Common Language and Methodology for Teaching Numeracy
St Luke’s Cluster

Contents **Page**

Introduction

A Learning and Teaching Scotland Numeracy project was set up to "raise standards of educational attainment for all in schools, especially in the core skills of literacy and numeracy". The project had three key areas of development:

- Expectation, continuity and progression in numeracy
- Numeracy for all
- Support for teachers of numeracy

Within East Renfrewshire the focus is on developing a common language and methodology for the teaching of numeracy. Within the St Luke's cluster a working party was set up to carry this out. The group consisted of Gerard O'Neil (St Luke's), Margaret Barr (St Thomas'), Laura Harold (St Mark's), Mark Ratter (Numeracy Driver) and Anne McLean (QIO).

Where possible the group has sought to make clear the correct use of language, and put in place a common use of language. Alongside that a common methodology has been designed for key elements of the 5-14 programme, across the primary and secondary schools within the cluster. The aim is to ensure continuity and progression for pupils, which should then impact on attainment.

It is expected that the work on Assessment is for Learning, Interactive maths, problem solving and mental maths would be integral to the delivery of the common language and methodology. See:

http://www.erc.schools/learnteach/Mathematics/mathematics.htm

Common Methodology for Algebra

Common Methodology - Algebra

Overview

Algebra is a way of thinking, i.e. a method of seeing and expressing relationships, and generalising patterns - it involves active exploration and conjecture. Algebraic thinking is not the formal manipulation of symbols.

Algebra is not simply a topic that pupils cover in Secondary school. From Primary One, staff are involved in helping pupils **lay the foundations for algebra**. This includes:

Level A, B and C

- Writing equations e.g. 16 add 8 equals?
- Solving equations e.g. $2 + \square = 7$
- Finding equivalent forms
  e.g. $24 = 20 + 4 = 30 - 6$
  $24 = 6 \times 4 = 3 \times 2 \times 2 \times 2$
- Using inverses or reversing e.g. $4 + 7 = 11 \rightarrow 11 - 7 = 4$
- Identifying number patterns

Level D, E and F

- Expressing relationships
- Drawing graphs
- Factorising numbers and expressions
- Understanding the commutative, associative and distributive laws

5-14 Arrangements – Functions and Equations

Pupils should be able to:

Level B	Level C	Level D	Level E	Level F
- find the missing numbers in statements where symbols are used for unknown numbers or operators.	- use a simple “function machine” for operations: - involving doubling, halving, adding and subtracting.	- recognise and explain simple relationships: - between 2 sets of numbers or objects.	- use a “function machine” in reverse for inverse operations;	- understand equivalence of expressions and use standard algebraic conventions to rearrange them;
			- solve simple equations and inequations;	- evaluate expressions using the conventions for order of operations in calculations;
			- use notation to describe general relationships between 2 sets of numbers;	- solve further equations and inequations;
			- use and devise simple rules.	- recognise simple relationships and construct/use simple formulae, equations and graphs (linear) to solve problems.

Level A

4 + 5 = 9 is the start of thinking about equations, as it is a statement of equality between two expressions.
Move from “makes” towards “equals” when concrete material is no longer necessary.
Pupils should become familiar with the different vocabulary for addition and subtraction as it is encountered. A wall display should be built up. See separate sections for more details of this.

Level B

Introduce the term “algebra” when symbols are used for unknown numbers or operators e.g.
2 + □ = 7
2 □ 6 = 8
6 = 3 + □

Use the word “something” or “what” to represent numbers or operators rather than the word “box” or “square” when solving these equations.

Level C - Function Machines

Use “in” and “out”, raising awareness of the terms “input” and “output”.

Level D – Recognise and explain simple relationships

Establish the operation(s) that are an option.

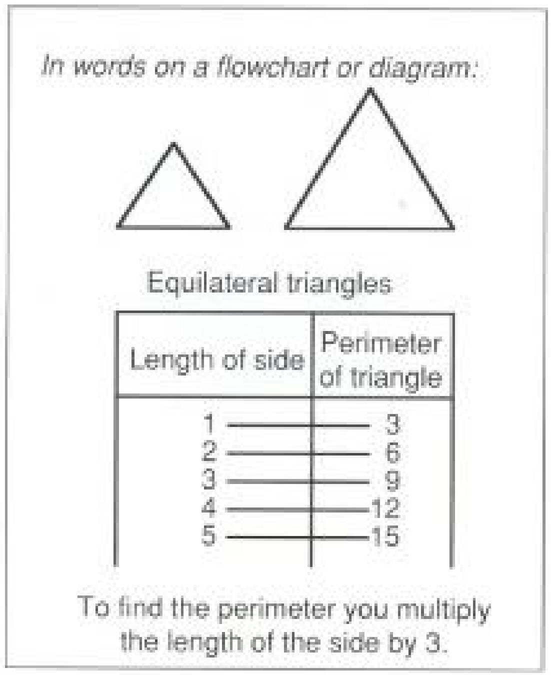
3→21
8→56
10→70

The outputs are larger than the input, so the options are either addition or multiplication.

18→9
14→7
6→3

The outputs are smaller than the input, so the options are either subtraction or division.

In this case, outputs are larger so the options are addition or multiplication. Add 2 works for the first one but
2 add 2 gives 4, we need the answer 6, addition does not work.
For multiplication, look at which table the output values are in.
Ensure you check the answer


In words on a flowchart or diagram:

Equilateral triangles

Length of side	Perimeter of triangle
1	3
2	6
3	9
4	12
5	15

To find the perimeter you multiply the length of the side by 3.

Level E – Collecting like terms (Simplifying Expressions)

The examples below are *expressions* not equations.

Have the pupils rewrite expressions with the like terms gathered together as in the second line of examples 2, 3 & 4 below, before they get to their final answer. The operator (+, −) and the term ($7x$) stay together at all times. It does not matter where the operator and term ($-7x$) are moved within the expression. (see example 3).

Example 1

Simplify

$$x+2x+5x$$
$$=8x$$

Example 2

Simplify

$$3a+2+6+7a$$
$$=3a+7a+2+6$$
$$=10a+8$$

The convention in Maths is to write the answer with the letters first, in alphabetical order, followed by number.

Example 3

Simplify

$$3+5x+4-7x \qquad\qquad 3+5x+4-7x$$
$$=5x-7x+3+4 \quad \text{or} \quad =3+4+5x-7x$$
$$=-2x+7 \qquad\qquad =7-2x$$

This is still correct although the first answer is preferable.

Example 4

Simplify

$$5m+3n-2m-n \qquad\qquad 5m+3n-2m-n$$
$$=5m-2m+3n-n \quad \text{or} \quad =3n-n+5m-3n$$
$$=3m+2n \qquad\qquad =2n+3m$$

Level E – Evaluating expressions

If $x = 2$, $y = 3$ and $z = -4$

Find the value of:

(*a*) $5x - 2y$
(*b*) $x + y - 2z$
(*c*) $2(x + z) - y$
(*d*) $x^2 + y^2 + z^2$

$$
\begin{aligned}
&a)\,5x - 2y\\
&= 5\times 2 - 2\times 3\\
&= 10 - 6\\
&= 4
\end{aligned}
$$

This line where the substitution takes place must be shown.
Marks are awarded in examinations for demonstrating this step.

$$
\begin{aligned}
&b)\,x + y - 2z\\
&= 2 + 3 - 2\times(-4)\\
&= 5 - (-8)\\
&= 13
\end{aligned}
$$

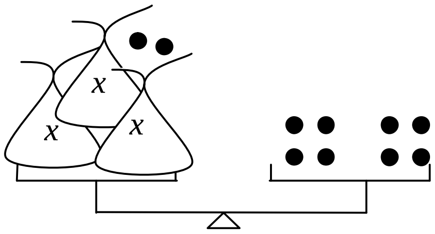
$$
\begin{aligned}
&c)\,2(x + z) - y\\
&= 2(2 + (-4)) - 3\\
&= 2\times(-2) - 3\\
&= -4 - 3\\
&= -7
\end{aligned}
$$

There is a bracket around the -4 as mathematicians do not write two operators side by side.

$$
\begin{aligned}
&d)\,x^2 + y^2 + z^2\\
&= 2^2 + 3^2 + (-4)^2\\
&= 4 + 9 + 16\\
&= 29
\end{aligned}
$$

Level E/F – Solve simple equations

The method used for solving equations is balancing. Each equation should be set out with a line down the right hand side where the method is written, as in the examples below. It is useful to use scales like the ones below to introduce this method as pupils can visibly see how the equation can be solved.



This represents the equation
$3x + 2 = 8$
See example 4 below

Example 1: Solve $x + 5 = 8$

$$\begin{array}{r|l} x + 5 = 8 & -5 \text{ from both sides} \\ \underline{\underline{x = 3}} & \end{array}$$

In the example shown pupils must state that they will "subtract 5 from both sides." If they only say, "Subtract five," ask them, "Where from?" and encourage them to tell you, "Both sides," on every occasion.

Pupils should be encouraged to check their answer mentally by substituting it back into the original equation.

Example 2: Solve $y - 3 = 6$

$$\begin{array}{r|l} y - 3 = 6 & +\ 3 \text{ to both sides} \\ \underline{\underline{y = 9}} & \end{array}$$

Example 3: Solve $4m = 20$

$$\begin{array}{r|l} 4m = 20 & \div \text{ by 4 on both sides} \\ \underline{\underline{m = 5}} & \end{array}$$

Example 4: Solve $3x + 2 = 8$

$$\begin{aligned} 3x + 2 &= 8 \\ 3x &= 6 \\ \underline{\underline{x = 2}} \end{aligned}$$

– 2 from both sides
÷ by 3 on both sides

Underline answer

The examples below are Level F. Examples of this type appear as part of the Level F programme and, as such, would not normally be attempted by pupils in the primary school.

Example 5: Solve $10 - 2x = 4$

$$\begin{aligned} 10 - 2x &= 4 \\ 10 &= 4 + 2x \\ 6 &= 2x \\ 3 &= x \\ \underline{\underline{x = 3}} \end{aligned}$$

+ $2x$ to both sides
– 4 from both sides
÷ by 2 on both sides

Write answer in the form $x =$

Example 6: Solve $3x + 2 = x + 14$

$$\begin{aligned} 3x + 2 &= x + 14 \\ 2x + 2 &= 14 \\ 2x &= 12 \\ \underline{\underline{x = 6}} \end{aligned}$$

– x from both sides
– 2 from both sides
÷ by 2 on both sides

NB Secondary:
Always deal with the variable before the constants, ensuring that the variable is written with a positive coefficient. This avoids errors when dividing by negatives and also avoids learning rules for dealing with inequations.

Other equations at this stage should include ones where x is a negative number or fraction. Pupils should be encouraged to write their answers as a fraction and not as a decimal. Use the language add, subtract, multiply (not times) and divide.
Also when referring to the number '−5' we say 'negative 5' **NOT** 'minus 5' as minus should be treated as an operation (verb).

Level E/F – Solve inequations

Example 1:

Solve the inequation $x + 3 > 6$ choosing solutions from $\{0, 1, 2, 3, 4, 5, 6\}$

$$\begin{array}{l|l} x + 3 > 6 & -3 \text{ from both sides} \\ x > 3 & \end{array}$$

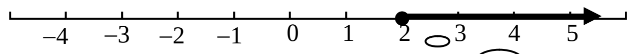
$\underline{\underline{x = \{4, 5, 6\}}}$

In the following examples a range of answers is not given therefore the answer should always be shown on a number line in preparation for more complex inequations at National Qualification level.

Example 2

Solve $x + 5 \geq 7$

$$\begin{array}{l|l} x + 5 \geq 7 & -5 \text{ from both sides} \\ \underline{\underline{x \geq 2}} & \end{array}$$

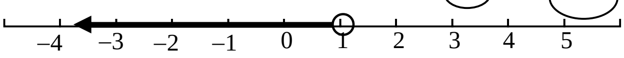


Note that if the value is included in the solution, i.e. 2 here. We represent this with a filled dot.

Example 3

Solve $x + 3 < 4$

$$\begin{array}{l|l} x + 3 < 4 & -3 \text{ from both sides} \\ \underline{\underline{x < 1}} & \end{array}$$



Note that if the value is not included in the solution, i.e. 1 here. We represent this with an open dot.

Level E – Using formulae

Pupils meet formulae in 'Area', 'Volume', 'Circle', 'Speed, Distance, Time' etc. In all circumstances, working must be shown which clearly demonstrates strategy, (i.e. selecting the correct formula), substitution and evaluation.

Example :

Find the area of a triangle with base 8cm and height 5cm.

$$A = \tfrac{1}{2}bh$$
$$A = \tfrac{1}{2} \times 8 \times 5$$
$$A = 20cm^2$$

Remember the importance of the substitution.

<u>Level E – Use and devise simple rules</u>

Pupils need to be able to use notation to describe general relationships between 2 sets of numbers, and then use and devise simple rules.

Pupils need to be able to deal with numbers set out in a table horizontally, set out in a table vertically or given as a sequence.
A method should be followed, rather than using "trial and error" to establish the rule.

At level D pupils have been asked to find the rule by establishing the single operation used. (+5, ×3, ÷2)

<u>Example 1:</u> Complete the following table, finding the n^{th} term.

In this example, the output values are still increasing, however, addition or multiplication on their own do not work, so this must be a two-step operation.

Input	1	2	3	4	5		n
Output	5	7	9	11	13		?

Look at the outputs. These are going up by 2 each time. This tells us that we are multiplying by 2. (This means × 2.)
Now ask:
1 multiplied by 2 is 2, how do we get to 5? Add 3.
2 multiplied by 2 is 4, how do we get to 7? Add 3.
This works, so the rule is:

Multiply by 2 then add 3.

Check using the input 5: $5 \times 2 + 3 = 13$

We use n to stand for any number
So the n^{th} term would be $\boldsymbol{n \times 2 + 3}$ which is rewritten as

$\boldsymbol{2n + 3}$

<u>Example 2:</u> Find the 20th term.

Input	Output	
1	7	+3
2	10	+3
3	13	
4	16	
5	19	
6	22	
n	$3n + 4$	
20		

To find the 20th term it is best to find the n^{th} first.

Look at the output values. These are going up by 3 each time. This tells us that we are multiplying by 3. (This means × 3.)
Now ask:
1 multiplied by 3 is 3, how do we get to 7? Add 4.
2 multiplied by 3 is 6, how do we get to 10? Add 4.
This works so the rule is

Multiply by 3 then add 4.

Check using 6: $6 \times 3 + 4 = 22$

We use n to stand for any number
So the n^{th} term would be $\boldsymbol{n \times 3 + 4}$ which is rewritten as

$\boldsymbol{3n + 4}$

To get the 20th term we substitute $n = 20$ into our formula.

$3n + 4$
$=3\times20 + 4$
$=60 + 4$
$=64$

Example 3:

For the following sequence find the term that produces an output of 90.

Input	Output
1	2
2	10
3	18
4	26
5	34
6	42
n	$8n - 6$
	90

+8
+8

We go through the same process as before to find the n^{th} term, which is $\mathbf{8n - 6}$.

Now we set up an equation.

$$\begin{array}{rl|l} 8n - 6 & = 90 & +6 \\ 8n & = 96 & \div \text{ by } 8 \\ n & = 12 & \end{array}$$

Therefore the 12th term produces an output of 90.

Common Methodology for Add and Subtract

5-14 Arrangements – Add and Subtract

Pupils should be able to add and subtract:

Level A	Level B	Level C	Level D	Level E	Level F
- mentally for numbers 0 to 10	- mentally for numbers 0 to 20, in some cases beyond 20	- mentally for 1 digit to or from whole numbers up to 3 digits beyond in some cases involving multiples of 10	- mentally for 2 digit whole numbers, beyond in some cases, involving multiples of 10 or 100	- mentally for 2 digit numbers including decimals	- mentally for 2 digit numbers including integers
- in applications in number, money and measurement, including payments and change to 10p	- without a calculator for numbers to 2 digits added to or subtracted from 3 digits	- mentally for subtraction by adding on	- without a calculator, for 4 digits with at most 2 decimal places (easy examples)	- without a calculator, for 4 digits with at most 2 decimal places	- without a calculator, for 4 digit numbers including decimals and integers
	- in applications in number, money and measurement, including payments and change up to £1	- without a calculator for whole numbers with 2 digits, added to or subtracted from 3 digits	- with a calculator for 4 digits with at most 2 decimal places	- with a calculator for any number of digits with at most 3 decimal places	- with a calculator for whole numbers, decimals and integers with any number of digits
		- with a calculator for 3 digit whole numbers	- in applications in number, money and measurement	- in applications in number, money and measurement	- in applications in number, money and measurement
		- in applications in number, money and measurement to £20		- positive and negative numbers in application such as rise in temperature	

Level A

- Use “maths” instead of “sums” as sum refers to addition. Use “show your working” or “written calculation” rather than “write out the sum”.
  Use the word “calculate.”

- Words for addition and subtraction
  Add: Plus, total, find the sum of
  Subtract: Take away moving towards subtract and minus when appropriate

- Avoid the use of “and” when meaning addition e.g. “4 and 2 is 6”

- Start addition and subtraction at the top and work downwards e.g.

$$\begin{array}{r} 5 \\ 4 \\ \underline{+7} \end{array}$$

is 5 add 4 add 7

For pupils who are secure in their number bonds encourage them to look for patterns in the number bonds e.g.

$$\begin{array}{r} 6 \\ 7 \\ \underline{+4} \end{array}$$

Pupils could either do 6 add 7 add 4 (down) or 6 add 4 add 7 (patterns)

- When one addition fact is known it is important to elicit the other three facts in terms of addition and subtraction.

$2 + 3 = 5$
$3 + 2 = 5$
$5 - 2 = 3$
$5 - 3 = 2$

Level B

- When “carrying”, lay out the algorithm as follows:

$$\begin{array}{r} 5\;\,6 \\ +\;3_1\,9 \\ \hline 9\;\,5 \\ \hline \end{array}$$

The “carry” digit always sits above the line.

Level E

- Say negative four for –4. Explain that in temperature “minus 4” is technically wrong even though it is widely used.

Common Methodology for Multiply and Divide

5-14 Arrangements – Multiply and Divide

Pupils should be able to multiply and divide:

Level B	Level C	Level D	Level E	Level F
- mentally by 2, 3, 4, 5, 10 within the confines of these tables	- mentally within the confines of all tables to 10	- mentally for whole numbers by single digits (easy examples)	- mentally for any whole number by a multiple of 10 or 100	- mentally for decimals and integers by single digits (easy examples)
- without a calculator for 2 digit numbers multiplied by 2, 3, 4, 5, 10	- mentally for any 2 or 3 digit number by 10	- mentally for 4 digit numbers including decimals by 10 or 100	- mentally for any numbers including decimals by 10, 100, 1000	- without a calculator for more complex examples
- with a calculator for 2 digit numbers multiplied and divided by any digit	- without a calculator for 2 digit whole numbers by any single digit whole number	- without a calculator for 4 digits with at most 2 decimal places by a single digit	- without a calculator for 4 digits with at most 2 decimal places by a single digit	- with a calculator for any pairs of numbers
- in applications in NMM to £1	- with a calculator for 2 or 3 digit whole numbers by a whole number with 1 or 2 digits	- with a calculator for 4 digits with at most 2 decimal places by a whole number with 2 digits	- with a calculator for any pair of numbers but at most 3 decimal places in the answer	- know that multiplication by a number less than one has a decreasing effect whereas division by a number less than one has an increasing effect
	- in applications in NMM to £20	- in applications in NMM	- in applications in NMM	- in applications in NMM

Level B

- Words for multiply and divide
  Multiply: Multiplied by, product
  Divide: Divided by, quotient

- Use "multiplication" tables rather than "times" tables.
  Use "multiplied by" instead of "times".

- Table facts e.g. $2 \times 5 = 10$ should be stated as "two fives are ten"

$$4\overline{)756}$$

- Say "this is 756 divided by 4". Start by saying "7 divided by 4". Support if necessary by saying "how many 4's in 7?"

- Avoid saying "4 goes into."

- When multiplying by one digit, lay out the algorithm as follows:

$$\begin{array}{r} 2\ 6 \\ \times_{2}\ 4 \\ \hline 1\ 0\ 4 \\ \hline \end{array}$$

The "carry" digit always sits above the line.

- When multiplying by two digits, lay out the algorithm as follows:

$$\begin{array}{r} 4\ 7 \\ \times 5_{4}\ 6 \\ \hline 2\ 8\ 2 \\ 2\ 3_{1}\,{}^{3}5\ 0 \\ \hline 2\ 6\ 3\ 2 \\ \hline \end{array}$$

It is important to emphasise the difference between the carrying digits so that when pupils are adding they only include the relevant digits.

Level D

- The decimal point stays fixed and the digits "move" when multiplying and dividing.

- Do not say "add on a zero" when multiplying by 10. Say "the digits move one place to the left."

Common Methodology for Information Handling

Information Handling

Discrete Data

Discrete data can only have a finite or limited number of possible values.
Shoe sizes are an example of **discrete** data because sizes 39 and 40 mean something, but size 39·2, for example, does not.

Continuous Data

Continuous data can have an infinite number of possible values within a selected range.
e.g. temperature, height, length.

Non-Numerical Data (Nominal Data)

Data which is non-numerical.
e.g. favourite TV programme, favourite flavour of crisps.

Tally Chart/Table (Frequency table)

A tally chart is used to collect and organise data prior to representing it in a graph.

Averages

Pupils should be aware that mean, median and mode are different types of average.

Mean: add up all the values and divide by the number of values.
Mode: is the value that occurs most often.
Median: is the middle value or the mean of the middle pair of an ordered set of values.

Pupils are introduced to the mean using the word average. In society average is commonly used to refer to the mean.

Range

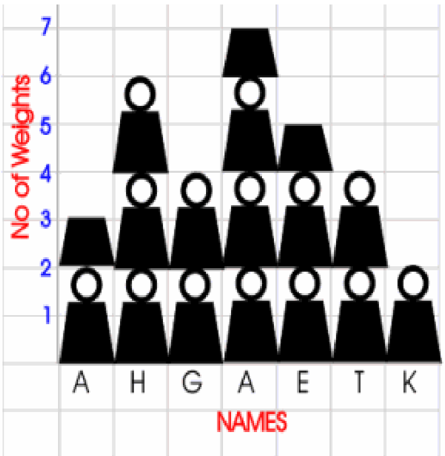
The difference between the highest and lowest value.

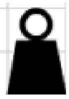
Pictogram/pictograph

A pictogram/pictograph should have a title and appropriate x and y-axis labels. If each picture represents a value of more than one, then a key should be used.

NAMES	WEIGHTS
ANDREW	3
HELEN	6
GARY	4
ALEX	7
ELAINE	5
THERESA	4
KEVIN	2
TOTAL	31
MEAN	4.4

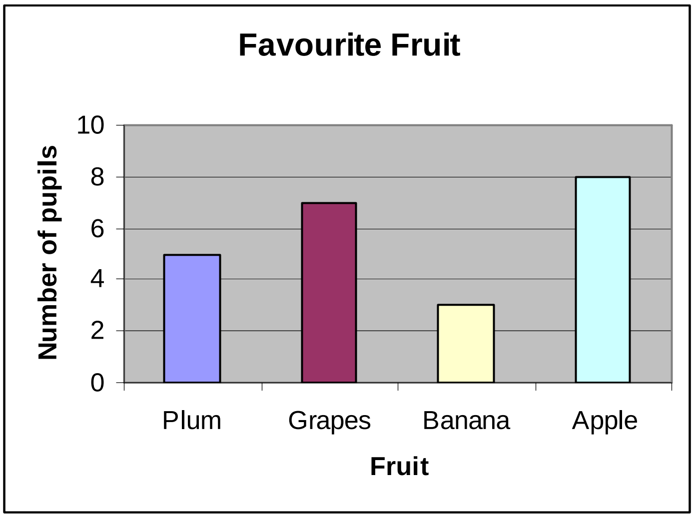
The weight each pupil managed to lift




represents two units

Bar Chart/Graph

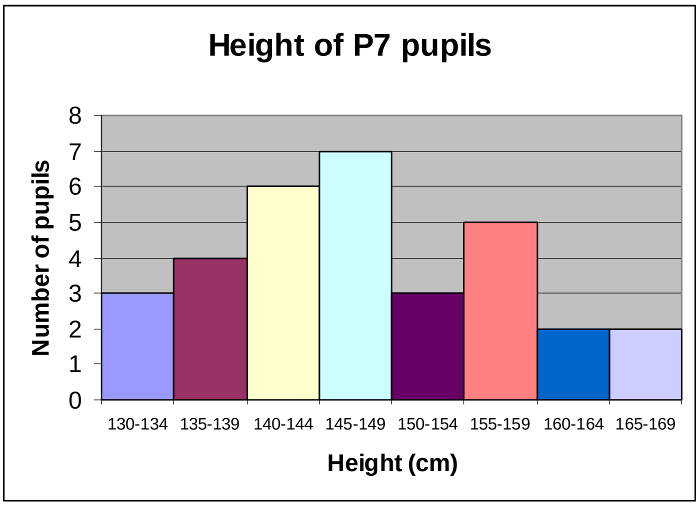
A bar chart is a way of displaying discrete or non-numerical data. A bar chart should have a title and appropriate *x* and *y*-axis labels. An even space should be between each bar and each bar should be of an equal width. Leave a space between the *y*-axis and the first bar. When using a graduated axis, the intervals must be evenly spaced.



Frequency diagrams

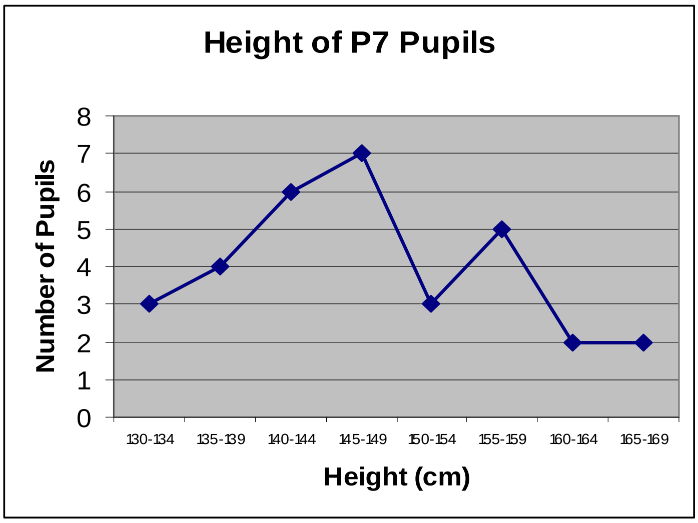
1. Histogram

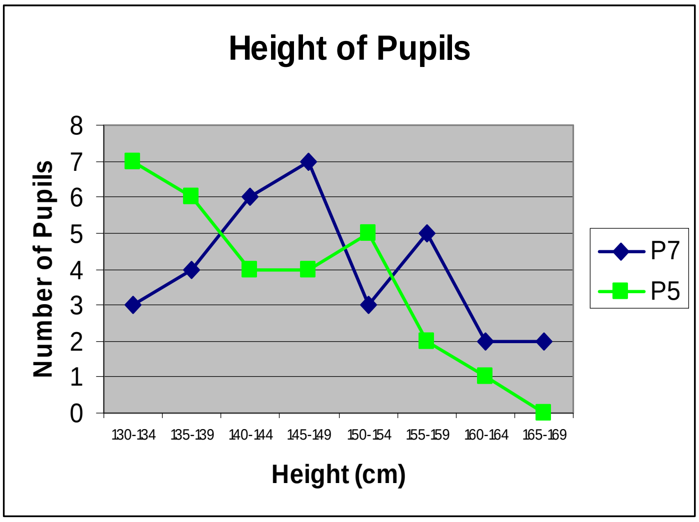
A histogram is a way of displaying grouped data. A histogram should have a title and appropriate *x* and *y*-axis labels. There should be no space between each bar. Each bar should be of an equal width. When using a graduated axis, the intervals must be evenly spaced.



2. Frequency Polygon

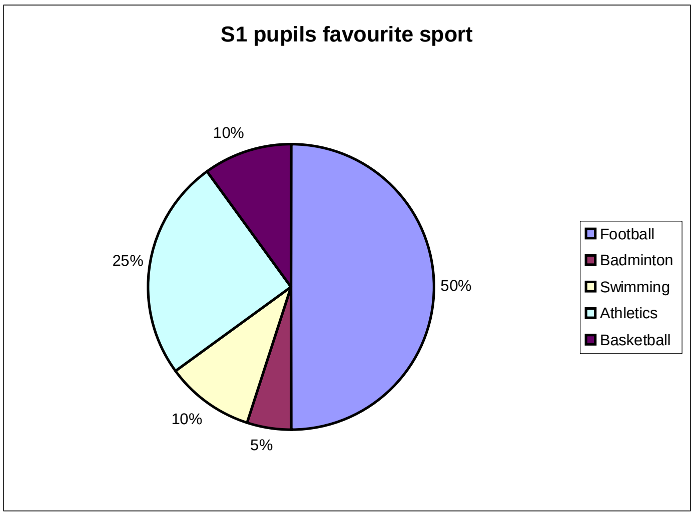
To draw a frequency polygon, draw a histogram then join the midpoints of the top of each bar. It is then optional, whether or not you remove the bars. Frequency polygons are useful when comparing two sets of data.





Pie Charts

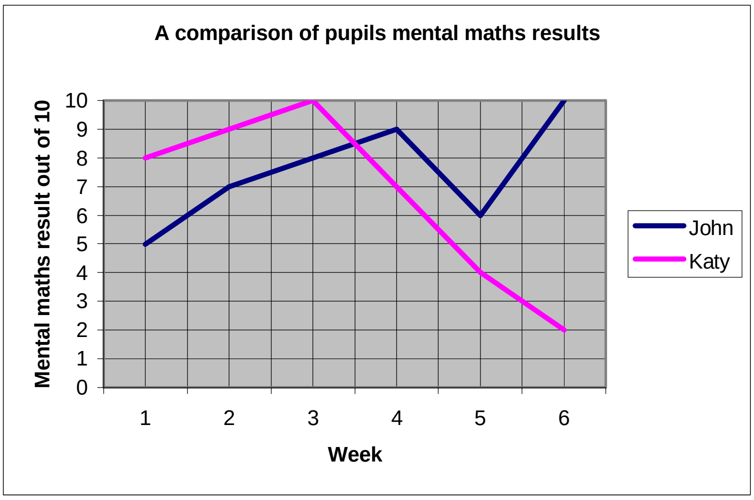
A pie chart is a way of displaying discrete or non-numerical data.
It uses percentages or fractions to compare the data. The whole circle (100% or one whole) is then split up into sections representing those percentages or fractions. A pie chart needs a title and a key.



Line Graphs

Line graphs compare two quantities (or variables). Each variable is plotted along an axis. A line graph has a vertical and horizontal axis. So, for example, if you wanted to graph the height of a ball after you have thrown it, you could put time along the horizontal, or *x*-axis, and height along the vertical, or *y*-axis.

A line graph needs a title and appropriate *x* and *y*-axis labels. If there is more than one line graph on the same axes, the graph needs a key.



Stem-and-leaf diagram

A stem-and-leaf diagram is another way of displaying discrete or continuous data. A stem-and-leaf diagram needs a title, a key and should be ordered. It is useful for finding the median and mode. If we have two sets of data to compare we can draw a back-to-back stem-and-leaf diagram.

Example: The following marks were obtained in a test marked out of 50. Draw a stem and leaf diagram to represent the data.

3, 23, 44, 41, 39, 29, 11, 18, 28, 48.

Split the data into a stem and a leaf. Here the tens part of the test mark is the stem. The units part of the test mark is called the leaf.

Unordered stem-and-leaf diagram showing test marks out of 50

0	3
1	1 8
2	3 9 8
3	9
4	4 1 8

1| 3 means 13 out of 50 n = 10

The diagram can be ordered to produce an ordered stem and leaf diagram.

Ordered stem-and-leaf diagram showing test marks out of 50

0	3
1	1 8
2	3 8 9
3	9
4	1 4 8

1| 3 means 13 out of 50 n = 10

Scattergraphs (Scatter diagrams)

A scattergraph allows you to compare two quantities (or variables). Each variable is plotted along an axis. A scattergraph has a vertical and horizontal axis. It needs a title and appropriate *x* and *y*-axis labels. For each piece of data a point is plotted on the diagram. The points are not joined up.

A scattergraph allows you to see if there is a connection (correlation) between the two quantities. There may be a positive correlation when the two quantities increase together e.g. sale of umbrellas and rainfall. There may be a negative correlation were as one quantity increases the other decreases e.g. price of a car and the age of the car. There may be no correlation e.g. distance pupils travel to school and pupils' heights.