Mathematics and Numeracy

ST LUKE'S HIGH SCHOOL

Supporting learning at home

HOW CAN YOU HELP?

- ☑ Practise the number facts regularly with your child
- ☑ Support them in the use of homework diary and class notes
- ☑ Ensure that they check off answers with small tick and complete any corrections
- ☑ Support their organisation - pencil/rubber/ruler/jotters and a scientific calculator to class (Casio FX83GTPlus Scientific Calculator is best)
- ☑ Encourage their best presentation
- ☑ Ensure they attempt all questions
- ☑ Step-by-step, vertical working where possible
- ☑ Support the use of the common language and methodology
- ☑ Regular revision and use of online learning resources including GLOW

- ☑ Time: Encourage your child to use a watch or clock to tell the time, provide timed activities and read timetables

- ☑ Calendars: Plan family birthdays on a calendar & do a birthday countdown.

- ☑ Measure: Take advantage of measure opportunities in the kitchen; weighing, timing and temperature

- ☑ Money: Talk about best deals with your child, budgeting pocket money or wages.

- ☑ Estimating: plan for activities in advance like calculating the number of rolls of paper or paint required to decorate a room, the length of time activities may take.

- ☑ Logical thinking: Ask your child to explain their thinking and consider consequences of actions. e.g. using the information gathered from reading newspapers, using the internet and watching TV to draw conclusions and make choices that involve numeracy.

MATHS PROPS FOR HOME

- A clock (analogue and digital if possible)
- A traditional wall calendar
- Board games that involve dice and spinners
- Pack of traditional playing cards
- A calculator
- Measuring jugs and scales
- A tape measure and a ruler
- Old-fashioned kitchen scales
- A dartboard (with velcro darts)
- Games with unusual dice
- Dominoes
- Guess Who?
- Yahtzee – the original dice scoring game
- An indoor/outdoor thermometer

MATHEMATICS SYMBOLS CHART

$+$	Add	$\rightarrow$	Tends to
$-$	Subtract	$\propto$	Is proportional to
$\times$	Multiply	∞	Infinity
$\div$	Divide	$\sum$	The sum of (sigma)
$=$	Equal to	$\parallel$	Parallel
$\pm$	Plus or minus	$\perp$	Perpendicular
$>$	Greater than	$\therefore$	Therefore
$<$	Less than	$\because$	Because
$\geq$	Greater than or equal to	π	Pi = 3.1415926…
$\leq$	Less than or equal to	$\angle$	Angle
$\neq$	Not equal to	$\in$	Set membership
$\approx$	Approximately equal to	$\mathbb{R}$	Real numbers
x^2	x squared	$\mathbb{Q}$	Rational numbers
$\sqrt{}$	Square root	$\mathbb{Z}$	Integers
$\Rightarrow$	Implies	$!$	Factorial
$\Leftrightarrow$	If and only if	P(..)	Probability of event

MATHEMATICS MARKING CODE

✓	Correct – One mark awarded
▁ ✗	First error underlined – No mark awarded
✓✗	Mark awarded – Correct working subsequent to error
∧	Crucial step missing- no mark awarded
✓✗✗	Correct working – inadequate to score marks, working has been eased
∼	Minor mistake that has not been penalised such as bad form
↓	Working which continues from previous attempt
T	Transcription Error (Only penalised at Higher and Advanced Higher)
St	Incorrect strategy
Sp	Incorrect spelling
R	Use a ruler
P	Presentation could improve
V	Show working vertically
W	Show your working

MULTIPLICATION TABLES

1				
1	x	1	=	1
1	x	2	=	2
1	x	3	=	3
1	x	4	=	4
1	x	5	=	5
1	x	6	=	6
1	x	7	=	7
1	x	8	=	8
1	x	9	=	9
1	x	10	=	10
1	x	11	=	11
1	x	12	=	12

2				
2	x	1	=	2
2	x	2	=	4
2	x	3	=	6
2	x	4	=	8
2	x	5	=	10
2	x	6	=	12
2	x	7	=	14
2	x	8	=	16
2	x	9	=	18
2	x	10	=	20
2	x	11	=	22
2	x	12	=	24

3				
3	x	1	=	3
3	x	2	=	6
3	x	3	=	9
3	x	4	=	12
3	x	5	=	15
3	x	6	=	18
3	x	7	=	21
3	x	8	=	24
3	x	9	=	27
3	x	10	=	30
3	x	11	=	33
3	x	12	=	36

4				
4	x	1	=	4
4	x	2	=	8
4	x	3	=	12
4	x	4	=	16
4	x	5	=	20
4	x	6	=	24
4	x	7	=	28
4	x	8	=	32
4	x	9	=	36
4	x	10	=	40
4	x	11	=	44
4	x	12	=	48

5				
5	x	1	=	5
5	x	2	=	10
5	x	3	=	15
5	x	4	=	20
5	x	5	=	25
5	x	6	=	30
5	x	7	=	35
5	x	8	=	40
5	x	9	=	45
5	x	10	=	50
5	x	11	=	55
5	x	12	=	60

6				
6	x	1	=	6
6	x	2	=	12
6	x	3	=	18
6	x	4	=	24
6	x	5	=	30
6	x	6	=	36
6	x	7	=	42
6	x	8	=	48
6	x	9	=	54
6	x	10	=	60
6	x	11	=	66
6	x	12	=	72

7				
7	x	1	=	7
7	x	2	=	14
7	x	3	=	21
7	x	4	=	28
7	x	5	=	35
7	x	6	=	42
7	x	7	=	49
7	x	8	=	56
7	x	9	=	63
7	x	10	=	70
7	x	11	=	77
7	x	12	=	84

8				
8	x	1	=	8
8	x	2	=	16
8	x	3	=	24
8	x	4	=	32
8	x	5	=	40
8	x	6	=	48
8	x	7	=	56
8	x	8	=	64
8	x	9	=	72
8	x	10	=	80
8	x	11	=	88
8	x	12	=	96

9				
9	x	1	=	9
9	x	2	=	18
9	x	3	=	27
9	x	4	=	36
9	x	5	=	45
9	x	6	=	54
9	x	7	=	63
9	x	8	=	72
9	x	9	=	81
9	x	10	=	90
9	x	11	=	99
9	x	12	=	108

10				
10	x	1	=	10
10	x	2	=	20
10	x	3	=	30
10	x	4	=	40
10	x	5	=	50
10	x	6	=	60
10	x	7	=	70
10	x	8	=	80
10	x	9	=	90
10	x	10	=	100
10	x	11	=	110
10	x	12	=	120

11				
11	x	1	=	11
11	x	2	=	22
11	x	3	=	33
11	x	4	=	44
11	x	5	=	55
11	x	6	=	66
11	x	7	=	77
11	x	8	=	88
11	x	9	=	99
11	x	10	=	110
11	x	11	=	121
11	x	12	=	132

12				
12	x	1	=	12
12	x	2	=	24
12	x	3	=	36
12	x	4	=	48
12	x	5	=	60
12	x	6	=	72
12	x	7	=	84
12	x	8	=	96
12	x	9	=	108
12	x	10	=	120
12	x	11	=	132
12	x	12	=	144

Pupils should learn the multiplication tables off by heart. It is not enough just to learn the stations eg. The stations for the 8 x table are 8, 16, 24, 32, etc

If you know 4 x 9 = 36
Then you also know the answers to

9 x 4, 36 ÷ 4 and 36 ÷9

<u>NUMBER BONDS / ADDITION FACTS</u>

Pupils should be able to recall addition and subtraction facts up to 20 and beyond

<u>Here is an example of the facts to 20</u>

Addition facts	**Subtraction facts**
1 + 19 = 20	20 – 1 = 19
2 + 18 = 20	20 – 2 = 18
3 + 17 = 20	20 – 3 = 17
4 + 16 = 20	20 – 4 = 16
5 + 15 = 20	20 – 5 = 15
6 + 14 = 20	20 – 6 = 14
7 + 13 = 20	20 – 7 = 13
8 + 12 = 20	20 – 8 = 12
9 + 11 = 20	20 – 9 = 11
10 + 10 =20	20 – 10 = 10

ESTIMATING MEASUREMENT

Pupils should always be encouraged to ESTIMATE-CALCULATE-CHECK their solutions

Pupils estimate best when comparing sizes to a known object or parameter.

Length

- height and length in cm and m.
- length of ruler = 30cm
- height of door = 2m

Area/Weight/Volume

- small weights, small areas, small volumes
- weight of bag of sugar = 1kg
- area of envelope = 10 x 8 = 80cm^2
- volume of lemonade bottle = 1 litre.

Progression in this topic will be able to estimate areas in m^2 and lengths in m and mm.

height of kitchen unit = 700mm
area of work surface = 6m^2

MENTAL CALCULATIONS

Pupils should be encouraged to complete simple calculations mentally, without a calculator

All pupils should be able to carry out the following processes mentally, although some will need time to arrive at an answer:

Estimating first ensures the final answer is sensible
Addition/Subtraction/Multiplication/Division

3456 + 975	Estimate 3500 + 1000 = 4500
6286 – 4857	Estimate 6300 – 4900 = 1400
357 × 8	Estimate 350 × 2 × 4 = 700 × 4 = 2800
810 ÷ 6	Estimate 800 ÷ 5 = 160

Pupils should also be encouraged to carry out other calculations mentally using a variety of strategies but there will be significant differences in their ability to do so.

e.g.

$34 + 28 = 34 + 30 - 2$

$163 - 47 = 163 - 50 + 3$

$23 \times 3 = (20 \times 3) + (3 \times 3)$

$68 + 4 = (68 + 2) + 2$

Decimal Notation

All pupils should be familiar with decimal notation for money although they may use incorrect notation.

✓	✗
£2·35	£2·35p
£4·60	£4·6
£0·25	0·25p
32p	0·32p

Pupils should also be familiar with the use of decimal notation for metric measures, but sometimes misinterpret the decimal part of the number. They may need to be reminded for example that 1·5 metres is 150 centimetres not 105 centimetres.

Pupils often read decimal numbers incorrectly e.g. 8·72 is often read as eight point seventy two instead of eight point seven two.

They may also have problems with comparing the size of decimal numbers and may believe that 2·36 is bigger than 2·8 because 36 is bigger than 8. If they need to compare numbers it may help to write all of the numbers to the same number of decimal places e.g. 2·36 and 2·80

ROUNDING ANSWERS

Pupils should round their answers to one more decimal place or significant figure than the numbers in the question

Rounding to nearest 10

73 to nearest 10 → 70
496 to nearest 10 → 500

Rounding to nearest whole number/ten/hundred

237.8 → 238 (to nearest whole number)
→ 240 (to nearest 10)
→ 200 (to nearest 100)

Rounding to 1 decimal place (to 1 d.p.)

5.31 → 5.3 (to 1 d.p.)
11.97 → 12.0 (to 1 d.p.)

Rounding to more than 1 decimal place/to significant figures (to s.f.)

6.2459 → 6.246 (to 3 d.p) → 6.25 (to 3 s.f)

STANDARD FORM, SCIENTIFIC NOTATION

Only pupils working at fourth level will be familiar with standard form or scientific notation, (both terms are in common use). They should be able to use calculators for standard form. Pupils often have difficulty in interpreting the result of a standard form calculation on some calculators and need to be encouraged to write for example $3{\cdot}42 \times 10^{8}$ and **not** 3.42^{08}, (i.e. the calculator display)

Very large or very small numbers can be written in standard form

$a \times 10^b$

where $1 \leq a < 10$

$2\ 000\ 000 = 2 \times 10^6$

$3\ 700\ 000\ 000 = 3{\cdot}7 \times 10^9$

$0{\cdot}00000009 = 9 \times 10^{-8}$

$0{\cdot}000000859 = 8{\cdot}59 \times 10^{-7}$

LANGUAGE OF ARITHMETIC

Avoid the use of the word 'sum' to mean a maths question.
Use the word '**calculation**'

Addition (+)
- sum of
- more than
- add
- total
- and
- plus
- increase

Subtraction (-)
- less than
- take away
- minus
- subtract
- difference between
- reduce
- decrease

Equals (=)
- is equal to
- same as
- makes
- will be

Multiplication (×)
- multiply
- times
- product
- of

Division (÷)
- divide
- share equally
- split equally
- groups of
- per
- quotient

WRITING CALCULATIONS

Pupils have a tendency to use the '=' sign incorrectly and write mathematical expressions that do not make sense.

e.g. $3 \times 2 = 6 + 4 = 10 - 7 = 3$ ✗

It is important that pupils write such calculations correctly.

e.g.

$$3 \times 2 = 6$$
$$6 + 4 = 10$$
$$10 - 7 = 3$$ ✓

The '=' sign should only be used when both sides of the equation have the same value. There is no problem with calculations such as:

$34 + 28 = 30 + 20 + 4 + 8 = 50 + 12 = 62$

because each part of the equation has the same value.

The '≈' (approximately equal to) sign should be used when pupils are estimating answers.

e.g. $1576 - 312 \approx 1600 - 300 = 1300$

ADD AND SUBTRACT

It may be best to avoid the use of “and” when meaning addition e.g. “4 and 2 is 6”
Subtraction is by ‘**decomposition**’
The algorithms can be laid out as follows:
The “carry” digit usually sits above the line. Some pupils carry below the line.

3 456 + 975

Estimate

3 500 + 1 000 = 4 500

$$
\begin{array}{ccccc}
 & 3 & 4 & 5 & 6 \\
+ & {}_{1} & 9_{1} & 7_{1} & 5 \\
\hline
 & 4 & 4 & 3 & 1 \\
\hline
\end{array}
$$

6 286 – 4 857

Estimate

6 300 – 4 900 = 1 400

$$
\begin{array}{ccccc}
 & {}^{5}\cancel{6} & {}^{1}2 & {}^{7}\cancel{8} & {}^{1}6 \\
- & 4 & 8 & 5 & 7 \\
\hline
 & 1 & 4 & 2 & 9 \\
\hline
\end{array}
$$

MULTIPLYING AND DIVIDING BY 10, 100, 1000

The rule for multiplying by 10 is that each of the digits moves one place to the left.
When multiplying by 100 each digit moves two places to the left and so on. In division, the digits move to the right.
This rule works for whole numbers and decimals. ***Decimal points do not move***.

e.g.

$23 \times 100 = 2\ 300$

Th	H	T	U
		2	3
2	3	0	0

Zeros are needed to fill the empty spaces in the tens and units columns, otherwise when the number is written without the column headings it will appear as a different number, i.e. 23 instead of 2 300

$3{\cdot}45 \times 10 = 34{\cdot}5$

Th	H	T	U	·	$\frac{1}{10}$	$\frac{1}{100}$
			3	·	4	5
		3	4	·	5	

Zeros are not generally needed in empty columns after the decimal point except in cases where a specified degree of accuracy is required, (significant figures).

$260 \div 10 = 26$

H	T	U	·	$\frac{1}{10}$	$\frac{1}{100}$
2	6	0	·		
	2	6	·		

$439 \div 100 = 4{\cdot}39$

H	T	U	·	$\frac{1}{10}$	$\frac{1}{100}$
4	3	9	·		
		4	·	3	9

Using the rule of adding or removing zeros can be confusing because it only works for some numbers.

Many pupils recognise that when multiplying a whole number by 10, a zero is added, by 100, two zeros are added etc.
Problems sometimes arise however with the less able student who applies this technique to decimals and writes $3{\cdot}5 \times 10 = 3{\cdot}50$.

MULTIPLY AND DIVIDE

357 × 8

Estimate

350 × 2 × 4 = 700 × 4 = 2 800

$$\begin{array}{ccccc} & & 3 & 5 & 7 \\ \times & & {}_{4} & {}_{5} & 8 \\ \hline & 2 & 8 & 5 & 6 \end{array}$$

810 ÷ 6

Estimate

800 ÷ 5 = 160

$$\begin{array}{r|lll} & 1 & 3 & 5 \\ \cline{2-4} 6 & 8 & {}^{2}1 & {}^{3}0 \end{array}$$

- This can be said as " 810 divided by 6". Start as "8 divided by 6". Can be supported if necessary by saying "how many 6's in 8?"
- The "carry" digit sits above the line.
- When multiplying by two digits, the algorithm can be laid out as follows:

$$\begin{array}{cccc} & & 4 & 7 \\ & \times & 5_{4} & 6 \\ \hline & 2 & 8_{3} & 2 \\ 2 & 3_{1} & 5 & 0 \\ \hline 2 & 6 & 3 & 2 \\ \hline \end{array}$$

ORDER OF OPERATIONS

BOMDAS: The order of operations

Brackets
Off
Multiply
Divide
Add
Subtract

Examples

$$30 - 4 \times 3$$
$$= 30 - 12$$
$$= 18$$

$$30 + 50 \div (2 + 8)$$
$$= 30 + 50 \div 10$$
$$= 30 + 5$$
$$= 35$$

The important *facts* are that **brackets** are done first, then **powers**, **multiplication** and **division** and finally **addition** and **subtraction**.

INTEGERS

Definition : An INTEGER is a negative or positive whole number, including 0.

A **negative number** is less than zero.

-100, -25, -12 and -4 are all negative numbers.

A **positive number** is more than zero.

4, 12, 89 and 568 are all positive numbers.

The **negative sign** goes in front of a negative number.

Negative 4 is the same as **-**4.

° is the symbol for degree.

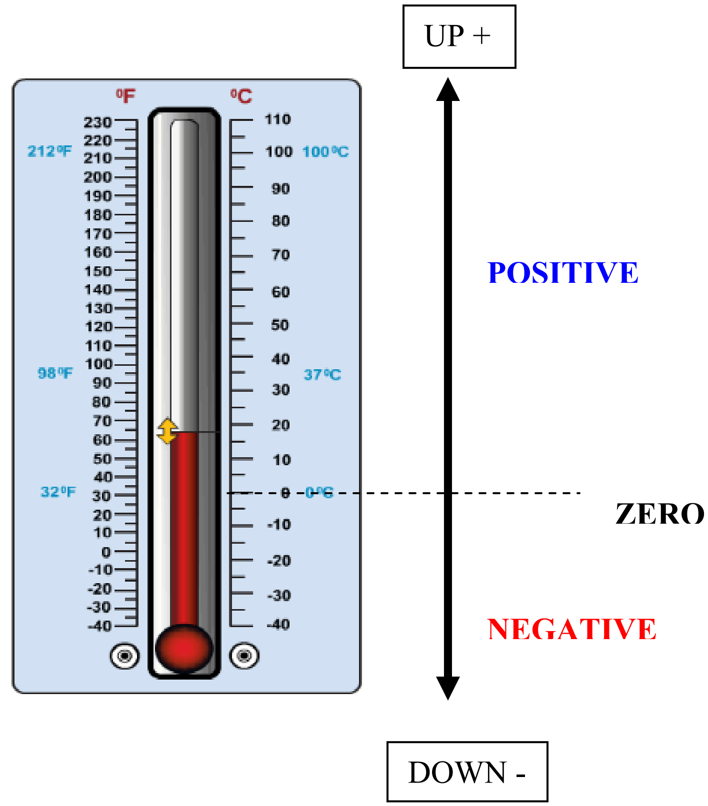
Celsius or **C** is a scale for measuring temperature.

6 degrees Celsius is the same as 6°C.

Negative temperatures are called **below freezing**.

Positive temperatures are called **above freezing**.

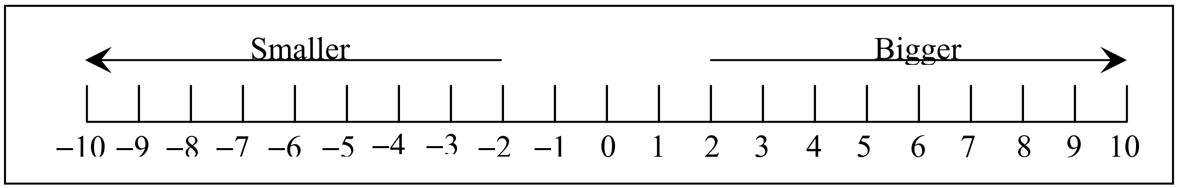
A negative number is less than zero



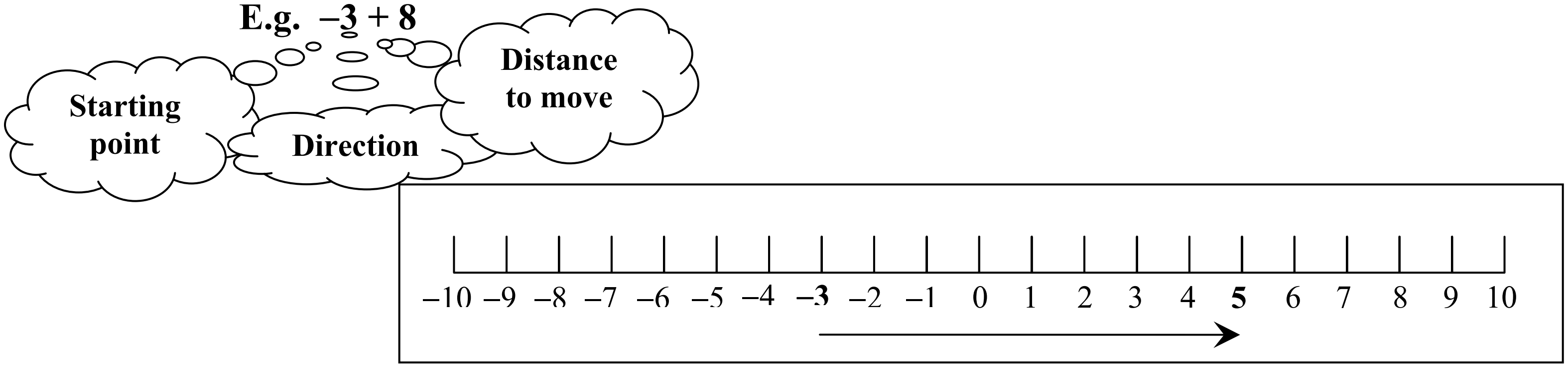
INTEGERS (2)

Some pupils have problems understanding the size of negative numbers and believe that –10 (read as '*negative 10*') is bigger than –5.

A number line can be of help when ordering numbers. Moving to the right, numbers getting bigger. Moving to the left, numbers getting smaller.



E.g. –3 + 8



Care should be taken with the use of language. '**Negative**' is used as an adjective to describe the sign of the number. '**Minus**' is used as a verb, indicating the operation of subtraction.

INTEGERS

So, + + or – – is the same as + and means a move to the right
and + – or – + is the same as – and means a move to the left.

e.g. (*i*) $-5 + (-6) = -5 - 6 = -11$

(*ii*) $6 - (-3) = 6 + 3 = 9$

Read as: '6 minus negative 3' or '6 subtract negative 3'

When multiplying positive and negative numbers, if both numbers are positive or both numbers are negative, the answer will be positive. If one of the numbers is positive and the other is negative, the answer will be negative.

(*i*) $(-3) \times (-2) = 9$ (*ii*) $(-8) \div 4 = -2$

Fraction of a Quantity

Unitary fraction of a quantity

$$\frac{1}{6} \text{ of } £18$$
$$= £18 \div 6$$

Simple fractions of a quantity

$$\frac{2}{3} \text{ of } 24$$
$$= 24 \div 3 \times 2$$
$$= 16$$

Simple fractions of a quantity

$$24 \times \frac{2}{3}$$
$$= 24 \div 3 \times 2$$
$$= 16$$

For a lower ability group

$$\frac{1}{3} \text{ of } 24$$
$$= 24 \div 3$$
$$= 8$$

then

$$\frac{2}{3} \text{ of } 24$$
$$= 8 \times 2$$
$$= 16$$

- **dividing by ½ is the same as multiplying by 2**
- **Numerator means <u>top</u> number**
- **Denominator means <u>bottom</u> number**

Rules of Fractions

Addition

- Find lowest common denominator
- Change tops (numerator)
- Add tops only

Example

$$2\frac{3}{4}+\frac{2}{3}$$
$$=\frac{11}{4}+\frac{2}{3}$$
$$=\frac{33}{12}+\frac{8}{12}$$
$$=\frac{41}{12}=3\frac{7}{12}$$

$$2\frac{3}{4}=\frac{2\times 4+3}{4}$$

This is known as converting from a mixed number to an improper fraction (top heavy fraction)

Subtraction

- Find lowest common denominator
- Change tops (numerator)
- Subtract tops only

Example

$$1\frac{7}{8}-\frac{5}{6}$$
$$=\frac{15}{8}-\frac{5}{6}$$
$$=\frac{45}{24}-\frac{20}{24}$$
$$=\frac{25}{24}=1\frac{1}{24}$$

Multiply

- Simplify where possible
- Multiply top with top
- Multiply bottom with bottom

Examples

$$\frac{3}{4}\times\frac{5}{6}$$
$$=\frac{1}{4}\times\frac{5}{2}$$
$$=\frac{5}{8}$$

$$5\times 3\frac{2}{3}$$
$$=15\frac{10}{3}$$
$$=18\frac{1}{3}$$

Divide

- Flip second
- Change to multiply

Example

$$\frac{8}{9}\div\frac{4}{3}$$
$$=\frac{8}{9}\times\frac{3}{4}$$
$$=\frac{2}{3}\times\frac{1}{1}$$
$$=\frac{2}{3}$$

Percentages
(**with** a calculator)

Pupils will be shown to set out examples in the following way:

$$\begin{aligned} & 27\% \text{ of } £469 \\ &= \frac{27}{100} \times £469 \\ &= 27 \div 100 \times £469 \\ &= 0.27 \times £469 \\ &= £126.63 \end{aligned}$$

The more able pupils are encouraged to convert the percentage to a decimal as the first line of working, for example

$$\begin{aligned} & 27\% \text{ of } £469 \\ &= 0.27 \times £469 \\ &= £126.63 \end{aligned}$$

- **percent means 'out of 100' and of means multiply**
- **Pupils benefit from showing working vertically with one '=' sign per line**
- **Pupils should be discouraged from using the % button on a calculator**

Percentages
(**without** a calculator)

Pupils will be shown to calculate amount using the following

1%	$\frac{1}{100}$
10%	$\frac{1}{10}$
20%	$\frac{1}{5}$
25%	$\frac{1}{4}$
50%	$\frac{1}{2}$
75%	$\frac{3}{4}$

Table 1

$33\frac{1}{3}\%$	$\frac{1}{3}$
$66\frac{2}{3}\%$	$\frac{2}{3}$

Table 2

5%	10% ÷ 2
2·5%	5% ÷ 2

Table 3

Lower ability pupils experience more success when they associate percentages with fractions of a pound in pence eg. 20p is 1/5 of a pound, there are 5 20 pences in £1.

Most percentages will be found using multiples of 10% and/or 1%
Examples

A. Find 11% of 560

10% is 56
1% is 5· 6
11% is 61·6

B. Find 17·5% of £220

10% is 22
5% is 11
2·5% is 5·5
17·5% is 38·5

C. Find 30% of 1800

10% is 180
30% is 180 × 3 = 540

- **It is useful to reinforce that 10% can be found by dividing by 10**
- **Many pupils try to divide by 1 to find 1%**

Ratio and Proportion (1)

Ratios are used to compare different quantities.

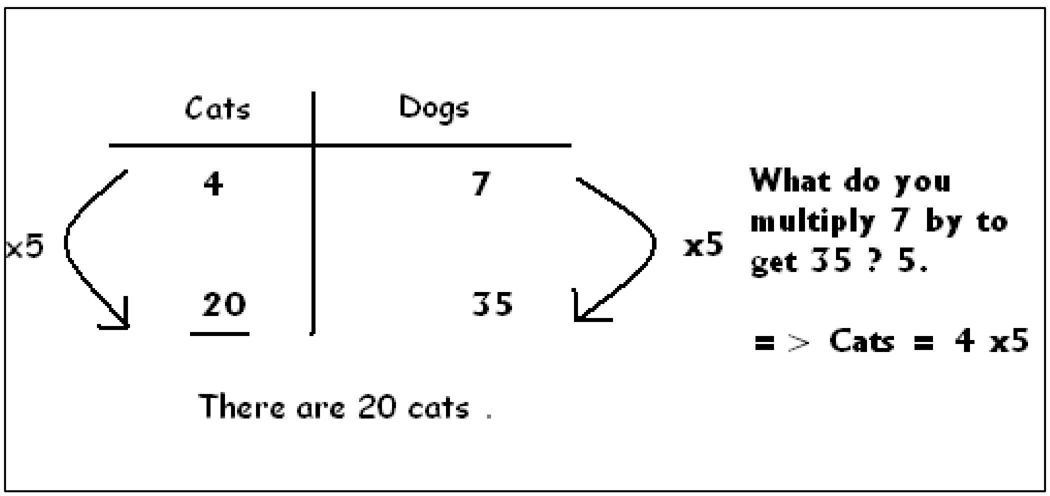
Pupils will be shown to simplify ratios like fractions.

Example simplify the ratio 10:25
What is the highest number that can be divided into 10 and 25?
It is 5 so divide 10 and 25 by 5.
= **2:5**

A ratio in which one of its values is '1', is called a unitary ratio e.g. 1:12

Ratio Calculations should be set out in a table and pupils should put their working at the side.

Example The ratio of cats to dogs in an animal shelter is 4:7. If there are 35 dogs in the shelter how many cats are there?



Ratio and Proportion (2)

If sharing money in given ratios pupils must:

- **Calculate** the number of shares by adding the parts of the ratio together.
- **Divide** the given quantity by the number of shares to find the value of one share.
- **Multiply** each ratio by the value of one share to find how the money has been split.

Example £35 is split between Jack and Jill in the ratio 3:2. How much does Jack receive and how much does Jill receive?

Solution	Number of shares	= 2 + 3	= 5
	value of 1 share	= £35 ÷ 5	= £7
	Jack's share	= 2 x £7	= £14
	Jill's share	= 3 x £7	= £ 21.

(Check by adding the value of the shares £14 + £21 = £35)

Ratio and Proportion (3)

Direct proportion

Pupils will be shown to

- **Set** working out in a table with clear headings.
- **Divide** by the given amount to find the unitary value.
- **Multiply** to find the required amount.
- **Write** final answer under the table.

Example 6 copies of a textbook cost £69. Find the cost of 4 textbooks.

Textbooks		Cost	
6	→	£69	(given in the question)
1	→	£69 ÷ 6 = £11.50	(find out the cost of 1 textbook)
4	→	£11.50 x 4 = £46.00	(multiply the cost of 1 by the number of books you are asked to find the cost of)

The cost of 4 textbooks = £46.00

Ratio and Proportion (4)

Inverse Proportion

With **direct proportion** as one quantity increases so does the other.
With **inverse proportion** as one quantity increases the other decreases.

Pupils will be shown to

- **Set** out working in a table with clear headings.
- **Multiply** to find out how long/much the unitary value will be.
- **Divide** to find the required amount
- **Write** the final answer underneath the table.

Example

If it takes 5 men 12 hours to paint a fence, how long would it take 6 men?

Solution

<u>Men</u>		<u>Hours</u>	
5	→	12	(given in the question)
1	→	5 x 12 = 60	(find out how long it would take 1 man)
6	→	60 ÷ 6 = 10	(divide to find out how long it would take 6 men).

It takes 6 men 10 hours to paint the fence.

Time – 12hour – 24hour

Pupils should be aware that using am and pm is very important when stating time in 12 hour notation.
When time is written in 24 hour notation am and pm should not be used. All times written in 24 hour notation should have 4 digits.

Converting 12 hr → 24 hr

e.g. 8.00am → 0800 hrs
10.47am → 1047 hrs
5.15pm → 1715 hrs (5 hours and 15 minutes after 1200, 12 noon)

Converting 24 hr → 12 hr

e.g. 0240 → 2.40am
2345 → 11.45pm

Time Intervals

Pupils will be shown how to work out time intervals by "counting on".

e.g. Calculate how long it is from 2.25pm to 7.25pm

2.25pm $\xrightarrow{\text{5 hours}}$ 7.25pm

e.g. Calculate how long it is from 8.40am to 3.23pm

8.40am $\xrightarrow{\text{20 mins}}$ 9.00am $\xrightarrow{\text{6 hours}}$ 3.00pm $\xrightarrow{\text{23 mins}}$ 3.23pm.

Total: 6 hours 43 mins

e.g. Calculate how long it is from 2250 to 0210

2250 $\xrightarrow{\text{10 mins}}$ 2300 $\xrightarrow{\text{3 hours}}$ 0200 $\xrightarrow{\text{10 mins}}$ 0210

Time – Converting units

Converting hours and mins ↔ decimals

Pupils should already know:

- 30 mins = $\frac{1}{2}$ hour = 0•5 hrs
- 15 mins = $\frac{1}{4}$ hour = 0•25 hrs
- 45 mins = $\frac{3}{4}$ hour = 0•75 hrs

Hours, mins → decimals

Rule

converting minutes to decimal hours → divide by 60

e.g. 24 minutes = 24/60 of an hour = 0•4 hr

3 hours 13 mins
= [3 + (13/60)]hrs
= [3 + 0•22]hrs
= 3•22hrs **(to 2 d.p.)**

Decimals → hours, mins

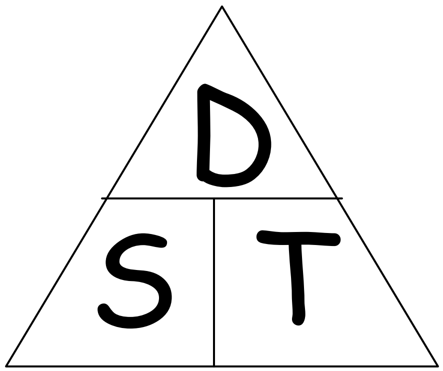
Rule

converting decimals hours to minutes → multiply by 60

e.g. 0•15 hr = (0•15 x 60) mins = 9 minutes.

3•4 hr
= 3 + (0•4 x 60) mins
= 3hrs + 24 mins
= 3 hrs 24 mins.

Time, Distance, Speed



$$\text{Average Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\text{Distance} = \text{Average Speed} \times \text{Time}$$

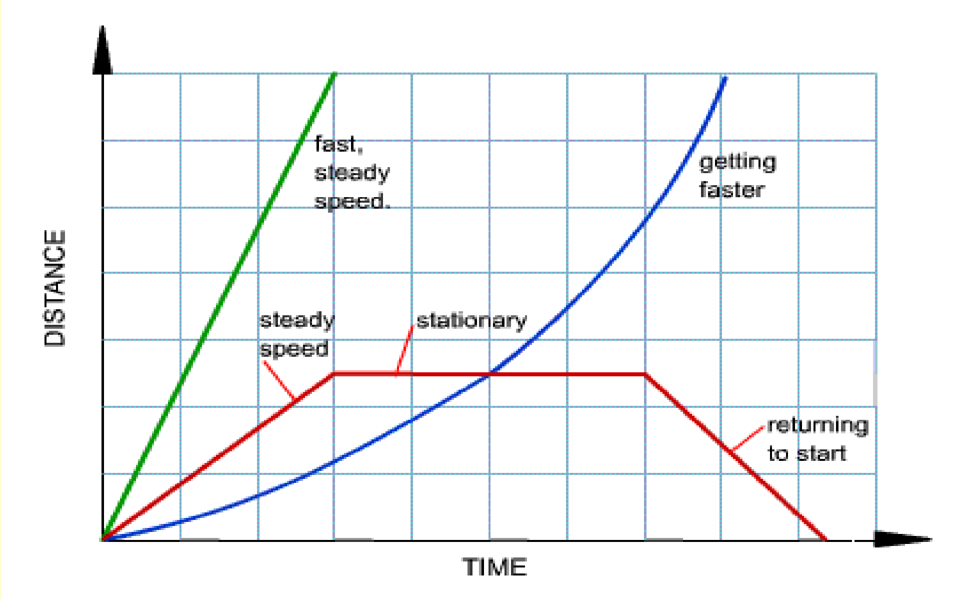
$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

Please NOTE – in Physics, pupils will use V instead of S for average speed. V is velocity which is speed and direction

Distance-Time Graphs

Interpreting graphs

Distance-time graph



A distance-time graph tells us how far an object has moved with time.

- The steeper the graph, the faster the motion.
- A horizontal line means the object is not changing its position - it is stationary.
- A downward sloping line means the object is returning to the start.

Algebra – Collecting Like Terms

The examples below are ***expressions*** not equations.

Have the pupils rewrite expressions with the like terms gathered together as in the second line of examples 2, 3 & 4 below, before they get to their final answer. The operator (+, –) and the term (7*x*) stay together at all times. It does not matter where the operator and term (–7*x*) are moved within the expression. (see example 3).

Example 1

Simplify

The convention in Maths is to write the answer with the letters first, in alphabetical

$$x + 2x + 5x$$
$$= 8x$$

Example 2

Simplify

$$3a + 2 + 6 + 7a$$
$$= 3a + 7a + 2 + 6$$
$$= 10a + 8$$

This is still correct although the first answer is preferable.

Example 3

Simplify

$$3 + 5x + 4 - 7x$$
$$= 5x - 7x + 3 + 4$$
$$= -2x + 7$$

or

$$3 + 5x + 4 - 7x$$
$$= 3 + 4 + 5x - 7x$$
$$= 7 - 2x$$

Example 4

Simplify

$$5m + 3n - 2m - n$$
$$= 5m - 2m + 3n - n$$
$$= 3m + 2n$$

or

$$5m + 3n - 2m - n$$
$$= 3n - n + 5m - 3n$$
$$= 2n + 3m$$

Algebra – Evaluating Expressions

If $x = 2$, $y = 3$ and $z = -4$

Find the value of: (a) $5x - 2y$ (b) $x + y - 2z$ (c) $2(x + z) - y$ (d) $x^2 + y^2 + z^2$

$$a)\ 5x - 2y$$
$$= 5 \times 2 - 2 \times 3$$
$$= 10 - 6$$
$$= 4$$

This line where the substitution takes place must be shown.
Marks are awarded in examinations for demonstrating this step.

$$b)\ x + y - 2z$$
$$= 2 + 3 - 2 \times (-4)$$
$$= 5 - (-8)$$
$$= 13$$

$$c)\ 2(x + z) - y$$
$$= 2(2 + (-4)) - 3$$
$$= 2 \times (-2) - 3$$
$$= -4 - 3$$
$$= -7$$

There is a bracket around the −4 as mathematicians do not write two operators side by side.

$$d)\ x^2 + y^2 + z^2$$
$$= 2^2 + 3^2 + (-4)^2$$
$$= 4 + 9 + 16$$
$$= 29$$

Multiplying Algebraic Expressions Involving Brackets

Example 1. Expand 5(x + 2)

5(x + 2)

= 5x + 10

Example 2. Multiply out 4(2p – 7)

4(2p – 7)

= 8p – 28

Example 3. Expand x(3x + 6)

x(3x + 6)

$= 3x^2 + 6x$

Example 4. Multiply out a(2a – 3m)

a(2a – 3m)

$= 2a^2 - 3am$

Multiplying Algebraic Expressions Involving Double Brackets

Example 1. Multiply $(x-3)(x-4)$

$= x^2 - 3x - 4x + 12$

$= x^2 - 7x + 12$

Example 2. Multiply $(2x+2)^2$

$(2x+2)^2 = (2x+2)(2x+2)$

$= 4x^2 + 4x + 4x + 4$

$= 4x^2 + 8x + 4$

Factorising Algebraic Expressions

The Common Factor

After 'multiplying out' brackets, we now turn to 'putting into' brackets. This process is called **factorising**

For example: $\boxed{2(x + 3) = 2x + 6}$ so, in reverse $\boxed{2x + 6 = 2(x + 3)}$

2 is the highest factor of $2x$ and 6, so 2 goes outside the bracket.

Also $\boxed{2a(a - 4) = 2a^2 - 8a}$ so, in reverse $\boxed{2a^2 - 8a = 2a(a - 4)}$

$2a$ is the highest factor of $2a^2$ and $8a$, so $2a$ goes outside the bracket.

$a - 4$ is then required inside the bracket.

Answers should always be checked by multiplying out the factorised answer.

Example 1. **Factorise $9x + 15$**

'What is the highest number to go into $9x$ and 15?' **3**
'Are there any letters common to $9x$ and 15?' **No**
So only 3 comes before a bracket. **3( )**

'What is required in the bracket so that the $9x$ can be found?' **$3x$**
$3(3x + \ )$

'What is required in the bracket so that the 15 can be found?' **5**
$3(3x + 5)$

$9x + 15$
$=$ **$3(3x + 5)$**
Check by multiplying out

Example 2. Factorise 18w2 – 12w

'What is the highest number to go into 18w2 and 12w?' **6** (not 3)
'Are there any letters common to 18w2 and 12w?' **Yes w**
So 6w comes before a bracket. **6w ( )**

'What is required in the bracket so that the 18w2 can be found?'**3w**
6w(3w –)
'What is required in the bracket so that the 12w can be found?' **2**
6w(3w – 2)

18w2 – 12w
= 6w(3w – 2)

Factorising Algebraic Expressions

Difference of Two Squares $(a^2 - b^2)$

Example 1. Factorise $w^2 - 9$

$$w^2 - 9$$
$$= \underline{(w+3)(w-3)}$$

Example 2. Factorise $x^2 - 4$

$$x^2 - 4$$
$$= \underline{(x+2)(x-2)}$$

Example 3. Factorise $b^2 - 25$

$$b^2 - 25$$
$$= (b+5)(b-5)$$

Example 4. Factorise $4x^2 - 25$

$$4x^2 - 25$$
$$= \underline{(2x+5)(2x-5)}$$

Example 5. Factorise $9y^2 - 4z^2$

$$9y^2 - 4z^2$$
$$= \underline{(3y+2z)(3y-2z)}$$

Example 6. Factorise $2x^2 - 32$

$$2x^2 - 32$$
$$= 2(x^2 - 16)$$
$$= 2(x+4)(x-4)$$

Trinomial Expressions (Quadratic Expressions)

Factorising ax2 + bx + c

Worked Example Factorise $x^2 + 5x + 6$

Draw up a table.

x	1	6	2	3
x	6	1	3	2

The x's are for the $x \times x = x^2$ term.
The numbers on the r.h.s. of the table are factors of the constant 6
(read vertically 1x6 6x1 2x3 3x2)

The middle term, the x term, has not been mentioned yet!
Find the combination of the vertical pairs of numbers that add together to give the value (coefficient) in front of the middle x.

In this case, 2+3 = 5 so the trinomial can be factorised as

$$(x+2)(x+5)$$

Example 1. Factorise $x^2 + 4x + 3$

x	3
x	1

$x^2 + 4x + 3$
$= \underline{(x+3)(x+1)}$

Example 2. Factorise $x^2 - 4x + 3$

x	-3
x	-1

$x^2 - 4x + 3$
$= \underline{(x-3)(x-4)}$

*These answers can be checked by multiplying out the brackets using the **box method**.*

Example 3. Factorise $x^2 - 4x - 5$

x	5	-5
x	-1	1

$x^2 - 4x - 5$
$= \underline{(x-5)(x+1)}$

Example 4. Factorise $x^2 + x - 12$

x	12	1	6	2	4
x	-1	-12	-2	-6	-3

$x^2 + x - 12$
$= \underline{(x+4)(x-3)}$

Trinomial Expressions (Quadratic Expressions): TYPE 2

Factorising ax2 + bx + c, a > 1

e.g. $\mathbf{3x^2 + 11x + 6}$,

$(3x + 2)(x + 3) = \mathbf{3x^2 + 11x + 6}$ so, in reverse $3x^2 + 11x + 6 = \mathbf{(3x + 2)(x + 3)}$

Example 1. Factorise $3x^2 + 11x + 6$

$$\begin{array}{c|cccc} 3x & 6 & 1 & 2 & 3 \\ \hline x & 1 & 6 & 3 & 2 \end{array}$$

Try $\begin{array}{c|c} 3x & 6 \\ \hline x & 1 \end{array}$

$3x + 6x = 9x$ ***NO***

Try $\begin{array}{c|c} 3x & 1 \\ \hline x & 6 \end{array}$

$18x + x = 19x$ ***NO***

Try $\begin{array}{c|c} 3x & 2 \\ \hline x & 3 \end{array}$

$9x + 2x = \mathbf{11x}$ ***YES***

$$3x^2 + 11x + 6$$
$$= (3x + 2)(x + 3)$$

Example 2. Factorise $6x^2 + 4x - 16$

Notice here that a common factor can be taken out FIRST!

$$6x^2 + 4x - 16$$
$$= 2(3x^2 + 2x - 8)$$

You can now factorise the trinomial in the bracket.

$$\begin{array}{c|cccc} 3x & -8 & -4 & -2 & -1 \\ \hline x & 1 & 2 & 4 & 8 \end{array}$$

From the table we can use:

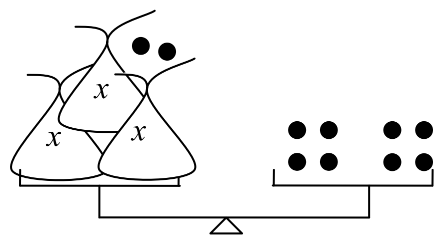
$$\begin{array}{c|c} 3x & -4 \\ \hline x & 2 \end{array}$$

$6x - 4x = 2x$ **YES**

$$6x^2 + 4x - 16$$
$$= 2(3x - 4)(x + 2)$$

Algebra – Solve Simple Equations

The method used for solving equations is balancing. Each equation should be set out with a line down the right hand side where the method is written, as in the examples below. It is useful to use scales like the ones below to introduce this method as pupils can visibly see how the equation can be solved.



This represents the equation
$3x + 2 = 8$
See example 4 below

Example 1: Solve $x + 5 = 8$

$$\begin{array}{r|l} x + 5 = 8 & -5 \text{ from both sides} \\ \underline{\underline{x = 3}} & \end{array}$$

In the example shown pupils must state that they will "subtract 5 from both sides." If they only say, "Subtract five," ask them, "Where from?" and encourage them to tell you, "Both sides," on every occasion.

Pupils should be encouraged to check their answer mentally by substituting it back into the original equation.

Example 2: Solve $y - 3 = 6$

$$y - 3 = 6 \quad \Big| \; + 3 \text{ to both sides}$$
$$\underline{\underline{y = 9}}$$

Example 3: Solve $4m = 20$

$$4m = 20 \quad \Big| \; \div \text{ by 4 on both sides}$$
$$\underline{\underline{m = 5}}$$

Example 4: Solve $3x + 2 = 8$

$$3x + 2 = 8 \quad \Big| \; - 2 \text{ from both sides}$$
$$3x = 6 \quad \Big| \; \div \text{ by 3 on both sides}$$
$$\underline{\underline{x = 2}}$$

Underline answer

Example 5: Solve $10-2x = 4$

$10 - 2x = 4$	$+ 2x$ to both sides
$10 = 4 + 2x$	$- 4$ from both sides
$6 = 2x$	$\div$ by 2 on both sides
$3 = x$	
$\underline{\underline{x = 3}}$	

Write answer in the form $x =$

Example 6: Solve $3x + 2 = x + 14$

$3x + 2 = x + 14$	$- x$ from both sides
$2x + 2 = 12$	$- 2$ from both sides
$2x = 12$	$\div$ by 2 on both sides
$\underline{\underline{x = 6}}$	

NB Secondary:
Always deal with the variable before the constants, ensuring that the variable is written with a positive coefficient. This avoids errors when dividing by negatives and also avoids learning rules for dealing with inequations.

Algebra – Solve Inequations

Terminology

$>$	Greater than
$<$	Less than
$\geq$	Greater than or equal to
$\leq$	Less than or equal to

Example 1:

Solve the inequation $x + 3 > 6$ choosing solutions from $\{0, 1, 2, 3, 4, 5, 6\}$

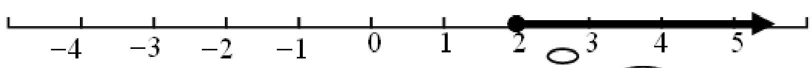
$x + 3 > 6$ | -3 from both sides
$x > 3$

$x = \{4, 5, 6\}$

Example 2

Solve $x + 5 \geq 7$

$$\begin{array}{l|l} x + 5 \geq 7 & -5 \text{ from both sides} \\ \underline{\underline{x \geq 2}} & \end{array}$$

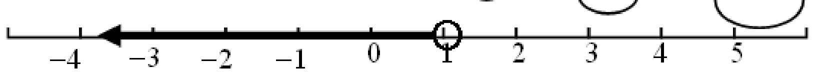


Note that if the value is included in the solution, i.e. 2 here. We represent this with a filled dot.

Example 3

Solve $x + 3 < 4$

$$\begin{array}{l|l} x + 3 < 4 & -3 \text{ from both sides} \\ \underline{\underline{x < 1}} & \end{array}$$



Note that if the value is not included in the solution, i.e. 1 here. We represent this with an open dot.

Use and devise simple rules

Pupils need to be able to use notation to describe general relationships between 2 sets of numbers, and then use and devise simple rules. Pupils need to be able to deal with numbers set out in a table horizontally, set out in a table vertically or given as a sequence. A method should be followed, rather than using “trial and error” to establish the rule.

Example 1: Complete the following table, finding the n^{th} term.

> In this example, the output values are still increasing, however, addition or multiplication on their own do not work, so this must be a two-step operation.

Input	1	2	3	4	5		n
Output	5	7	9	11	13		?

Look at the outputs. These are going up by 2 each time. This tells us that we are multiplying by 2. (This means × 2.)
Now ask:
1 multiplied by 2 is 2, how do we get to 5? Add 3.
2 multiplied by 2 is 4, how do we get to 7? Add 3.
This works, so the rule is:

Multiply by 2 then add 3.

Check using the input 5: $5 \times 2 + 3 = 13$

We use n to stand for any number
So the n^{th} term would be $\boldsymbol{n \times 2 + 3}$ which is rewritten as

$\boldsymbol{2n + 3}$

Example 2: Find the 20th term.

Input	Output	
1	7	+3
2	10	+3
3	13	
4	16	
5	19	
6	22	
n	$3n + 4$	
20		

To find the 20th term it is best to find the n^{th} first.

Look at the output values. These are going up by 3 each time. This tells us that we are multiplying by 3. (This means $\times$ 3.)
Now ask:
1 multiplied by 3 is 3, how do we get to 7? Add 4.
2 multiplied by 3 is 6, how do we get to 10? Add 4.
This works so the rule is

Multiply by 3 then add 4.

Check using 6: $6 \times 3 + 4 = 22$

We use n to stand for any number
So the n^{th} term would be $\boldsymbol{n} \times \mathbf{3} + \mathbf{4}$ **which is rewritten as**

$\mathbf{3}\boldsymbol{n} + \mathbf{4}$

To get the 20th term we substitute $n = 20$ into our formula.

$3n + 4$
$=3\times20 + 4$
$=60 + 4$
$=64$

Example 3:

For the following sequence find the term that produces an output of 90.

Input	Output
1	2
2	10
3	18
4	26
5	34
6	42
n	$8n - 6$
	90

+8
+8

We go through the same process as before to find the n^{th} term, which is $\mathbf{8}\boldsymbol{n} - \mathbf{6}$.

Now we set up an equation.

$$\begin{array}{rl|l} 8n - 6 &= 90 & +6 \\ 8n &= 96 & \div \text{ by } 8 \\ n &= 12 & \end{array}$$

Therefore the 12$^{\text{th}}$ term produces an output of 90.

Using Formulae

Pupils meet formulae in '**Area**', '**Volume**', '**Circle**', '**Speed**, **Distance**, **Time**' etc. In all circumstances, working must be shown which clearly demonstrates strategy, (i.e. selecting the correct formula), substitution and evaluation.

Example :

Find the area of a triangle with base 8cm and height 5cm.

$$A = \tfrac{1}{2}bh$$

$$A = \tfrac{1}{2} \times 8 \times 5$$

Remember the importance of the substitution.

$$A = 20cm^2$$

Rearranging Formulae

Pupils are taught the balance method of transforming formulae which involves carrying out the same operation on both sides of the equation. The operation being completed is written to the right of the line of working.

e.g. (*i*) $V = IR$. Change the subject of the formula to R.

$$\begin{array}{rl|l} V &= IR & \\ \dfrac{V}{I} &= R & \div I \end{array}$$

(*ii*) $v^2 = u^2 + 2as$. Make *a* the subject of the formula.

$$\begin{array}{rl|l} v^2 &= u^2 + 2as & \\ v^2 - u^2 &= 2as & -u^2 \\ \dfrac{v^2 - u^2}{2s} &= a & \div 2s \end{array}$$

When using a formula, pupils may find it easier to substitute known values before carrying out the transformation.

e.g. The volume of a sphere is given by the formula $V = \frac{4}{3}\pi r^3$.

Find the radius of the sphere when the volume is 75 cm^3.

$$
\begin{array}{rl|l}
V = & \tfrac{4}{3}\pi r^3 & \\
75 = & \tfrac{4}{3}\pi r^3 & \\
225 = & 4\pi r^3 & \times 3 \\
\dfrac{225}{4\pi} = & r^3 & \div 4\pi \\
\sqrt[3]{\dfrac{225}{4\pi}} = & r & \sqrt[3]{\ } \\
\end{array}
$$

$$\underline{\underline{r \approx 2 \cdot 62 \text{ cm}}}$$

Length, Mass and Capacity

All pupils should be familiar with metric units of length, mass and capacity but some pupils will have difficulty with notation and converting from one unit to another.

They should know the following facts:

Length	**Mass**	**Capacity**
10 mm = 1 cm	**1 000 g = 1 kg**	**1 000 ml = 1 litre**
100 cm = 1 m	**1 000 kg = 1 tonne**	**1 cm^3 = 1 ml**
1 000 m = 1 km		

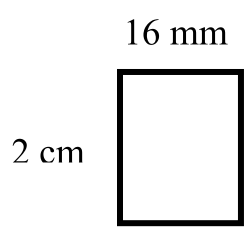
Pupils are encouraged to develop an awareness of the sizes of units and an ability to make estimates in everyday contexts. They are also expected to consider the appropriateness of their solution to a problem in the particular context of the question.

Reading Scales

Pupils of lower ability often have difficulty in reading scales on graphs. They are inclined to assume that one division on the scale represents one unit. It may help if they begin by counting the number of divisions between each number on the scale and then determine what each division of the scale represents.

Area and Volume

Problems can arise when calculating areas and volumes if the lengths given are not in the same units. It is usually easiest to begin by converting all of the units of length to the units that are required for the answer, before doing any calculation.



Either
Area = $16 \times 20 = 320 \text{ mm}^2$
or
Area = $1{\cdot}6 \times 2 = 3{\cdot}2 \text{ cm}^2$

When converting between units of length, area and volume, it is important to note the following facts.

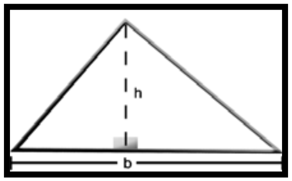
Length	Area	Volume
1km= 1,000 m **1m = 100 cm** **1cm=10 mm**	**1km^2 = 1 000 000 m^2** **1m^2 = 10 000 cm^2** **1cm^2 = 100 mm^2**	**1km^3=1 000 000 000 m^3** **1m^3 = 1 000 000 cm^3** **1cm^3 = 1 000 mm^3**

NOTE: km means kilometre, m is metre, cm is centimetre
m^2 is read as square metres NOT metres squared
m^3 is read as cubic metres NOT metres cubed

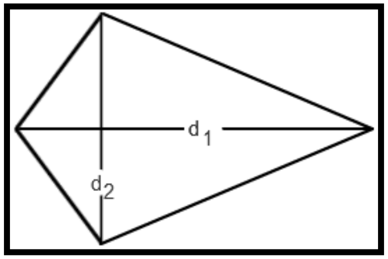
Area and Volume (2)

$\text{Area}_{rectangle}$ = length x breadth = L x B = LB

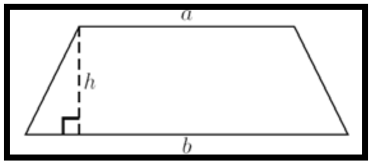
$\text{Area}_{triangle}$ = ½ base x height



$\text{Area}_{kite\ or\ rhombus}$ = $\frac{1}{2} \times d_1 \times d_2$

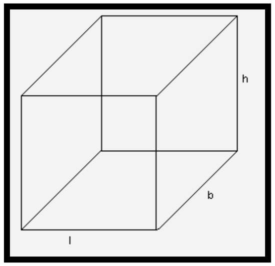


$\text{Area}_{trapezium}$ = $\frac{1}{2}(a+b)h$

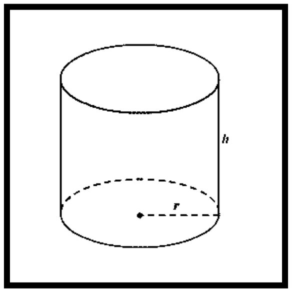


Volume_{cube} = $\text{length}^3 = L^3$

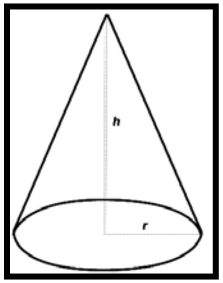
Volume_{cuboid} = length x breadth x height = LBH



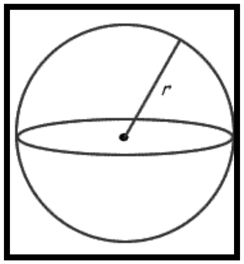
$\text{Volume}_{cylinder}$ = $\pi r^2 h$



Volume_{cone} = $\frac{1}{3}\pi r^2 h$



Volume_{sphere} = $\frac{4}{3}\pi r^3$

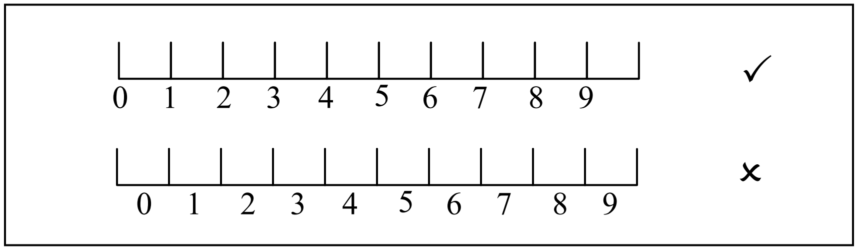


Volume_{prism} = Area of cross-section x height = Ah

Plotting Points and Drawing Graphs

When drawing a diagram on which points are to be plotted, some pupils will need to be reminded that numbers on the axes are written on the lines not in the spaces.

e.g.



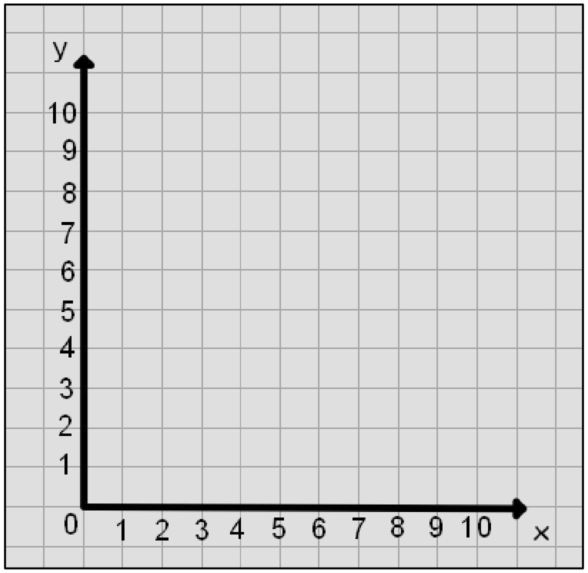
When drawing graphs for experimental data it is customary to use the horizontal axes for the variable which has a regular interval.

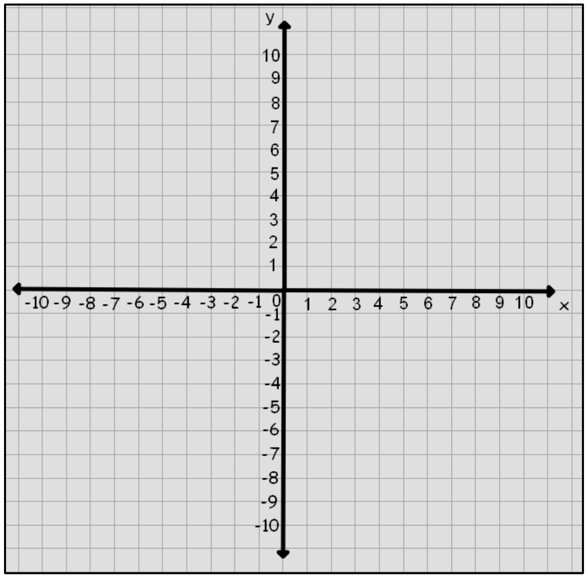
e.g. (*i*) In an experiment in which the temperature is taken every 5 minutes the horizontal axis would be used for time and the vertical axis for temperature.

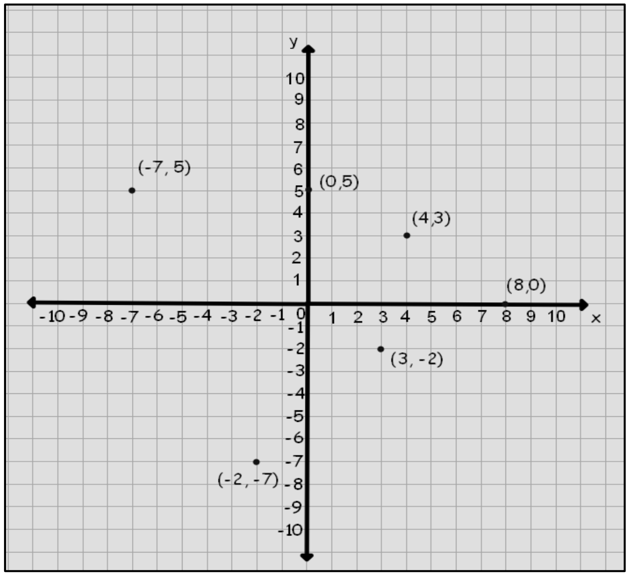
(*ii*) If the depth of a river is measured every metre, the horizontal axis would be used for the distance from the bank and the vertical axis for the depth.

Coordinates

- Practice drawing vertical and horizontal lines on the existing lines using a ruler and pencil
- Mark a dot at the centre of the paper (the origin)
- Draw a horizontal line (x-axis) and vertical line (y-axis) through the origin and extend in both directions to form 4 quadrants (-10 to 10)
- Label axes
- Write y-axes scale on left of axis
- Write x-axis scale below x-axis
- A coordinate is plotted by identifying position on the x-axis then the position on the y-axis (along the corridor, up the stairs OR horizontal then vertical OR right/left then up/down)
- A coordinate is stated in this form (x, y)







Information Handling

Terminology and Methodology

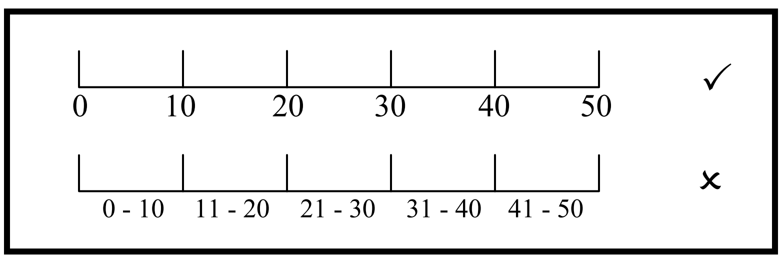
Discrete Data

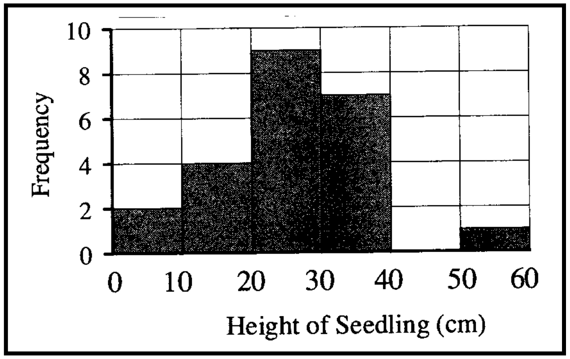
Discrete data can only have a finite or limited number of possible values.
Shoe sizes are an example of **discrete** data because sizes 39 and 40 mean something, but size 39·2, for example, does not.

Continuous Data

Continuous data can have an infinite number of possible values within a selected range
e.g. temperature, height, length.

Where data are continuous e.g. lengths, the horizontal scale should be like the scale used for a graph on which points are plotted.





Non-Numerical Data (Nominal Data)

Data which is non-numerical.
e.g. favourite TV programme, favourite flavour of crisps.

Tally Chart/Table (Frequency table)

A tally chart is used to collect and organise data prior to representing it in a graph.

Averages : Pupils should be aware that **mean**, **median** and **mode** are different types of average.

Mean: add up all the values and divide by the number of values.
Mode: is the value that occurs most often.
Median: is the middle value or the mean of the middle pair of an ordered set of values.

Pupils are introduced to the mean using the word average. In society average is commonly used to refer to the mean.

e.g. for the values: 3, 2, 5, 8, 4, 3, 6, 3, 2

- **Mean** $= \frac{3+2+5+8+4+3+6+3+2}{9} = \frac{36}{9} = 4$
- The **median** is 3 because 3 is in the middle when the values are put in order.

  2 , 2 , 3 , 3 , $\boxed{3}$, 4 , 5 , 6 , 8
- The **mode** is 3 because 3 is the value which occurs most often.

Non-Numerical Data (Nominal Data)

Range

The difference between the highest and lowest value.
In the example above the range = 8 – 2 = 6

Standard Deviation

The Standard Deviation indicates how much, on average, the data points differ from the mean. A set of data that has a small range will have a small standard deviation and vice versa.

Type of data	Best measure of central tendency
Nominal (can count but not order or measure) when the data is about personal opinions or results where the most common is the important conclusion to draw, rather an accurate exact "average"	**Mode**
Ordinal (can count, order but not measure) when the data has a bell shape, with the middle values being the most common	**Median**
Interval/ratio when the data are all as expected, without any outliers (not skewed)	**Mean**
Interval/ratio (skewed)	**Median**

Mean from a frequency table

In a survey, the number of books carried by each girl in a group of students was recorded.
The results are shown in the frequency table below.

Number of books	*Frequency*
0	1
1	2
2	3
3	5
4	5
5	6
6	2
7	1

Calculate the **mean number of books** carried by the girls in the group.
To calculate the mean, we add a column to the table and multiply together the numbers in the first two columns. We label these two columns x and "f". We label the third column "fx".
Then we add up the numbers in the second and third columns (Σx and Σf).
Then we divide Σfx by Σf to calculate the mean.

No. of books (x)	Frequency (f)	fx
0	1	0
1	2	2
2	3	6
3	5	15
4	5	20
5	6	30
6	2	12
7	1	7
	$\Sigma f = \mathbf{25}$	$\Sigma fx = \mathbf{93}$

Mean = $\frac{93}{25}$

Mean = 3.7 (to 1 dp)

Median from a frequency table

In a survey, the number of books carried by each girl in a group of students was recorded.
The results are shown in the frequency table below.

Number of books	*Frequency*
0	1
1	2
2	3
3	5
4	5
5	6
6	2
7	1

Calculate the **median number** of books carried by the girls in the group.
To calculate the median, we add a column to the table called "**cumulative frequency (cf)**" which is a running total of frequencies.

No. of books	Frequency (f)	**cf**
0	1	1
1	2	3
2	3	6
3	5	11
4	5	16
5	6	22
6	2	24
7	1	25

The median is the middle value when all the values are placed in order, as they are in the table.

Median = $\frac{n+1}{2}$ (where n is the number of values)

There are 25 girls in the group.

The median is the 13^{th} girl.

The 13^{th} value (girl) is between the 11^{th} and 16^{th} value in the cf column, therefore it falls in the highlighted row.

Therefore the median number of books is 4.

Standard Deviation

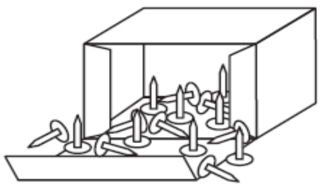
Standard Deviation

The Standard Deviation indicates how much, on average, the data points differ from the mean. A set of data that has a small range will have a small standard deviation and vice versa.

The two formulae for standard deviation are given in National 5 exam papers, they do not need to be memorised. Pupils should practice **using both formulae**.

A sample of six boxes contains the following numbers of pins per box.

43 39 41 40 39 44



(*a*) For the above data, calculate:

(i) the mean;

(ii) the standard deviation.

$$s = \sqrt{\frac{\sum (x-\bar{x})^2}{n-1}} = \sqrt{\frac{\sum x^2 - (\sum x)^2/n}{n-1}},$$ where n is the sample size.

Method 1 (formula 1)

(i) $\bar{x} = \frac{\sum x}{n} = \frac{246}{6} = 41$

(ii) Make a table with three columns

x	$x - \bar{x}$	$(x - \bar{x})^2$
39	-2	4
39	-2	4
40	-1	1
41	0	0
43	2	4
44	3	9

$\sum(x - \bar{x})^2 = 22$

$S = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}}$

$S = \sqrt{\frac{22}{5}}$

$S = \sqrt{4.4}$

$S = 2.1$ (to 1 dp)

Method 2 (formula 2)

(i) $\bar{x} = \frac{\sum x}{n} = \frac{246}{6} = 41$

(ii) Make a table with two columns

x	x^2
39	1521
39	1521
40	1600
41	1681
43	1849
44	1936

$\sum x = 246$ $\sum x^2 = 10108$

$S = \sqrt{\frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}}$

$S = \sqrt{\frac{10108 - \frac{(246)^2}{6}}{5}}$

$S = \sqrt{\frac{10108 - 10086}{5}}$

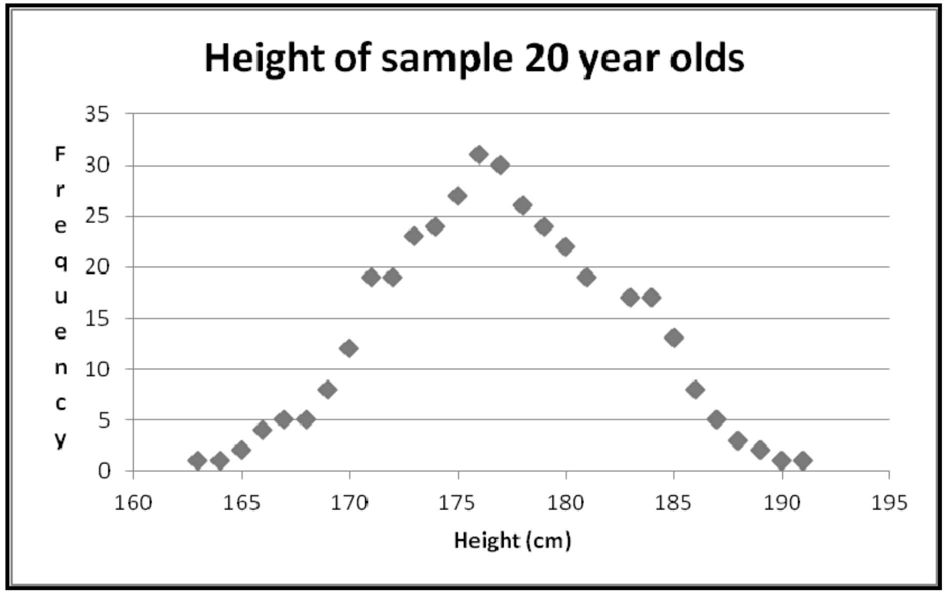
$S = \sqrt{4.4}$

$S = 2.1$ (to 1 dp)

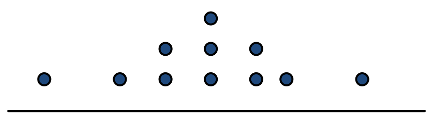
Normal Distribution

A probability distribution that plots all of its data values in a symmetrical fashion and most of the results are situated around the probability's mean. Sometimes known as a "**bell curve**".

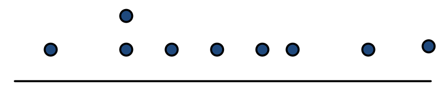
Examples:
Heights of people
Errors in measurements
blood pressure



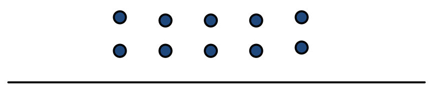
Describing the distribution of Data



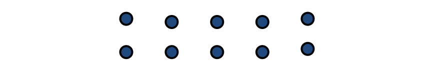
Symmetrical distribution



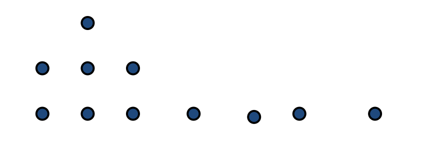
Widely spread distribution



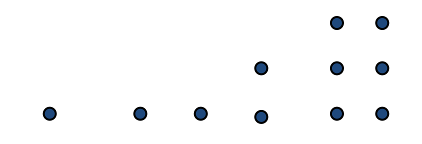
Uniform distribution



Tightly clustered distribution



Skewed to the right distribution

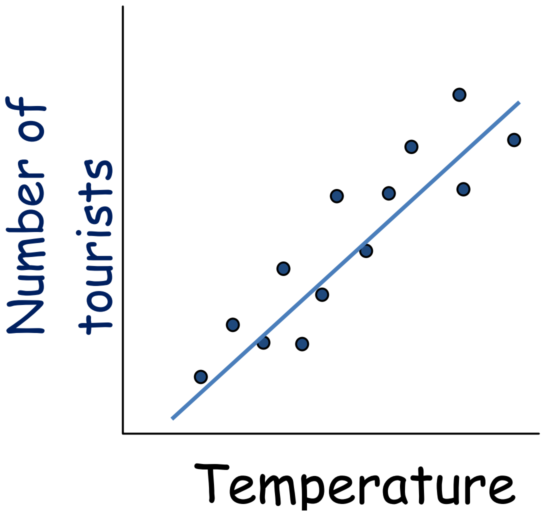


Skewed to the left distribution

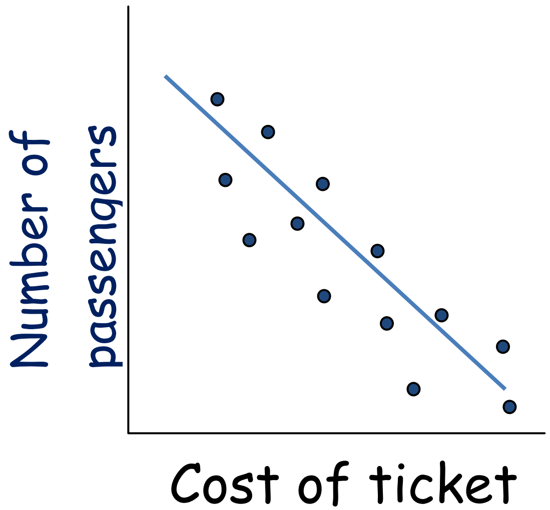
Scattergraphs

Scatter graphs are used to show whether there is a relationship between two sets of data. The relationship between the data can be described as:

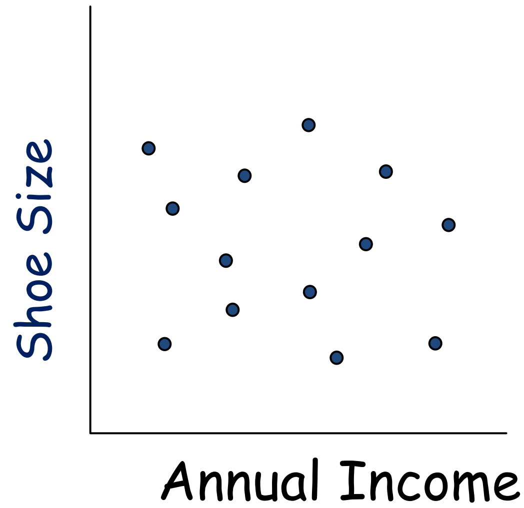
A **positive** correlation. As one quantity increases so does the other
A **negative** correlation. As one quantity increases the other decreases.
No correlation. Both quantities vary with no clear relationship.



Positive Correlation



Negative correlation

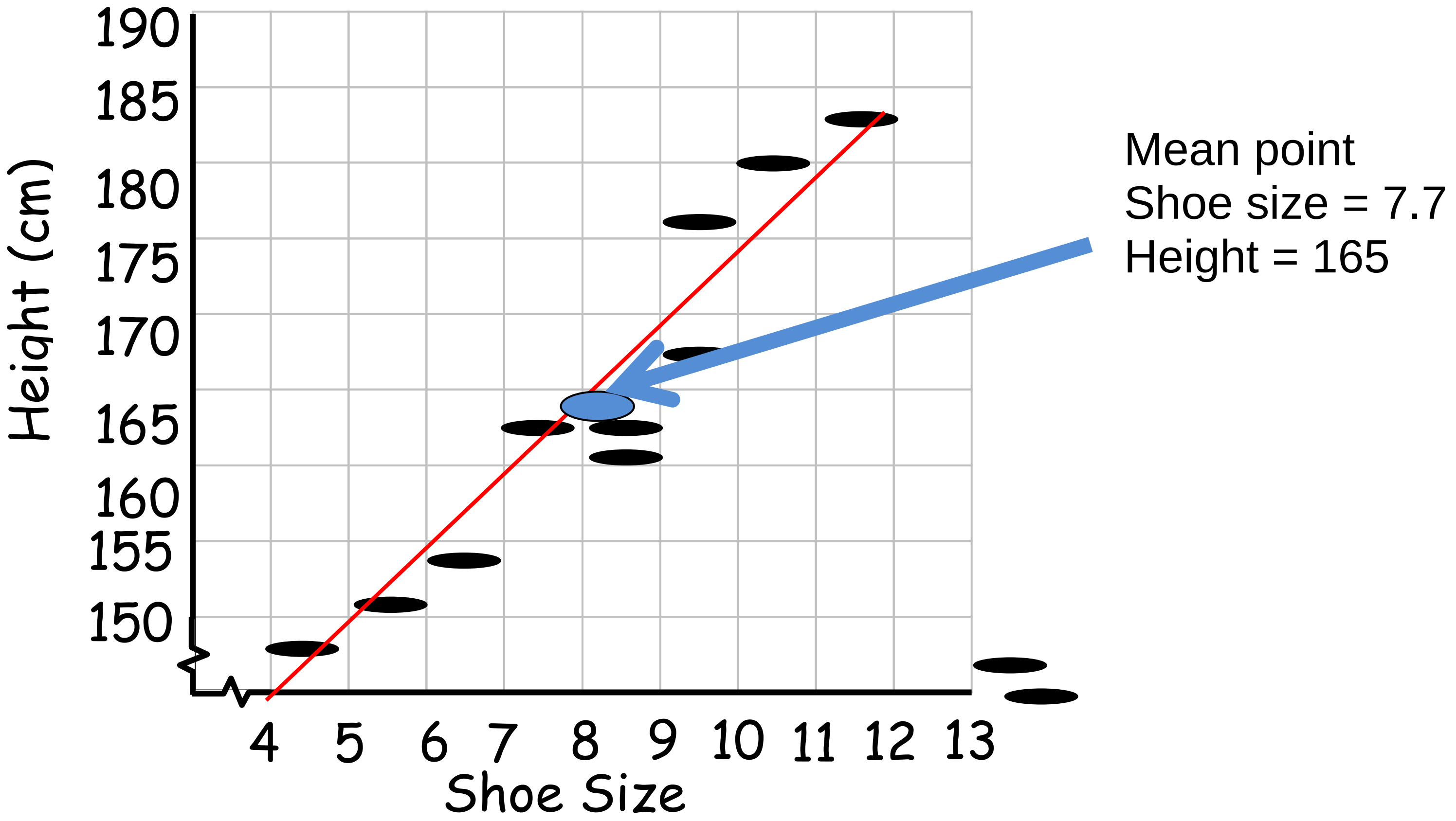


No correlation

Drawing a Line of Best Fit

A line of best fit can be drawn to data that shows a correlation. The stronger the correlation between the data, the easier it is to draw the line. The line should go through the mean point and should have approximately the same number of data points on either side.

Shoe Size	6	9	7	10	4	9	8	11	5	8
Height cm	155	177	159	180	150	168	162	183	152	164

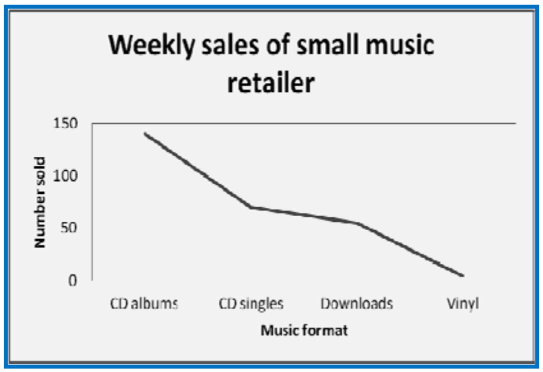


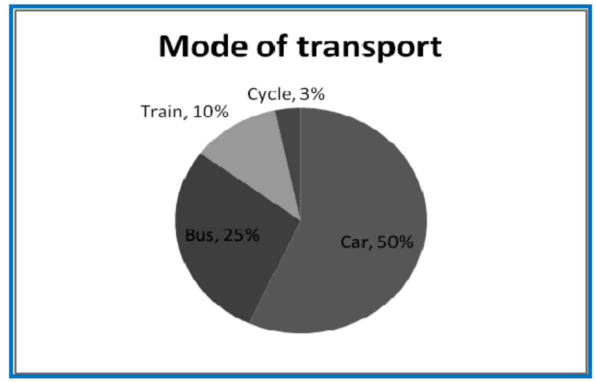
Graphs, charts and tables

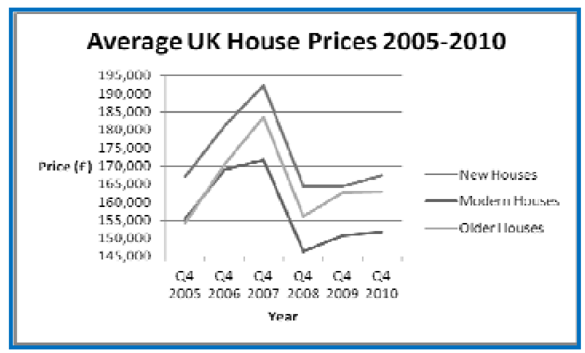
Good graphs, charts and tables are powerful tools for displaying large quantities of data and help turn information into knowledge.

Some graphs can be misleading if the designer chooses to give readers the impression of better or worse results than is actually the case or by selecting a poor choice of a graph or poor graph construction.

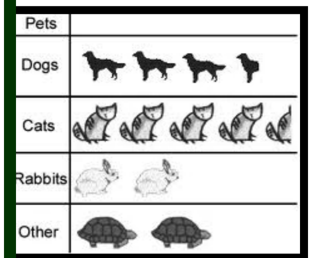
Why are these graphs misleading?

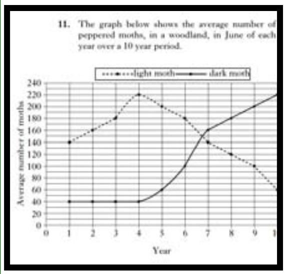


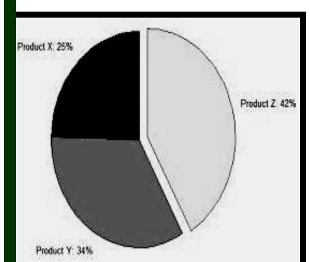


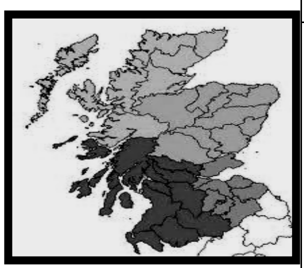


Graphs, Charts and Tables

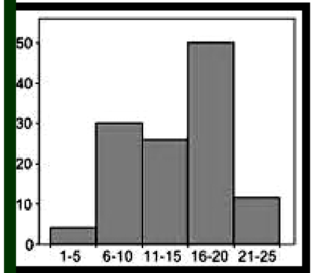
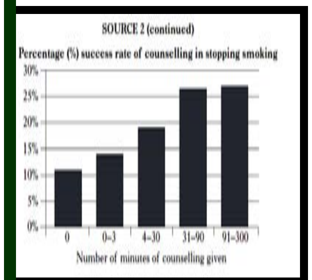
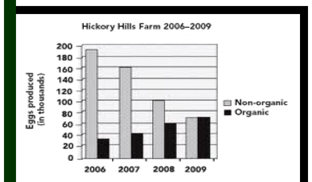
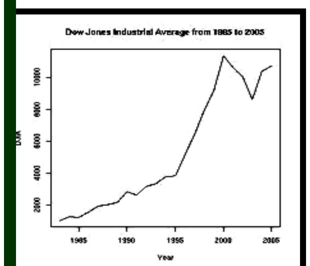
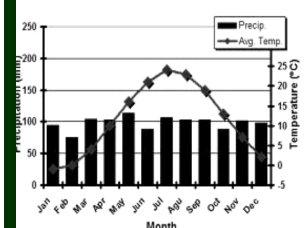


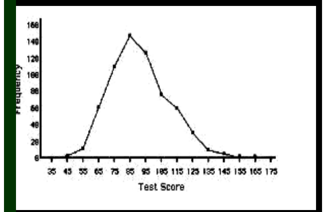
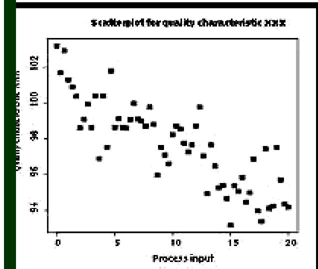
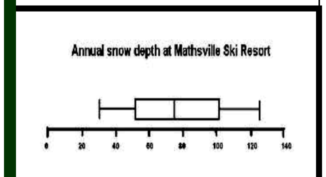






Graph	Advantages	Disadvantages
Pictograms A pictogram uses an icon to represent a quantity of data values in order to decrease the size of the graph. A key must be used to explain the icon.	Easy to read Visually appealing Handles large data sets easily	Hard to quantify partial icons Icons must be of consistent size Best for only 2-6 categories Very simplistic
Linegraph A line graph can be used as an initial record of discrete data values. The range determines a number line which is then plotted with X's for each data value.	Quick analysis of data Shows range, minimum & maximum, gaps & clusters, and outliers easily Exact values retained	Not as visually appealing Best for under 50 data values Needs small range of data
Piechart A piechart displays data as a percentage of the whole. Each pie section should have a label and percentage. A total data number should be included.	Visually appealing Shows percent of total for each category	No exact numerical data Hard to compare 2 data sets "Other" category can be a problem Total unknown unless specified Best for 3 to 7 categories Use only with discrete data
Map Chart A map chart displays data by shading sections of a map, and must include a key. A total data number should be included.	Good visual appeal Overall trends show well	Needs limited categories No exact numerical values Colour key can skew visual interpretation

	**Histogram** A histogram displays continuous data in ordered columns. Categories are of continuous measure such as time, inches, temperature, etc.	Visually strong Can compare to normal curve Usually vertical axis is a frequency count of items falling into each category	Cannot read exact values because data is grouped into categories More difficult to compare two data sets Use only with continuous data
	**Bar graph** A bar graph displays discrete data in separate columns. A double bar graph can be used to compare two data sets. Categories are considered unordered and can be rearranged alphabetically, by size, etc.	Visually strong Can easily compare two or three data sets	Graph categories can be reordered to emphasize certain effects Use only with discrete data
	**Compound/ Comparative Bar graph**	a bar graph that compares two or more quantities simultaneously	Graph categories can be reordered to emphasize certain effects Use only with discrete data
	**Line Graph** A line graph plots continuous data as points and then joins them with a line. Multiple data sets can be graphed together, but a key must be used.	Can compare multiple continuous data sets easily Interim data can be inferred from graph line	Use only with continuous data Can sometimes have two different vertical scales to represent the different quantities
	**Combination of Bar Graph and Line Graph**	Can illustrate continuous data sets and discrete data set on same graph Can demonstrate the correlation of two variables such as temperature and rainfall	

	**Frequency Polygon** A frequency polygon can be made from a line graph by shading in the area beneath the graph. It can be made from a histogram by joining midpoints of each column.	Visually appealing	Anchors at both ends may imply zero as data points Use only with continuous data
	**Scattergraph** A scattergraph displays the relationship between two factors of the experiment. A trend line is used to determine positive, negative, or no correlation.	Shows a trend in the data relationship Retains exact data values and sample size Shows minimum/maximum and outliers	Hard to visualize results in large data sets Flat trend line gives inconclusive results Data on both axes should be continuous
1 | 9 2 | 2 5 6 7 8 9 3 | 0 4 6 7 4 | 2 3 4 6 8 8 9 5 | 2 3 5 7 8 6 | 2	**Stem and Leaf Plot** Stem and leaf plots record data values in rows, and can easily be made into a histogram. Large data sets can be accommodated by splitting stems.	Concise representation of data Shows range, minimum & maximum, gaps & clusters, and outliers easily Can handle extremely large data sets	Not visually appealing Does not easily indicate measures of centrality for large data sets
	**Box plot** A boxplot is a concise graph showing the five point summary. Multiple boxplots can be drawn side by side to compare more than one data set.	Shows 5-point summary and outliers Easily compares two or more data sets Handles extremely large data sets easily	Not as visually appealing as other graphs Exact values not retained

Probability : Terminology and Methodology

This is the **chance** that a particular outcome will occur, measured as a ratio of the total possible outcomes.

It can be thought of as a simple fraction or a decimal fraction.

It always lies between **0** and **1**

0 meaning **impossible** (could not happen)

1 meaning **certain** (will definitely happen)

Probability of an event happening = $\dfrac{\text{number of } \textbf{favourable outcomes}}{\text{number of } \textbf{possible outcomes}}$

Calculating Probability

Example
A boy tossed a coin.

What is the probability that it is heads?

P (heads) = 1/2

Example
There are 3 red and 4 green balls in a bag.

What is the probability a green ball is picked?

P(green) = 4 / 3+4 = **4 / 7**

Speed Distance and Time

The study of the relationships between **speed**, **distance** and **time**.
There are three formulas that are used for calculating Speed, Distance and Time. We would use the substitution method again when taking on a problem in this field.

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} \qquad \text{Distance} = \text{Speed} \times \text{Time} \qquad \text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

Example

Find the average speed, when someone travels 20 miles in 4 hours.

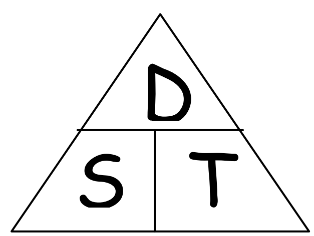
$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\text{Speed} = \frac{20}{4}$$

<u>Speed = 5 miles per hour</u>

It is important to remember to check the units being used in these calculations, mph, seconds, etc as this will change the difficulty of the question.

Sometimes **Speed, Distance** , **Time** can be represented by a formula triangle which is a way of remembering the three formulae



Trigonometry

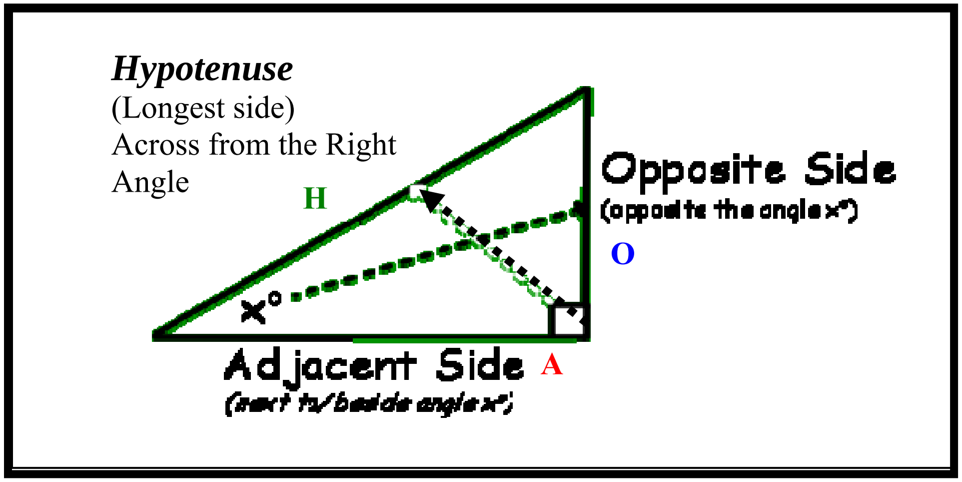
This branch of mathematics studies **triangles** and the <u>relationship between the measurement of its sides and the angles between those sides.</u>

There are three main formulas used in Right Angled triangle Trigonometry, and these are known as the Trig ratios.

$$\text{Sin } x^o = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

$$\text{Cos } x^o = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

$$\text{Tan } x^o = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

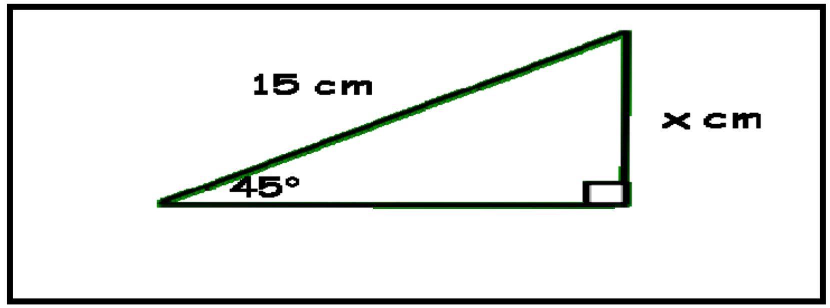


A common Aide Memoir for these ratios is ***SOH, CAH, TOA***, where each letter represents the first letter of each part of the ratio. (Highlighted above.)

Questions will be based upon finding the size of an angle or finding the length of a side. We would use the substitution and rearranging method as seen in the following examples.

Trigonometry

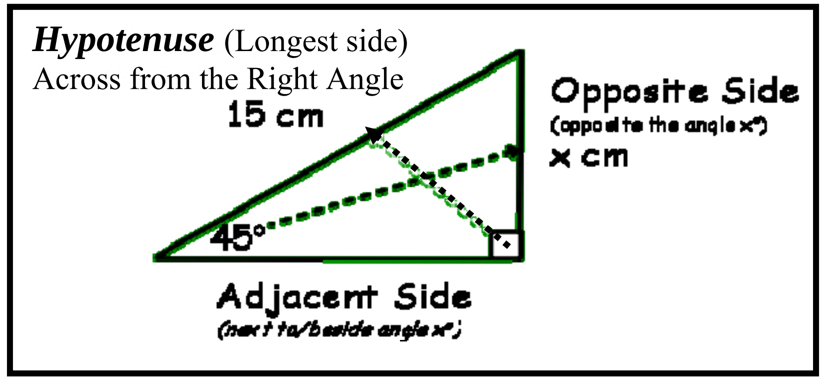
Worked Example 1: Finding the length of a side



Find the length of the missing side in this triangle.

To solve this type of problem there are a number of steps we must follow.

Step 1: Mark out the sides.



Step 2: Highlight the sides which have information.

Step 3 : Write down the 3 trig ratios, and identify which one is the correct one to be using.

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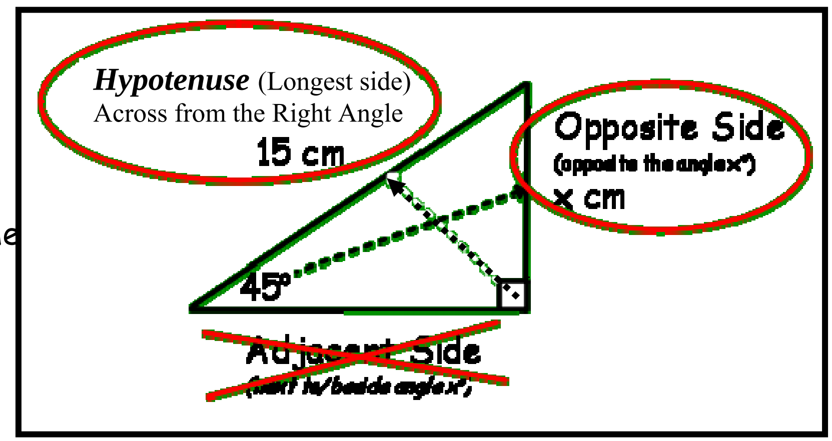
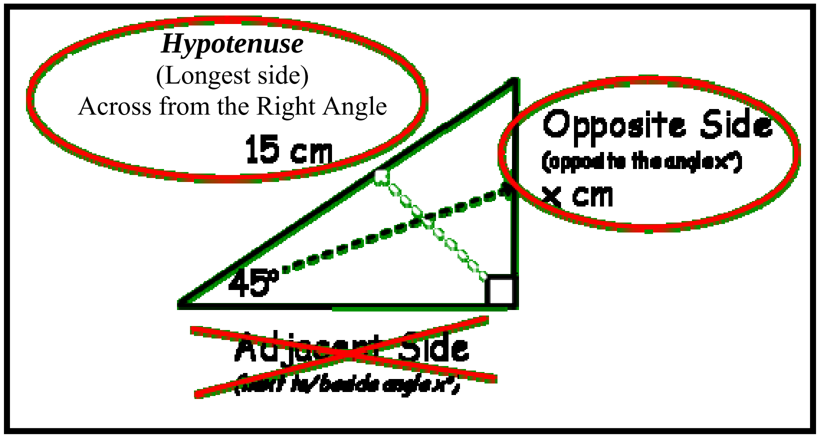


This process helps us identify which ratio we will use to solve the question. As we can see in the 3rd diagram the two sides with information are the opposite and Hypotenuse. We have ticked both of these in the one ratio, confirming that we will use Sin.

Trigonometry

Step 4: Laying out the correct working

$$\text{Sin } x^o = \frac{\text{Opposite}}{\text{Hypotenuse}}$$



We can now substitute the correct information into our formula.

$$\text{Sin } 45^o = \frac{x}{15}$$

We must now rearrange the formula so that the unknown x is what we are trying to find. We can do this by using simple rules of algebra. We have an equation with a fraction here, which can be removed if we **balance** the equation and multiply both sides b the denominator.

$$\text{Sin } 45^o = \frac{X}{15} \quad \Big| \quad \text{X 15 both sides}$$

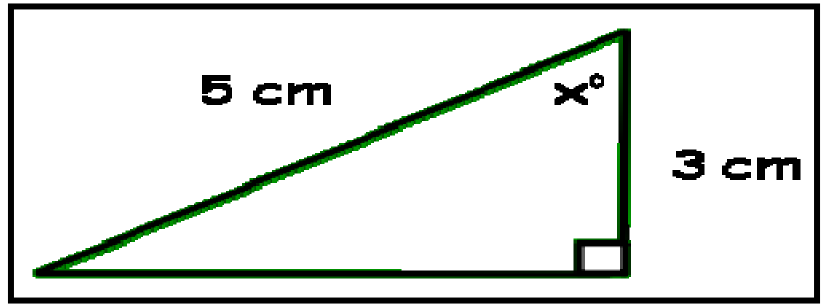
This will give

15 x sin 45 = *X* If we put this into the calculator we shall find the length of the side x.

$\underline{\underline{X = 10.6 \text{ cm}}}$

Trigonometry

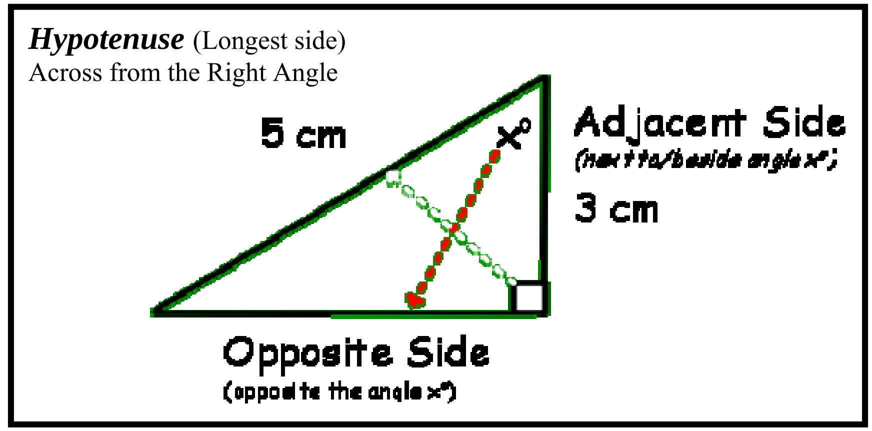
<u>Worked Example 2: Finding the size of an Angle</u>



Find the missing angle in this triangle

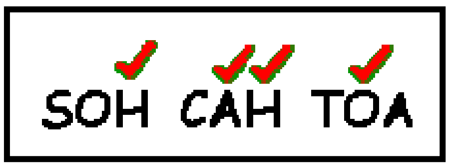
Step 1: Mark out the sides.

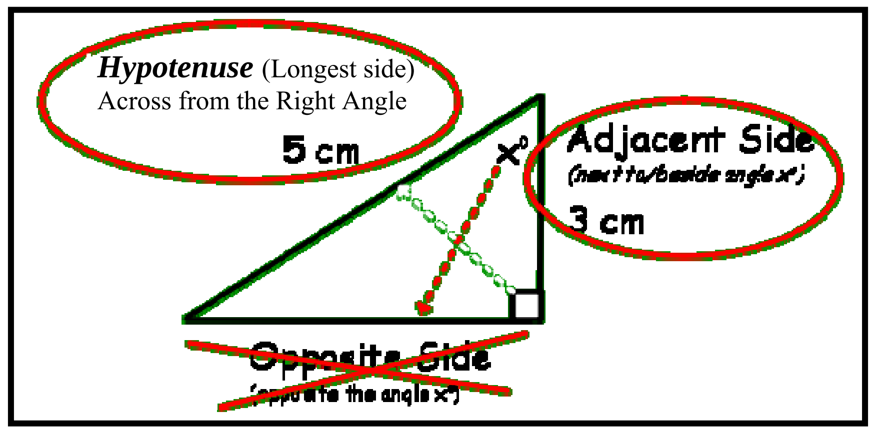
(N.B. Notice as the angle is in a different position, the sides change according to it)



Step 2: Highlight the sides which have information.

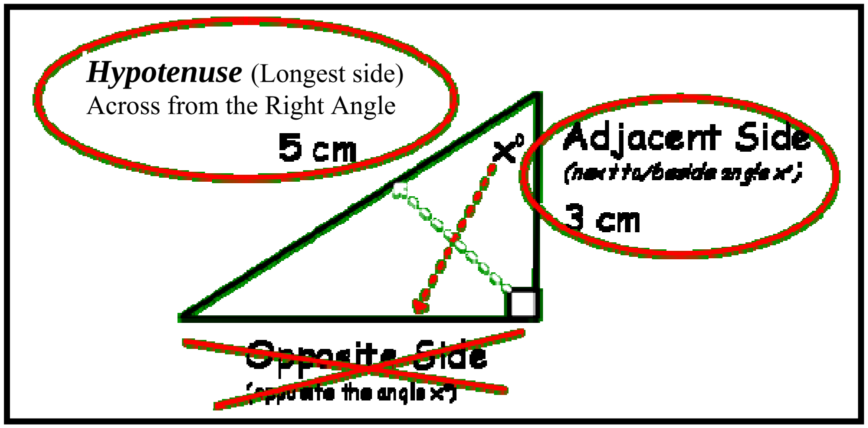
Step 3 : Write down the 3 trig ratios, and identify which one is the correct one to be using.





This process helps us identify which ratio we will use to solve the question. As we can see in the 3rd diagram the two sides with information are the adjacent and Hypotenuse. We have ticked both of these in the one ratio, confirming that we will use Cos.

Trigonometry



Step 4: Laying out the correct working

$$\text{Cos } x^{o} = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

We can now substitute in the information into the formula

$$\text{Cos } x^{o} = \frac{3}{5}$$

We will now divide 3 into 5

$$\text{Cos } x^{o} = 0.6$$

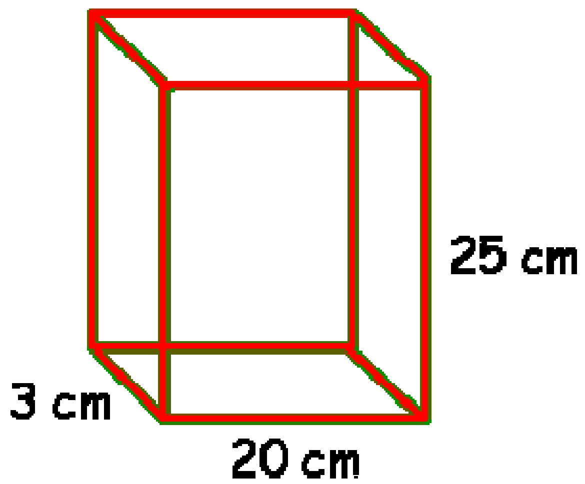
At this stage we have what Cos x is equal to, we want to find x so we must find the inverse of Cos.
This can be done easily using the calculator Shift/2nd function button must be pressed before Cos to give the correct answer.

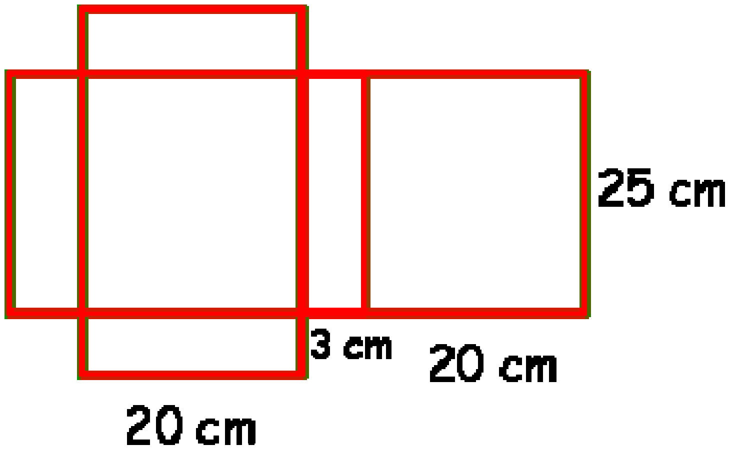
$$x^{o} = \text{Cos}^{-1}\, 0.6$$

$$x^{o} = 53.1^{o}$$

Surface area and Volume

The surface area of a solid is the total area of all the faces of the net





Front = 20 x 25 = 500 cm ^{2}

Back = 20 x 25 = 500 cm ^{2}

Top = 3 x 20 = 60 cm ^{2}

Bottom = 3 x 20 = 60 cm ^{2}

Left Side = 3 x 25 = 75 cm ^{2}

Right Side = 3 x 25 = 75 cm ^{2}

Total Surface Area = 1270 cm ^{2}

Volume of a Cuboid = Length x Breadth x Height

lbh

= 20 x 3 x 25 = 1500 cm^3