St Ninian's HS

Higher Physics

Our Dynamic Universe

Pupil Notes

SQA Course Specification Higher

Motion – equations and graphs

- Use of appropriate relationships to solve problems involving distance, displacement, speed, velocity, and acceleration for objects moving with constant acceleration in a straight line.

- Interpretation and drawing of motion-time graphs for motion with constant acceleration in a straight line, including graphs for bouncing objects and objects thrown vertically upwards.

- Knowledge of the interrelationship of displacement-time, velocity-time, and acceleration-time graphs.

- Calculation of distance, displacement, speed, velocity, and acceleration from appropriate graphs (graphs restricted to constant acceleration in one dimension, inclusive of change of direction).

- Description of an experiment to measure the acceleration of an object down a slope.

For an object travelling at a **constant speed**, or which has a motion represented by an average speed:

Average speed can be calculated using:

$$d = \bar{v}t$$

distance (m)

time (s)

average speed (ms^{-1})

Or, taking direction into account:

Average velocity can be calculated using:

$$s = \bar{v}t$$

displacement (m)

time (s)

average velocity (ms^{-1})

Equations of Motion

For an object that experiences a **constant acceleration** in a straight line, we use the equations of motion (often referred to as "*suvat*"):

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

where:

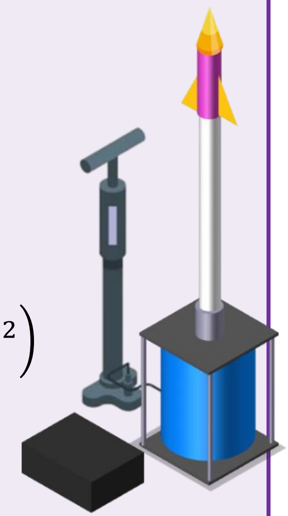
s is the displacement (m)
u is the initial velocity (ms^{-1})
v is the final velocity (ms^{-1})
a is the acceleration (ms^{-2})
t is the time (m)

s, u, v, and a are all **vectors** so **directions matters**!

We need to decide on a positive direction and stick with it (e.g. anything upwards is positive, and anything downwards is negative).

Worked Example: The Paper Rocket

A physics student launches a paper rocket vertically upwards with an initial velocity of 25 ms^{-1}. It reaches it maximum height after 2.6 seconds. Assume friction is negligible (acceleration is under the influence of gravity alone, so has a magnitude of 9.8 ms^{-2}).



Calculate the maximum height reached by the paper rocket.

+ve

$$s = ?$$
$$u = 25\ ms^{-1}$$
$$v =$$
$$a = -9.8\ ms^{-2}$$
$$t = 2.6\ s$$

Note: since we have defined **upwards as** the **positive** direction, the **downwards** acceleration due to gravity **is negative**.

$$s = ut + \frac{1}{2}at^2$$

$$s = (25 \times 2.6) + \left(\frac{1}{2} \times (-9.8) \times 2.6^2\right)$$

$$s = 31.9\ m$$

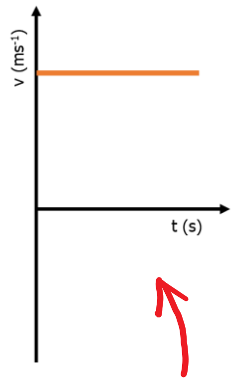
Since the number is **positive**, the displacement is **upwards** (which makes sense within the context)

Graphs of Motion

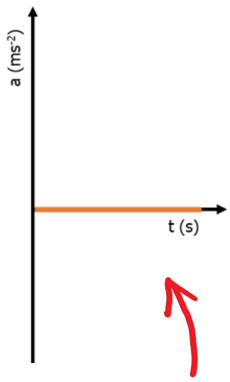
We can use motion-time graphs to represent the motion of an object over a period of time. In National 5 we looked at velocity-time graphs, but in Higher Physics we will expand our knowledge to the corresponding displacement-time graphs, and acceleration-time graphs.

Situation 1: Constant Velocity

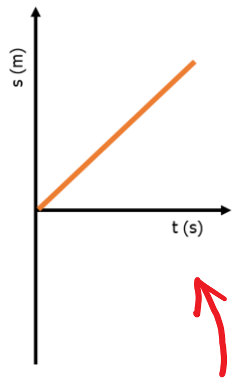
Constant velocity means the velocity **does not change** over time (both magnitude and direction). The motion-time graphs could look like:



This represents a constant velocity in whichever direction has been defined as positive.



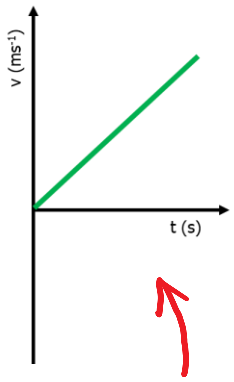
Since **acceleration** is the **change in velocity per second**, the value of acceleration is 0 ms^{-2} for this period of time.



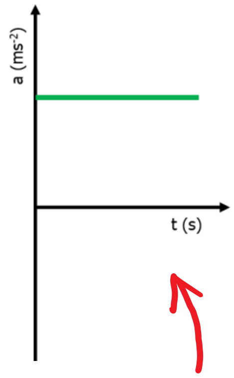
Constant velocity means the **number of metres** travelled **each second** remains consistent. Change in position from starting point increase gradually.

Situation 2: Constant Acceleration (Positive)

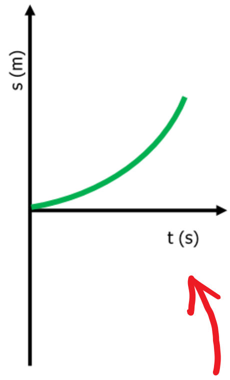
Constant acceleration in the positive direction means the **velocity consistently changes over time**, tending towards the positive direction. The motion-time graphs could look like:



As time passes, the **velocity increases** in the **positive direction** (magnitude gets further from 0 ms^{-1}) by a consistent amount each second.



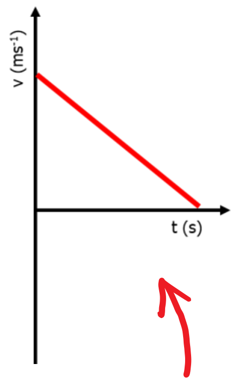
Since the **velocity** in tending towards the **positive direction** over time, the **acceleration** is **positive**.



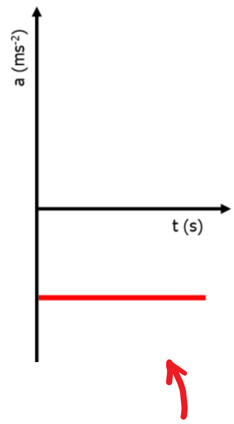
An **increasing velocity** means the **number of metres** travelled **each second increases** over time. The line gets steeper the faster the object goes.

Situation 3: Constant Acceleration (Negative)

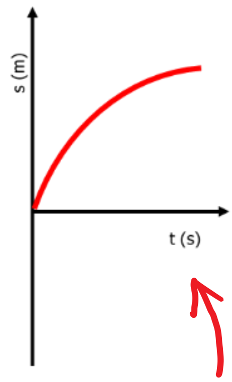
AKA constant deceleration. **Constant acceleration** in the negative direction means the **velocity consistently changes over time**, tending towards the negative direction (not necessarily slowing down). The motion-time graphs could look like:



As time passes, the **velocity** tends towards the negative direction (here, **decreasing** in the **positive direction**) by a consistent amount each second.



Since the **velocity** in tending towards the **negative direction** over time, the **acceleration** is **negative**.

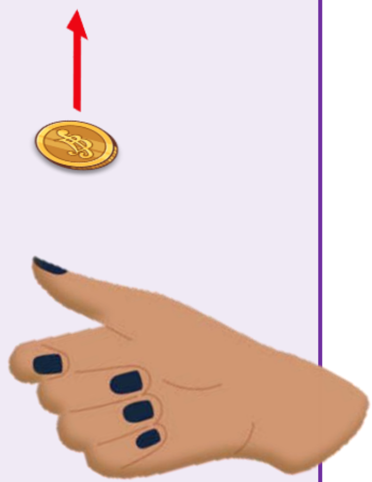


A **decreasing velocity** means the **number of metres** travelled **each second decreases** over time. The line gets less steep the slower the object goes.

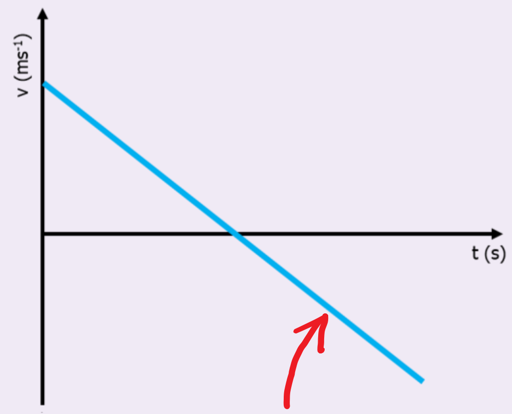
Worked Example: An Object Thrown Vertically Upwards

A coin is tossed vertically upwards, and caught in the same position it started. Air resistance is negligible.

A velocity time graph representing the coin's motion is shown below:



The coin is tossed **upwards** and the initial velocity is **positive**, so **upwards** must be the **positive direction**.

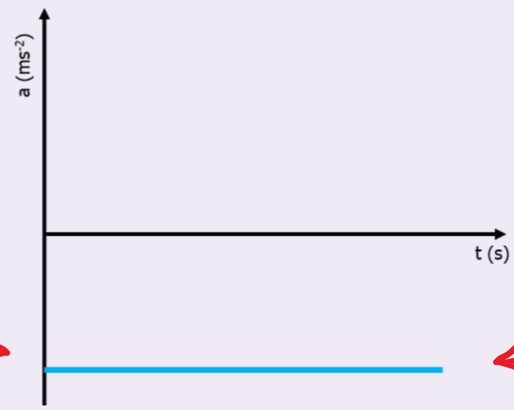


The **negative velocities** mean the coin is moving **downwards**.

Sketch the corresponding acceleration-time graph, and a displacement-time graphs. Numerical values are not required.

For the entire journey, the coin tends towards the negative direction (downwards). The acceleration-time graph looks like:

Even though there are positive and negative velocities on the graph above, since the line is straight there is only one value of acceleration.

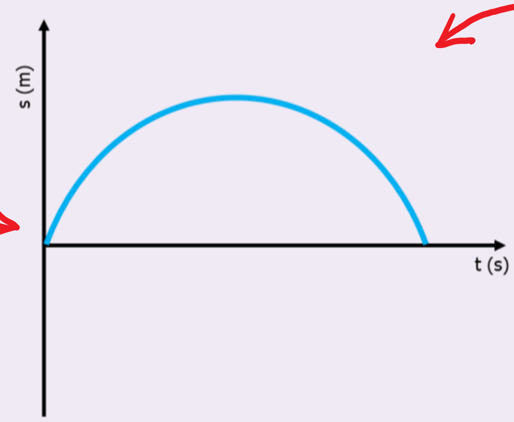


Since air resistance is negligible, acceleration of an object in free fall applies here ($a = -9.8\ ms^{-2}$).

The displacement-time graph shows how far away the coin is from its starting point at any moment:

In the first half of the journey the coin's **speed is reducing over time** → the s-t line gets **less steep** (the coin travels away from its starting point, in the positive direction.

$s = 0\ m$ is where the journey starts.



Then in the second half of the journey the coin's **speed is increasing over time** → the s-t line gets **steeper** (the coin moves back towards the starting point, since velocity is now negative).

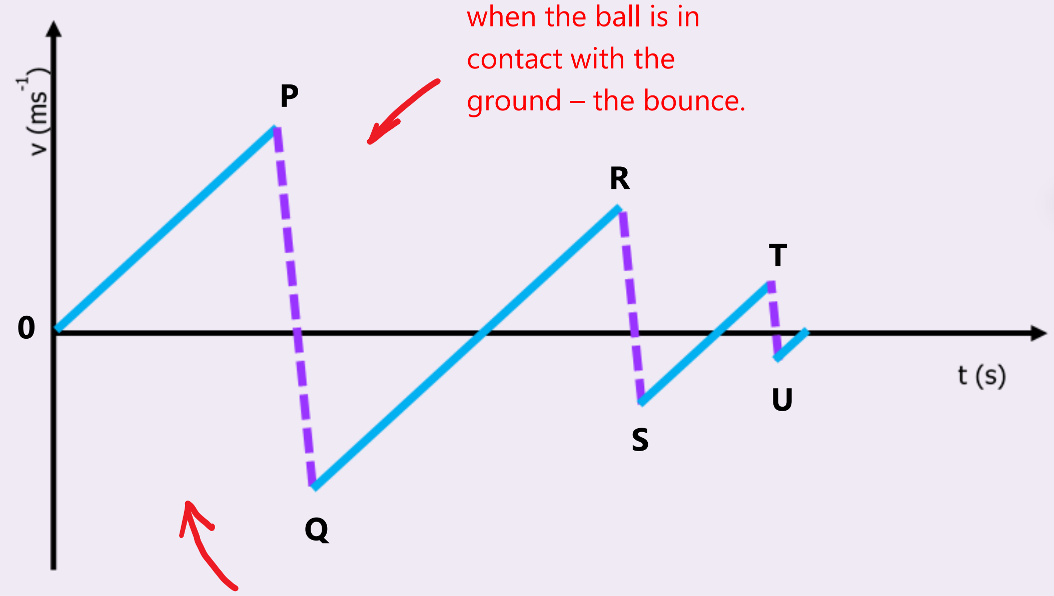
Worked Example: A Bouncing Ball

A ball is dropped from rest and allowed to bounce several times.

The graph shows how the velocity of the ball varies with time.

Assume air resistance is negligible.

The dotted lines show when the ball is in contact with the ground – the bounce.



Remember: **positive and negative** velocities inform us which **direction** the ball is moving in. Since the ball is "dropped from rest" its **initial motion** will be a **downwards acceleration from zero**.

∴ any **positive velocity** is a **downwards** velocity (and any **negative** velocity is **upwards**)

(a) **Describe** the motion of the ball between 0 and P.

constant acceleration downwards from rest

(b) **Explain** what causes the rapid change in velocity during section P and Q.

During the "bounce" (the collision between the ball and the ground) the ground exerts a force on the ball (larger than the weight of the ball). An upwards unbalanced force causes an upwards acceleration.

(c) **Compare** the velocity of the ball at P and the velocity of the ball at Q.

At P: velocity is positive (downwards) and at its maximum value.
At Q: velocity is negative (upwards), and has a lower magnitude than at P.

The magnitude of velocity is less at Q because the ball has lost energy to the surroundings during the bounce.

(d) **State** which direction the ball is moving at S.

At S the velocity is negative ∴ the ball is moving upwards.

Calculations Using Data from Graphs of Motion

Information can be taken from motion-time graphs and used to find other quantities – distance, displacement, speed, velocity, or acceleration.

In Higher Physics, these are limited to objects experiencing a constant acceleration in one plane (e.g. up/down) where opposite directions are described as positive and negative.

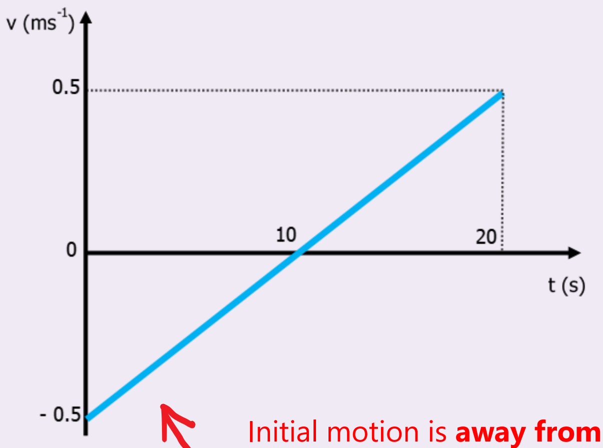
Quantity	***Symbol***	***Definition***
distance	d	path length travelled, regardless of direction
displacement	s	change in position from between identified initial and final points, requires a direction (generally positive or negative)
time	t	the time between identified initial and final points of an objects motion (between u and v)
speed (initial)	u	distance travelled (generally in metres) per second, regardless of direction, at the start of its motion
speed (final)	v	distance travelled (generally in metres) per second, regardless of direction, at the end of its motion
velocity (initial)	u	displacement per second, at the start of an objects motion, requires a direction (generally positive or negative)
velocity (final)	v	displacement per second, at the end of an objects motion, requires a direction (generally positive or negative)
acceleration	a	change in velocity per second, requires a direction (generally positive or negative)

Worked Example: The Astronaut

An astronaut wears a SAFER unit during a spacewalk to maneuver if they become untethered from the space station. The SAFER unit provides a constant acceleration towards the station,

The v-t graph represents the motion of the astronaut from when from the moment they lose contact with the station, to the moment they reattach.

Calculate the maximum distance betweent the astronaut and the space station.



Initial motion is **away from the station** ∴ any **negative velocity** is the astronaut moving **away**.

0→10s astronaut moves away from station,
10→20s astronaut moves towards station

$s = ?$
$u = -0.5\ ms^{-1}$
$v = 0\ ms^{-1}$
$a =$
$t = \ 10\ s$

$$s = \frac{1}{2}(u+v)t$$

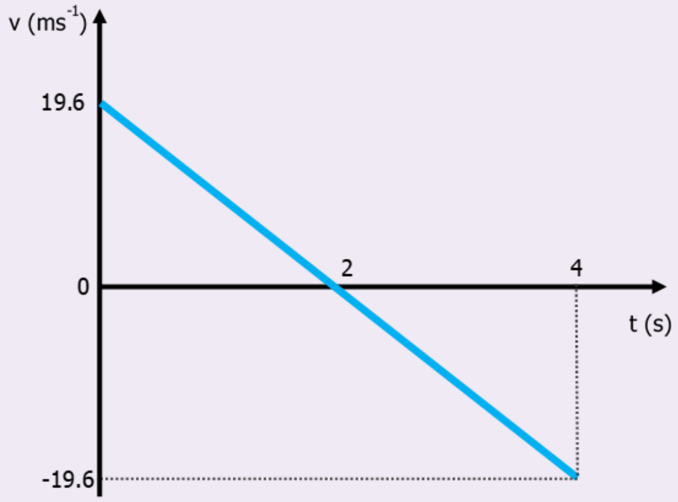
$$s = \frac{1}{2}(-0.5+0)10$$

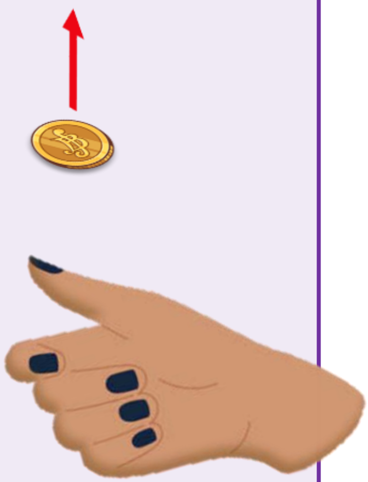
$$s = -2.5\ m$$

$$\therefore d_{MAX} = 2.5\ m$$

Worked Example: The Coin Flip

Let's look again at the v-t graph representing the motion of the coin:





(a) **Calculate** the acceleration of the coin between 0 s and 4 s.

There is one straight line on the graph between 0 and 4s (even though that line crosses the x-axis), ∴ one value of acceleration:

$s =$
$u = 19.6\ ms^{-1}$
$v = -19.6\ ms^{-1}$
$a = ?$
$t = 4\ s$

$$v = u + at$$

$$-19.6 = 19.6 + (a \times 4)$$

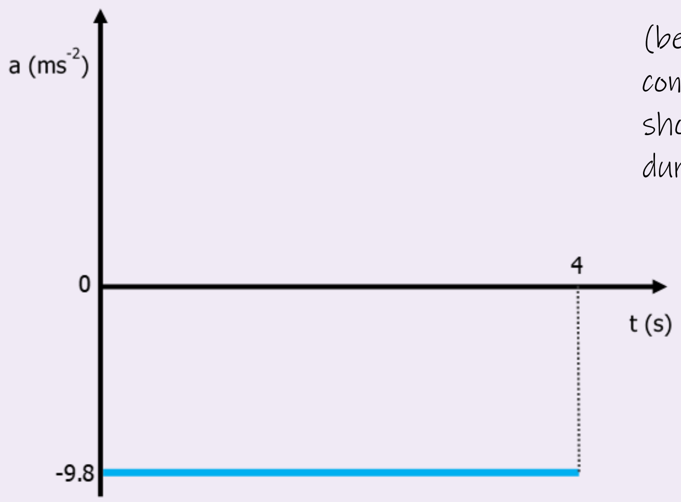
$$a = \frac{-19.6 - 19.6}{4}$$

$$a = -9.8\ ms^{-2}$$

Remember PEMDAS/BODMAS to get the correct order of operations when rearranging!

(the same calculation using values from 0→2s or 2→4s would have given the same answer)

(b) **Sketch** an acceleration-time graph which represents this motion.



(between 0 and 4 seconds, the coin experiences a constant acceleration of -9.8 ms^{-2}, so the graph shows a horizontal line at this value for the whole duration)

A ***sketched*** graph doesn't require a lot of detail, only:

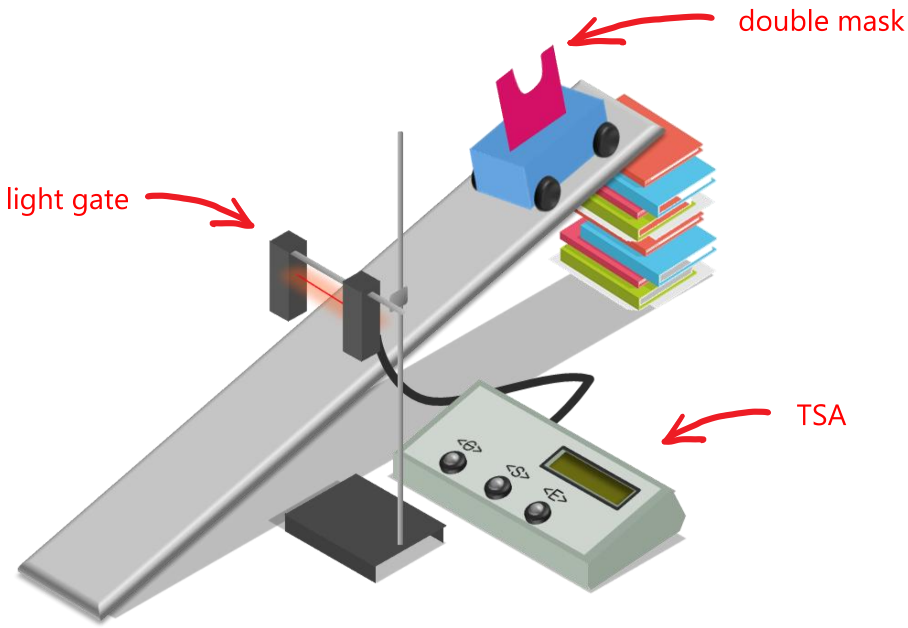
- labels with units
- correct shape
- important values (often including 0)

Measuring the Acceleration of an Object down a Slope

The accelation of an object travelling down a slope can be calculated using the equipment shown below.

Understanding the experiment:

- The TSA is connected to a light gate, and can only **directly** measure the time that the laser is blocked for.
- Inputting the length of the mask allows the TSA to convert this into an instantaneous velocity.
- Using the **double mask**/card allows us to record **two instantaneous velocities** (u and v), as well as the time between them (t).
- With this setup the TSA will provide the acceleration of the object.



If we assume friction and air resistance are negligible, then the value of acceleration should be the same at any point on the slope (though the *speed* will increase as the object moves further down the slope).

In reality, as an object's speed increases the magnitude of air resistance it experiences will also increase. At greater speeds this may affect the resultant acceleration.

SQA Course Specification Higher

Forces, energy and power

- Use of vector addition and appropriate relationships to solve problems involving balanced and unbalanced forces, mass, acceleration, and gravitational field strength.

- Knowledge of the effects of friction on a moving object (no reference to static and dynamic friction).

- Explanation, in terms of forces, of an object moving with terminal velocity.

- Interpretation of velocity-time graphs for a falling object when air resistance is taken into account.

- Use of Newton's first and second laws to explain the motion of an object.

- Use of free body diagrams and appropriate relationships to solve problems involving friction and tension.

- Resolution of a vector into two perpendicular components.

- Resolutions of the weight of an object on a slope into component forces parallel and normal to the surface of the slope.

- Use of the principle of conservation of energy and appropriate relationships to solve problems involving work done, potential energy, kinetic energy, and power.

Forces

A force can be considered as an action (a push or a pull) which can affect the motion (or shape) of an object. All forces are measured in Newtons (N).

Some common forces are listed below:

Force	***Symbol***	***Definition***
Weight	W	The force exerted on a body due to gravity.
Friction	F_f	A contact force between two surfaces that are sliding (or trying to slide) across each other. Always opposes motion.
Air resistance	F_{AR}	AKA drag. A form of friction, caused by an object colliding with air particles as it moves through the air.
Thrust	F_{th}	Force generated by engines to propel an object (e.g. a rocket) in the desired direction. Explained using Newton's 3rd Law as the reaction force that exhaust gases exert on the object being propelled.
Tension	T	The pulling force exerted on an object by a string/rope/cable. Applies to both ends of a cable (as seen in internal forces problems).
Reaction	R	The reaction force from the ground/surface, exerted on an object which rests upon it (equal and opposite to weight if on a level surface and velocity is constant).
Lift	F_L	Upwards force produced when an object moves through the air (e.g. plane wings or helicopter blades).
Buoyancy	F_B	Upwards force acting on an object which is submerged (partially or fully) in a fluid (generally water). Caused by differences in fluid pressure at different depths.

Applying Newton's 1st or 2nd Law will allow us to solve most force problems:

Newton's 1st Law

balanced forces ↔ constant velocity

Newton's 2nd Law

unbalanced force ↔ acceleration in that direction

Free Body Diagrams

A strategy that will prove useful for *all* forces problems is using a free body diagram.

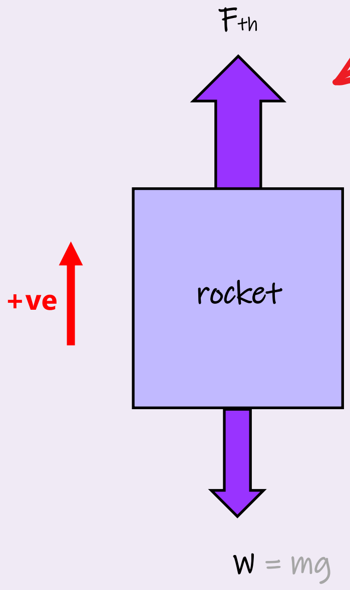
To break down that phrase: a *free body* is a single object, represented alone (not even including the ground it sits on); a *diagram* is just a picture.

This allows us to express clearly what forces are acting on an object (considering both magnitude and direction), and determine the resulting motion.

Worked Example: A Rocket at Launch

At launch, a rocket has a total mass of 20,000 kg, and the engines produce a constant thrust of 436 kN.

(a) **Sketch** a free body diagram, showing the names and directions of the forces acting on the rocket at launch (once it has *just* left the ground).



It can be useful for our problem-solving process to represent the relative sizes of the forces on the diagram (especially if we don't yet have all values).

There will be no reaction force from the ground, since the question specified that the rocket has *just* left the ground.

If the rocket is *just* off the ground, we can also assume its speed is low ∴ air resistance at this point will be negligible.

(b) **Calculate** the initial acceleration of the rocket at launch.

$$a = ?$$
$$m = 20{,}000\ kg$$
$$F_{th} = 436 \times 10^3\ N$$
$$W = mg$$
$$g = 9.8\ Nkg^{-1}$$

$$F_{un} = ma$$

$$F_{th} - mg = ma$$

$$4.36 \times 10^3 - (20{,}000 \times 9.8) = 20{,}000 \times a$$

$$a = 12\ ms^{-2}$$

Remember PEMDAS/BODMAS to get the correct order of operations when rearranging!

(c) **Explain** why the acceleration of the rocket may change during its journey upwards.

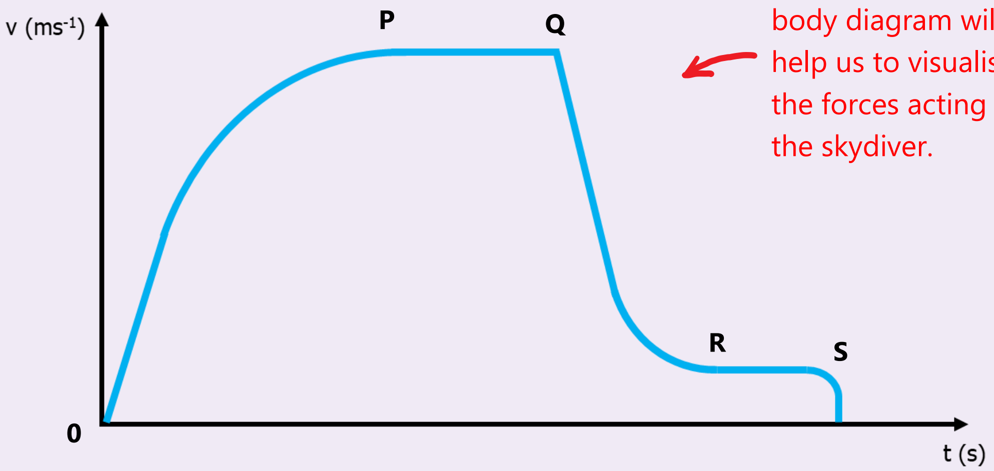
As the rocket travels upwards:

- It will burn fuel ∴ mass decreases ∴ weight decreases ∴ greater upwards unbalanced force ∴ greater upwards acceleration.
- It's upwards velocity increases ∴ magnitude of air resistance increases ∴ smaller upwards unbalanced force ∴ smaller upwards acceleration.

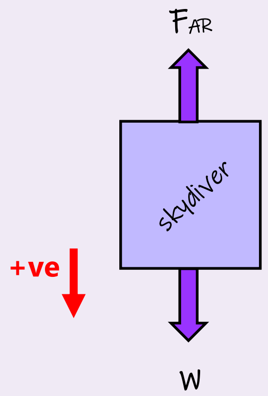
(How the acceleration is affected will depend on the balance of weight and air resistance.)

Worked Example: The Skydiver

The v-t graph below represents the vertical motion of a skydiver, from the moment they jump from the plane to the moment they land safely on the ground.



Drawing a free body diagram will help us to visualise the forces acting on the skydiver.



(a) **Describe** and **explain** the motion of the skydiver between 0 and P.

Between 0 and P, the skydiver is **accelerating downwards**, due to a **downwards unbalanced force** ($W \gg F_{AR}$).

Since the line gets less steep over time, the value of acceleration decreases as they fall. At first they are essentially in free-fall (air resistance will be negligible) and so their acceleration will be close to 9.8 ms^{-2}. Here, the unbalanced force is equivalent to their weight.
As their speed increases, air resistance increases ∴ the downwards unbalanced force decreases ∴ downwards acceleration decreases.

(b) **Describe** and **explain** the motion of the skydiver between P and Q.

Between P and Q the skydiver travels downwards at a **constant velocity**. The **forces** (weight downwards and air resistance upwards) acting on the skydiver **are balanced**.

This is referred to as their ***terminal velocity.***

(c) **Describe** and **explain** the motion of the skydiver between Q and R.

Between Q and R the skydiver **decelerates** whilst travelling **downwards**, due to an **upwards unbalanced force** ($F_{AR} \gg W$). They pull their parachute at Q. The increased surface area increases the magnitude of air resistance.

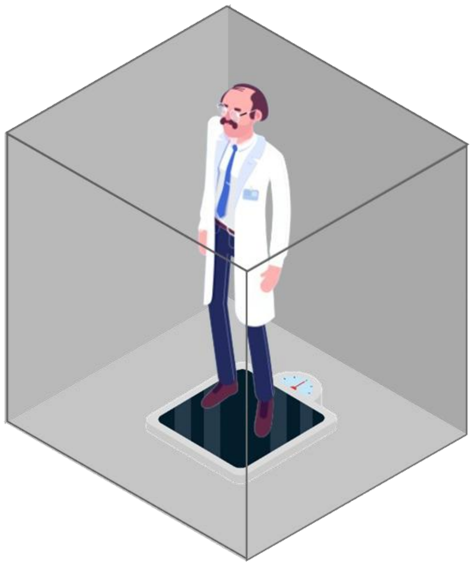
(d) **Describe** and **explain** the motion of the skydiver between R and S.

Between R and S the skydiver is travelling downwards at a **constant velocity**. The **forces** (weight downwards and air resistance upwards) acting on the skydiver **are balanced**.

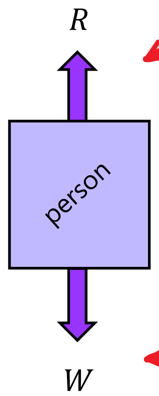
Lift Problems

Lifts (as in elevators) are an interesting application of Newton's Laws considered in Higher Physics. If you were to stand on a set of scales (AKA a "balance" in science) in a lift, it won't *always* show you your true mass/weight. This is because the scales directly measure the force *they* exert on *you* – the **reaction force** – not *your* weight.

Consider a person standing in a lift, on top of a set of scales.



Now, consider the forces acting on the person in the lift which can affect their motion. It's best to use a free body diagram for this:

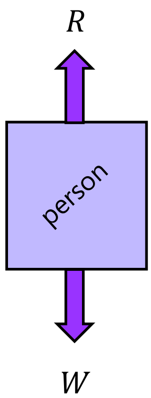


The only thing in *contact* with the person are the scales, so the **reaction** force from the scales are the only contact force acting on the person.

Since they are within the Earth's gravitational field, **weight** pulls them downwards, towards the Earth's centre.

Whether the person in the lift is accelerating, decelerating, or moving at a constant velocity (remember "stationary" is a constant velocity of 0 ms^{-1}) depends on the balance of these two forces and its initial motion.

If $R = W$:



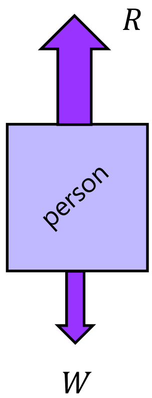
The **forces** acting on the person are **balanced**

∴ the person continues to move at a **constant velocity.**

Note: since weight is a product of mass and gravitational field strength, it will not change.

Motion will therefore depend on the magnitude of the upwards force alone.

If $R > W$:



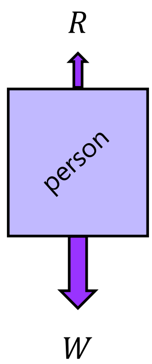
There is an **upwards unbalanced force**

∴ there is an **upwards acceleration**

If initially moving upwards, the persons speed will increase.

If initially moving downwards, the persons speed will decrease.

If $R < W$:



There is a **downwards unbalanced force**

∴ there is a **downwards acceleration**

If initially moving upwards, the persons speed will decrease.

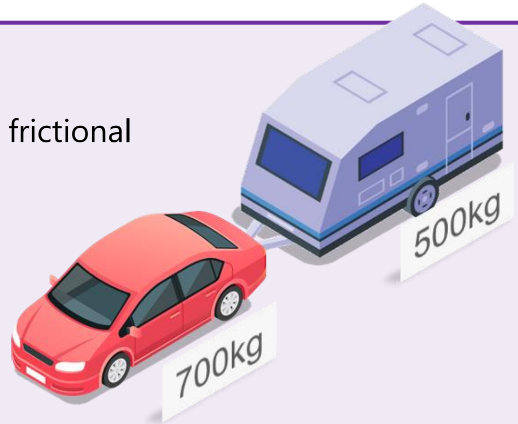
If initially moving downwards, the persons speed will increase.

Internal Forces

When *multiple objects* are moving *together*, there are forces acting *between them* that we need to consider.

To determine the magnitude of a force acting between two grouped objects, the problem-solving strategy is the same as always: draw a free body diagram to clearly show what forces are acting on your chosen object; apply Newton's 2nd Law ($F_{un} = ma$) to solve for the unknown value.

Worked Example: Tension in the Towbar



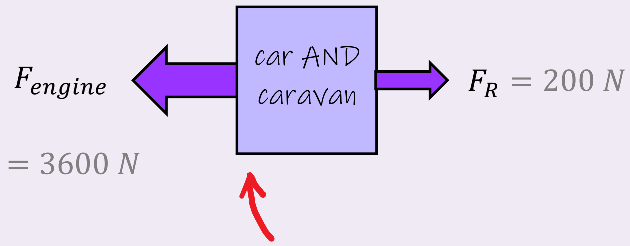
Consider a car, towing a caravan behind it. Each experiences a frictional force of 100 N.

The car has a mass of 700 kg, and the caravan a mass of 500 kg.

The car's engine provides a forwards force of 3600 N.

a) **Calculate** the acceleration of the whole system.

First, consider the two as one object:



If we need to find the acceleration of the system, we generally need to clump the objects together to do this (this way we needn't consider the internal forces as they act *within* the object, not *on* the object).

Then, apply $F_{un} = ma$:

+ve

$$F_{un} = ma$$

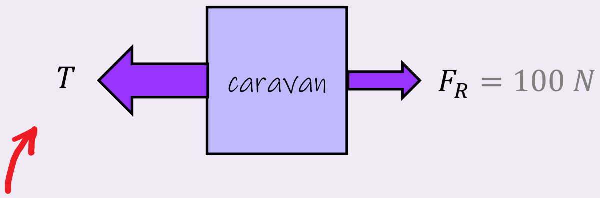
$$F_{engine} - F_R = ma$$

$$3600 - 200 = 1200 \times a$$

$$a = 2.83\ ms^{-2}$$

b) **Calculate** the tension in the towbar.

Choosing one of the objects (works with either):



The forwards force being applied to the *caravan* comes from the towbar – not the car engine. Since there is no *contact* between the car engine and the caravan, the force cannot be applied directly.

Then, apply $F_{un} = ma$:

+ve

$$F_{un} = ma$$

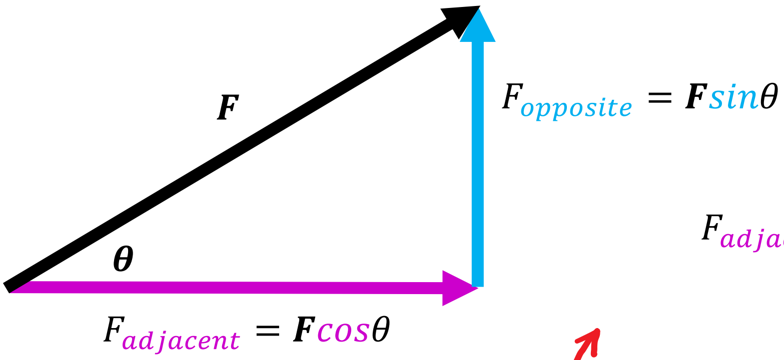
$$T - F_R = ma$$

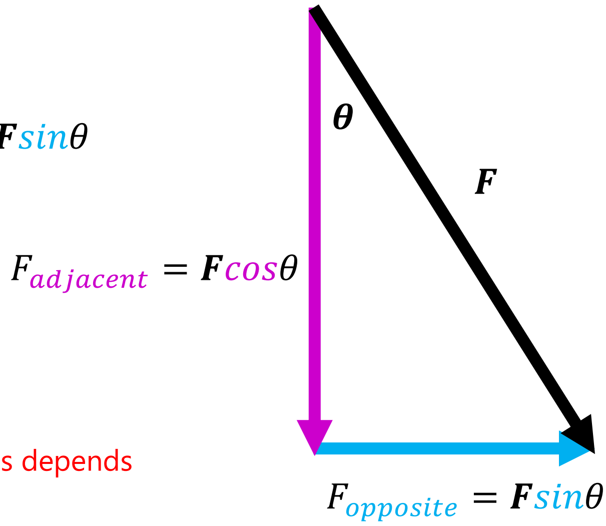
$$T - 100 = 500 \times 2.83$$

$$T = 1520\ N$$

Resolving Forces at an Angle

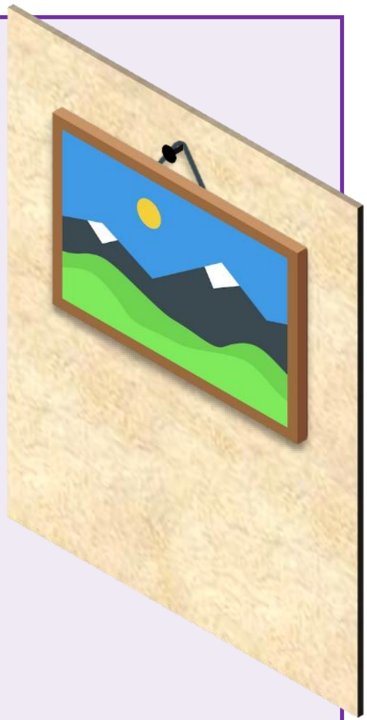
If a problem involves forces applied at an angle, we can use trigonometry to turn 1 force applied at an angle into 2 equivalent forces (one horizontal and one vertical). From there, we would follow the same process as usual: draw a free body diagram, apply Newton's Laws.





Note that the choice of sin/cos depends on the position of the angle!

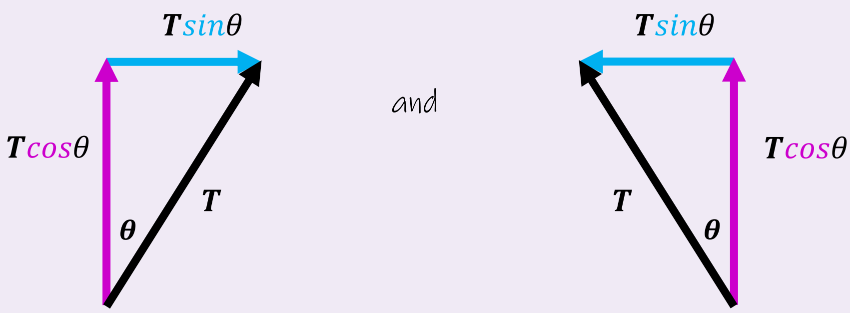
Worked Example: The Picture Frame



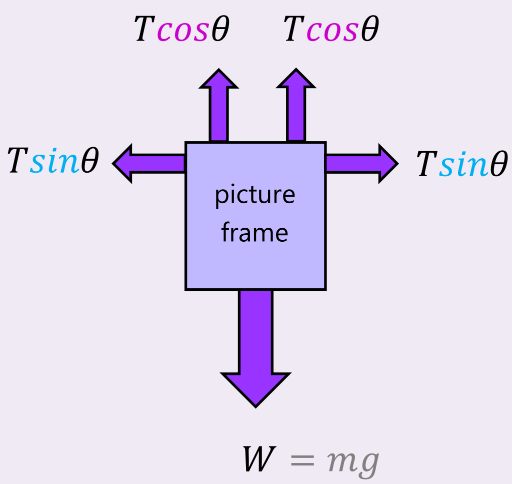
A 3 kg picture frame hangs from a wall, suspended by two cables, each at an angle of 30° to the vertical plane.

Calculate the tension in each cable.

First resolve the forces at angles into horizontal and vertical components:



Then draw a free body diagram and apply Newton's Laws:



The picture frame is **stationary**, so **forces are balanced** both horizontally and vertically.

Vertically:

$$mg = 2 \times T\cos\theta$$

$$3 \times 9.8 = 2 \times T\cos(30)$$

$$T = 17\ N$$

Components of Weight

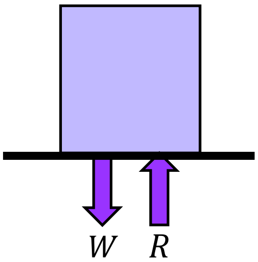
Weight always acts on an object vertically downwards. If an object is on a flat surface this is more straightforward, but if the object rests on a slope it becomes more complicated. We just need to use a bit of trigonometry to make it simpler for ourselves!

Object on a flat surface:

Weight acts directly down.

Weight is perpendicular to the surface.

Reaction force from the surface, $R = W$.



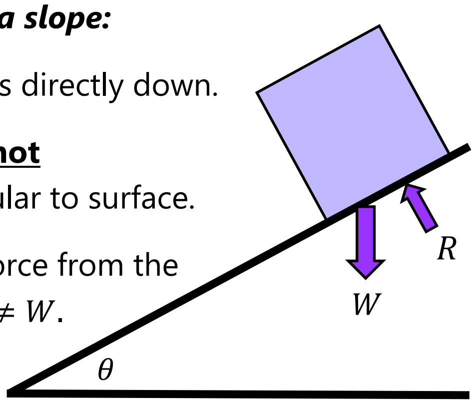
Both forces act in the same plane (up/down) so are easy to deal with

Object on a slope:

Weight acts directly down.

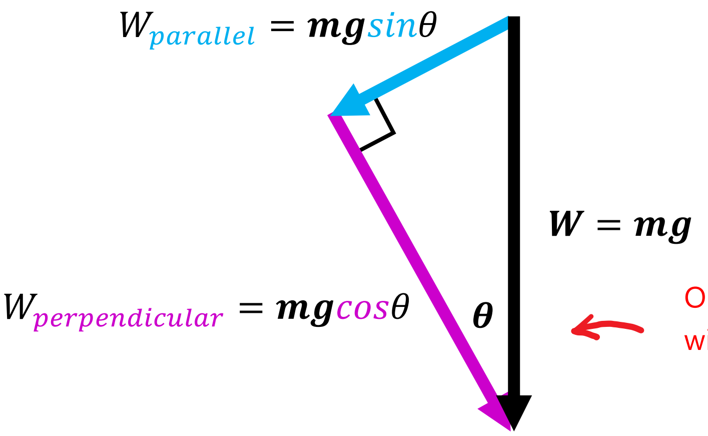
Weight **is not** perpendicular to surface.

Reaction force from the surface, $R \neq W$.



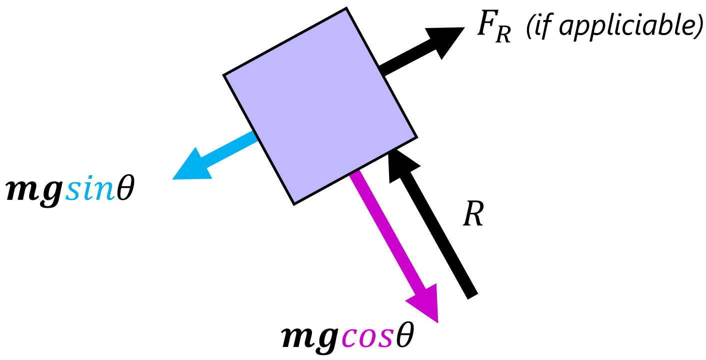
We need to resolve weight into two components – parallel and perpendicular to the slope.

One component of the objects weight act down (parallel to) the slope – this is why an object on a slope will slide down. The other component will act into (perpendicular to) the slope:



Our angle changes place with some clever geometry!

A free body diagram would look like…



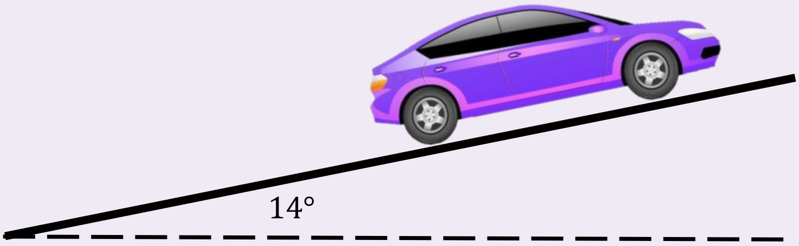
Motion is typically *down* the slope, not *through* the slope, so *mgsinθ* is more useful.

Remember: ***sin*** makes it ***slide*** down the ***slope!***

…and with no forces applied at angles that aren't perpendicular we can apply Newton's Laws like any other forces problem!

Worked Example: The Uphill Drive

A professor of engineering at the University of Strathclyde drives her car (1200 kg) up Montrose Street (which has an incline of 14°), maintaining a constant velocity. The car's engine provides a constant forwards force of 3200 N.



a) **Show that** the component of the car's weight acting parallel to the slope is 2800 N.

parallel component of weight
(makes it slide down the slope)

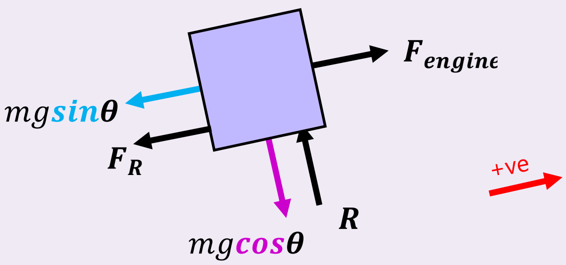
$$= mg\sin\theta$$

$$= 1200 \times 9.8 \times \sin(14)$$

$$= 2800\ N$$ as required

b) **Determine** the force of friction acting on the car as it is driven up the slope.

First, draw a free body diagram to show the forces acting on the car:



Since the car is moving at a constant velocity parallel to the slope, the forces parallel to the slope are balanced:

$$F_{engine} = mg\sin\theta + F_R$$

Then substitute known values and solve for friction:

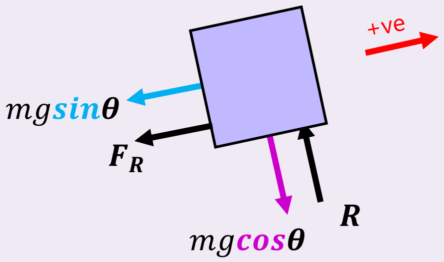
$$3200 = 2800 + F_R$$

$$F_R = 400\ N$$

c) The car's engine suddenly switches off.

Calculate the acceleration of the car immediately after the engine switches off.

If the engine is off:



Acceleration is parallel to the slope so we can ignore R and mgcosθ, and apply Newton's 2nd Law:

$$F_{un} = ma$$

$$-(mg\sin\theta + F_R) = 1200 \times a$$

$$-(2800 + 400) = 1200 \times a$$

$$a = -2.67\ ms^{-2}$$

"immediately after the engine switches off" tells us that the speed won't have changed yet ∴ the magnitude of friction is still 400 N

Energy & Power

Energy can be considered a property of all matter – it can be stored, transformed from one form to another, or transferred from one object to another.

The **Law of Conservation of Energy** states that energy can neither be created nor destroyed – only converted from one form to another. The **total energy** of an isolated system is ***always* conserved**.

$$\sum E_{before} = \sum E_{before}$$

here the Greek letter sigma (Σ) represents summation – read as "the total energy before equals the total energy after"

The following equations can be used to calculate various forms of energy:

$$E_w = Fd$$

$$E_p = mgh$$

$$E_k = \frac{1}{2}mv^2$$

where:

E_w	is the work done (energy transferred to an object by an applied force (J))
F	is the force applied to an object (N)
d	is the distance travelled by object due to applied force (m)
E_p	is the gravitational potential energy (J)
m	is the mass of an object (kg)
g	is the gravitational field strength (Nkg^{-1})
h	is the height gained/lost by an object (m)
E_k	is the kinetic energy (J)
v	is the speed of an object (ms^{-1})

Power is the rate of energy transfer – the amount of energy transferred or converted per second.

Power can be calculated using:

energy transferred (J)

power (W)

$$P = \frac{E}{t}$$

time (s)

Worked Example: The Downhill Skier

A skier of mass 60 kg slides from rest down a slope of length 20 m.
The skiers position at the top of the slope was 10 m higher than the bottom of the slope. The final speed of the skier at the bottom of the slope is 13 ms^{-1}.

(a) **Calculate** the potential energy lost by the skier on their journey down the slop.

The potential energy lost by the skier is proportional to height lost (10 m):

$$E_p = mgh$$

$$E_p = 60 \times 9.8 \times 10$$

$$E_p = 5880\, J$$

(b) **Calculate** the kinetic energy of the skier at the bottom of the ski slope.

Their kinetic energy at the bottom depends on their speed:

$$E_k = \frac{1}{2}mv^2$$

$$E_k = \frac{1}{2} \times 60 \times 13^2$$

$$E_k = 5070\, J$$

(c) **Determine** the work done by friction during their journey down the ski slope.

Apply the *Law of Conservation of Energy* to the skiers journey down the slope:

$$\sum E_{before} = \sum E_{after}$$

here we assume that the only energy lost from this system (the skier) is the energy lost to the surroundings via friction

since the skier is "at rest" at the top of the slope, their initial kinetic energy was 0 J (so E_p only before)

$$E_p = E_k + E_w$$

$$5880 = 5070 + E_w$$

$$E_w = 810\, J$$

(d) **Calculate** the average force of friction acting on the skier.

"average" here suggest that the magnitude of friction may actually change at different points in the journey

$$E_w = Fd$$

$$810 = F \times 20$$

$$F = 40.5\, N$$

SQA Course Specification Higher

Collisions, explosions, and impulse

- Use of the principles of conservation of momentum and an appropriate relationship to solve problems involving the momentum, mass and velocity of objects interacting in one dimension.

- Knowledge of interactions involving the total kinetic energy of systems of objects undergoing inelastic collisions, elastic collisions, and explosions.

- Use of an appropriate relationship to solve problems involving the total kinetic energy of systems of interacting objects.

- Use of Newton's third law to explain the motion of objects involved in interactions.

- Interpretation of force-time graphs involving interacting objects.

- Knowledge that the impulse of a force is equal to the area under a force-time graph and is equal to the change in momentum of an object involved in the interaction.

- Use of data from a force-time graph to solve problems involving the impulse of a force, the average force and its duration.

- Use of an appropriate relationship to solve problems involving mass, change in velocity, average force and duration of the force for an object involved in an interaction.

Momentum

Momentum is a measure of an objects motion – the product of its mass and velocity.

For an object:

velocity (ms^{-1})

$$p = mv$$

momentum $(kgms^{-1})$

mass (kg)

Momentum is a vector quantity. Since it relies on an object's velocity (not speed), it requires a direction – usually positive or negative.

Conservation of Momentum

For any two objects interacting in one dimension (e.g. forwards/backwards, or up/down), the *Law of Conservation of Momentum* applies.

The **Law of Conservation of Momentum** states that, **for an isolated system**:

$$\sum p_{before} = \sum p_{before}$$

The **total momentum before** an event is **equal to** the **total momentum after** an event, **in the absence of external forces**.

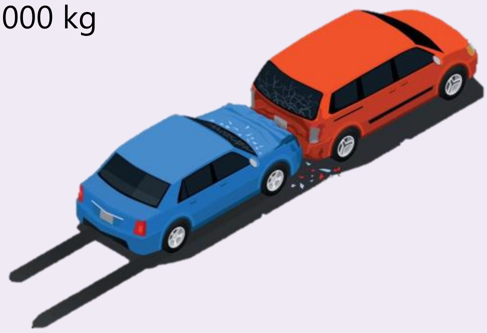
friction would generally be considered an "external force" – so this law only applies if friction is negligible

There are two types of events to consider:

- Collisions – an interaction in which two objects come together, exerting opposite forces on each other in a short period of time. After the collision the two objects may move off separately or together.
- Explosions – an interaction where an object breaks into two pieces (which move off in opposite directions) due to internal forces.

Both types of events involve 2 objects, as well as a *before* and an *after* – this means 2 masses (m_1 and m_2) 2 initial velocities (u_1 and u_2) and 2 final velocities (v_1 and v_2) to consider.

Worked Example: The Car Crash



Two cars are stuck in traffic. The red car (in front) has a mass of 2000 kg and the blue car (behind) has a mass of 1700 kg.

Before the collision, the red car travels forwards at 1.5 ms^{-1}.

The driver of the blue car, not paying attention, travels forwards at 2ms^{-1}. After the collision the blue car is stationary.

Friction is negligible.

Calculate the velocity of the red car after the collision.

Drawing a diagram of the two objects before and after will help to keep the data from getting mixed up:

+ve

$m_1 = 1700$ kg	$m_2 = 2000$ kg		$m_1 = 1700$ kg	$m_2 = 2000$ kg
$u_1 = 2\ ms^{-1}$	$u_2 = 1.5\ ms^{-1}$		$v_1 = 0\ ms^{-1}$	$v_2 = ?$

since forwards the positive direction here, these velocities are all positive (anything moving backwards would be negative)

The two objects are colliding, and "friction is negligible" tells us this happens in the absence of external forces ∴ the Law of Conservation of Momentum applies:

$$\sum p_{before} = \sum p_{after}$$

this line comes from the momentum ($p = mv$) of each object, before and after the collision

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$(1700 \times 2) + (2000 \times 1.5) = (1700 \times 0) + 2000v_2$$

$$v_2 = \frac{(1700 \times 2) + (2000 \times 1.5)}{2000}$$

$$v_2 = 3.2\ ms^{-1}$$

this is a forwards velocity (as it is positive) – perhaps bad news for the driver of the car in front if they are too close!

Worked Example: The Cannon

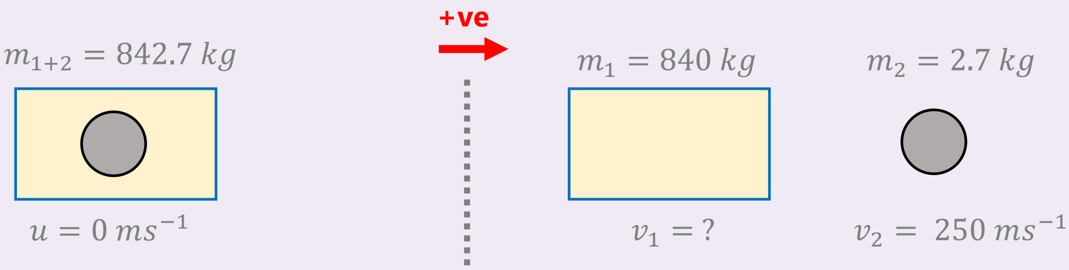
A 2.7 kg cannonball is fired from a 840 kg cannon with a forwards velocity of 250 ms^{-1}.

Before firing the cannon was stationary.

Friction is negligible.

Calculate the recoil velocity of the cannon immediately after it was fired.

Draw a diagram of the two objects before and after will help to organise the information.



since the cannon is initially stationary, and the cannonball would be *inside* the cannon they can be considered as one object, of total mass 842.7 kg and velocity 0ms^{-1}

This is an explosion – one object splitting into two parts moving in opposite directions, and "friction is negligible" tells us this happens in the absence of external forces ∴ the Law of Conservation of Momentum applies:

$$\sum p_{before} = \sum p_{after}$$

$$m_{1+2}u = m_1v_1 + m_2v_2$$

$$(842.7 \times 0) = 840v_1 + (2.7 \times 250)$$

$$v_1 = \frac{0 - (2.7 \times 250)}{840}$$

$$v_1 = -0.80\ ms^{-1}$$

this is a backwards velocity (as it is negative) – this makes sense within the context of the cannon experiencing "recoil"

Elastic Collisions

When two objects collide the **total energy** is **always conserved** – some energy may be lost to the surroundings but it has not been *destroyed*.

The **total <u>kinetic</u> energy** of the objects involved in a collision may or may not be conserved – this determines whether a collision is considered **elastic or inelastic**.

A collision can be described as **elastic** if::

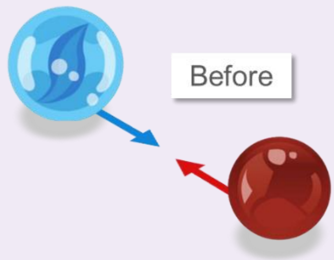
$$\sum E_{k_{before}} = \sum E_{k_{after}}$$

and **inelastic** if **kinetic energy is <u>lost</u>**:

$$\sum E_{k_{before}} > \sum E_{k_{after}}$$

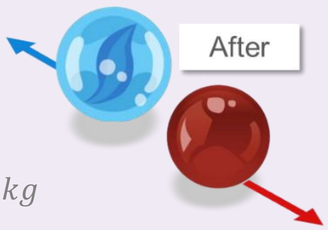
note: it is insufficient to say "$E_{k_{before}} \neq E_{k_{after}}$",
as this does not necessarily imply E_k has been <u>lost</u>

Worked Example: The Marbles



A 120 g marble, travelling on a frictionless surface at 1.0 ms^{-1} collides with a 95 g marble travelling in the opposite direction at 0.5 ms^{-1}.

After the collision, the 120 g marble bounces back with a speed of 0.2 ms^{-1}.



(a) **Calculate** the final velocity of the 95 g marble.

+ve

$m_1 = 0.12\ kg$ $m_2 = 0.095\ kg$ | $m_1 = 0.12\ kg$ $m_2 = 0.095\ kg$

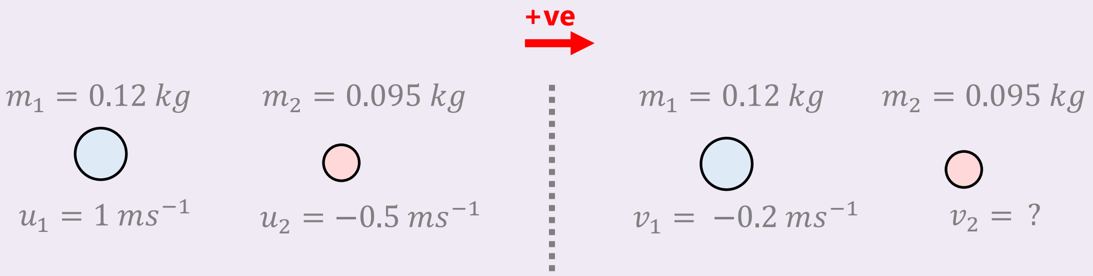
$u_1 = 1\ ms^{-1}$ $u_2 = -0.5\ ms^{-1}$ | $v_1 = -0.2\ ms^{-1}$ $v_2 = ?$

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$(0.12 \times 1.0) + (0.095 \times (-0.5)) = (0.12 \times (-0.2)) + 0.095v_2$$

$$v_2 = 1.0\ ms^{-1}$$

Worked Example: The Marbles (continued)



Show that the collision is inelastic.

The collision is inelastic if $\sum E_{k_{before}} > \sum E_{k_{after}}$

Kinetic energy can be calculated using $E_k = \frac{1}{2}mv^2$ for each object, before and after the collision.

energy is a scalar value and depends on speed, not velocity, so the direction (positive or negative) has no impact on this calculation

$\sum E_{k_{before}}$	$\sum E_{k_{after}}$
$= \left(\frac{1}{2}m_1u_1^2\right) + \left(\frac{1}{2}m_2u_2^2\right)$	$= \left(\frac{1}{2}m_1v_1^2\right) + \left(\frac{1}{2}m_2v_2^2\right)$
$= \left(\frac{1}{2} \times 0.12 \times 1.0^2\right) + \left(\frac{1}{2} \times 0.095 \times 0.5^2\right)$	$= \left(\frac{1}{2} \times 0.12 \times 0.2^2\right) + \left(\frac{1}{2} \times 0.095 \times 1.0^2\right)$
$= 0.072\ J$	$= 0.050\ J$

$\sum E_{k_{before}} > \sum E_{k_{after}}$ $\therefore$ the collision is inelastic

Impulse

Impulse is equivalent to the change in momentum of an object, caused by an applied force acting over the time of contact.

For an object:

$$\underbrace{Ft}_{\text{this part is the ``impulse'' applied to object (Ns)}} = \underbrace{mv - mu}_{\text{this part is referred to as the ``change in momentum'' of the object (kgms}^{-1}\text{)}}$$

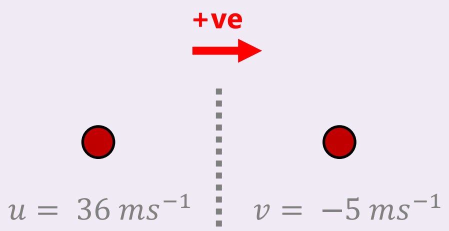
where:

F	is the average force exerted on the object (N)
t	is the contact time (s)
m	is the mass of the object (kg)
v	is the final velocity of the object (ms^{-1})
u	is the initial velocity of the object (ms^{-1})

Worked Example: The Cricket Ball

A 160 g cricket ball is moving towards the batter at 36 ms^{-1}. After colliding with the bat, the ball moves away from the batter at a speed of 5 ms^{-1}. The time of contact between the bat and ball was 7 ms.

(a) **Calculate** the average force that the bat exerts on the ball.



$$Ft = mv - mu$$

$$F \times (7 \times 10^{-3}) = \big(0.16 \times (-5)\big) - (0.16 \times 36)$$

$$F = \frac{\big(0.16 \times (-5)\big) - (0.16 \times 36)}{(7 \times 10^{-3})}$$

$$F = -940\ N$$

the negative means the force is applied *away from* the batter (left), which makes sense within the context

(b) **State** the average force that the ball exerts on the bat.

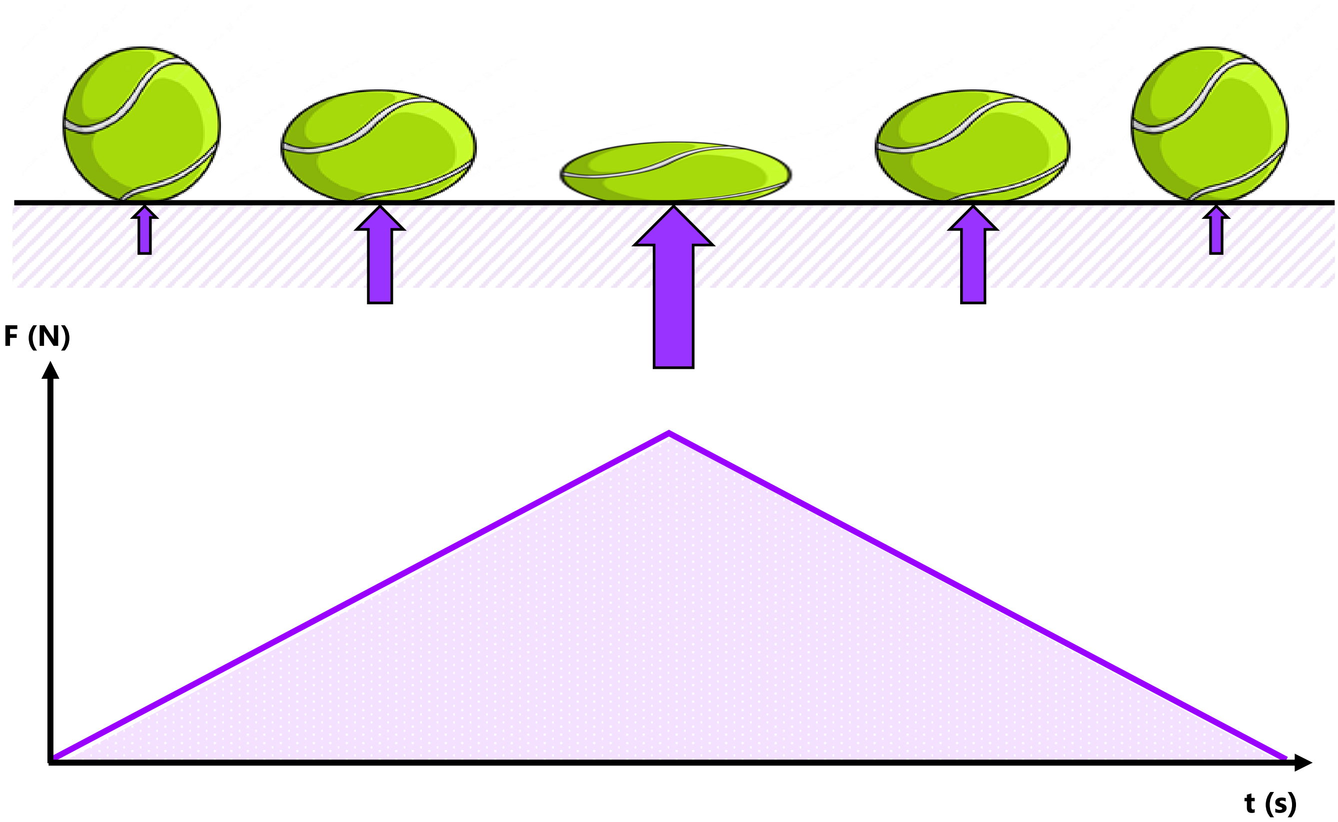
According to Newton's 3rd Law, the ball exerts and equal and opposite reaction force on the bat.

$$\therefore \quad F = 940\ N$$

Force-time Graphs

When using the equation above we referred to the "average force" – we use this term because when objects collide, the magnitude of the forces between them varies over the time they are in contact.

Consider a ball, as it bounces off the ground:

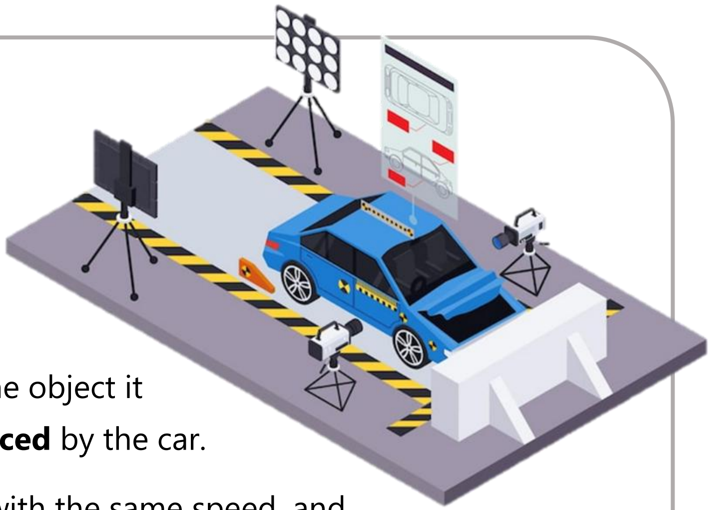


During a collision, the two objects involved (here the tennis ball and the ground) are only in contact for a brief period of time:

- The moment they come into contact the magnitude of force between them increases from zero, and will increase as the ball deforms and slows.
- The maximum force is applied when the ball is most deformed, and has reached a speed of 0 ms^{-1}.
- The ball is accelerates upwards and the two objects begin to separate. As contact decreases, the force decreases back to zero.

The **area under a force-time graph** is equivalent to the **impulse**.

For any collision with a **fixed change in momentum**, the **applied force** and **time of contact** are ***inversely proportional*** – if one goes up, the other goes down.

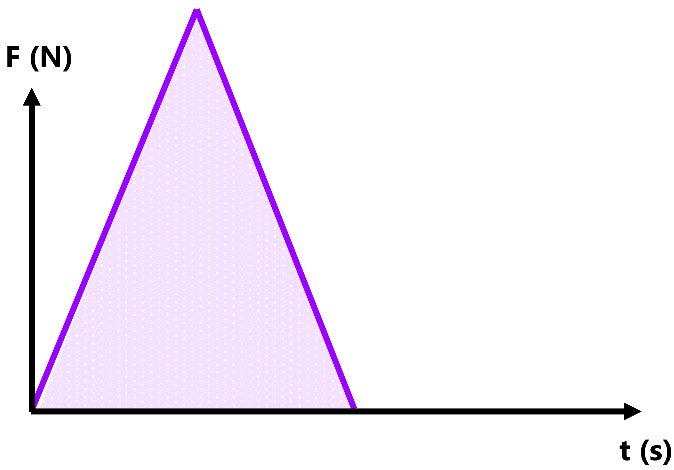


Consider a car that is being crash tested.

One of the safety features of modern cars is a "crumple zone" – a part of the car that is designed to deform during a collision.

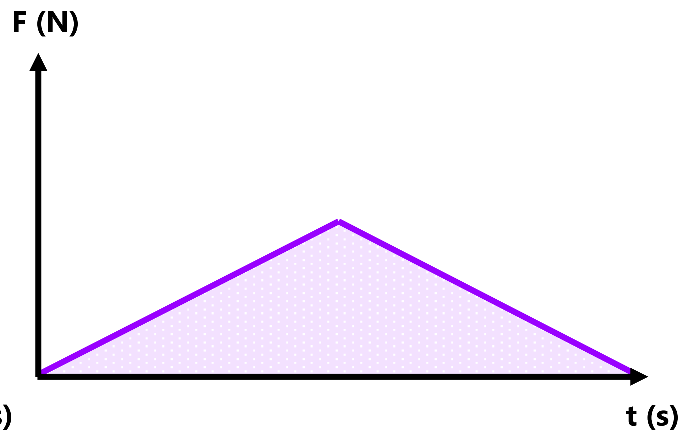
This extends the duration of the collision (a longer **time of contact** between the car and the object it collides with) ∴ decreasing the **force experienced** by the car.

For two cars of equal mass, hitting the barrier with the same speed, and both coming to rest after the collision:



F-t graph for car **without** a crumple zone:

- Same change of momentum $(mv - mu)$
- Same impulse (Ft)
- Same $Area$ Under Graph
- Lower contact time
- Greater average and peak force
- Greater risk to occupants



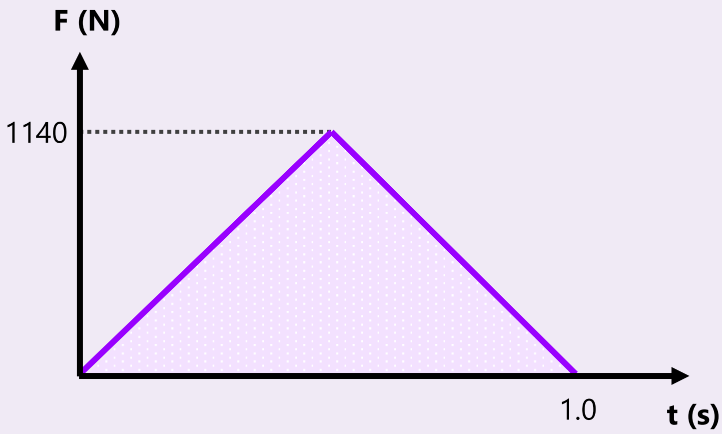
F-t graph for car **with** a crumple zone:

- Same change of momentum $(mv - mu)$
- Same impulse (Ft)
- Same $Area$ Under Graph
- Greater contact time
- Lower average and peak force
- Less risk to occupants

Worked Example: The Rock Climber

A bouldering-enthusiast falls off a climbing wall.

The F-t graph represents their collision with the crash mat:



a) **Determine** the change in the momentum the bouldering-enthusiast experiences during their collision with the crash mat.

$$Ft = mv - mu = Area\ Under\ Graph$$

$$mv - mu = \frac{1}{2} \times 1.0 \times 1140$$

$$mv - mu = 570\ kgms^{-1}$$

if we wanted a shorter way to refer to "change in momentum" we could use Δp

b) **Explain** why the bouldering-enthusiast would be more likely to injure themselves if they had landed on a concrete surface rather than the crash mat.

If the landing surface was the only thing to change the following would remain the same:

- Mass of the person, m
- Initial speed of the person before the collision, u (based on the height they fell from)
- Final speed of the person after the collision, v (they would still come to rest)
- Change in momentum experienced by the person, $mv - mu$ (based on the above)

The crash mat extends the duration of the collision, so without it:

- The contact time, t, would be shorter
- The average/peak force experienced by the person, F, would be less

$$\underbrace{Ft}_{t\ \text{decreases}\ \therefore\ F\ \text{increases}} = \underbrace{mv - mu}_{\text{this bit doesn't change}}$$

SQA Course Specification Higher

Gravitation

- Description of an experiment to measure the acceleration of a falling object.

- Knowledge that the horizontal motion and the vertical motion of a projectile are independent of each other.

- Knowledge that satellites are in free fall around a planet/star.

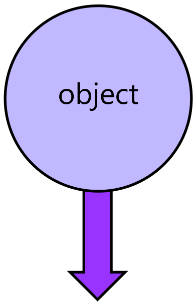
- Resolution of the initial velocity of a projectile into horizontal and vertical components and their use in calculations.

- Use of resolution of vectors, vector addition, and appropriate relationships to solve problems involving projectiles.

- Use of Newton's Law of Universal Gravitation to solve problems involving force, masses and their separation.

Free fall

An object is considered to be "in free fall" if the influence of gravity is the only thing affecting its motion. In free fall air resistance, lift, or any other applied force is negligible, and **weight** is the only force acting on the object.

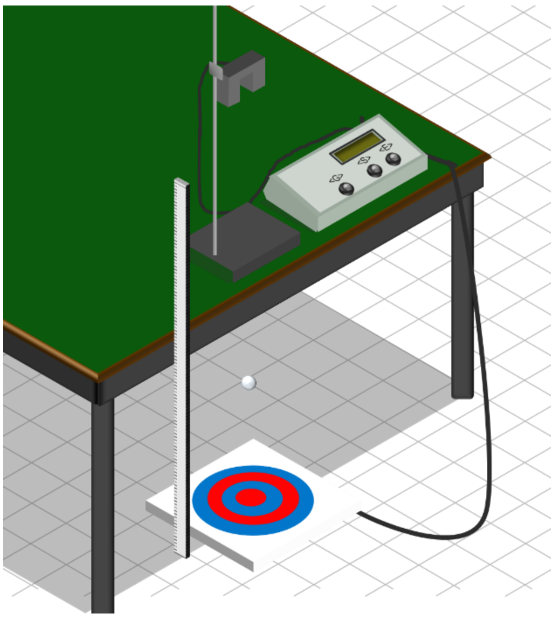


Any object in **free fall**, regardless of its mass, weight, or shape, will **accelerate** towards the ground at a rate of **9.8 ms^{-2} downwards**.

This doesn't necessarily mean it will fall straight down – the trajectory of any object in free fall depends on its initial motion.

Measuring the Acceleration of a Falling Object

If air resistance is negligible, an object is considered to be "in free fall" and therefore experiences a constant downwards acceleration of 9.8 ms^{-2}. We can test this out in the lab using the equipment shown below:



For an object experiencing a constant experiencing acceleration in one dimension, our equations of motion apply:

s is the displacement of the ball, from the height it was released to the timing plate (measured with metre stick)

u = 0 ms^{-1} (released from rest)

v is unknown

a is the acceleration of the ball – determined experimentally from a graph

t is the time of fall between release and the ball hitting the timing plate (measured with TSA)

With the quantities above we can make use of $s = ut + \frac{1}{2}at^2$. Since $u = 0\ ms^{-1}$ this can be simplified to $s = \frac{1}{2}at^2$. Plotting a graph of $2s$ vs t^2 would allow us to calculate a from the gradient.

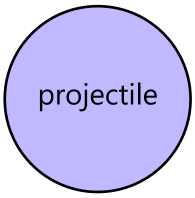
Projectiles

Another term used to describe objects which move freely, under the influence of gravity alone, is a **projectile**.

Projectile motion describes the movement of objects which are thrown or launched (or "projected") into the air, which typically follow a curved path ("trajectory"). We can understand and analyse projectile motion by breaking it down into **two independent components** – **horizontal motion** and **vertical motion**.

Horizontal motion:

Air resistance is negligible, no other applied forces during flight.



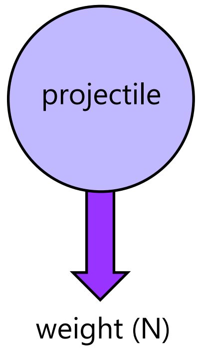
∴ horizontal forces are balanced

∴ horizontal velocity is constant

∴ apply $d = vt$

Vertical motion:

Air resistance is negligible, weight acts downwards, no other forces applied.



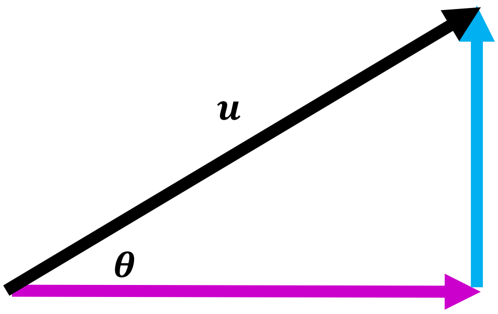
∴ vertical forces are unbalanced

∴ vertical acceleration

∴ apply $suvat$

At Higher level, we consider objects that are **projected at an angle**, as well as horizontally. Typically, a projectile will be launched at an angle. To analyse the motion of the projectile we must **split this initial velocity** into to two independent components – a **constant horizontal velocity**, and an **<u>initial</u> vertical velocity**.

Using trigonometry to split the launch velocity, $\boldsymbol{v}$, into horizontal and vertical components:



$$\boldsymbol{u}_V = \boldsymbol{u}_{opposite} = \boldsymbol{u}sin\theta$$

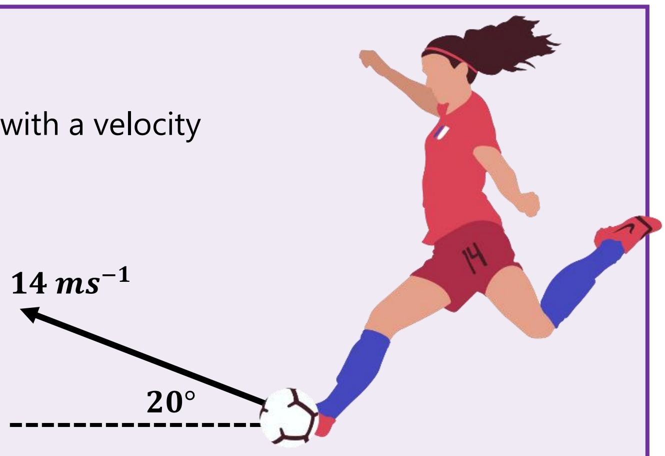
here u_V is used to represent "initial vertical velocity" – since the vertical velocity will change over time it is important to use u

$$\boldsymbol{v}_H = \boldsymbol{u}_{adjacent} = \boldsymbol{u}cos\theta$$

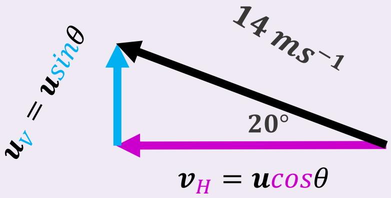
here v_H is used to represent "constant horizontal velocity" – since this component does not change there is no need to use u and v to denote initial and final velocities

Worked Example: The Football

A footballer kicks a stationary ball. The ball is launched with a velocity of 14 ms^{-1} at an angle of 20° above the horizontal.



(a) **Calculate** the horizontal component of the initial velocity of the ball.



$$v_H = 14\cos(20°)$$

$$v_H = 13\ ms^{-1}$$

(b) **Calculate** the vertical component of the initial velocity of the ball.

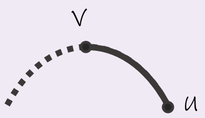
$$u_V = 14\sin(20°)$$

$$u_V = 4.8\ ms^{-1}$$

once u has been resolved into v_H and u_V we won't use it again

(c) **Calculate** the time taken for the ball to reach its maximum height.

The maximum height is at the turning point – where the vertical velocity is 0 ms^{-1}.
Since we're working vertically we'll use suvat:



$s =$
$u = 4.8\ ms^{-1}$
$v = 0\ ms^{-1}$
$a = -9.8 ms^{-2}$
$t = ?$

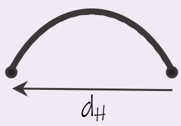
$$v = u + at$$

$$0 = 4.8 + (-9.8)t$$

$$t = \frac{-4.8}{-9.8}$$

$$t = 0.49\ s$$

(d) **Calculate** the horizontal distance travelled by the ball before it lands.



$$d = vt$$

$$d_H = 13 \times (2 \times 0.49)$$

$$d_H = 13\ m$$

assuming the pitch is level, the trajectory is symmetrical – it takes the same amount of time to come down as it did to get up

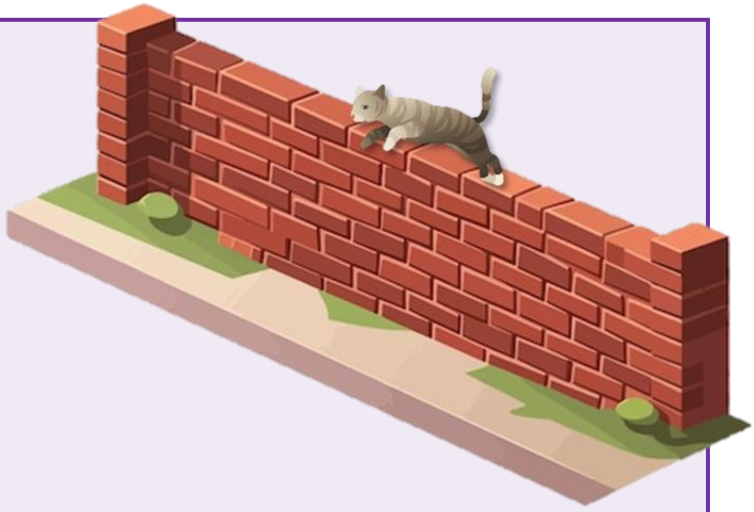
(e) The football coach offers the footballer the following advice:

"If you want the ball to travel further, you can kick it just as hard, but increase the size of the angle a little"

Explain why the coach's statement is correct.

Increasing the angle a little (not above 45°) gives the ball a greater initial vertical velocity, which will increase the time of flight. If it's in the air for longer, it will travel farther (horizontally) before it hits the ground.

Worked Example: The Cat

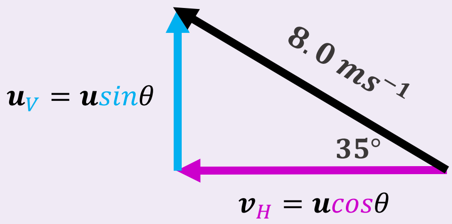


A cat jumps from a 1.2 metre wall with a velocity of 8.0 ms^{-1} at an angle of 35° above the horizontal.

It lands on the ground 1.2 seconds later.

Friction is negligible.

(a) **Calculate** the horizontal component of the initial velocity of the cat.



$$v_H = 8.0\cos(35°)$$

$$v_H = 6.6\ ms^{-1}$$

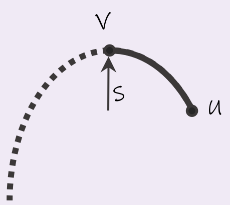
(b) **Calculate** the vertical component of the initial velocity of the cat.

$$u_V = 8.0\sin(35°)$$

$$u_V = 4.6\ ms^{-1}$$

(c) **Calculate** the maximum height, from the ground, the cat reaches.

The maximum height is at the turning point – where the vertical velocity is 0 ms^{-1}.
Since we're working vertically we'll use suvat:



$s = ?$
$u = 4.6\ ms^{-1}$
$v = 0\ ms^{-1}$
$a = -9.8 ms^{-2}$
$t =$

$$v^2 = u^2 + 2as$$

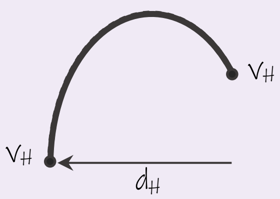
$$0^2 = 4.6^2 + (2 \times (-9.8) \times s)$$

$$s = 1.1\ m$$

The maximum height from the ground: $h_{max} = h_{wall} + s = 1.2 + 1.1 = 2.3\ m$

(d) **Calculate** the horizontal distance travelled by the cat from when it jumps off the wall to when it reaches the ground.

The horizontal velocity is constant so we can use d= vt:



$$d = vt$$

$$d_H = 6.6 \times 1.2$$

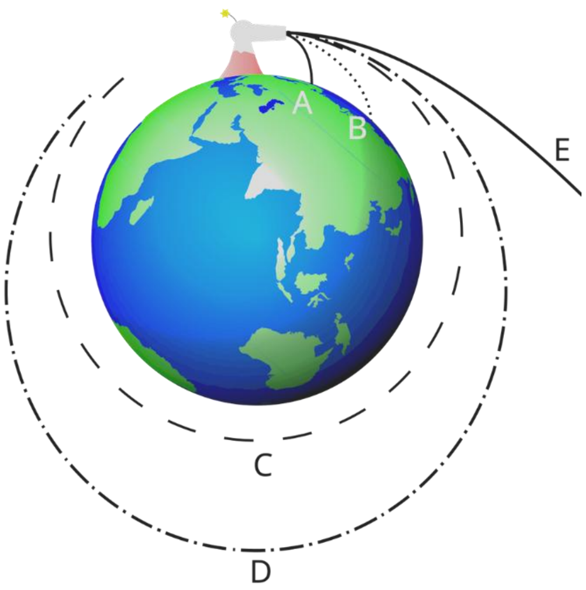
$$d_H = 7.9\ m$$

Satellite Motion

Understanding projectile motion is key to understanding satellite motion – or indeed any orbital motion (a satellite orbiting a planet, a planet orbiting a star, etc.).

All projectiles follow a curved (parabolic) path since they move with constant horizontal velocity while experiencing constant vertical acceleration due to gravity. This is true for satellites as well, although at this scale the acceleration can be considered "towards the centre of the Earth", rather than "downwards".

The image below represents "Newton's Cannon" – a thought experiment which help us connect the standard projectile motion we're familiar with to satellite motion.



A cannon on top of a *very* high mountain shoots a cannonball horizontally (launch speed increases from A to E):

A, B — Launch speed too slow: standard projectile motion. Cannonball in free fall until it hits the ground.

C, D — Launch speed just right: projectile travels so fast *horizontally* that the Earth's surface curves away from it as it falls. The projectile is in **continuous free fall**.

E — Launch speed too high: escape velocity. Object escapes the planets gravitational pull.

Newton's thought experiment described above shows us that **satellite/orbital motion** involves an object being in **continuous free fall**. To orbit something is to continuously fall towards some large mass but never reach it:

- A **satellite** (like a moon or the ISS) is in **continuous free fall** around a **planet**.
- A **planet** is in **continuous free fall** around a **star**.
- A **star** is in **continuous free fall** around the **centre of mass of a galaxy**.

Newton's Law of Universal Gravitation

In N5 Physics we learned that weight is a force an objects experiences, which is caused by the pull/attraction of a large body – e.g. the *force* with which the *Earth* pulls *you* towards *it*:

The weight of an object:

gravitational field strength at a planet's surface (N/kg)

$$W = mg$$

weight of an object (kg)

mass of an object (kg)

We learned that the gravitational field strength is different for different planets – this is because the *mass* of each planet is different. In Higher Physics we develop a deeper understanding of gravity.

Gravitation is one of the four fundamental forces (these are explored further in *Unit 2: Particles & Waves*) which are responsible for shaping the universe we inhabit. **Gravitation** means that **any** two **objects with mass** exert an **attractive force** on **each other**. The *Earth* pulls *you* towards *it*; *you* pull the *Earth* towards *you*. Both are true – you are just easier to move that the Earth is.

Newton's Law of Universal Gravitation describes the attractive force acting between any two masses:

mass of one object (kg)

mass of other object (kg)

gravitational force (N)

$$F = G\frac{m_1 m_2}{r^2}$$

Universal Constant of Gravitation ($6.67 \times 10^{-11}\ m^3 kg^{-1} s^{-2}$) [on data sheet]

distance between the centre of mass of the two objects (m)

This relationship is sometimes referred to as the *inverse square law of gravitation* ($\boldsymbol{F}$ is **inversely proportional** to $\boldsymbol{r^2}$) – e.g. if the distance between two masses is doubled, the gravitational force between them is quartered.

Worked Example: The Scientist

Earth has a mass of 5.97 × 10^{24} kg and a radius of 6370 km.

Let Newton have a mass of 75 kg.

(a) **Calculate** the magnitude of gravitational force between Earth and Newton.

$$F = G\frac{m_1 m_2}{r^2}$$

$$F = \frac{(6.67 \times 10^{-11}) \times (5.97 \times 10^{24}) \times 75}{(6370 \times 10^3)^2}$$

$$F = 740\ N$$

here the radius of the Earth is equivalent to the distance between centres of masses – note also that distance is measure in METRES (not km), and that it is squared (forgetting this is a common mistake!)

(b) **Calculate** the weight of Newton on Earth.

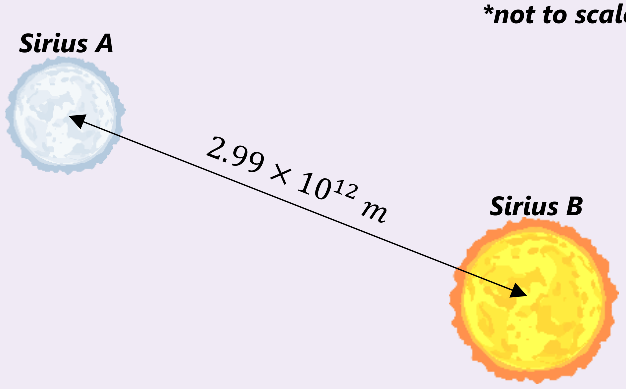
$$W = mg$$

$$W = 75 \times 9.8$$

$$W = 740\ N$$

notice than W = F_{grav} because weight IS the gravitational force that the Earth exerts on Newton

Worked Example: The Binary Stars



In a binary star system, two stars orbit around each other. The distance between the centre of mass of each is 2.99 × 10^{12} m.

Sirius A has a mass of 1.99 × 10^{30} kg and Sirius B has a mass of 4.38 × 10^{30} kg.

(a) **Calculate** the gravitational force between the two stars.

$$F = G\frac{m_1 m_2}{r^2}$$

$$F = \frac{(6.67 \times 10^{-11}) \times (1.99 \times 10^{30}) \times (4.38 \times 10^{30})}{(2.99 \times 10^{12})^2}$$

$$F = 6.5 \times 10^{25}\ N$$

(b) **Compare** how the gravitational force between these two stars would change if the distance between them was halved.

If r is **halved**, then F will be **four times greater than** its original value (since r is squared).

if unsure of this (or how you would word an answer) you can always *justify by calculation* – just use half the original value of r and do the calculation again to see what happens

SQA Course Specification Higher

Special relativity

- Knowledge that the speed of light in a vacuum is the same for all observers.

- Knowledge that measurements of space, time and distance for a moving observer are changed relative to those for a stationary observer, giving rise to time dilation and length contraction.

- Use of appropriate relationships to solve problems involving time dilation, length contraction and speed.

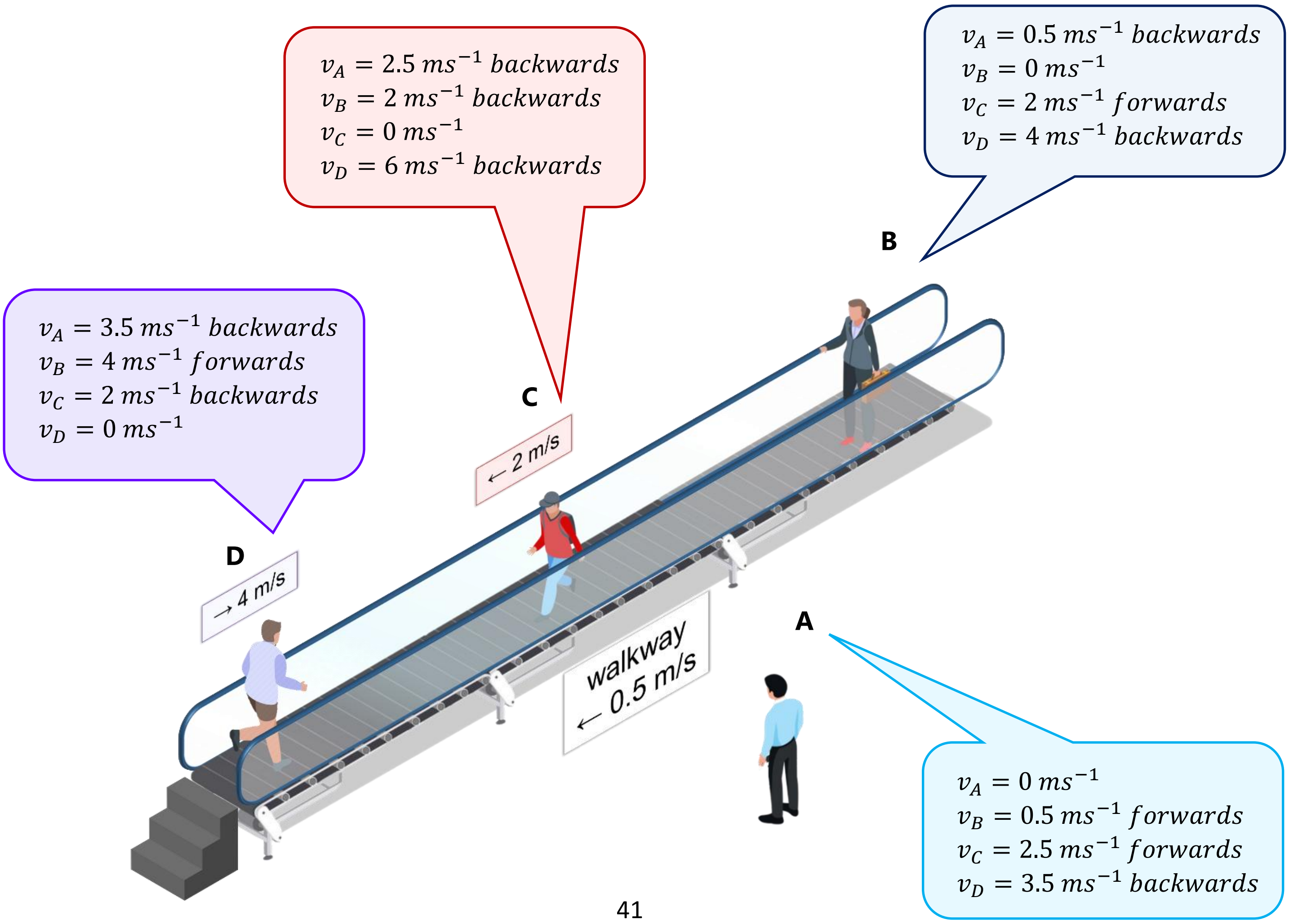
Frames of Reference

In Physics, relativity is about observing *events* (and measuring their physical quantities like distance and time) from different *frames of reference*. A *frame of reference* is like the perspective of an observer (sometimes a person) making a measurement.

Consider these two people, standing at opposite sides of the desk and reading the number drawn on the desk:

Each person reads a different number – but **both are correct** from their own **frame of reference**. This is a key aspect of relativity – two different measurements of the same event, each from a different frame of reference, can both be correct.

Consider a moving walkway and the people on/around it. The walkway itself travels forwards at a speed of 0.5 ms^{-1}. How fast each person is moving depends on the frame of reference the measurement is taken from – but all are correct in their own frame of reference.



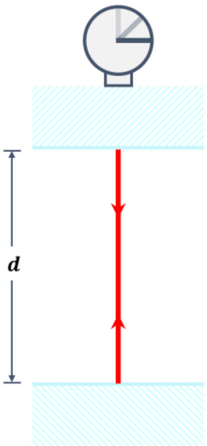
Special Relativity

So *relativity* is about measuring things from different *frames of reference*.

Special relativity means those measurements are taken under certain conditions (observers maintain a *constant speed*).

When the different observers are moving at **relativistic speeds** (speeds close to the speed of light) then strange effects occur.

Consider a theoretical device known as a *light clock* - two mirrors a constant distance, d, apart, with a pulse of light bouncing between them. Each time the light reaches the top mirror, a certain amount of time, t, has passed by:



An observer in the *same frame of reference* as the light clock (e.g. someone standing right next to it/travelling *with* it at the same speed) would measure things as expected:

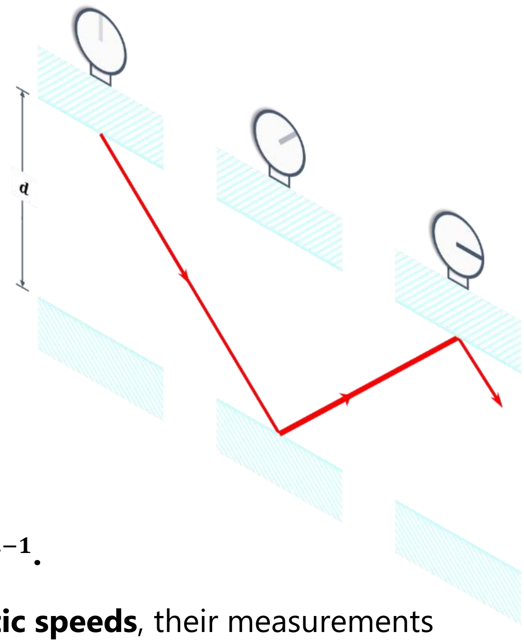
- The speed of light in a vacuum, known as c, would be $3 \times 10^8\ ms^{-1}$
- The distance the light travels between each mirror is d.
- The time taken for the light to return to the top mirror is t.

Now imagine an observer in a *different frame of reference* – someone who is moving *relative to the light clock*. Let's say the light clock is on one spaceship, and this observer is on another spaceship, travelling past in the opposite direction.

This observer looks into the other spaceship and sees the moving light clock, and as the light bounces between the mirrors it must have to follow the path shown:

This observer, in a *different reference frame* from the light clock would measure things differently:

- The speed of light in a vacuum, c, is $3 \times 10^8\ ms^{-1}$.
- The distance travelled by the light between each mirror must be greater than d.
- If c is constant, the time taken for the light to return to the top mirror must change too.



The speed of light in a vacuum is ALWAYS $3 \times 10^8\ ms^{-1}$.

Because of this, when observers/events travel at **relativistic speeds**, their measurements of **space and time are relative**. Two observers might measure the length of the same object differently – both are correct.

Time Dilation & Length Contraction

Since the **speed of light in a vacuum, c, is *always* $3 \times 10^8\ ms^{-1}$**, measurements of time and length for the same event can be different depending on the observer.

We generally consider an event, and two observers of that event:

- One observer in the **same frame of reference** as the event. They are not moving relative to the event. Their measurements are called the **proper time**, t, or the **proper length**, l.
- One observer in a **different frame of reference** as the event. They are moving relative to the event. Their measurement is the **dilated time**, t', or the **contracted length**, l'.

Both are correct. The closer to c the relative velocity between the two frames of reference, the greater the measurement of dilated time will be/smaller the measurements of contracted length will be.

For two observers in different *frames of reference* measuring the time taken for the same event:

proper time (consistent units)

$$t' = \frac{t}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

dilated time (consistent units)

relative velocity between frames of reference

speed of light in a vacuum ($3 \times 10^8\ ms^{-1}$)

For two observers in different *frames of reference* measuring the length/distance of the same event:

$$l' = l\sqrt{1 - \left(\frac{v}{c}\right)^2}$$

relative velocity between frames of reference

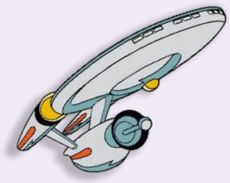
contracted length (consistent units)

proper length (consistent units)

speed of light in a vacuum ($3 \times 10^8\ ms^{-1}$)

Try to remember what the words "dilated" and "contracted" mean more generally. To "dilate" is to expand, so the dilated time should always be larger. To "contract" is to get smaller, so the contracted length should always be smaller.

Worked Example: The Starship



A starship is travelling at a speed of $0.300c$ relative to the planet. On the way past, the starship emits a signal for 10.0 seconds as measured by the technician on the starship.

(a) **Calculate** the duration of the signal as measured by an observer on the planet.

The signal emitted by the starship is the event.

The proper time, $t = 10.0\ s$ (since the technician is <u>on</u> the starship they are in the <u>same</u> frame of reference as the event).

The dilated time, t', is measured by the observer on the planet (moving relative to the starship).

$$t' = \frac{t}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

$$t' = \frac{10.0}{\sqrt{1 - \left(\frac{0.300c}{c}\right)^2}} \qquad = \frac{10.0}{\sqrt{1 - (0.300)^2}}$$

we can simplify $\left(\frac{0.300c}{c}\right)$ by cancelling out c on the top and bottom of the fraction to be left with (0.300)

$$t' = 10.5\ s$$

(b) On the way past the starship measures the diameter of the planet to be approximately $16\ 400\ km$.

Calculate the diameter of the planet as measured by an observer on the planet. .

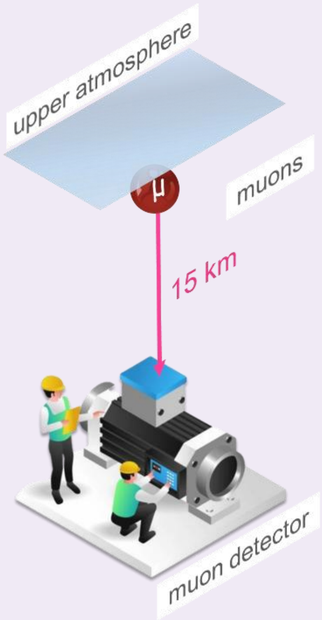
The starship is moving relative to the planet (which is being measured) so it is in a <u>different</u> frame of reference. $l' = 16\ 400\ km$. The observer <u>on</u> the planet is in the <u>same</u> frame of reference (<u>not</u> moving compared to the planet).

$$l' = l\sqrt{1 - \left(\frac{v}{c}\right)^2}$$

$$16\ 400 = l\sqrt{1 - (0.300)^2}$$

$$l = 17\ 200\ km$$

Worked Example: The Muon Detector



Muons are particles that can be produced when cosmic rays collide with the atoms of Earth's upper atmosphere, 15 km above the ground. They travel at speeds very close to c.

Muons have an average lifetime of 2.2 μs (in their own frame of reference), after which they decay into other particles. Without taking special relativity into account, they should only be able to travel an average of 650 metres before decaying.

However, we can actually detect some of these muons on the ground. Since they travel at relativistic speeds and are moving relative to the Earth, the distance between the Earth's upper atmosphere and the ground is less than 15 km (from the perspective of the muons).

(a) **Show** that muons which travel at $0.995c$ travel $1.5\ km$ between the upper atmosphere and the ground.

15 km is the length between the upper atmosphere and the Earth – as measured by someone on the Earth itself (not moving compared the measured length) so it is the proper length.

Since the muons are moving when compared to the thing being measured, they experience the contracted length.

$$l' = l\sqrt{1 - \left(\frac{v}{c}\right)^2}$$

$$l' = 15\sqrt{1 - (0.995)^2}$$

$$l' = 1.5\ km \quad \text{as required}$$

(b) A physicist states "if we measured the average lifetime of a muon at rest in our lab, we'd get 2.2 μs, but when if we measured the average lifetime of these muons coming from the upper atmosphere it would be much longer".

Explain why the statement made by the physicist is correct.

The two measurements would be taken in different frames of reference from the event.

2.2 μs would be taken from the same frame of reference – no time dilation occurs.

The measurement of the muons coming from the upper atmosphere is taken from a different frame of reference (since the physicist is moving relative to these muons) – time dilation occurs.

SQA Course Specification Higher

The expanding Universe

- Knowledge that the Doppler effect causes shifts in wavelength of sound and light.

- Use of an appropriate relationship to solve problems involving the observed frequency, source frequency, source speed and wave speed.

- Knowledge that the light from objects moving away from us is shifted to longer wavelengths (redshift).

- Knowledge that the redshift of a galaxy is the change in wavelength divided by the emitted wavelength. For slowly moving galaxies, redshift is the ratio of the recessional velocity of the galaxy to the velocity of light.

- Use of appropriate relationships to solve problems involving redshift, observed wavelength, emitted wavelength, and recessional velocity.

- Use of an appropriate relationship to solve problems involving the Hubble constant, the recessional velocity of a galaxy and its distance from us.

- Knowledge that the Hubble-Lemaître Law allows us to estimate the age of the Universe.

- Knowledge that measurements of the velocities of galaxies and their distance from us lead to the theory of the expanding Universe.

- Knowledge that the mass of a galaxy can be estimated by the orbital speed of stars within it.

- Knowledge that evidence supporting the existence of dark matter comes from estimations of the mass of galaxies.

- Knowledge that the evidence supporting the existence of dark energy comes from the accelerating rate of expansion of the Universe.

- Knowledge that the temperature of stellar objects is related to the distribution of emitted radiation over a wide range of wavelengths.

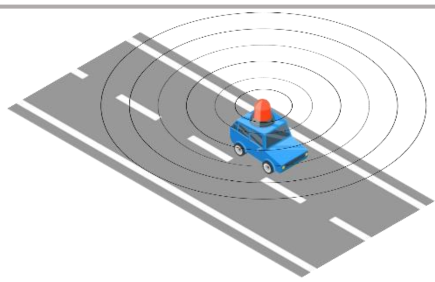
- Knowledge that the peak wavelength of this distribution is shorter for hotter objects than for cooler objects.

- Knowledge that hotter objects emit more radiation per unit surface area per unit time than cooler objects.

- Knowledge of evidence supporting the Big Bang theory and subsequent expansion of the Universe: cosmic microwave background radiation, the abundance of the elements hydrogen and helium, the darkness of the sky (Olbers' paradox) and the large number of galaxies showing redshift rather than blueshift.

The Doppler Effect

The Doppler effect is the name for the **change in observed frequency** of a wave, **when the source is moving relative to the observer**. All waves are subject to the Doppler effect, but it may be most familiar as sound.

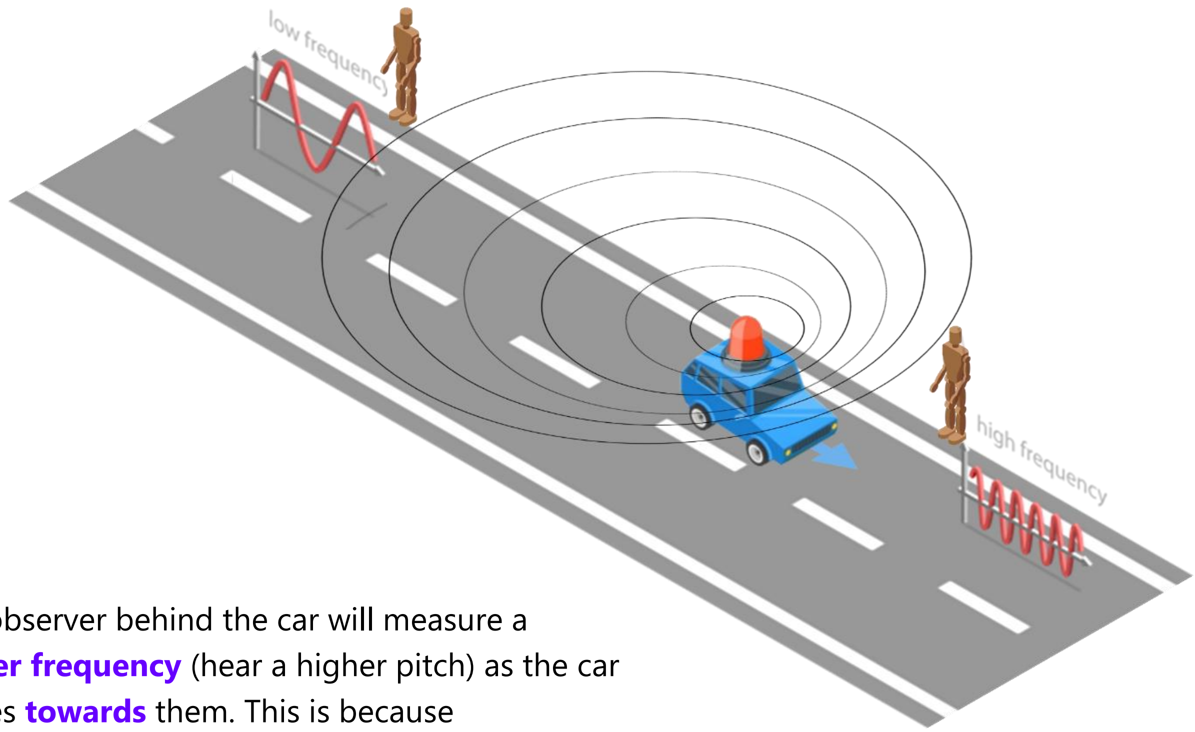


Consider a noisy race car. The engine emits sounds waves at a constant frequency (constant pitch). The driver, or any other observer who is not moving relative to the car (*in the same frame of reference*), will experience the sound as a constant frequency.

To someone standing beside the race track when the car zooms past (who *is* moving relative to the car – *different frame of reference*) the frequency of the sound varies. As the car moves **towards the observer**, the **frequency** of the sound **increases**. As the car moves **away from the observer**, the **frequency** of the sound **decreases**.

The observer behind the car will measure a **lower frequency** (hear a lower pitch) as the car moves **away from** them. This is because

- as each complete wave is emitted, the car is further away from the observer,
- the **wavefronts are further apart**/the **wavelength** of the observed sound wave has **increased**,
- the **wavefronts arrive** at the observer **less frequently**.



The observer behind the car will measure a **higher frequency** (hear a higher pitch) as the car moves **towards** them. This is because

- as each complete wave is emitted, the car is closer to the observer,
- the **wavefronts are closer together**/the **wavelength** of the observed sound wave has **decreased**,
- the **wavefronts arrive** at the observer **more frequently**.

To calculate the observed frequency, from the perspective of an observer moving relative to the source of the waves:

frequency of the source (Hz)

$$f_o = f_s\left(\frac{v}{v \pm v_s}\right)$$

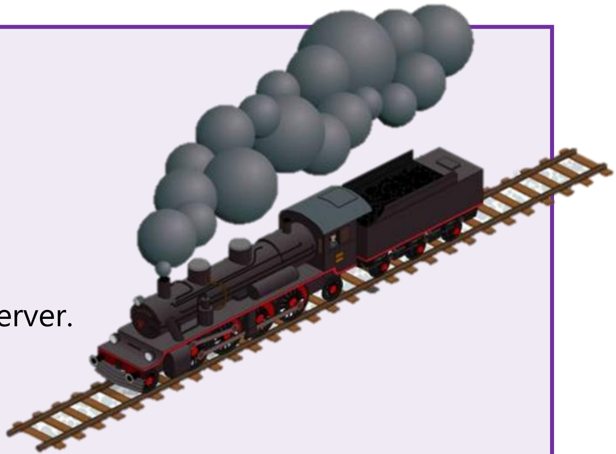
observed frequency (Hz)

speed of the source (ms^{-1})

speed of the wave (ms^{-1})
(e.g. 340 ms^{-1} for sound waves in air)

The denominator is "plus or minus" – this depends on whether the source if moving away from the observer or towards them. Remember: **towards**, **takeaway**; **away from**, **add**.

Worked Example: The Steam Train



An observer is watching a steam train approach at 23 ms^{-1}.
The whistle on the train emits a sound of frequency 450 Hz.
The speed of sound in air is 340 ms^{-1}.

(a) **Calculate** the frequency of the sound heard by the observer.

The train is "approaching" – moving toward the observer so the ± would be a takeaway.

$$f_o = f_s\left(\frac{v}{v \pm v_s}\right)$$

$$f_o = 450\left(\frac{340}{340 - 23}\right)$$

$$f_o = 480\ Hz$$

(b) **State** the frequency of the whistle heard by the train driver. **Justify** your answer.

450 Hz because they are not moving relative to the source of the sound waves.

(c) The train continues to sound its whistle as is passes the observer.
Explain why the frequency of the sound heard by the observer decreases as the train moves away from them.

As the train moves away from the observer, the wavefronts are further apart so they arrive less frequently. Since $f = \frac{N}{t}$, the observed frequency decreases.

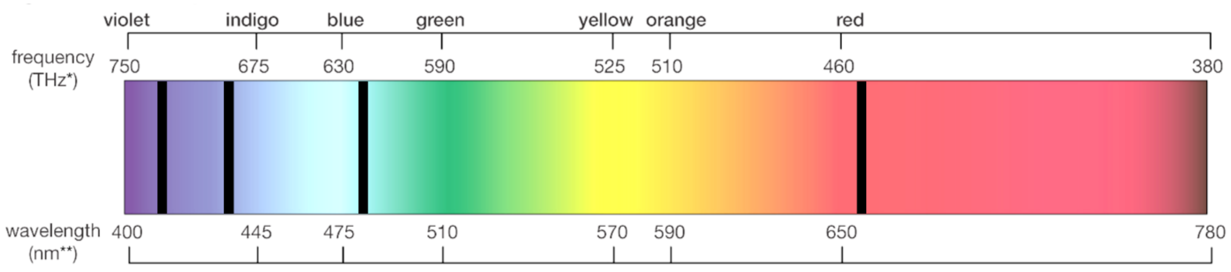
Redshift

The Doppler effect also applies to light waves. Similar changes in frequency/wavelength are observed when we look at light from stars/galaxies that are *moving relative to us*.

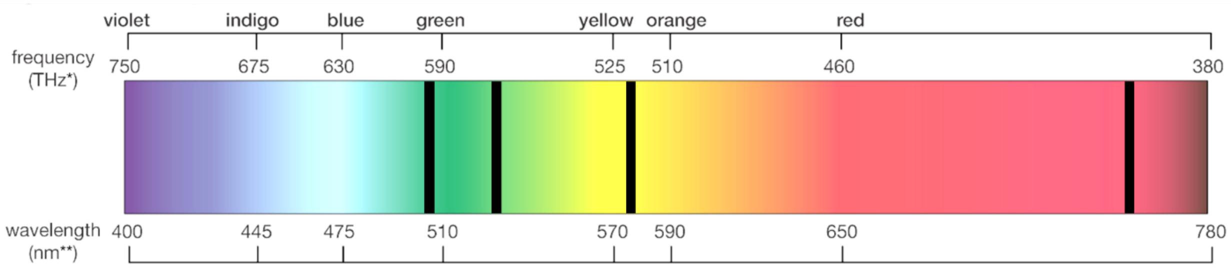
As we learned in National 5 Physics, each element in the period table emits/absorbs a unique set of wavelengths of visible light (known as a line spectrum). By observing the spectral lines from a star, and comparing those lines with that of known elements (such as hydrogen) we can determine what elements make up a star.

If a distant star is moving relative to us, we can use these spectral lines to determine the relative velocity between us and the star, and whether it is moving away from us or towards us.

When we look at the spectral lines of hydrogen on Earth (source not moving relative to observer) we may see:



Look at the spectral lines for a distant star we may see:



The pattern of lines is the same (matching with what we know are the wavelengths absorbed by hydrogen) *however* for each line in the spectrum the wavelength has increased. An **increase in observed wavelength** (or **decrease in observed frequency**) means the source of the waves must be **moving *away*** from the observer.

For a source of light moving *away* from us, all wavelengths appear longer, *shifted* towards the *red* side of the visible spectrum – this is known as *redshift*. *Blueshift* could also be used to describe a source that is moving *towards* an observer, but this is less frequently observed – the majority of stars/galaxies in the observable universe are moving *away* from us.

Redshift as a quantity describes the relative change in wavelength observed, and can be calculated by:

wavelength observed from moving source (m or other consistent unit)

$$z = \frac{\lambda_{observed} - \lambda_{rest}}{\lambda_{rest}}$$

redshift (no units)

known wavelength at rest (m or other consistent unit)

Positive values of redshift mean the source is moving *away* from the observer. Negative values of redshift means the source if moving *towards* the observer.

For "slowly" moving ($v \ll c$) objects, we can determine their relative velocities using:

velocity of star/galaxy (ms^{-1})

$$z = \frac{v}{c}$$

redshift (no units)

speed of light in a vacuum ($3.00 \times 10^8\ ms^{-1}$)

v is often referred to as the "recessional velocity" as most galaxies tend to be *receding* (moving away) from us.

Why are most galaxies moving away from us?

This observation is our first piece of evidence to suggest that the *universe is expanding* – galaxies move farther apart from each other because the very fabric if space is stretching as the universe expands.

Worked Example: The Runaway Galaxy

Messier 101, known as the Pinwheel Galaxy, is approximately 20 million light-years away from Earth.

An astrophysicist identifies a hydrogen spectral line emitted by Messier 101 at 672 nm. The same line on a hydrogen spectrum emitted by the Sun is 656 nm.

Calculate the recessional velocity of Messier 101.

Since the Sun is not moving away from/towards the observer, the wavelength of light from the Sun (656 nm) is the rest wavelength.

Since Messier 101 is moving relative to the observer, 672 nm is the wavelength of light observed from a moving source.

With this data we can calculate the value of redshift:

$$z = \frac{\lambda_o - \lambda_r}{\lambda_r}$$

since these values of wavelength are ALL in nanometres, we don't need to convert into metres here

$$z = \frac{672 - 656}{656}$$

$$z = 0.024 \ldots$$

since redshift is not the final answer, this number should remain unrounded (the "..." communicates that this number has not been rounded)

Then use this redshift to determine the recessional velocity of Messier 101:

$$z = \frac{v}{c}$$

again the "..." communicates that we have used the unrounded value of redshift

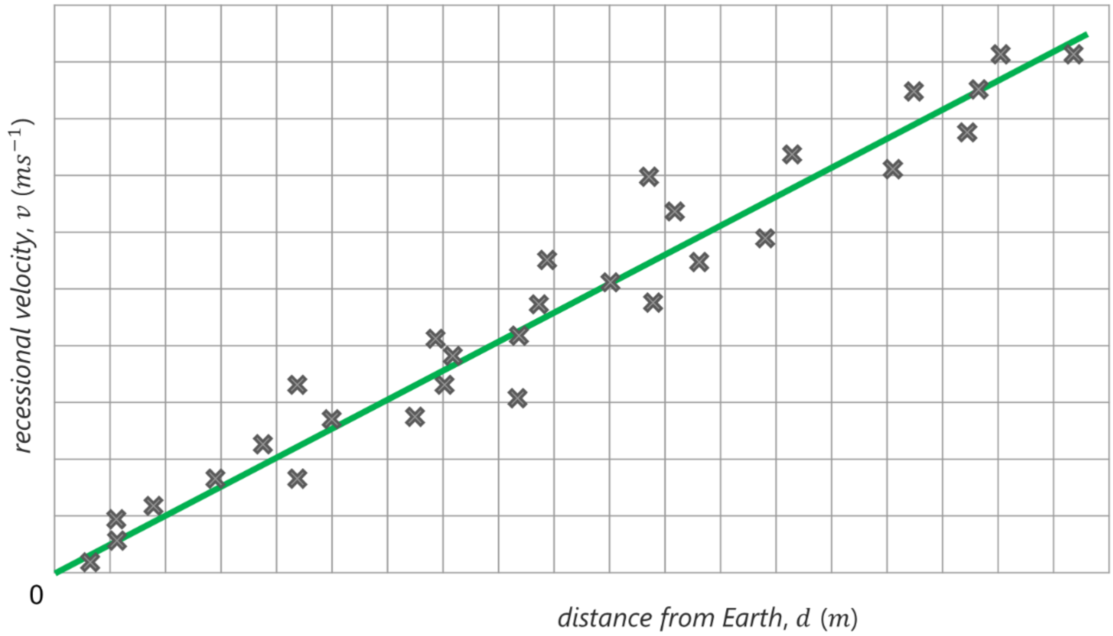
$$0.024 \ldots = \frac{v}{3 \times 10^8}$$

$$v = 7.32 \times 10^6\ ms^{-1}$$

Since the value of redshift (and recessional velocity) is positive, we know that Messier 101 is moving away from us at this velocity.

Hubble's Law

Edwin Hubble was an American astronomer who measured the recessional velocity of galaxies against the distance of each galaxy from Earth. Plotting recessional velocity against distance from Earth produces a graph like the one shown below:



From this graph we can observe:

- The general trend is that galaxies farther from Earth have greater recessional velocities.
- Since the *line of best fit is a straight line* we could say that recessional velocity of galaxies and their distance from Earth have a *linear relationship*.
- Since the *line of best fit is a <u>straight line that intersects the origin</u>* we could say that that recessional velocity of galaxies and their distance from Earth have are *<u>directly proportional</u>*.

From the final analysis point, we can apply $y = mx$ to this data to give $v = md$. The gradient, m, of this line of best fit is known as Hubble's constant. The relationship is known as Hubble's Law, or the Hubble-Lemaître Law (Lemaître was a Belgian astronomer who independently came to the same conclusion as Hubble).

Hubble's Law is given as:

$$v = H_0 d$$

recessional velocity of galaxy (ms^{-1})

distance of galaxy from Earth (m)

Hubble's constant $(2.3 \times 10^{-18}\ s^{-1})$ [on data sheet]

What does this tell us about the universe?

Hubble's Law shows that the farther away from Earth a galaxy is, the faster it is moving – further supporting the scientific theory that the *universe is expanding*.

There are a few common analogies that can help us to visualise and understand the expansion of the universe:

A balloon:

Inflation of the balloon represents the expansion of universe.

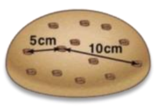
Good things: the balloon stretches, as does space. The distance between the galaxies on the balloon increases, as it does in reality. The EM waves drawn on the balloon stretch – reflecting the increase in observed wavelength we see in the light from distant galaxies.

Limitations: the galaxies themselves stretch, which does not happen in reality. The balloon has a limit – blow it up too much and it will burst – whereas there's nothing to suggest this will happen to the universe.

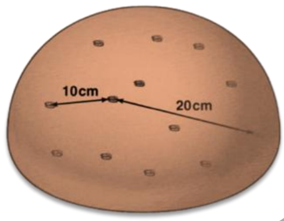
A loaf of bread:

Imagine an unbaked loaf of raisin bread. The dough represents space and the raisins are the galaxies. When the bread is baked, the loaf expands. The raisins spread out but remain the same size.

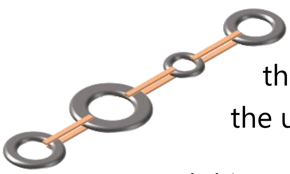
Good things: like the balloon, this shows the expansion of the universe. This could be considered a better analogy since the galaxies spread apart without changing size themselves. Every raisin/galaxy experiences the same thing – every other raisin moving away from it, the more distant, the faster they recede.



Limitations: similar to the balloon, the bread expands to take up the space of the air/oven around it. The universe *is everything that exists* so it doesn't expand into anything, it *just expands*.

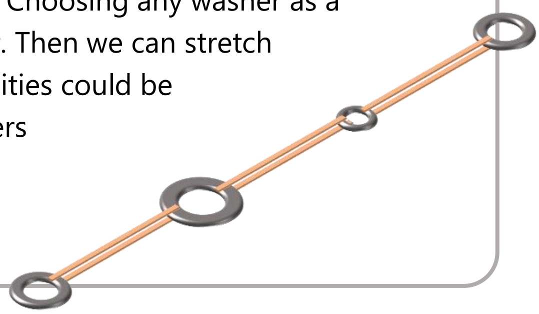


Elastic bands and washers:



The washers represent galaxies and the rubber bands the space between them. Pulling the washers apart from each end shows the stretching of space as the universe expands.

Good things: This model allows us to make measurements. Choosing any washer as a reference point, we can measure the distance to any other washer. Then we can stretch the system and measure the increased distance. Recessional velocities could be calculated using $v = \frac{\Delta d}{t}$ – this would show that more distant washers (representing galaxies) have greater recessional velocities.



Limitations: this model is one-dimensional, space is not.

Calculating the age of the universe

So as time passes, the universe expands – if we could turn back time all the way back that would suggest that the universe expanded from a single point (known as a singularity). Using the distance from Earth and recessional speed of any galaxy within the data set we can calculate the age of the universe.

A distant galaxy is a distance, d, from Earth, and has a recessional velocity, v.

We need to make the assumption that this recessional velocity, v, has been constant for all of time.

How long has it been since the distant galaxy was in the same place as the Earth (when time began, as the singularity)? We can calculate the time taken for that journey using:

$$t = \frac{d}{v}$$

We know d and v are related by Hubble's Law, so we can substitute in $v = H_0 d$:

$$t = \frac{d}{H_0 d}$$

...and simplify...

$$t = \frac{1}{H_0}$$

Then substituting in the value of Hubble's constant:

$$t = \frac{1}{2.3 \times 10^{-18}} = 4.35 \times 10^{17} s$$

Or, expressed in years:

$$t = 4.35 \times 10^{17} s$$

$$t = \left(\frac{4.35 \times 10^{17}}{60 \times 60 \times 24 \times 365.25}\right) years = 1.38 \times 10^{10}\ years$$

Which can be written as 13.8 *billion years* – the *estimated* age of the universe.

This method has limitations – we've assumed a constant recessional velocity (and therefore a constant rate of expansion but physicists now think this is not the case).

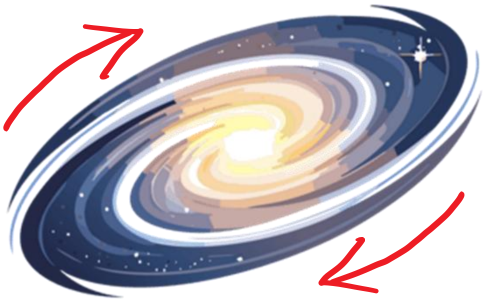
Dark Matter & Dark Energy

Dark Energy

Hubble's and Lemaître's observations led to the understanding that the universe is expanding, but teams of scientists later observed that the *rate* of expansion is increasing. Something must *cause* this ***accelerated rate expansion*** – something known as ***dark energy***.

The nature of dark energy is still a bit of an unsolved mystery of modern physics. We know what is does – drives the accelerating rate of expansion – but not what it *is*. As the name *dark* matter suggests, we cannot observe it directly as it does not interact with light.

Dark Matter



Consider our galaxy, the Milky Way. Physicists currently estimate that it is made up of around 100-400 billion stars. The galaxy is spinning - the stars that make up the Milky Way orbit the centre of mass.

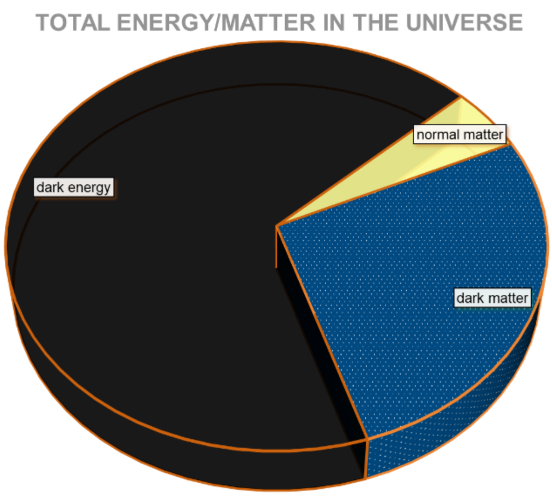
The ***mass of a galaxy*** is related to the ***orbital speed of stars*** within it – galaxies of larger mass tend to rotate faster. However, the Milky Way (and all other galaxies) seem to rotate faster (and therefore must have more mass) than we would expect them to based on the mass we can observe. This leads to the conclusions that there must be mass present that we cannot observe – ***dark matter***.

How much of the universe is a mystery?

A lot.

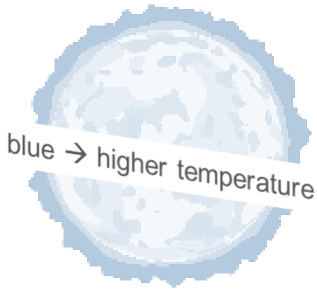
From what physicists know about the early universe (and the *shape* of the universe!), there is a specific total amount of matter and energy that exists.

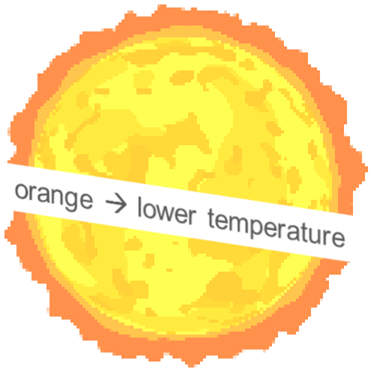
What we can *observe* (the bits that interact with light) makes up around 5%. Measurements of dark matter (using methods such as galaxy rotation described above) make up around 23%. The rest – 68% - must be dark energy.



Stellar Temperature

We can figure out a lot about the universe just by looking at the stars – all of this information comes from the light we observe.

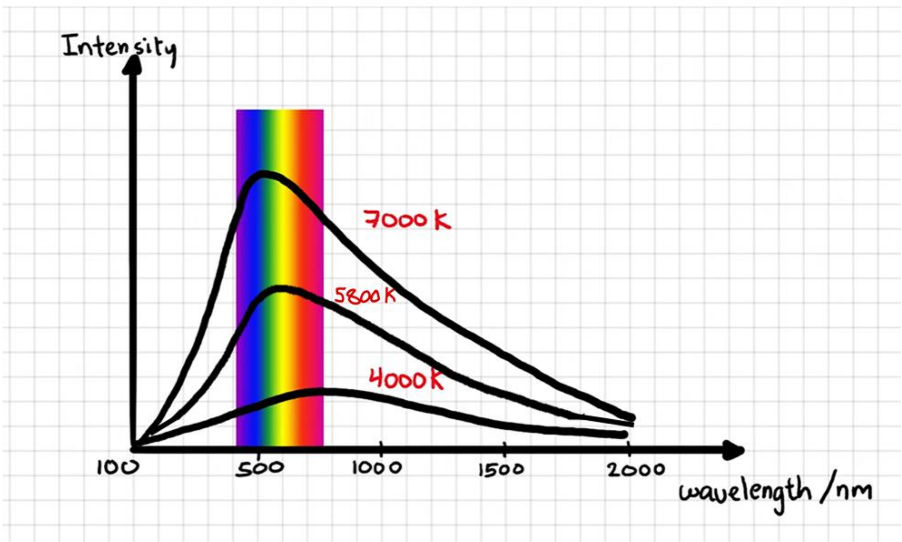




The light (visible and other EM radiation) that a star emits can also tell us a lot about the star itself Consider the way metals glow "red hot" when heated. Further heating would cause a metal to glow orange, then yellow, then white. Or a Bunsen burner – the roaring blue flame is hotter than the yellow safety flame.

Stars are similar – there is a relationship between the temperature of a star and the wavelengths (colours) of light it emits. This means we can look at the light emitted from a star to determine its temperature.

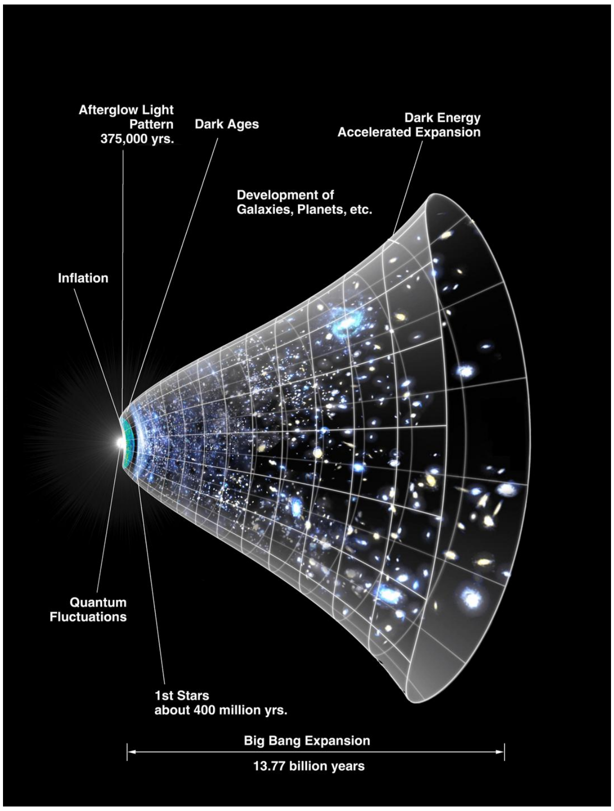
The graph below shows the intensity of radiation emitted (sometimes also referred to as "radiation per unit surface area per unit time") varies with the wavelength of the emitted radiation for 3 different stars (one star with a surface temperature of 4000 K, one of 5800 K, and one of 7000 K):



From this graph we can conclude:

- Stars emit radiation over a wide range of wavelengths, including but not limited to the visible spectrum.
- For ***all* wavelengths**, the **intensity** of emitted radiation is **greater for hotter stars** *(e.g. the 7000 K is above the 5800 K line at all points).*
- The **peak intensity** of emitted radiation is at **shorter wavelengths for hotter stars** *(e.g. the highest point the 7000 K line is further left on the graph than the highest point on the 5800 K line).*

The Big Bang Theory



The Big Bang Theory is the leading scientific explanation for the origin of the universe. It describes that universe expanded rapidly outwards from an initial state of high density and temperatures (a singularity).

A theory, in science, is more than just a guess. A scientific theory is a possible explanation for something – one which is based on strong evidence and is thoroughly tested.

Some evidence that supports the Big Bang Theory:

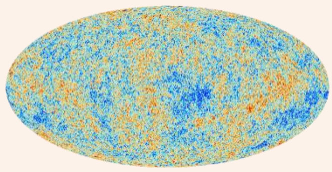
- The expansion of the universe (evidenced by galactic redshift).
- Cosmic microwave background radiation.
- The abundance of hydrogen and helium.
- The darkness of the sky (Olbers' paradox).

The expanding universe…

…which in turn is evidenced by the majority of galaxies moving *away* from us (showing redshift) rather than *towards* us (showing blueshift). Winding the clock back all the way would suggest that the universe expanded from a single point (known as a singularity).

Cosmic Microwave Background Radiation (CMBR)…

…which is EM radiation leftover from the Big Bang. In every direction we look, we can observe microwaves. This fits with the theory of the early universe consisting of hot, dense plasma emitted EM radiation, which has now been stretched to microwaves as the universe has expanded.



The abundance of hydrogen and helium…

…which are detected in amounts that are only possible due to something like the Big Bang. We observe much more helium in the universe than the calculated amount produced by stars – this is possible if the early universe was in a hot, dense state.

The darkness of the sky…

…which only makes sense if the universe *had a beginning*. In an infinite, eternal universe, a dark sky would be impossible. Light from infinite stars in every direction would have had infinite amount of time to reach us.

BUT the universe has a finite age. Light only travels so fast, and we are only able to observe light from stars that has had enough time to reach the Earth.