National
Qualifications
2017

2017 Mathematics of Mechanics

Advanced Higher

Finalised Marking Instructions

General marking principles for Advanced Higher Mathematics of Mechanics

This information is provided to help you understand the general principles you must apply when marking candidate responses to questions in this Paper. These principles must be read in conjunction with the detailed marking instructions, which identify the key features required in candidate responses.

For each question the marking instructions are generally in two sections, namely Illustrative Scheme and Generic Scheme. The illustrative scheme covers methods which are commonly seen throughout the marking. The generic scheme indicates the rationale for which each mark is awarded. In general, markers should use the illustrative scheme and only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.

(a) Marks for each candidate response must always be assigned in line with these general marking principles and the detailed marking instructions for this assessment.

(b) Marking should always be positive. This means that, for each candidate response, marks are accumulated for the demonstration of relevant skills, knowledge and understanding: they are not deducted from a maximum on the basis of errors or omissions.

(c) If a specific candidate response does not seem to be covered by either the principles or detailed Marking Instructions, and you are uncertain how to assess it, you must seek guidance from your Team Leader.

(d) Credit must be assigned in accordance with the specific assessment guidelines.

(e) One mark is available for each •. There are no half marks.

(f) Working subsequent to an error must be **followed through**, with possible credit for the subsequent working, provided that the level of difficulty involved is approximately similar. Where, subsequent to an error, the working for a follow through mark has been eased, the follow through mark cannot be awarded.

(g) As indicated on the front of the question paper, full credit should only be given where the solution contains appropriate working. Unless specifically mentioned in the marking instructions, a correct answer with no working receives no credit.

(h) Candidates may use any mathematically correct method to answer questions except in cases where a particular method is specified or excluded.

(i) As a consequence of an error perceived to be trivial, casual or insignificant, eg $6\times 6=12$ candidates lose the opportunity of gaining a mark. However, note the second example in comment **(j)**.

(j) Where a transcription error (paper to script or within script) occurs, the candidate should normally lose the opportunity to be awarded the next process mark, eg

This is a transcription error and so the mark is not awarded.

Eased as no longer a solution of a quadratic equation so mark is not awarded.

$$x^2+5x+7=9x+4$$
$$x-4x+3=0$$
$$x=1$$

Exceptionally this error is not treated as a transcription error as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.

$$x^2+5x+7=9x+4$$
$$x-4x+3=0$$
$$(x-3)(x-1)=0$$
$$x=1 \text{ or } 3$$

(k) **Horizontal/vertical marking**

Where a question results in two pairs of solutions, this technique should be applied, but only if indicated in the detailed marking instructions for the question.

Example:

	$\bullet^5$	$\bullet^6$
$\bullet^5$	$x=2$	$x=-4$
$\bullet^6$	$y=5$	$y=-7$

Horizontal: $\bullet^5$ $x=2$ and $x=-4$ Vertical: $\bullet^5$ $x=2$ and $y=5$
$\bullet^6$ $y=5$ and $y=-7$ $\bullet^6$ $x=-4$ and $y=-7$

Markers should choose whichever method benefits the candidate, but **not** a combination of both.

(l) In final answers, unless specifically mentioned in the detailed marking instructions, numerical values should be simplified as far as possible, eg:

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$ $\qquad$ $\frac{43}{1}$ must be simplified to 43

$\frac{15}{0\cdot3}$ must be simplified to 50 $\qquad$ $\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*

*The square root of perfect squares up to and including 100 must be known.

(m) Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

(n) Unless specifically mentioned in the marking instructions, the following should not be penalised:

- Working subsequent to a correct answer
- Correct working in the wrong part of a question
- Legitimate variations in numerical answers/algebraic expressions, eg angles in degrees rounded to nearest degree
- Omission of units
- Bad form (bad form only becomes bad form if subsequent working is correct), eg $(x^3+2x^2+3x+2)(2x+1)$ written as $(x^3+2x^2+3x+2)\times 2x+1$

  $2x^4+4x^3+6x^2+4x+x^3+2x^2+3x+2$ written as $2x^4+5x^3+8x^2+7x+2$ gains full credit

- Repeated error within a question, but not between questions or papers

(o) In any 'Show that…' question, where the candidate has to arrive at a required result, the last mark of that part is not available as a follow-through from a previous error unless specified in the detailed marking instructions.

(p) All working should be carefully checked, even where a fundamental misunderstanding is apparent early in the candidate's response. Marks may still be available later in the question so reference must be made continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that the candidate has gained all the available marks.

(q) Scored-out working which has not been replaced should be marked where still legible. However, if the scored out working has been replaced, only the work which has not been scored out should be marked.

(r) Where a candidate has made multiple attempts using the same strategy and not identified their final answer, mark all attempts and award the lowest mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

(s) Any rounded answer should be accurate to three significant figures (or one decimal place for angles given in degrees) unless otherwise stated. If an answer differs due to rounding or prior rounding the candidate may be penalised. Only penalise one mark in any question.

Detailed marking instructions for each question

Question			Generic scheme	Illustrative scheme	Max mark
1.			•¹ Resolve forces in two perpendicular directions •² Use $F = ma$ with substitution •³ Find acceleration on slope •⁴ Use equations of motion to find velocity after 75 metres	•¹ $R = mg\cos\theta$ $mg\sin\theta - \mu R = ma$ •² $F = ma:\ mg\sin\theta - \mu mg\cos\theta = ma$ $\frac{g}{4} - \frac{g}{8}\times\frac{\sqrt{15}}{4} = a$ •³ $a = 1\cdot264 \quad [1\cdot26]$ •⁴ $v^2 = u^2 + 2as$ $= 2\times1\cdot264\times75$ $v = 13\cdot8\,\mathrm{ms}^{-1}$	4

Notes:

•⁴ **Accept** $13\cdot7\,\mathrm{ms}^{-1}$

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
Alternative Solution:					
1.			•[1] State initial values of kinetic energy and potential energy •[2] Calculate work done against friction •[3] Use work energy principle with substitution •[4] Value of speed after 75 metres	•[1] At top of the slope $\varepsilon_k + \varepsilon_P$ $0 + m \times g \times 75 \sin\theta$ $= \frac{75mg}{4}$ At bottom of slope $\varepsilon_K = \frac{1}{2}mv^2$ •[2] $W = \mu N \times 75$ $N = mg\cos\theta$ •[3] $\frac{1}{8} \times \frac{\sqrt{15}mg}{4} \times 75 = \frac{75\sqrt{15}}{32}$ $\frac{1}{2}mv^2 = \frac{75mg}{4} - \frac{75\sqrt{15}}{32}$ •[4] $v^2 = \frac{75g}{2} - \frac{75\sqrt{15}g}{16}$ $v = 13 \cdot 8\,\mathrm{ms}^{-1}$	4

Notes:

•[3] and •[4] can be found using definite integration

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
2.	(a)		**Quotient Rule:** •¹ correct use of quotient rule with one term correct •² numerator and denominator correct •³ **fully** simplify	•¹ $f'(x)=\dfrac{2x^2\times\frac{1}{x}-\ldots\ldots\ldots\ldots}{\ldots\ldots\ldots\ldots\ldots}$ •² $f'(x)=\dfrac{2x^2\times\frac{1}{x}-\ln x\times(4x)}{4x^4}$ •³ $f'(x)=\dfrac{2x-4x\ln x}{4x^4}=\dfrac{1-2\ln x}{2x^3}$	**3**
Notes:					
Commonly Observed Responses:					
Alternative solution					
			Product rule: •¹ express as product and start differentiation correctly •² complete differentiation correctly •³ **fully** simplify	•¹ $f(x)=\frac{1}{2}(\ln x)\left(x^{-2}\right)$ $f'(x)=\frac{1}{2}(\ln x)(-2x^{-3})\ldots\ldots\ldots$ •² $f'(x)=\frac{1}{2}(\ln x)(-2x^{-3})+\frac{1}{2}x^{-2}\left(\frac{1}{x}\right)$ •³ $f'(x)=\left(\frac{-\ln x}{x^3}\right)+\frac{1}{2}\left(\frac{1}{x^3}\right)=\frac{1-2\ln x}{2x^3}$	
Notes:					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
2.	(b)		•¹ 1st application of chain rule •² 2nd application of chain rule •³ Substitution for y and complete solution	•¹ $\frac{dy}{dx} = 2\text{cosec}3x \times 3.......$ •² $\frac{dy}{dx} = 2\text{cosec}3x \times 3 \times -\text{cosec}3x\cot 3x$ $\frac{dy}{dx} = -6y\cos\text{ec}^2 3x\cot 3x$ •³ $= -6y\cot 3x$ $\frac{dy}{dx} + 6y\cot 3x = 0$	3

Notes:

Accept $\frac{dy}{dx} = -6y\cot 3x$

Commonly Observed Responses:

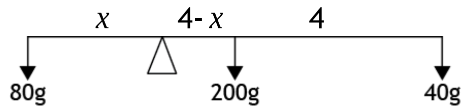
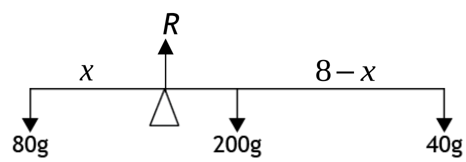
Question			Generic scheme	Illustrative scheme	Max mark
3.			•[1] Evidence of differentiation to give expression for acceleration •[2] Correct expression •[3] Substitution to give acceleration in vector form •[4] Magnitude of acceleration	•[1] $\mathbf{v} = (3\sin 2t)\mathbf{i} + (\cos 2t - 3)\mathbf{j}$ $\mathbf{a} = \frac{d\mathbf{v}}{dt} = \ldots\mathbf{i} + \ldots\mathbf{j}$ •[2] $\mathbf{a} = \frac{d\mathbf{v}}{dt} = (6\cos 2t)\mathbf{i} - (2\sin 2t)\mathbf{j}$ •[3] $t = \frac{\pi}{6} : \mathbf{a} = 6\cos\frac{\pi}{3}\mathbf{i} - 2\sin\frac{\pi}{3}\mathbf{j}$ $[\mathbf{a} = 3\mathbf{i} - \sqrt{3}\mathbf{j}]$ •[4] $\lvert\mathbf{a}\rvert = \sqrt{12} = 2\sqrt{3}\,\text{ms}^{-2}$ $[3\cdot 46\,\text{ms}^{-2}]$	4

Notes

•[1] can be implied in •[2]

•[4] Accept $\sqrt{12}\,\text{ms}^{-2}$

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
4.			•1 model the situation by considering the forces acting on the beam and the distances of each from the pivot	•1 	4
			•2 state moments about pivot anticlockwise	•2 $80gx$	
			•3 state moments about pivot clockwise, and equate	•3 $80gx = 200g(4-x) + 40g(8-x)$	
			•4 solve and interpret answer	•4 $320gx = 1120g$ $[320x = 1120]$ $x = 3{\cdot}5$ The support is positioned $3{\cdot}5\,\mathrm{m}$ from A	
Notes:					
Commonly Observed Responses:					
Alternative Solution:					
			•1 model the situation by considering the forces acting on the beam and the distances of each from the pivot	•1 	4
			•2 Use vertical equilibrium	•2 $R = 80g + 200g + 40g$ $R = 320g$	
			•3 Taking moments about *A*	•3 $320g \times x = 200g \times 4 + 40g \times 8$	
			•4 solve and interpret answer	•4 $320x = 800 + 320$ $x = 3{\cdot}5$ The support is positioned $3{\cdot}5\,\mathrm{m}$ from A	
Notes:					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
5.			•¹ Write as a sum of fractions	•¹ $\frac{A}{x-3}+\frac{Bx+C}{x^2+5}$ $A\left(x^2+5\right)+\left(Bx+c\right)\left(x-3\right)=3x^2+4x+17$	**4**
			•² Rewrite equation with no denominator	•² $A=4$	
			•³ Calculate two constants	•³ $B=-1 \ or \ C=1$	
			•⁴ Calculate final value and rewrite original function as sum of partial fractions	•⁴ $\frac{4}{x-3}+\frac{1-x}{x^2+5}$	

Notes:

•¹ if incorrect can only achieve 2 marks

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
6.			•[1] consider vertical forces •[2] consider forces radially •[3] combine equations and substitute for known quantities •[4] find the value of the coefficient of friction	•[1] Diagram to show forces acting or $F = mg = \mu R$ •[2] $\mu R = mg$ $R = mr\omega^2$ •[3] $\mu mr\omega^2 = mg$ $\mu(3\cdot5)(4)^2 = g$ •[4] $\mu = 0\cdot175$	**4**

Notes:
•[1] can be implied by •[2] or •[3]

Commonly Observed Responses:
A diagram was drawn showing balanced forces. This would not allow for a centripetal force.

Question			Generic scheme	Illustrative scheme	Max mark
7.			•[1] Use range to find expression for time of flight	•[1] $R = u\cos\theta \times t$ $60 = u\cos 28° \times t$	5
			•[2] use equations of motion with constant acceleration vertically	•[2] $s = ut + \frac{1}{2}at^2$ $0 = u\sin 28° - \frac{g}{2}t^2$	
			•[3] Rearrange to give expression for *t* and substitution	•[3] $t = \frac{60}{u\cos 28°}$ $0 = u\sin 28° \times \frac{60}{u\cos 28°} - \frac{g}{2}\left(\frac{60}{u\cos 28°}\right)^2$	
			•[4] process algebra	•[4] $\frac{g}{2}\left(\frac{60}{u\cos 28°}\right)^2 = 60\tan 28°$	
			•[5] find initial speed	•[5] $u = 26\cdot6\,\mathrm{ms}^{-1}$	

Notes:	
Commonly Observed Responses:	

Question			Generic scheme	Illustrative scheme	Max mark
8.			•[1] Find expression for momentum before collision	•[1] $0 \cdot 2 \times 6 + 0 \cdot 5 \times 3$	6
			•[2] apply conservation of linear momentum	•[2] $0 \cdot 2 \times 6 + 0 \cdot 5 \times 3 = 0 \cdot 5 \times v_y$	
			•[3] calculate speed of Y after collision	•[3] $v_y = 5 \cdot 4 \, \mathrm{ms}^{-1}$	
			•[4] use Newton's second law to calculate deceleration of Y	•[4] $-mg \sin 30 = ma$ $a = -4 \cdot 9 \, \mathrm{ms}^{-2}$ $-mg \sin 30 = ma$	
			•[5] use appropriate equation of motion and substitute	•[5] $0^2 = 5 \cdot 4^2 + 2 \times (-4 \cdot 9) \times s$	
			•[6] calculate distance travelled before coming to rest and communicate result relative to B	•[6] $s = 2 \cdot 98 \, \mathrm{m}$ which is 52cm below B	

Notes
For mark •[6] conclusion must be stated

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
9.	(a)		•[1] Find resultant force •[2] Work done by variable force with substitution •[3] Integrate function •[4] Calculate work done	•[1] Resultant force = $(249-50\sqrt{x})-\mu R$ $=249-50\sqrt{x}-0\cdot25\times20g)$ $=200-50\sqrt{x}$ •[2] Work done = $\int_0^{10}(200-50\sqrt{x})dx$ •[3] $\left[200x-\frac{100}{3}x^{\frac{3}{2}}\right]_0^{10}$ •[4] 946 J	**4**
Notes					
Commonly Observed Responses:					
	(b)		•[5] Work done equated to change in energy with substitution •[6] Value of speed after 10 metres	•[5] $\frac{1}{2}(20)v_{10}{}^2-\frac{1}{2}(20)12^2=945\cdot9$ •[6] $v_{10}=15\cdot4\,\text{ms}^{-1}$	
Notes					
Commonly Observed Responses:					

Question	Generic scheme	Illustrative scheme	Max mark
Alternative Solution: This solution does the question in reverse and so cannot be split into (a) and (b)			
9. (a) +(b)	•¹ Find resultant force	•¹ Resultant force = $(249-50\sqrt{x})-\mu R$ $=249-50\sqrt{x}-0\cdot25\times20g)$ $=200-50\sqrt{x}$	**6**
	•² Set up differential equation	•² $F=ma:\ 20v\dfrac{dv}{dx}=(200-50\sqrt{x})$	
	•³ separate the variables	•³ $\int vdv=\int(10-\frac{5}{2}\sqrt{x})dx$	
	•⁴ Obtain the general equation for velocity at any time	•⁴ $\dfrac{v^2}{2}=10x-\dfrac{5}{3}x^{\frac{3}{2}}+c$ $x=0,v=12\Rightarrow c=72$ $\dfrac{v^2}{2}=10x-\dfrac{5}{3}x^{\frac{3}{2}}+72$	
	•⁵ Value of speed after 10 metres	•⁵ $v_{10}=15\cdot4\,\text{ms}^{-1}$	
	•⁶ Work done equated to change in energy with substitution	•⁶ $\frac{1}{2}(20)v_{10}{}^2-\frac{1}{2}(20)12^2=945\cdot9\,\text{J}$	

Notes:

•³ and •⁴ can be found using definite integration

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
10.			•1 Integrate one function and differentiate other •2 Correct choice of functions for the process •3 Correct expression for integral •4 Second integration by parts •5 Substitution and final answer	$f(x)=x^2 \quad g'(x)=\sin 5x$ •1 •2 $f'(x)=2x \quad g(x)=-\frac{1}{5}\cos 5x$ •3 $I=-\frac{x^2}{5}\cos 5x+\frac{2}{5}\int x\cos 5x dx$ •4 $\int x\cos 5x dx=\frac{x}{5}\sin 5x-\frac{1}{5}\int \sin 5x dx$ $=\frac{x}{5}\sin 5x+\frac{1}{25}\cos 5x dx$ •5 $I=\frac{-x^2}{2}\cos 5x+\frac{2}{5}\left(\frac{x}{5}\sin 5x+\frac{1}{25}\cos 5x\right)+c$ $=\left[\left(\frac{2}{125}-\frac{x^2}{5}\right)\cos 5x+\frac{2x}{25}\sin 5x+c\right]$	**5**

Notes:
•5 Do not penalise omission of constant

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
11.			•[1] Differentiate $3y^2$ and 4 •[2] Differentiate product •[3] Expression for derivative •[4] Substitute for x and find **two** values of y •[5] Choose correct value for y and substitute x value and y value to obtain gradient.	•[1] $6y\frac{dy}{dx}$ and 0 •[2] $-2xy - x^2\frac{dy}{dx}$ •[3] $\frac{dy}{dx} = \frac{2xy}{6y - x^2}$ $x = 2 \quad 3y^2 - 4y - 4 = 0$ •[4] $y = -\frac{2}{3};\ y = 2$ •[5] $\frac{dy}{dx} = \frac{2\cdot 22}{6\cdot 2 - 2^2} = 1$	**5**

Notes:

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
12.	(a)		•[1] Consider body in equilibrium and Hooke's Law •[2] Evaluate equilibrium extension	•[1] $T = mg : \frac{150e}{0\cdot5} = 0\cdot75g$ •[2] $e = 0\cdot0245 = 2\cdot45\,\text{cm}$	**2**
Notes:					
Commonly Observed Responses:					
	(b)	(i)	•[3] Apply $F = ma$ vertically •[4] Apply Hooke's Law in extension with substitution •[5] Complete to prove SHM	•[3] $mg - T = m\ddot{x}$ •[4] $mg - \frac{150(0\cdot0245 + x)}{0\cdot5} = m\ddot{x}$ $g - 400(0\cdot0245 + x) = \ddot{x}$ •[5] $-400x = \ddot{x}$ [SHM with $\omega = 20$]	**3**
Notes:					
Commonly Observed Responses:					
		(ii)	•[6] Use correct equation for speed with substitution •[7] Find the value of speed	•[6] $a = 0\cdot02 \quad \omega = 20 \quad x = 0\cdot015$ $v^2 = \omega^2(a^2 - x^2) \Rightarrow v^2 = 400(0\cdot02^2 - 0\cdot015^2)$ •[7] $v = 0\cdot265[\text{ms}^{-1}]$	**2**
Notes:					
Commonly Observed Responses:					
	(c)		•[8] Statement about extension that allows tension in the string	•[8] $3\,\text{cm} > 2\cdot45\,\text{cm}$ so string is not in tension throughout.	**1**
Notes:					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
13.	(a)		•[1] Use Newton's law of gravitation and $\boldsymbol{F = ma}$ at earth's surface •[2] Create second equation for satellite and combine •[3] Interpret condition for gravity •[4] Find expression for height of satellite	•[1] $mg = \frac{GMm}{R^2} \quad \Rightarrow GM = gR^2$ •[2] $GM = g_s(R+h)^2 \quad g_s(R+h)^2 = gR^2$ •[3] $g_s = \frac{1}{9}g$ $\frac{1}{9}g(R+h)^2 = gR^2 \Rightarrow r = 3R$ $\frac{1}{9}g(R+h)^2 = gR^2 \quad (R+h)^2 = 9R^2$ •[4] $R+h = 3R$ $h = 2R$	4
Notes:					
Commonly Observed Responses:					
Alternative solution					
			•[1] Use Newton's inverse square law and $\boldsymbol{F = ma}$ at earth's surface •[2] Create second equation for satellite and combine •[3] Interpret condition for gravity •[4] Find expression for height of satellite	•[1] $a = \frac{l}{r^2} : \ g = \frac{k}{R^2} \quad k = gR^2$ •[2] $\frac{1}{9}g = \frac{k}{(R+h)^2}$ •[3] $\frac{1}{9}g = \frac{gR^2}{(R+h)^2}$ $\frac{1}{9}g(R+h)^2 = gR^2 \quad (R+h)^2 = 9R^2$ •[4] $R+h = 3R$ $h = 2R$	
Notes:					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
13.	(b)		•[5] Use Newton's Law of Gravitation and circular motion at surface •[6] Equate with expression from (a) and substitute for r •[7] Complete proof	•[5] $\frac{GMm}{r^2} = mr\omega^2 \Rightarrow GM = r^3\omega^2$ •[6] $gR^2 = r^3\omega^2$ $gR^2 = (4R)^3\omega^2$ •[7] $\omega^2 = \frac{gR^2}{64R^3} \Rightarrow \omega = \frac{1}{8}\sqrt{\frac{g}{R}}$	**3**
Notes					
Commonly Observed Responses:					
Alternative solution					
			•[5] Use Newton's inverse square law and circular motion at surface •[6] Equate with expression from (a) and substitute for r •[7] Complete proof	•[5] $g = \frac{k}{R^2}$ $k = gR^2$ •[6] $a = \frac{k}{(4R)^2} = \omega^2(4R)$ •[7] $\omega^2 = \frac{gR^2}{64R^3} \Rightarrow \omega = \frac{1}{8}\sqrt{\frac{g}{R}}$	
Notes:					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
14.	(a)	(i)	•[1] Comment about **i** and **j** as **unit** vectors •[2] Specify **i** as in direction of East and **j** as in direction of North	•[1] As per Generic Scheme •[2] As per Generic Scheme	**2**
Notes					
Commonly Observed Responses:					
		(ii)	•[3] obtain equations for the velocity of boat A and boat B •[4] state initial positions and obtain equations for the positions of boat A and boat B at time t •[5] obtain equation for position of boat A relative to boat B	•[3] $v_A = 10\sin 60\mathbf{i} + 10\cos 60\mathbf{j} = 5\sqrt{3}\mathbf{i} + 5\mathbf{j}$ $v_B = -10\sqrt{3}\sin 30\mathbf{i} + 10\sqrt{3}\cos 30\mathbf{j} = -5\sqrt{3}\mathbf{i} + 15\mathbf{j}$ •[4] $r_A = 0\mathbf{i}$ and $r_B = 12\mathbf{i}$ $\Rightarrow$ after time t $r_A = 5\sqrt{3}t\mathbf{i} + 5t\mathbf{j}$ $r_B = (12 - 5\sqrt{3}t)\mathbf{i} + 15t\mathbf{j}$ •[5] ${}_Ar_B = r_A - r_B$ ${}_Ar_B = (5\sqrt{3}t - (12 - 5\sqrt{3}t))\mathbf{i} + (5t - 15t)\mathbf{j}$ $= (10\sqrt{3}t - 12)\mathbf{i} - 10t\mathbf{j}$	**3**
Notes					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
14.	(b)		•6 obtain expression for the magnitude relative distance between boats A and B •7 Equate distance expression to 7km •8 Obtain quadratic equation in standard form •9 Solve quadratic equation to find values for t •10 State time interval rounded to nearest minute	•6 $\lvert {}_{A}r_{B} \rvert = \sqrt{(10\sqrt{3}t-12)^2+(10t)^2}$ •7 $400t^2-240\sqrt{3}t+144<49$ $400t^2-240\sqrt{3}t+144=49$ •8 $400t^2-240\sqrt{3}t+95=0$ •9 $t=\dfrac{240\sqrt{3}\pm\sqrt{20800}}{800}$ $t=0\cdot339\,\text{hours}\ [20\cdot4\,\text{mins}]$ $t=0\cdot700\,\text{hours}\ [42\cdot0\,\text{mins}]$ •10 22 minutes	**5**
Notes					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
15.	(a)		•[1] Statement of the total force	•[1] $\frac{P}{v}-\frac{mkv^2}{6}$	**2**
			•[2] Use of $F=ma$ with use of $mv\frac{dv}{dx}$	•[2] $mv\frac{dv}{dx}=\frac{6P-mkv^3}{6v}$	
Notes					
Commonly Observed Responses:					
	(b)		•[3] Separation of variables to prepare for integration	•[3] $\int dx=\int\frac{6mv^2}{6P-mkv^3}dv$	**4**
			•[4] explicit term for x	•[4] $x=\frac{-2}{k}\ln\left\|6P-mkv^3\right\|+c$	
			•[5] Substitute initial values to find c or use definite integral	•[5] $0=\frac{-2}{k}\ln(6P)+c$ $c=\frac{2}{k}\ln 6P$	
			•[6] Expression for the displacement	•[6] $x=\frac{2}{k}\ln 6P-\frac{2}{k}\ln(\left\|6P-mkv^3\right\|$ $\left[x=\frac{2}{k}\ln\left\|\frac{6P}{6P-mkv^3}\right\|\right]$	
Notes:					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
16.			•¹ identify $\int P dt$ and its integration	•¹ $\int \frac{-1}{t} dt = -\ln t$	5
			•² integrating factor	•² $e^{-\ln t} = e^{\ln\frac{1}{t}} = \frac{1}{t}$	
			•³ multiply through by IF and state derivative	•³ $\frac{1}{t}\frac{dv}{dt} - \frac{v}{t^2} = \frac{3}{t}$ $\frac{d}{dt}(\frac{v}{t}) = \frac{3}{t}$	
			•⁴ integrate to give expression for v	•⁴ $\frac{v}{t} = 3\ln t + c$ $[v = 3t\ln t + ct]$	
			•⁵ use initial conditions to find c and state full expression for velocity	•⁵ $c = 5 \Rightarrow v = 3t\ln t + 5t$	

Notes

•⁴ must include constant.

•⁵ must be expression for v and not $\frac{v}{t}$.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
17.	(a)		•[1] resolve forces perpendicular to the slope	•[1] $R = mg\cos\theta \quad [R = 12g\frac{\sqrt{7}}{4} = 3\sqrt{7}g]$	**5**
			•[2] resolve forces parallel to the slope	•[2] $ma = mg\sin\theta - \mu R$	
			•[3] combine equations to give an expression for the acceleration.	•[3] $a = g\sin\theta - \mu g\cos\theta$ $a = \frac{(3-\sqrt{7}\mu)g}{4}$	
			•[4] use appropriate equation of motion with substitution.	•[4] $v^2 = u^2 + 2as$ $100 = 25 + \frac{2(3-\sqrt{7}\mu)gs}{4}$	
			•[5] algebraic manipulation to give the required expression.	•[5] $\frac{(3-\sqrt{7}\mu)gs}{2} = 75$ $s = \frac{150}{(3-\sqrt{7}\mu)g}$	

Notes

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
17.	(b)		•[6] consider motion of the body down slope with resisting forces	•[6] $ma = mg\sin\theta - \mu R - 260\cos\theta$	**6**
			•[7] consider equilibrium perpendicular to slope	•[7] $R = 260\sin\theta + 12g\cos\theta$	
			•[8] combine equations and substitute in values to get an expression for acceleration	•[8] $12a = 12g\sin\theta - \mu(260\sin\theta + 12g\cos\theta) - 260\cos\theta$ $12a = -83\cdot8 - 273\mu$ $a = -6\cdot98 - 22\cdot75\mu$	
			•[9] substitute expression for acceleration into equation of motion with original distance now halved	•[9] $v^2 = u^2 + 2as$ $0 = 100 + 2(-6\cdot98 - 22\cdot75\mu)\times\dfrac{75}{(3-\sqrt{7}\mu)g}$	
			•[10] Simplify equation	•[10] $-100(3-\sqrt{7}\mu)g = 150(-6\cdot98 - 22\cdot75\mu)$	
			•[11] complete solution to find value of μ	•[11] $\mu = 0\cdot32$	

Notes

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
Alternative Solution:					
17.	(b)		•¹ Statement of force acting down slope	•¹ $F=(mg\sin\theta-\mu mg\cos\theta)$	**6**
			•² Change in kinetic energy	•² $\frac{1}{2}mv^2-\frac{1}{2}mu^2=450$	
			•³ use of work/energy principle	•³ $(mg\sin\theta-\mu mg\cos\theta)\times s=450$	
			•⁴ substitution of exact values	$450=(9g-3\sqrt{7}\mu g)\times s$ •⁴ $s=\dfrac{450}{(9-3\sqrt{7}\mu)g}=\dfrac{150}{g(3-\sqrt{7}\mu)g}$	
			•⁵ algebraic manipulation to give required answer	$(3-\sqrt{7}\mu)gs=150$ •⁵ $s=\dfrac{150}{(3-\sqrt{7}\mu)g}$	
			•⁶ Resolve forces acting down the slope	•⁶ $(mg\sin\theta-260\cos\theta-\mu R)$	
			•⁷ Equilibrium of forces perpendicular to slope to give expression for R with substitution	•⁷ $R=260\sin\theta+mg\cos\theta$ $(mg\sin\theta-260\cos\theta-260\mu\sin\theta-\mu mg\cos\theta)\times\frac{1}{2}s=-600$	
			•⁸ set up equation from the work/energy principle	•⁸ $(mg\sin\theta-260\cos\theta-\mu R)\times\frac{1}{2}s=\frac{1}{2}mv^2-\frac{1}{2}mu^2$	

Question			Generic scheme	Illustrative scheme	Max mark
Alternative Solution continued:					
17.	(b)		•⁹ substitute in expression for displacement down slope	•⁹ $(-83\cdot8-273\mu)\times\dfrac{75}{(3-\sqrt{7}\mu)g}=-600$	
			•¹⁰ Process algebra	•¹⁰ $75(-83\cdot8-273\mu)=-600(3-\sqrt{7}\mu)g$	
			•¹¹ calculate value of μ	•¹¹ $\mu=0\cdot32$	
Notes					
Commonly Observed Responses:					

[END OF MARKING INSTRUCTIONS]