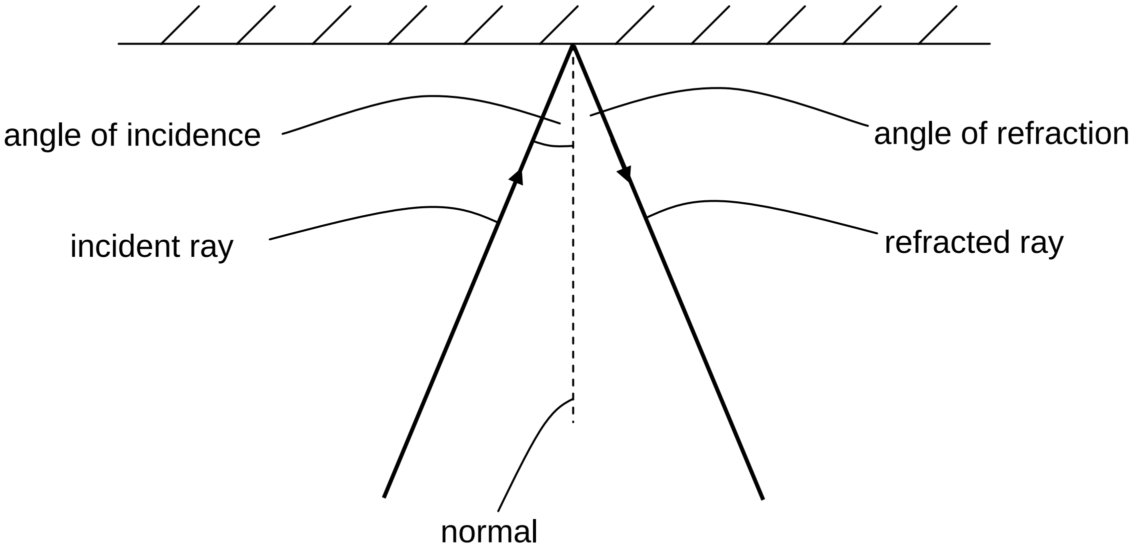
The **speed** of the wave is the distance travelled by any part of the wave each second. It is given the symbol v and is measured in metres per second (m s^{-1}).

$$v = \frac{s}{t} \quad \text{and} \quad v = f\lambda$$

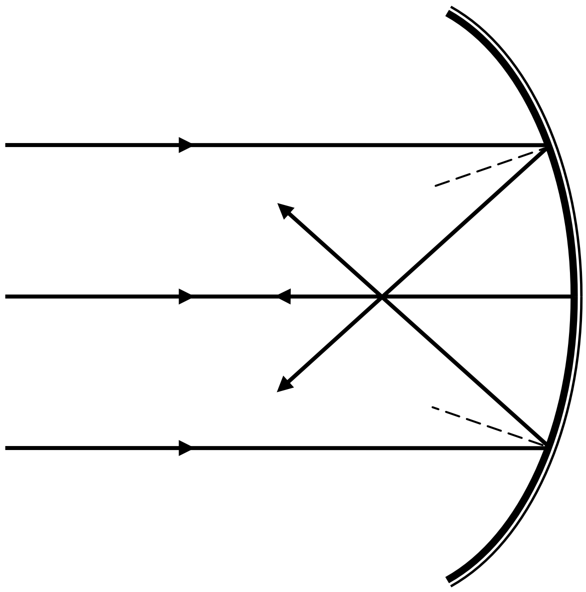
All waves exhibit four properties: reflection; refraction; diffraction and interference.

Reflection

Plane reflector



Concave curved reflector

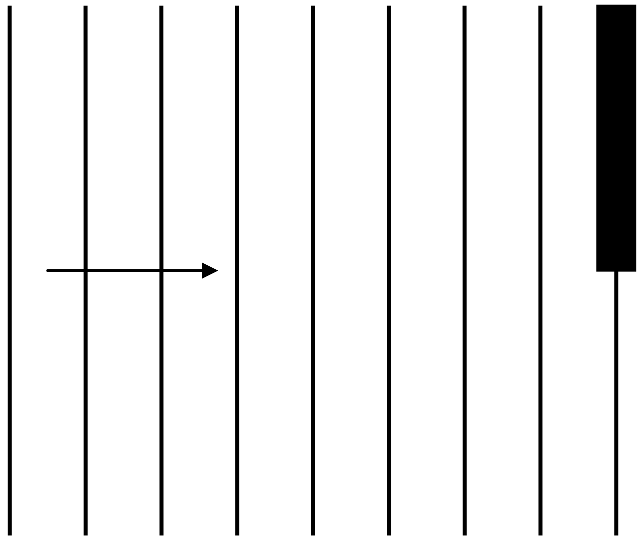


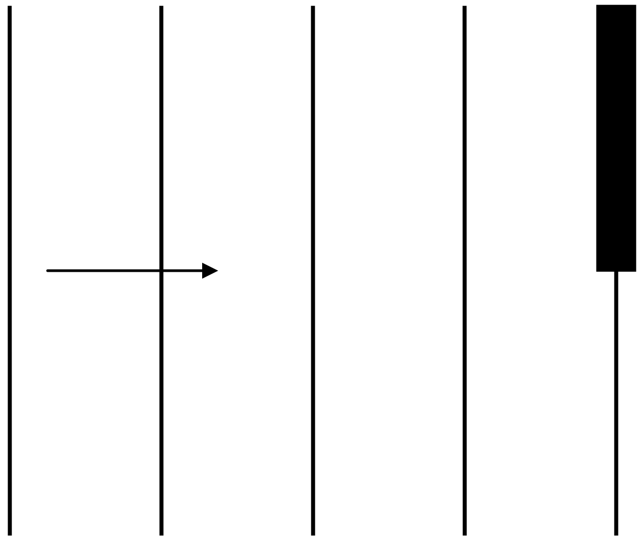
The angle of incidence always equals the angle of reflection.

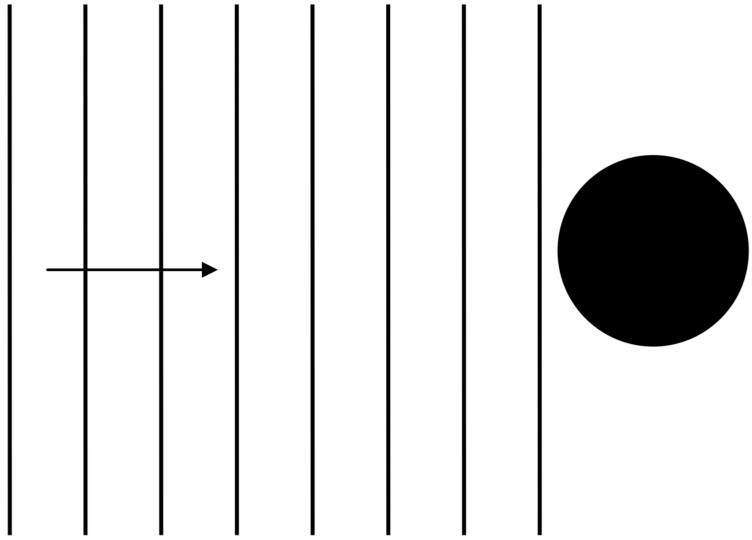
Interference and Diffraction

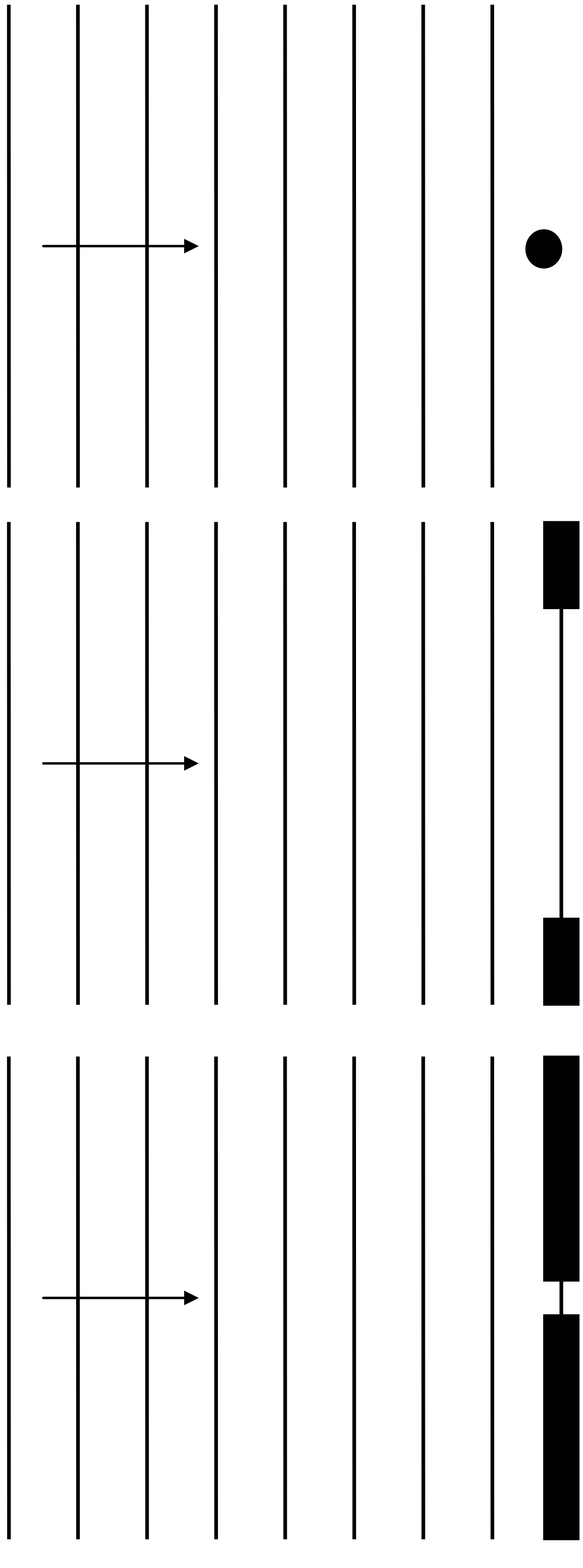
Diffraction

Diffraction is the bending of waves around obstacles or barriers.







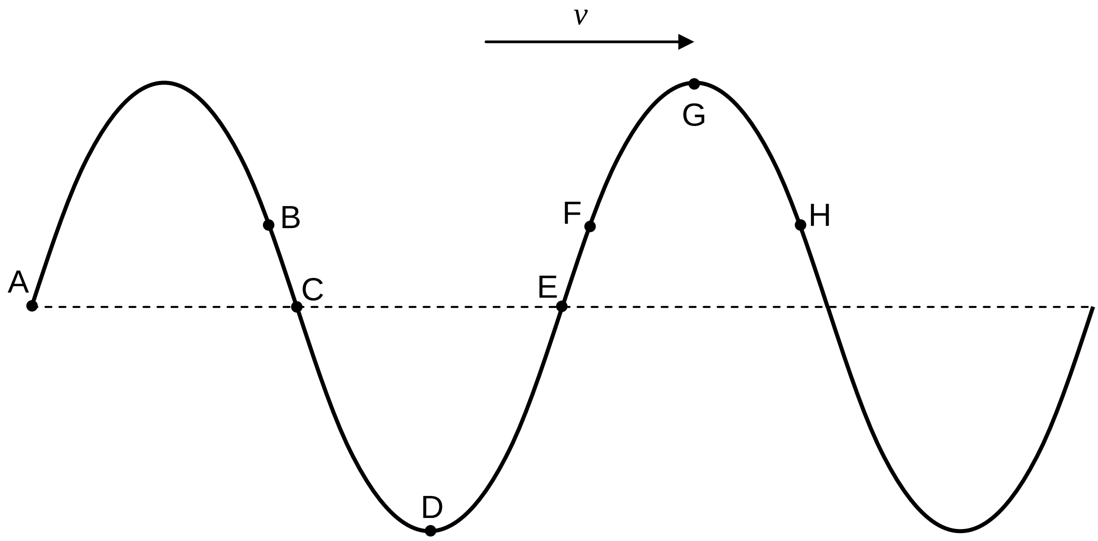


Phase and Coherence

Phase

Two points on a wave that are vibrating in exactly the same way, at the same time, are said to be **in phase**, e.g. two crests, or two troughs.

Two points that are vibrating in exactly the opposite way, at the same time, are said to be **exactly out of phase**, or **180^o out of phase**, e.g. a crest and a trough.



Points ___ & ___ or ___ & ___ are in phase.

Points ___ & ___ or ___ & ___ or ___ & ___ are exactly out of phase.

Points ___ & ___ or ___ & ___ or ___ & ___ are 90° out of phase.

Points ___ and ___ are at present stationary.

Points ___, ___ and ___ are at present rising.

Points ___, ___ and ___ are at present dropping.

Coherent Sources

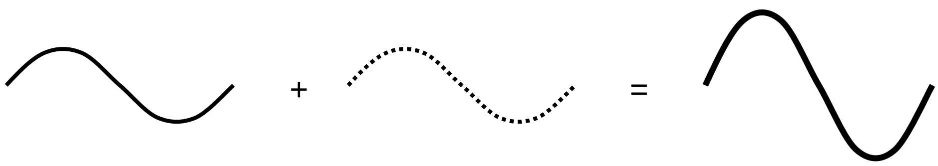
Two sources that are oscillating with a constant phase relationship are said to be **coherent**. This means the two sources also have the same frequency. Interesting interference effects can be observed when waves with a similar amplitude and come from coherent sources meet.

Interference

When two, or more, waves meet superposition, or adding, of the waves occurs resulting in one waveform.

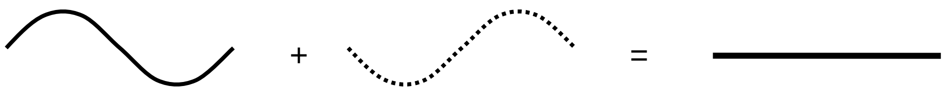
Constructive Interference

When the two waves are in phase constructive interference occurs.



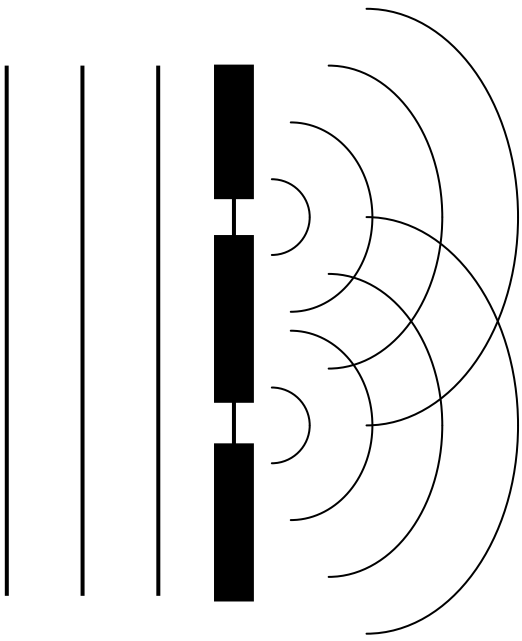
Destructive Interference

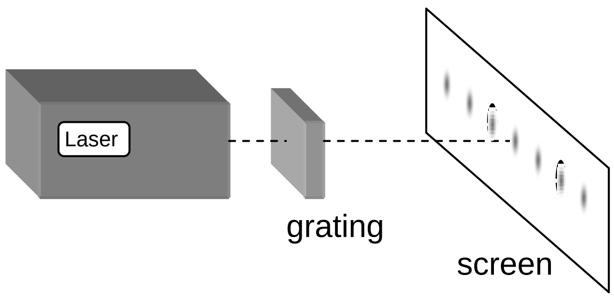
When the two waves are exactly out of phase destructive interference occurs.

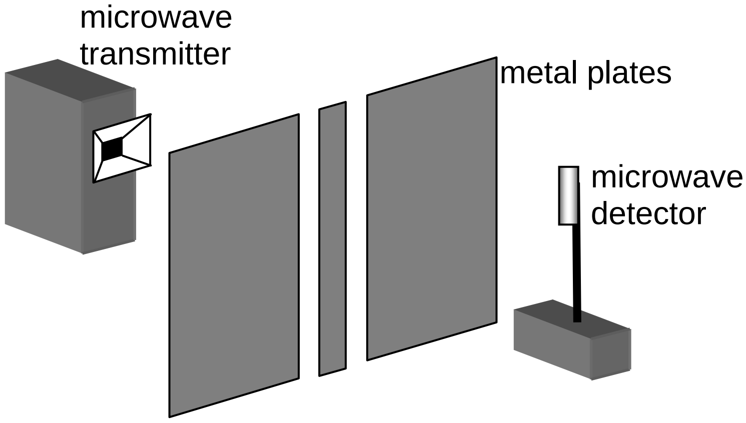


Only waves show this property of interference. Therefore interference is the test for a wave.

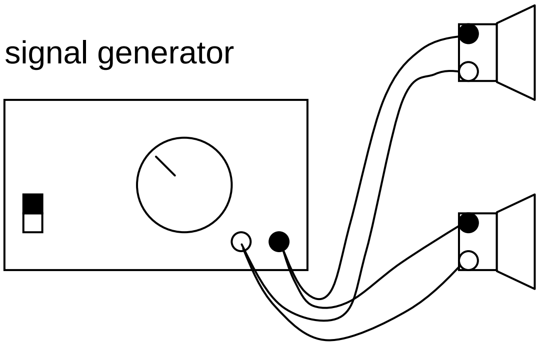
Interference can be demonstrated by allowing waves from one source to diffract through two narrow slits in a barrier. This can be done with water waves in a ripple tank, microwaves and light.





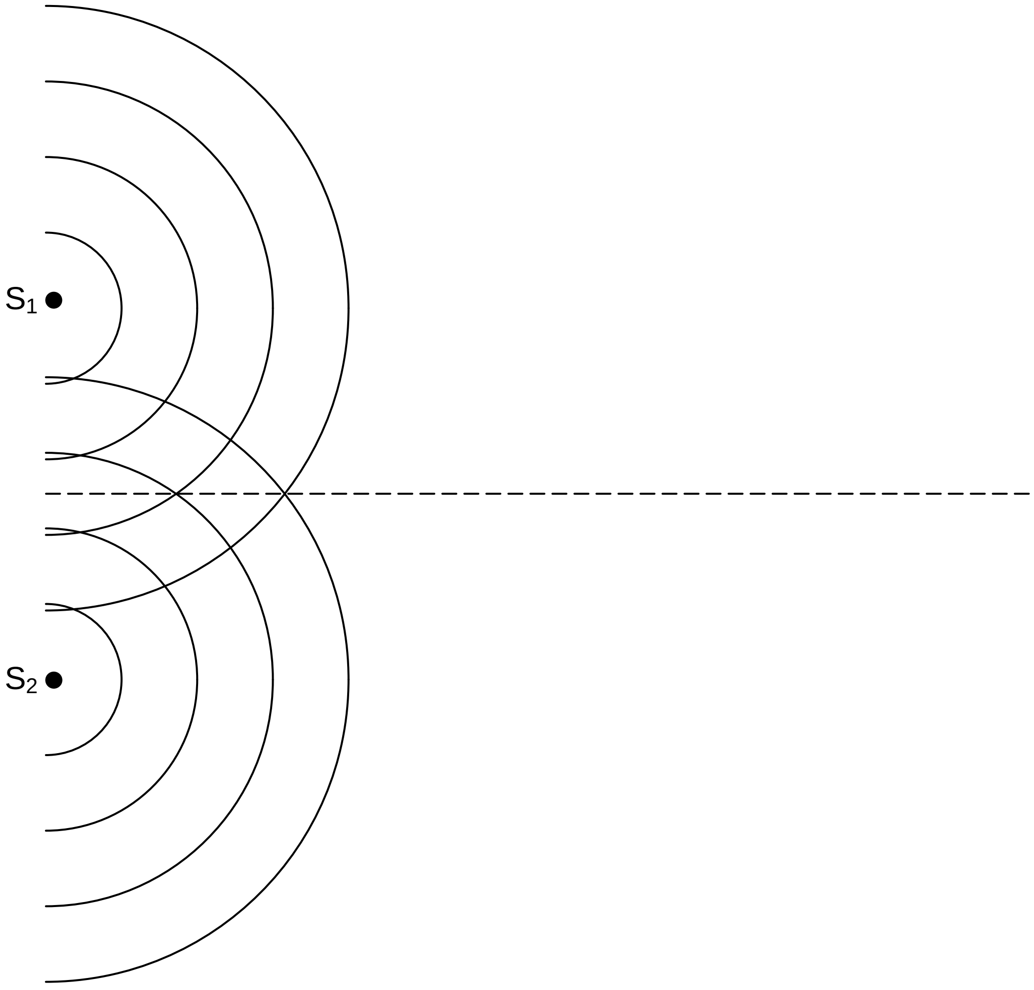


Alternatively, for some types of wave, to connect two transducers to the same a.c. signal. Such as with two loudspeakers connected to the same signal generator.

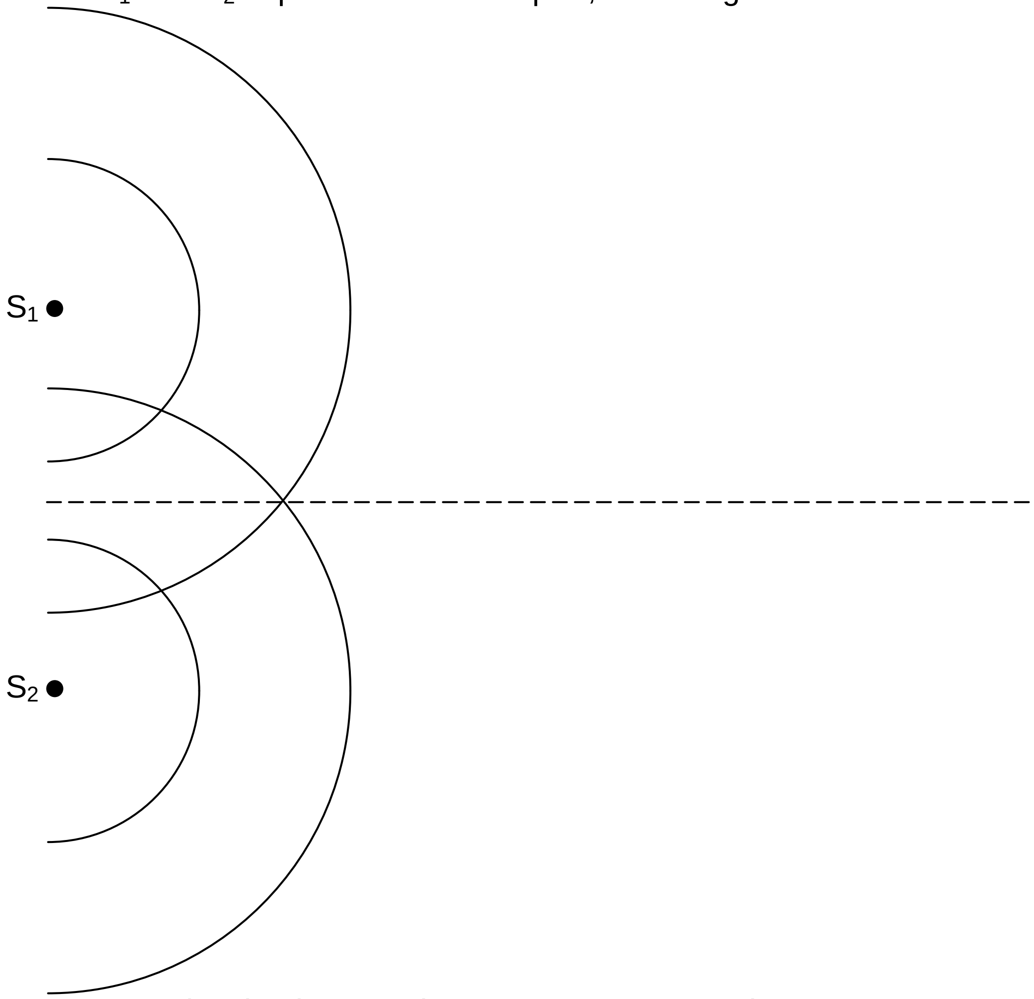


Interference Patterns

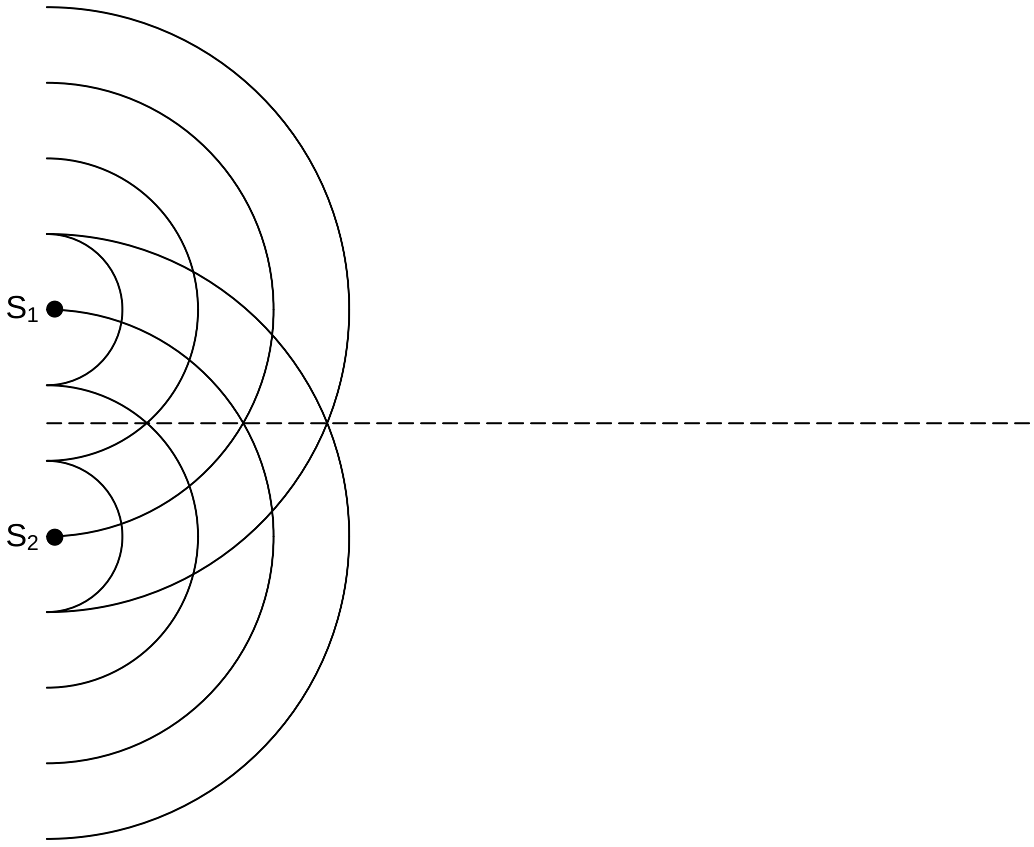
Sources S_1 and S_2 in phase and 5 cm apart, wavelength 1 cm.



Sources S_1 and S_2 in phase and 5 cm apart, wavelength 2 cm.



Sources S_1 and S_2 in phase and 3 cm apart, wavelength 1 cm.



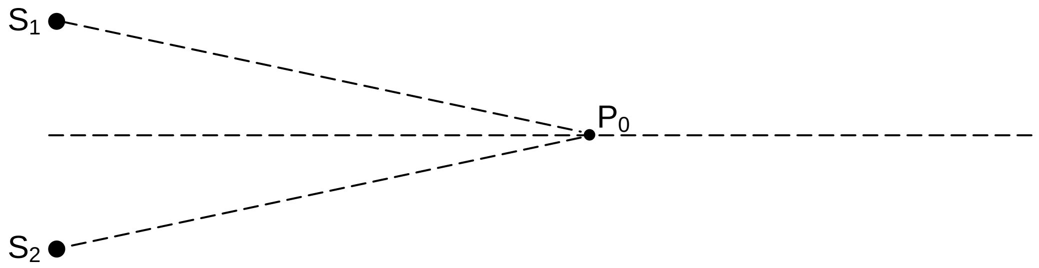
(a) Decreasing the separation of the sources S_1 and S_2 increases the spaces between the lines of interference.

(b) Increasing the wavelength (i.e. decreasing the frequency) of the waves increases the spaces between the lines of interference.

(c) Observing the interference pattern at an increased distance from the sources increases the spaces between the lines of interference.

Interference and Path Difference

Two sources S_1 and S_2 in phase and 3 cm apart, wavelength 1 cm.

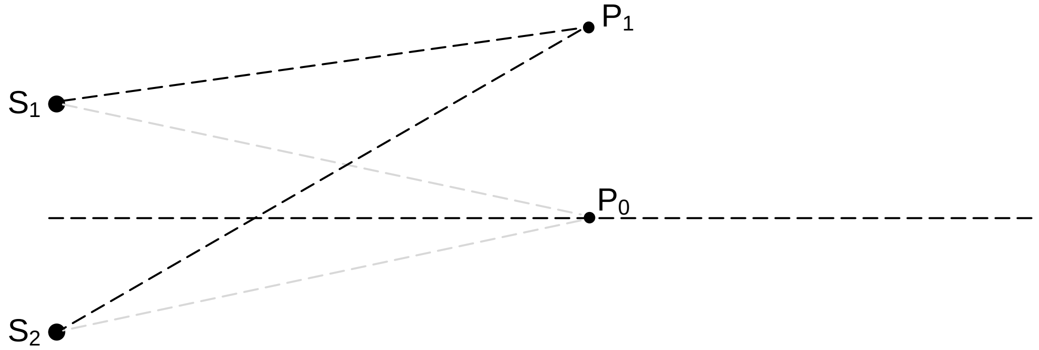


P_0 is a point on the centre line of the interference pattern.

P_0 is the same distance from S_1 as it is from S_2.

The path difference between S_1P_0 and $S_2P_0 = 0$

Waves arrive at P_0 in phase and therefore constructive interference occurs.

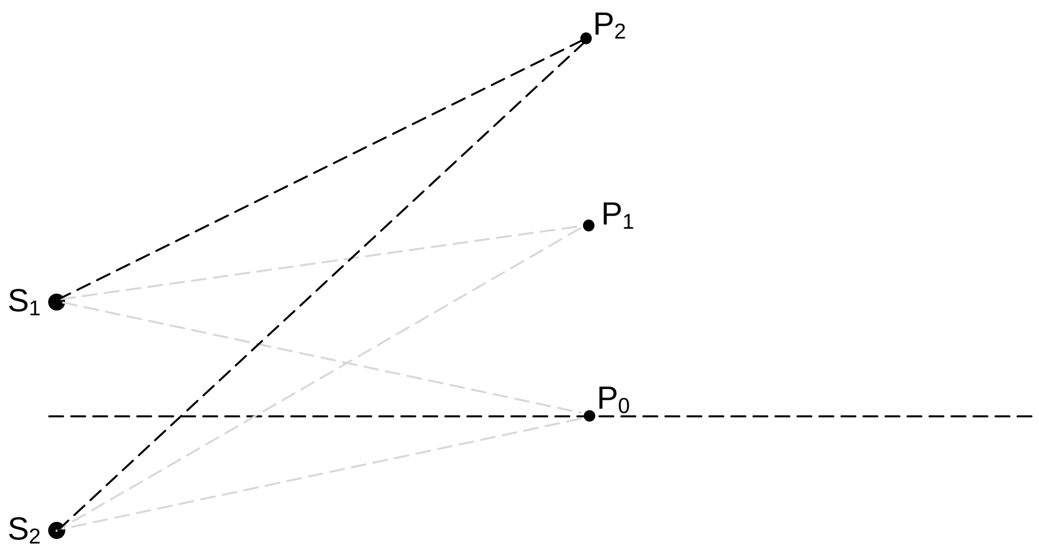


P_1 is a point on the first line of constructive interference out from the centre line of the interference pattern.

P_1 is one wavelength further from S_2 than it is from S_1.

The path difference between S_1P_1 and $S_2P_1 = 1 \times \lambda$

Waves arrive at P_1 in phase and therefore constructive interference occurs.



P_2 is a point on the second line of constructive interference out from the centre line of the interference pattern.

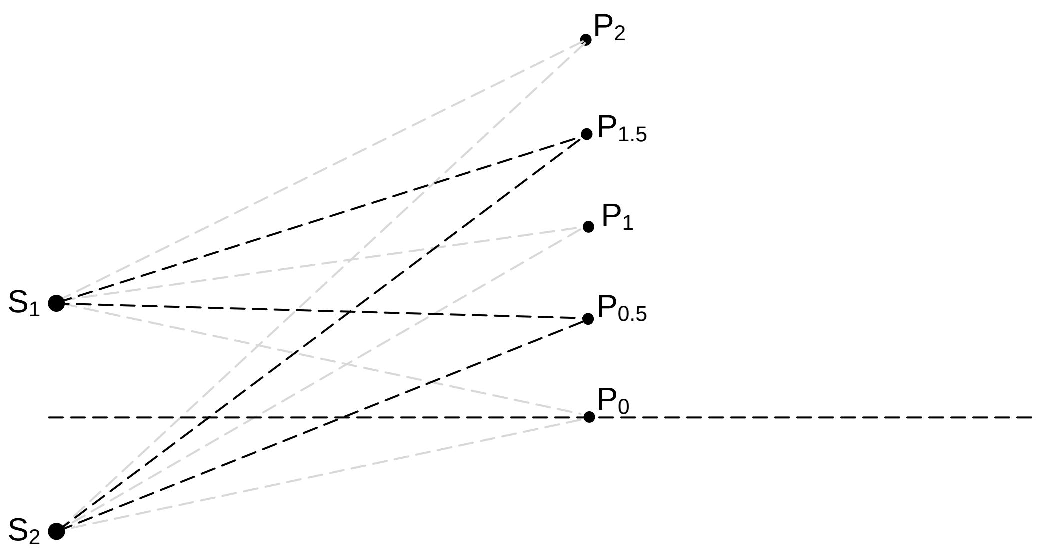
P_2 is one wavelength further from S_2 than it is from S_1.

The path difference between S_1P_2 and S_2P_2 = $2 \times \lambda$

Waves arrive at P_2 in phase and therefore constructive interference occurs.

Constructive interference occurs when:

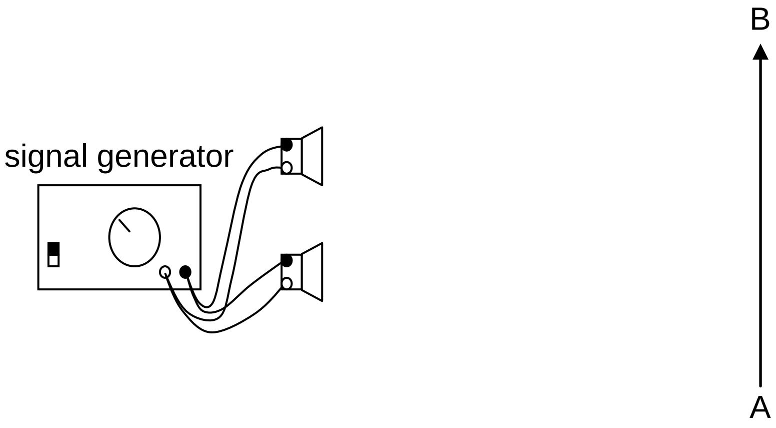
$$\text{path difference} = m\lambda \quad \text{where m is an integer}$$



Destructive interference occurs when:

$$\text{path difference} = (m + \tfrac{1}{2})\lambda \quad \text{where m is an integer}$$

Example: A student sets up two loudspeaker a distance of 1·0 m apart in a large room. The loudspeakers are connected in parallel to the same signal generator so that they vibrate at the same frequency and in phase.



The student walks from A and B in front of the loudspeakers and hears a series of loud and quiet sounds.

(a) Explain why the student hears the series of loud and quiet sounds.
(b) The signal generator is set at a frequency of 500 Hz. The speed of sound in the room is 340 m s^{-1}. Calculate the wavelength of the sound waves from the loudspeakers.
(c) The student stands at a point 4·76 m from loudspeaker and 5·78 m from the other loudspeaker. State the loudness of the sound heard by the student at that point. Justify your answer.
(d) Explain why it is better to conduct this experiment in a large room rather than a small room.

Solution:

(a) The student hears a series of loud and quiet sounds due to interference of the two sets of sound waves from the loudspeakers. When the two waves are in phase there is constructive interference and a loud sound. When the two waves are exactly out of phase there is destructive interference and a quiet sound.

(b) $v = f\lambda$

$340 = 500 \times \lambda$

$\lambda =$ <u>0·68 m</u>

(c) Path difference = 5·78 – 4·76 = 1·02 m

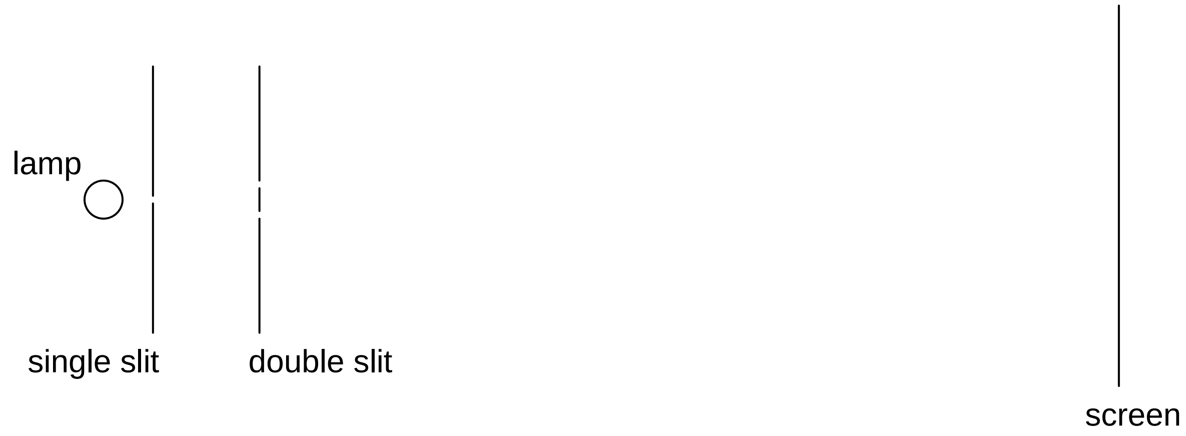
Number of wavelengths = 1·02/0·68 = 1·5λ

A path difference of 1·5λ means the waves are exactly out of phase.
The student hears a quiet sound.

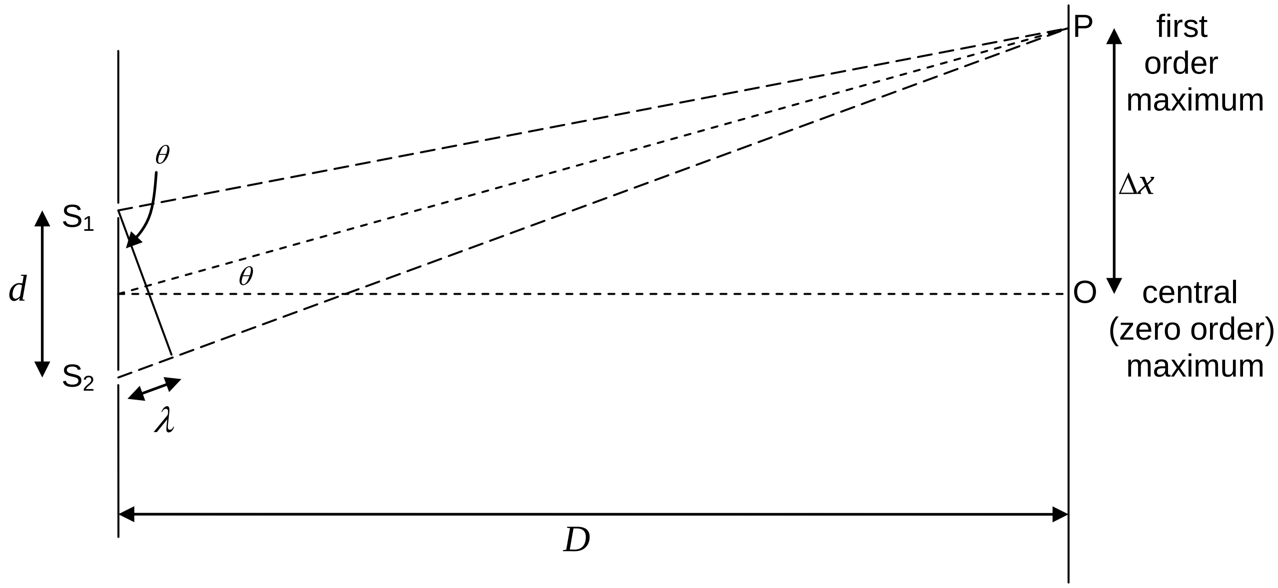
(d) In a small room, sound waves will reflect off the walls and therefore other sound waves will also interfere with the waves coming directly from the loudspeakers.

Young's Double Slit Experiment

In 1801 Thomas Young showed that an interference pattern could be produced using light. At the time this settled the long running debate about the nature of light in favour of light being a wave.



Passing light from the lamp through the single slit ensures the light passing through the double slit is coherent. An interference pattern is observed on the screen.



The path difference between S_1P and S_2P is one wavelength.

As the wavelength of light λ is very small the slits separation d must be very small and much smaller than the slits to screen distance D. Angle θ between the central axis and the direction to the first order maximum is therefore very small. For small angles $\sin\theta$ is approximately equal to $\tan\theta$, and the angle θ itself if measured in radians.

Hence from the two similar triangles:

$$\theta = \sin\theta = \frac{\lambda}{d} \qquad \text{and} \qquad \theta = \tan\theta = \frac{\Delta x}{D}$$

Therefore:

$$\frac{\lambda}{d} = \frac{\Delta x}{D}$$

Resulting in the expression for the fringe spacing:

$$\Delta x = \frac{\lambda D}{d}$$

To produce a widely spaced fringe pattern:

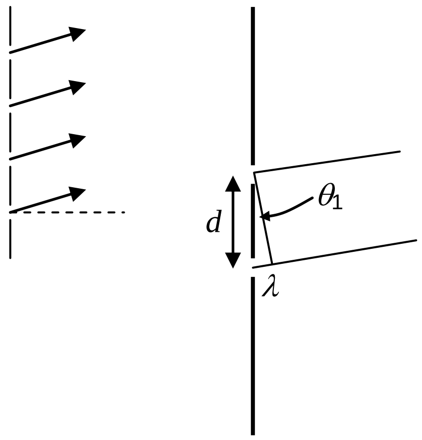
(a) Very closely separated slits should be used since $\Delta x \propto 1/d$.

(b) A long wavelength light should be used, i.e. red, since $\Delta x \propto \lambda$.
(Wavelength of red light is approximately $7{\cdot}0 \times 10^{-7}$ m, green light approximately $5{\cdot}5 \times 10^{-7}$ m and blue light approximately $4{\cdot}5 \times 10^{-7}$ m.)

(c) A large slit to screen distance should be used since $\Delta x \propto D$.

Gratings

A double slit gives a very dim interference pattern since very little light can pass through the two narrow slits. Using more slits allows more light through to produce brighter and sharper fringes.



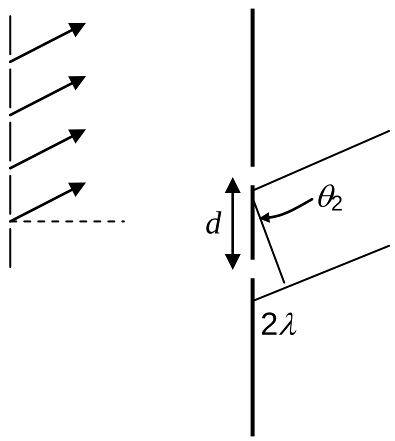
As in Young's Double Slit Experiment the first order bright fringe is obtained when the path difference between adjacent slits is one wavelength λ.

Therefore:

$$\sin\theta_1 = \frac{\lambda}{d}$$

and:

$$\lambda = d\sin\theta_1$$



The second order bright fringe is obtained when the path difference between adjacent slits is two wavelengths 2λ.

Therefore:

$$\sin\theta_2 = \frac{2\lambda}{d}$$

and:

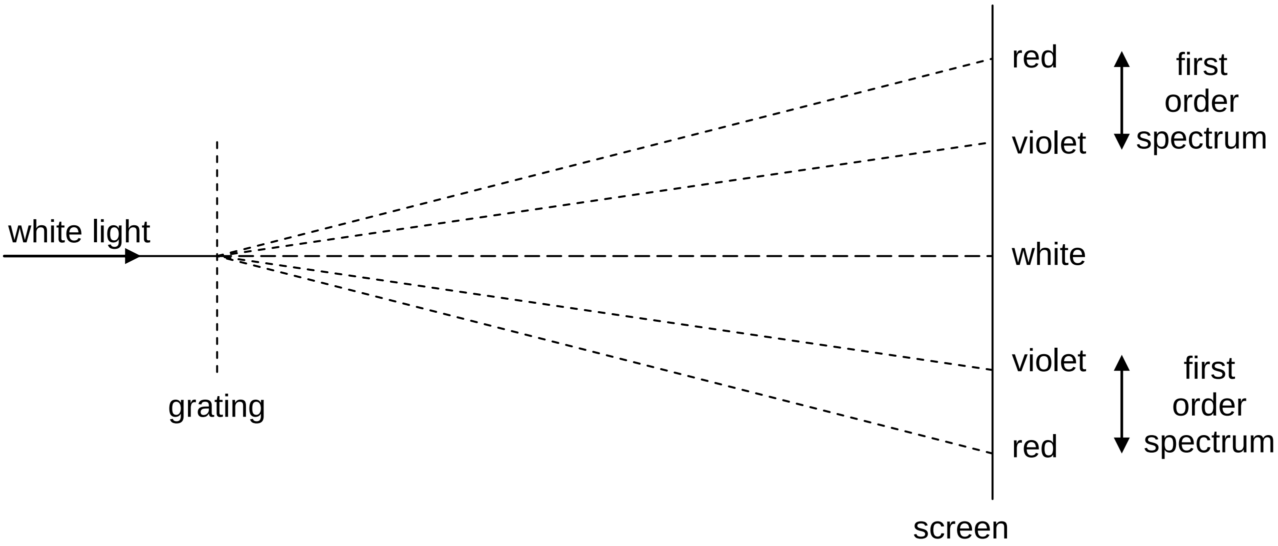
$$2\lambda = d\sin\theta_2$$

The general formula for the mth order spectrum is:

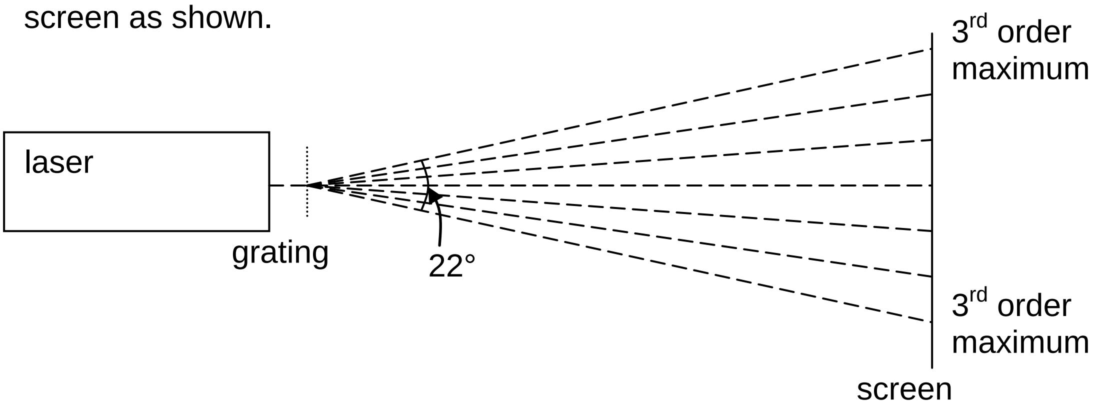
$m\lambda = d\sin\theta_m$ where m is an integer.

When white light passes through a grating a series of visible spectra are observed either side of a central white maximum.

At the central maximum all wavelengths of light are in phase so all wavelengths interfere constructively. All colours mix to produce white light.



Example: Monochromatic light from a laser is directed through a grating and on to a screen as shown.



The grating has 100 lines per millimetre.

Calculate the wavelength of the laser light.

Solution:

$n = 3$
$\theta = 22/2 = 11°$
100 lines per millimetre = 100 000 lines per metre, $d = \frac{1}{100\ 000} = 1{\cdot}00 \times 10^{-5}$ m

$n\lambda = d \sin \theta$
$3 \times \lambda = 1{\cdot}00 \times 10^{-5} \times \sin 11°$

$$\lambda = \frac{1{\cdot}00 \times 10^{-5} \times \sin 11°}{3}$$

$\lambda = \underline{6{\cdot}36 \times 10^{-7}\ \text{m}}$

Refraction of Light

Refraction

Refraction is the property of light which occurs when it passes from one medium to another. While in one medium the light travels in a straight line. Light, and other forms of electromagnetic radiation, do not require a medium through which to travel.

Light travels at its greatest speed in a vacuum. Light also travels at almost this speed in gases such as air. The speed of any electromagnetic radiation in space or a vacuum is $3{\cdot}00 \times 10^8$ m s^{-1}.

Whenever light passes from a vacuum to any other medium its speed decreases. Unless the light is travelling perpendicular to the boundary between the media this then results in a change in direction.

It is the change in the speed of the light that causes refraction. The greater the change in speed the greater the amount of refraction.

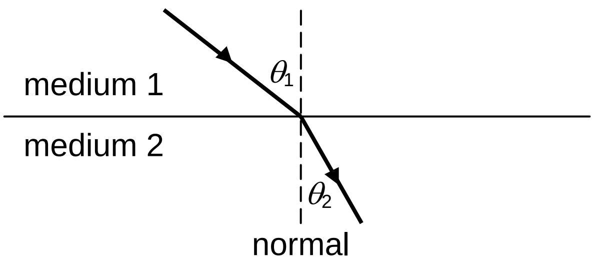
Media such as glass, perspex, water and diamond are optically more dense than a vacuum. Air is only marginally more dense than a vacuum when considering its optical properties.

$v_{air} \approx c = 3{\cdot}00 \times 10^8$ m s^{-1} (where c is the speed of light in a vacuum)

Refractive Index

The refractive index of a material (or medium) is a measure of how much the material slows down light passing through that material. It therefore also gives a measure of how much the direction of the light changes as it passes from one material to another.

The absolute refractive index of a material, n, is the refractive index of that material compared to the refractive index of a vacuum. The absolute refractive index of a vacuum (and therefore also air) is 1·0.



Snell's Law:

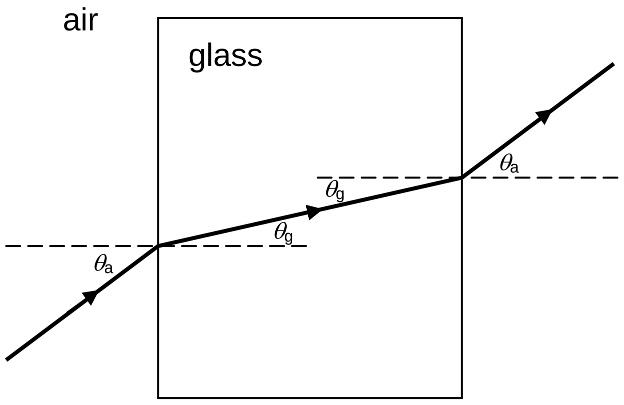
$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

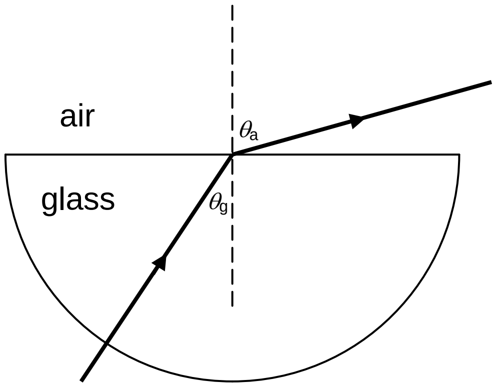
where medium 1 is a vacuum or air, and therefore $n_1 = 1{\cdot}0$, this simplifies to:

$$\sin \theta_1 = n_2 \sin \theta_2 \qquad \text{or} \qquad n_2 = \frac{\sin \theta_1}{\sin \theta_2}$$

Measuring the Refractive Index of Glass

The refractive index of glass can be measured by directing a ray of light through optical blocks and measuring the appropriate angles in the glass and the surrounding air.





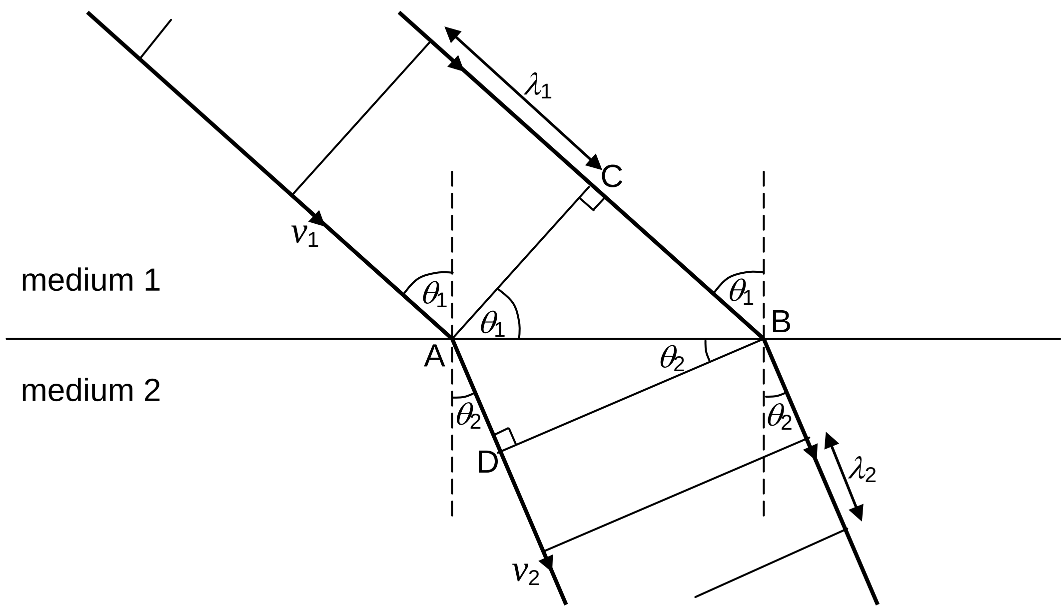
$$n_g = \frac{\sin \theta_a}{\sin \theta_g}$$

Refractive Index and Waves

When light waves pass from one medium to another the frequency of the waves do not change. The number of wavelengths leaving one medium per second must equal the number of waves entering the other medium per second. The wave is continuous and energy must be conserved.

Since $v = f\lambda$, v is directly proportional to λ. Therefore if the waves pass into an optically more dense medium the speed of the waves must decrease and therefore the wavelength of the waves must also decrease with the frequency remaining constant.

Consider the wavefronts of a parallel sided beam of light entering an optically more dense medium, i.e. one with a higher refractive index, as shown:



The relative refractive index is the ratio of the speed of light in the two media:

$$_1n_2 = \frac{v_1}{v_2}$$

The distance the light travels in the time of one period, T, in medium 1 is BC. Hence:

$$BC = v_1T \text{ and therefore } v_1 = \frac{BC}{T}$$

Likewise, the distance the light travels in one period, T, in medium 2 is AD. Hence:

$$AD = v_2T \text{ and therefore } v_2 = \frac{AD}{T}$$

Therefore:

$$_1n_2 = \frac{v_1}{v_2} = \frac{BC}{AD} = \frac{\lambda_1}{\lambda_2}$$

But from the triangle ABC:

$$\sin \theta_1 = \frac{BC}{AB}$$

and from the triangle ABD:

$$\sin \theta_2 = \frac{AD}{AB}$$

Therefore:

$$_1n_2 = \frac{v_1}{v_2} = \frac{BC}{AD} = \frac{\sin \theta_1}{\sin \theta_2}$$

In summary, the refractive index of medium 2 relative to medium 1 can be determined from:

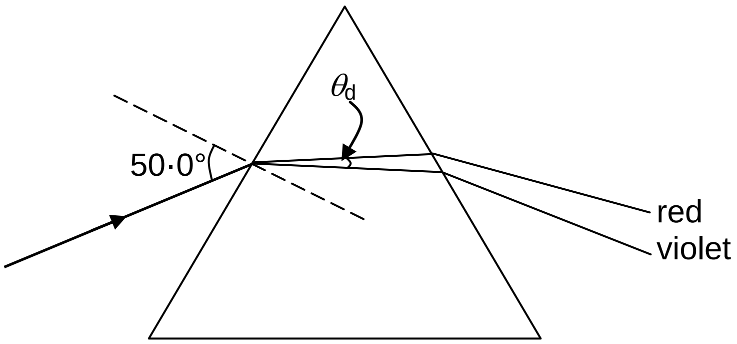
- the ratio of the speeds in the two media
- the ratio of the wavelengths in the two media
- the ratio of the sines of the angles in the two media.

$$_1n_2 = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} = \frac{\sin \theta_1}{\sin \theta_2}$$ but f is always constant.

As the refractive index of a medium is only a ratio it does not have a unit. The absolute refractive index of all media is greater than 1·00 as light slows down in all media compared with a vacuum.

The refractive index of a medium decreases as the wavelength of the light increases. Long wavelength red light is refracted less than other colours. As a result when white light passes through a prism the white light is dispersed into a visible spectrum.

Example: A narrow ray of white light is shone through a glass prism as shown.



The ray disperses into the visible spectrum. The glass has a refractive index of 1·47 for red light and 1·51 for violet light.

(a) Calculate the angle of dispersion θ_d in the glass.
(b) Calculate speed of the red light in the glass prism.

Solution:

(a) Red:

$$n_g = \frac{\sin \theta_a}{\sin \theta_g}$$

$$1{\cdot}47 = \frac{\sin 50{\cdot}0}{\sin \theta_R}$$

$$\sin \theta_R = \frac{\sin 50{\cdot}0}{1{\cdot}47}$$

$$\theta_R = 31{\cdot}4°$$

Violet

$$1{\cdot}51 = \frac{\sin 50{\cdot}0}{\sin \theta_V}$$

$$\sin \theta_V = \frac{\sin 50{\cdot}0}{1{\cdot}51}$$

$$\theta_V = 30{\cdot}5°$$

$$\theta_d = 31{\cdot}4° - 30{\cdot}5° = \underline{0{\cdot}9°}$$

(b) $n_2 = \frac{v_1}{v_2}$

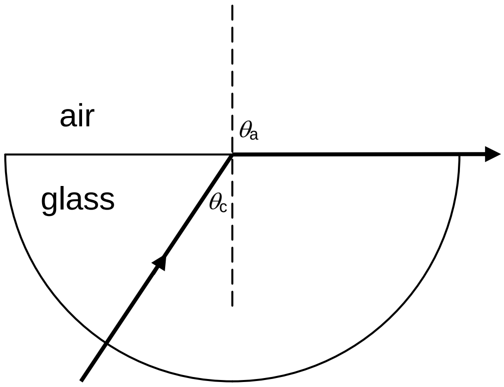
$$1{\cdot}47 = \frac{3{\cdot}00 \times 10^8}{v_1}$$

$$v_1 = \frac{3{\cdot}00 \times 10^8}{1{\cdot}47}$$

$$v_1 = \underline{2{\cdot}04 \times 10^8 \text{ m s}^{-1}}$$

Critical Angle and Total Internal Reflection

When a ray of light is passing through a material with a high refractive index and strikes a boundary with a material of lower refractive index there is an angle of incidence that results in the refracted ray exiting along the boundary at 90° to the normal. This angle of incidence is called the critical angle, θ_c.



$$n = \frac{\sin \theta_a}{\sin \theta_c}$$

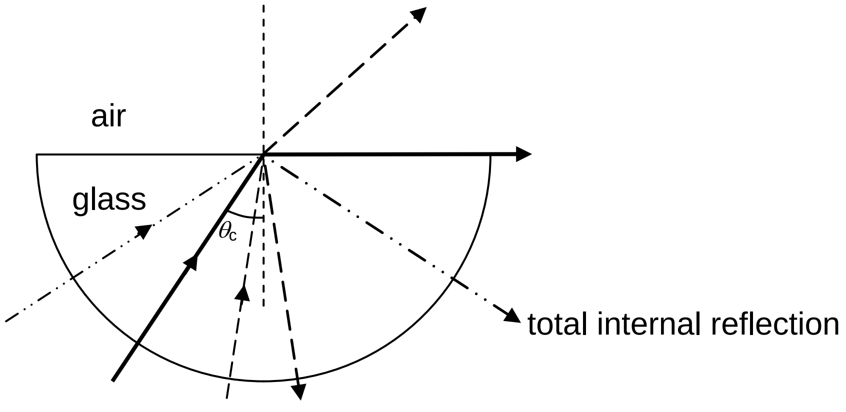
$$n = \frac{\sin 90}{\sin \theta_c}$$

$$n = \frac{1}{\sin \theta_c}$$

$$\sin \theta_c = \frac{1}{n}$$

For angles of incidence less than the critical angle some reflection and some refraction occur. The energy of the light is split along two paths.

For angles of incidence greater than the critical angle only reflection occurs, i.e. total internal reflection, and all of the energy of the light is reflected inside the material.



Total internal reflection allows light signals to be sent large distances through optical fibres. Very pure, high quality glass absorbs very little of the energy of the light making fibre optic transmission very efficient.

Spectra

Irradiance and the Inverse Square Law

The irradiance of light I is defined as the amount of light energy incident on every square metre of a surface per second.

The equation for irradiance is therefore: $I = \frac{E}{A\,t}$

This can be reduced to: $I = \frac{P}{A}$

If light from a point source spreads out in all directions, at a distance r from the source, it strikes the inside of a sphere of area $A = 4\pi r^2$.

Therefore at a distance of 1 m from the source the light strikes an area:

$$A_1 = 4\pi \times 1^2 = 4\pi \text{ m}^2$$

At a distance of 2 m from the source the light strikes an area:

$$A_2 = 4\pi \times 2^2 = 16\pi \text{ m}^2 = 4A_1$$

At a distance of 3 m from the source the light strikes an area:

$$A_3 = 4\pi \times 3^2 = 36\pi \text{ m}^2 = 9A_1$$

The area the light strikes increases with the square of the distance from the source:

$$A \propto r^2$$

However since:

$$I = \frac{P}{A}$$

Therefore:

$$I \propto \frac{1}{r^2}$$

Which gives the equation:

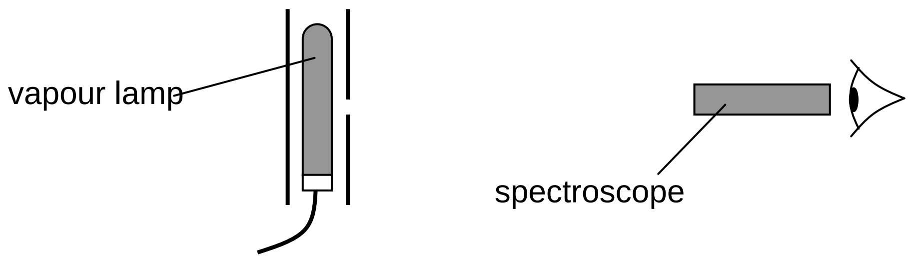
$$I_1 r_1^2 = I_2 r_2^2$$

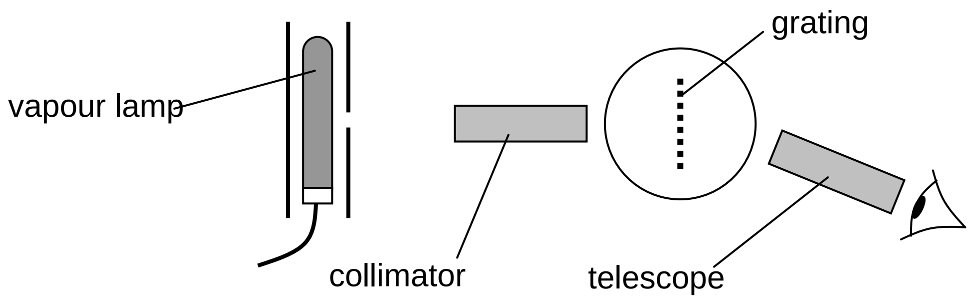
Spectra

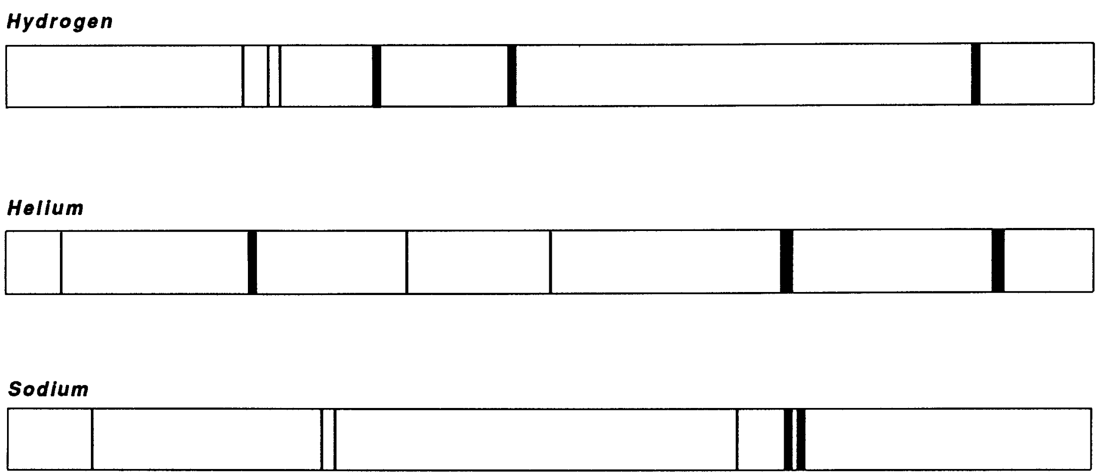
Line Emission Spectra

A line spectrum is emitted by excited atoms in a low pressure gas. Each element emits its own unique line spectrum that can be used to identify that element. The spectrum of helium was first observed in light from the sun (Greek - helios), and only then was helium searched for and identified on Earth.

A line emission spectrum can be observed using either a spectroscope or a spectrometer using a grating or prism.



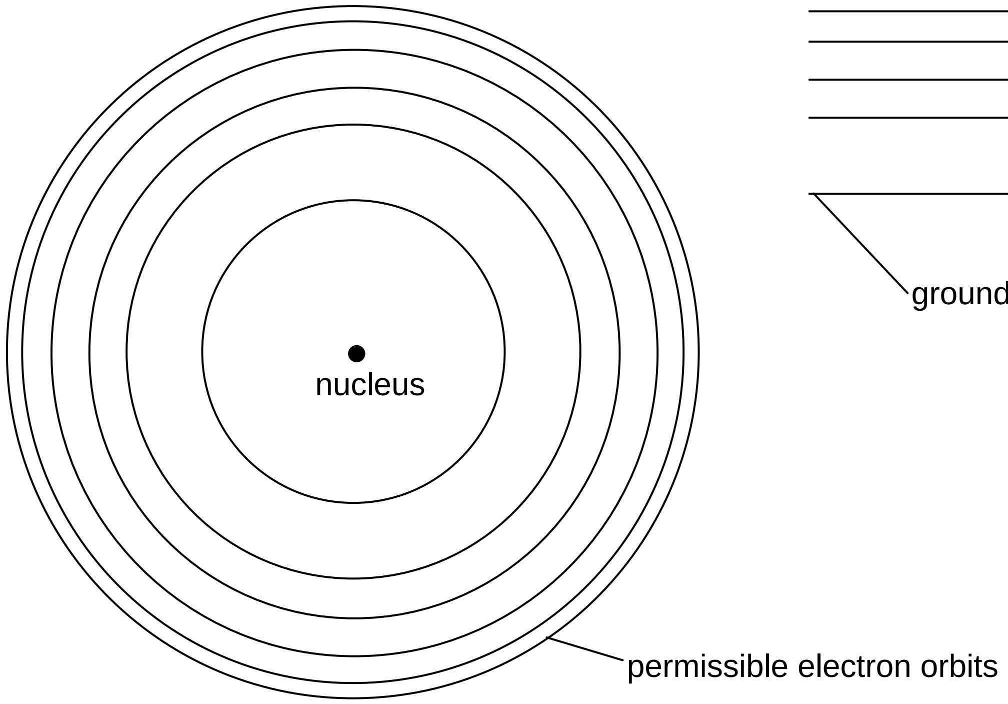




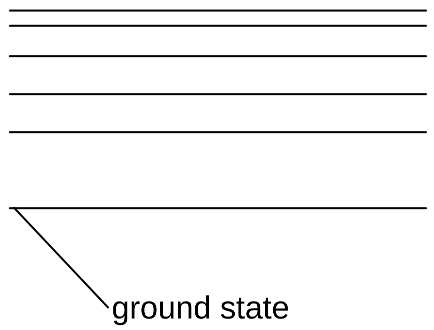
As with the photoelectric effect, line emission spectra cannot be explained by the wave theory of light. In 1913, Neils Bohr, a Danish physicist, first explained the production of line emission spectra. This explanation depends on the behaviour of both the electrons in atoms and of light to be quantised.

The electrons in a *free* atom are restricted to particular radii of orbits. A free atom does not experience forces due to other surrounding atoms. Each orbit has a discrete energy associated with it and as a result they are often referred to as energy levels.

Bohr model of the atom



Energy level diagram

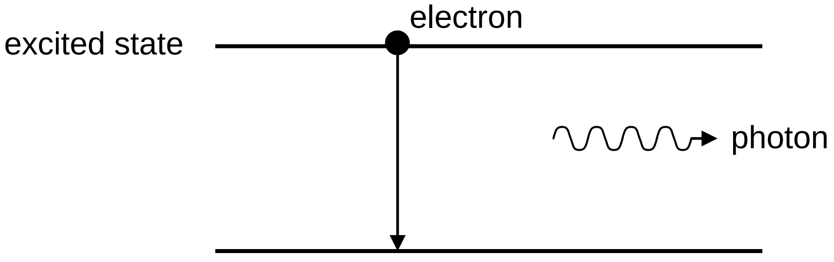


When an electron is at the **ground state** it has its lowest energy. When an electron gains energy it moves to a higher energy level. If an electron gains sufficient energy it can escape from the atom completely - the **ionisation level**.

By convention, the electron is said to have zero energy when it has escaped the atom. Therefore the energy levels in the atom have negative energy levels. The ground state is the level with the most negative energy. When an electron moves to a higher energy level it gains energy and moves to a less negative energy level.

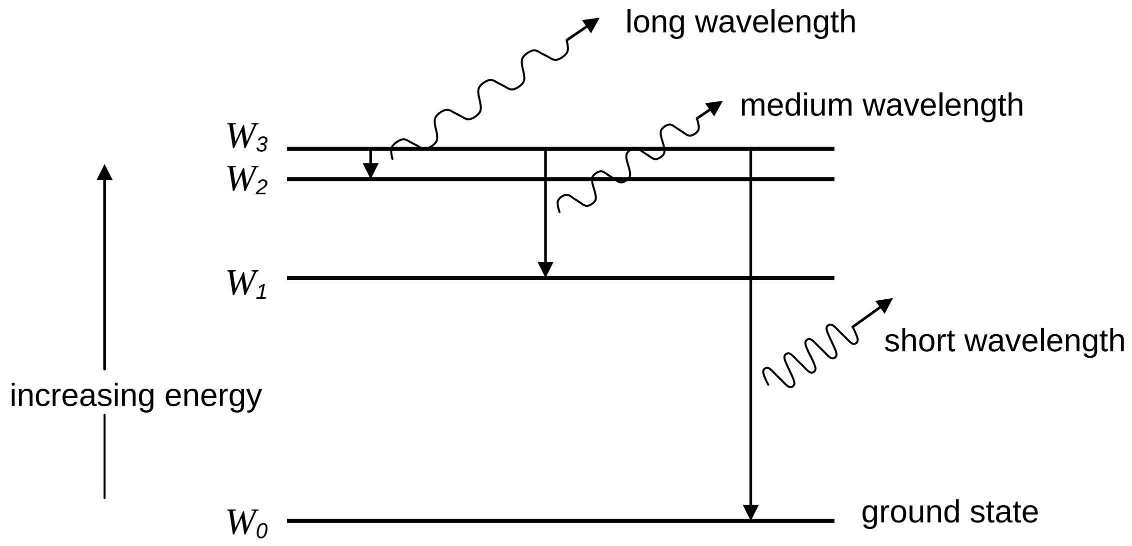
The electrons move between the energy levels by absorbing or emitting a photon of electromagnetic radiation with just the correct energy to match the gap between energy levels. As a result only a few frequencies of light are emitted as there are a limited number of possible energy jumps or transitions.

The lines on an emission spectrum are made by electrons making the transition from high energy levels (excited states) to lower energy levels (less excited states).



When the electron drops the energy is released in the form of a photon where its energy and frequency are related by:

$$E = hf$$



- The photons emitted may not all be in the visible wavelength.

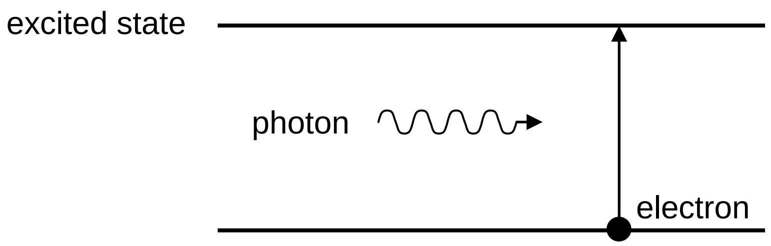
- The larger the number of excited electrons that make a particular transition, the more photons are emitted and the brighter the line in the spectrum.

The Continuous Spectrum

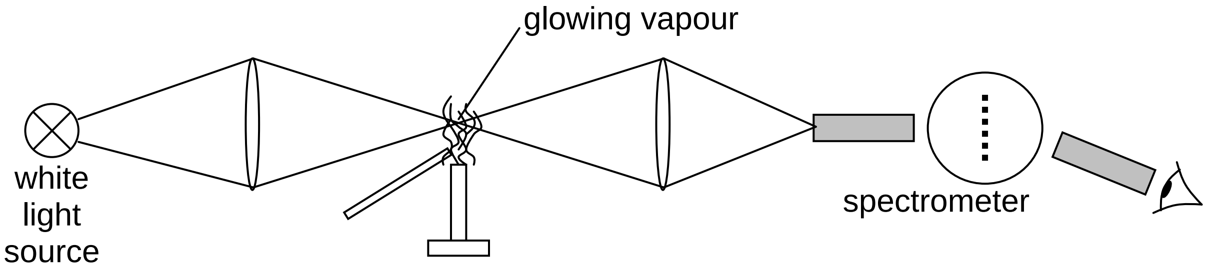
A continuous visible spectrum consists of all wavelengths of light from violet (~400 nm) to red (~700 nm). Such spectra are emitted by glowing solids (a tungsten filament in a lamp), glowing liquids or gases under high pressure (stars). In these materials the electrons are not *free*. The electrons are shared between atoms resulting in a large number of possible energy levels and transitions.

Absorption Spectra

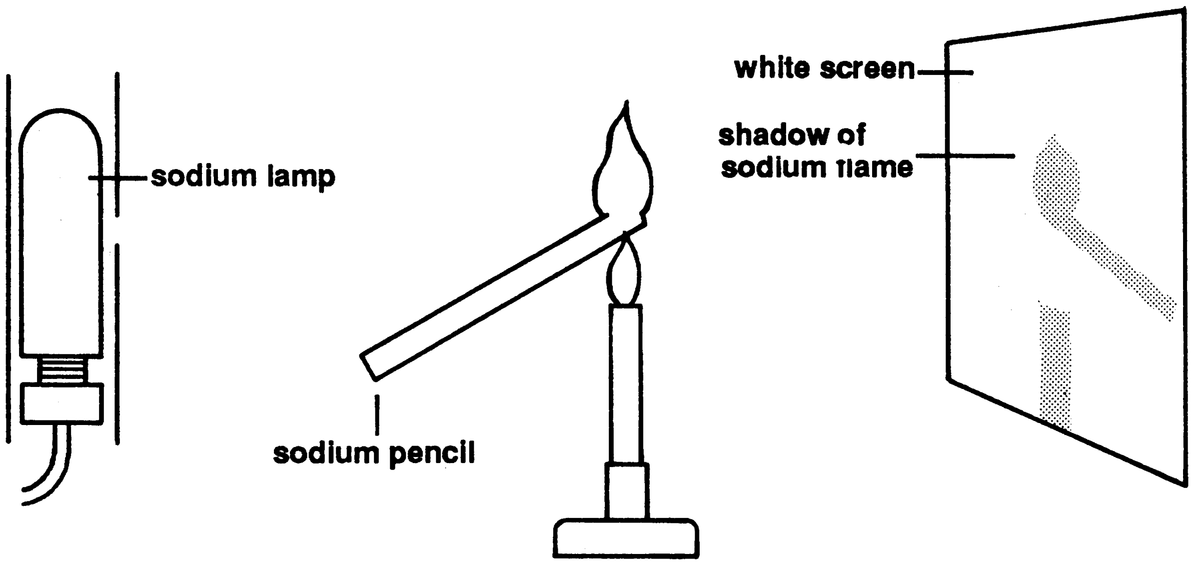
An electron may also make a transition from a lower energy level to a higher energy level. The electron must gain energy corresponding to the energy level gap. It can do this by absorbing a photon of exactly the correct frequency.



When white light is passed through a colour filter, a dye in solution or a glowing vapour, the frequencies of light corresponding to the energy level gaps are absorbed. This gives dark absorption lines across the otherwise continuous spectrum.



The fact that the frequencies of light that are absorbed by the glowing vapour match exactly those emitted can be demonstrated by the fact that a sodium vapour casts a shadow when illuminated with sodium light.

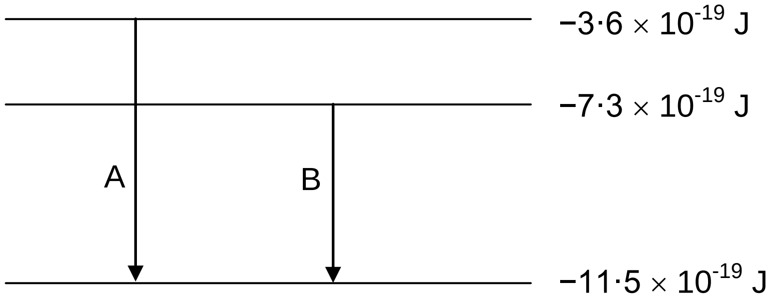


Absorption Lines in Sunlight

The white light produced in the centre of the Sun passes through the relatively cooler gases in the outer layer of the Sun's atmosphere. After passing through these layers, certain frequencies of light are missing. This gives dark lines (Fraunhofer lines) that correspond to the frequencies that have been absorbed.

The lines correspond to the bright emission lines in the spectra of certain gases. This allows the elements that make up the Sun to be identified.

Example: The diagram below shows two energy transitions within an atom.



(a) Determine the energy of the photons emitted during transitions A and B.
(b) Calculate the frequency of the emission line produced by transition A.
(c) Determine the wavelength of the remaining spectral line due to transitions between these energy levels.

Solution:

(a) A:

$\Delta E = (-11{\cdot}5 \times 10^{-19}) - (-3{\cdot}6 \times 10^{-19})$

$\Delta E = -7{\cdot}9 \times 10^{-19}$ J

energy of photon A = $7{\cdot}9 \times 10^{-19}$ J

B:

$\Delta E = (-11{\cdot}5 \times 10^{-19}) - (-7{\cdot}3 \times 10^{-19})$

$\Delta E = -4{\cdot}2 \times 10^{-19}$ J

energy of photon B = $4{\cdot}2 \times 10^{-19}$ J

(b) $E = hf$

$7{\cdot}9 \times 10^{-19} = 6{\cdot}63 \times 10^{-34} \times f$

$$f = \frac{7{\cdot}9 \times 10^{-19}}{6{\cdot}63 \times 10^{-34}}$$

$f = \underline{1{\cdot}2 \times 10^{15}\ \text{Hz}}$

(c) $\Delta E = (-7{\cdot}3 \times 10^{-19}) - (-3{\cdot}6 \times 10^{-19})$

$\Delta E = -3{\cdot}7 \times 10^{-19}$ J

$E = hf$

$3{\cdot}7 \times 10^{-19} = 6{\cdot}63 \times 10^{-34} \times f$

$$f = \frac{3{\cdot}7 \times 10^{-19}}{6{\cdot}63 \times 10^{-34}}$$

$f = 1{\cdot}09 \times 10^{15}$ Hz

$v = f\lambda$

$3{\cdot}00 \times 10^{8} = 1{\cdot}09 \times 10^{15} \times \lambda$

$\lambda = 2{\cdot}75 \times 10^{-7}$ m