2022 Mechanics

① a) $m = 8kg$ $\quad a = \frac{65}{8} = 8.125 \, ms^{-2}$

$F = 65N$

$t = 1.2 \, sec$ $\quad V = u + at$

$u = 0 \, ms^{-1}$ $\quad v = 8.125 \times 1.2$

$$\underline{\underline{V = 9.75 \, ms^{-1}}}$$

b)

$p = mV$ $\quad p = -78 \, kgms^{-1}$

$p = 8 \times 9.75$

$p = 78 \, kgms^{-1}$

$\therefore \underline{\underline{J = 156 \, kgms^{-1}}}$

② $f(x) = \dfrac{2 - 3x - x^2}{(1+x)(1-x)^2} = \dfrac{A}{1+x} + \dfrac{B}{1-x} + \dfrac{C}{(1-x)^2}$

$$2 - 3x - x^2 = A(1-x)^2 + B(1+x)(1-x) + C(1+x)$$

let $x = 1$ $\quad 2 - 3 - 1 = C(1+1)$

$$2C = -2$$

$$\underline{\underline{C = -1}}$$

let $x = -1$ $\quad 2 + 3 - 1 = A(4)$

$$4A = 4$$

$$\underline{\underline{A = 1}}$$

$$2 - 3x - x^2 = (1-x)^2 + B(1-x)(1+x) + (-1)(1+x)$$

$$= 1 - 2x + x^2 + B(1-x^2) - x - 1$$

$$= (1-B)x^2 + (-3x) + (B)$$

$$\underline{\underline{B = 2}}$$

$$h(x) = \frac{1}{1+x} + \frac{2}{1-x} - \frac{1}{(1-x)^2}$$

③

a) $u_v = 25 \sin 30° = \frac{25}{2}$

$u_h = 25 \cos 30° = \frac{25\sqrt{3}}{2}$

At max $V_v = 0$

$$V^2 = u^2 + 2as$$

$$0 = \left(\frac{25}{2}\right)^2 - 2gs$$

$$s = \frac{\left(\frac{25}{2}\right)^2}{2g}$$

$$s = 7.971\ldots$$

$$\underline{\underline{s = 7.97\,m}}$$

b) $s_y = 1m$

$s = ut + \frac{1}{2}at^2$

$1 = \frac{25}{2}t - \frac{g}{2}t^2$

$4.9t^2 - \frac{25}{2}t + 1 = 0$

$t = \dfrac{\frac{25}{2} \pm \sqrt{\left(-\frac{25}{2}\right)^2 - (4 \times 4.9 \times 1)}}{9.8}$

$t = 2.468\ldots$ or $0.0826\ldots$

$= 2.47\,s$ will be time as it falls.

$S_x = 25\cos 30° \times 2.47$

$= 53.4411\ldots$

$= 53.4\,m$

④ $V_{max} = 6ms^{-1}$ $t = 4secs$ a) period $= 16$ secs

b) $V_{max} = \omega A$ $\omega = \frac{2\pi}{T} = \frac{2\pi}{16} = \frac{\pi}{8}$

$6 = \frac{\pi}{8}A$

$A = \frac{8 \times 6}{\pi} = 15.278\ldots$

$= 15.3\,m$

⑤ $u = 5\,ms^{-1}$ $t = 0$

$$\frac{d^2x}{dt^2} + \frac{dx}{dt} - 6x = 0$$

Aux Equ. $m^2 + m - 6 = 0$

$(m+3)(m-2) = 0$

$m = -3$ $m = 2$

$$x = Ae^{-3t} + Be^{2t}$$

$$\frac{dx}{dt} = -3Ae^{-3t} + 2Be^{2t}$$

$$\frac{d^2x}{dt^2} = 9Ae^{-3t} + 4Be^{2t}$$

$\cancel{9Ae^{-3x}}$ $\frac{dx}{dt} = u$ when $t = 0$.

$\Rightarrow -3Ae^{-3(0)} + 2Be^{2(0)} = 5$

$-3A + 2B = 5.$ ①

$x = 0$ when $t = 0$

$0 = Ae^{-3(0)} + Be^{2(0)}$

$A + B = 0.$ ② $\times 3$

$3A + 3B = 0$ ③

① + ③

$5B = 5$

$B = 1$

$\Rightarrow A = -1$

$$x = -e^{-3t} + e^{2t}$$

$\cancel{x = \frac{1}{?} e^{-3t} + e^{2t}}$.

(6) $u = u$
$a = a$
$t = 0$

a) $V = \int a \, dt$

$V = at + C$

$u = a \cdot 0 + C$

$C = u$

$\Rightarrow \underline{\underline{V = u + at}}$ as required.

b) $s = \int_0^t v \, dt$

$s = \int_0^t u + at \, dt$

$s = \left[ut + \frac{1}{2}at^2 \right]_0^t$

$s = ut + \frac{1}{2}at^2 - 0$

$\underline{\underline{s = ut + \frac{1}{2}at^2}}$

(7) $\int 18x \sin 3x \, dx$

$u = 18x$ $\quad v' = \sin 3x$

$u' = 18$ $\quad v = -\frac{1}{3} \cos 3x$

$= 18x\left(-\frac{1}{3}\cos 3x\right) - \int 18 \cdot -\frac{1}{3}\cos 3x \, dx$

$= -6x\cos 3x + 6 \int \cos 3x \, dx$

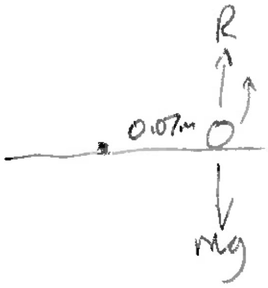
$= -6x\cos 3x + 6\left(\frac{1}{3}\sin 3x\right) + C$

$\underline{\underline{= -6x\cos 3x + 2\sin 3x + C}}$

(8) 15 rps = 900 rpm $\qquad r = 0.07\,m$

$$\omega = \frac{900 \times 2\pi}{60}$$

$$\omega = 30\pi \text{ rad/s}$$



$$F_r = \mu R = \mu mg$$

$$F_c = m\omega^2 r$$
$$= m(30\pi)^2 \cdot 0.07$$

In equilibrium

$$F_c = F_r$$

$$\cancel{m}(30\pi)^2 \cdot 0.07 = \mu \cancel{m} g$$

$$\mu = \frac{(30\pi)^2 \cdot 0.07}{g}$$

$$= 0.63447$$

$$\underline{\underline{\mu = 0.634}}$$

(9) $y = \dfrac{6x}{3x^3-1}$ $\quad x=1 \quad x=2$

$$V = \int \pi y^2 \, dx$$

$$V = \int_1^2 \pi \left(\frac{6x}{3x^3-1}\right)^2 dx$$

$$V = \int_1^2 \pi \frac{36x^2}{(3x^3-1)^2} \, dx$$

$u = 3x^3-1 \quad \dfrac{du}{dx} = 9x^2 \quad x=1 \quad u = 3-1 = 2$

$dx = \dfrac{du}{9x^2} \quad x=2 \quad u = 3 \cdot 8 - 1 = 23$

$$V = \int_{\cancel{1}\,2}^{\cancel{2}\,23} \pi \frac{\overset{4}{\cancel{36}}\cancel{x^2}}{u^2} \cdot \frac{du}{\cancel{9}\cancel{x^2}}$$

$$= \int_2^{23} \pi \cdot \frac{4}{u^2} \, du$$

$$= 4\pi \int_2^{23} u^{-2} \, du$$

$$= 4\pi \left[-\frac{1}{u}\right]_2^{23}$$

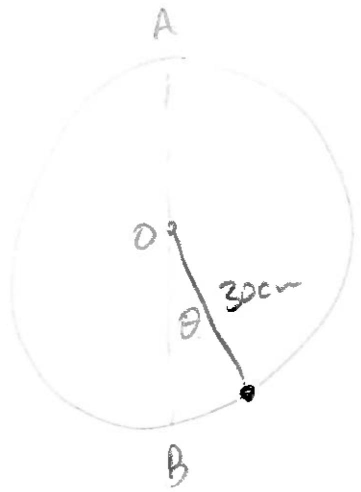
$$= 4\pi \left[-\frac{1}{23} + \frac{1}{2}\right]$$

$$= 4\pi \left[-\frac{2}{46} + \frac{23}{46}\right)$$

$$= \frac{4\pi \cdot 21}{46} = \frac{42\pi}{23} \text{ units}^3$$

(10)

a)



$m = 0.1 kg$

$r = 30 cm = 0.3 m$

$V_A = 1.2 ms^{-1}$

At A

Total Energy $= 0.1 \times g \times 0.6 + \frac{1}{2} 0.1 \times 1.2^2$

$= 0.66 J.$

A B $E_p = 0$

$\Rightarrow \frac{1}{2} m v^2 = 0.66$

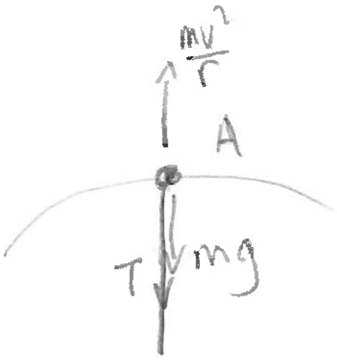
$\frac{1}{2} \times 0.1 \times v^2 = 0.66$

$v^2 = \frac{0.66}{\frac{1}{2} \times 0.1}$

$v = \sqrt{\frac{0.66}{\frac{1}{2} \times 0.1}}$

$V = \frac{\sqrt{330}}{5} = \underline{\underline{3.63 ms^{-1}}}$

b)



$T + mg = \frac{mv^2}{r}$

$T = \frac{mv^2}{r} - mg$

$T = \frac{0.1 \times 1.2^2}{0.3} - 0.1 \times 9.8$

$\underline{T = -\frac{1}{2} N}$

The rod is in compression as the downward force is greater the the centrifugal force

c) $T = 0.$

$$\Rightarrow \quad mg\cos\theta + \frac{mv^2}{r} = 0$$

$$\Rightarrow \quad \frac{\cancel{m}v^2}{r} = -\cancel{m}g\cos\theta$$

$$v^2 = -rg\cos\theta$$

At C $\quad E_k = \frac{1}{2}mv^2 \qquad E_p = mgh$

$x = 0.3\cos\theta$

$h = 0.3 - x$

$= 0.3 - 0.3\cos\theta$

$= 0.3(1-\cos\theta)$

$= 0.3mg(1-\cos\theta)$

At C $\quad \frac{1}{2}\cancel{m}v^2 + 0.3\cancel{m}g(1-\cos\theta) = \frac{1}{2}\cancel{m}v_B^2$

$$0.3g(1-\cos\theta) = \frac{1}{2}v_B^2 - \frac{1}{2}v^2$$

$$0.3g(1-\cos\theta) = \frac{1}{2}v_B^2 + \frac{1}{2}(rg\cos\theta)$$

$$0.6g - 0.6g\cos\theta = v_B^2 + rg\cos\theta$$

$$rg\cos\theta + 0.6g\cos\theta = 0.6g - v_B^2$$

$$(rg + 0.6g)\cos\theta = 0.6g - v_B^2$$

$$\cos\theta = \frac{0.6g - v_B^2}{rg + 0.6g}$$

$$= \frac{0.6g - 3.63^2}{0.3g + 0.6g}$$

$$= -0.8273\ldots$$

$$\theta = 145.82\ldots = 146^\circ$$

11

$$\frac{dy}{dx} - \frac{y}{x} = xe^{2x}$$

$$I(x) = e^{\int -\frac{1}{x}dx} = e^{-\ln x} = e^{\ln x^{-1}} = \frac{1}{x} \qquad Q(x) = xe^{2x}$$

General Solution

$$I(x)\,y = \int I(x)\,Q(x)\,dx$$

$$\frac{1}{x}y = \int \frac{1}{x}\cdot xe^{2x}\,dx$$

$$= \int e^{2x}\,dx$$

$$\frac{1}{x}y = \frac{1}{2}e^{2x} + C$$

$$y = \frac{3}{2}e^{2} \qquad x = 1$$

$$\Rightarrow \frac{1}{1}\cdot\frac{3}{2}e^{2} = \frac{1}{2}e^{2} + C$$

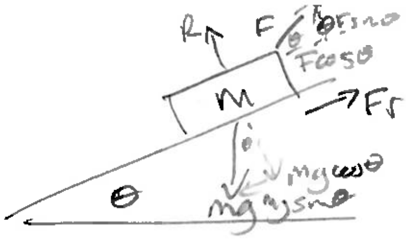
$$C = e^{2}$$

$$\frac{1}{x}y = \frac{1}{2}e^{2x} + e^{2}$$

$$\underline{y = \frac{x}{2}e^{2x} + xe^{2}}$$

(12)

a)



<u>Perp</u>

$$R + F\sin\theta = mg\cos\theta$$

<u>Parallel</u>

$F = \frac{1}{2}mg$

$$mg\sin\theta = F\cos\theta + \mu R$$

$$mg\sin\theta = F\cos\theta + \mu\left(mg\cos\theta - F\sin\theta\right)$$

$$mg\sin\theta = \frac{1}{2}mg\cos\theta + \mu\left(mg\cos\theta - \frac{1}{2}mg\sin\theta\right)$$

$$\mu = \frac{mg\sin\theta - \frac{1}{2}mg\cos\theta}{mg\cos\theta - \frac{1}{2}mg\sin\theta} \quad \times 2$$

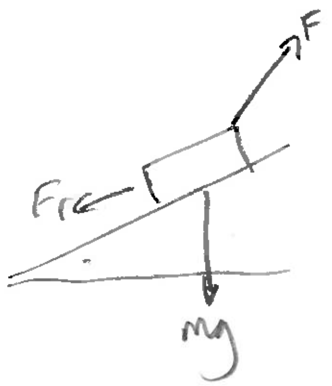
$$= \frac{2\sin\theta - \cos\theta}{2\cos\theta - \sin\theta} \quad \div \cos\theta$$

$$= \frac{2\tan\theta - 1}{2 - \tan\theta} \quad \text{as required}$$

b) $\theta = 30°$

$$\mu = \frac{2\tan 30° - 1}{2 - \tan 30}$$

$$= 0.10874\ldots = 0.109$$



<u>Perp</u> $R + F\sin\theta = mg\cos\theta$

<u>Parallel</u> $F\cos\theta = mg\sin\theta + \mu R$.

$$F\cos\theta = mg\sin\theta + \mu\left(mg\cos\theta - F\sin\theta\right)$$

$$F\cos\theta + \mu F\sin\theta = mg\sin\theta + \mu\, mg\cos\theta$$

$$F\left(\cos\theta + \mu\sin\theta\right) = mg\sin\theta + \mu\, mg\cos\theta$$

$$F = \frac{mg(\sin\theta + \mu\cos\theta)}{\cos\theta + \mu\sin\theta}$$

$$F = \frac{mg(\sin 30^\circ + 0.109 \times \cos 30)}{\cos 30 + 0.109 \times \sin 30}$$

$$\underline{\underline{F = 0.646\,mg}}$$

(13) a) $f(x) = \dfrac{\sec x}{\tan x + 1} \qquad 0 \leq x \leq \dfrac{\pi}{2}$.

$$f'(x) = \frac{(\tan x + 1)\sec x \tan x - \sec x \sec^2 x}{(\tan x + 1)^2}$$

$$= \frac{\sec x(\tan^2 x + \tan x - \sec^2 x)}{(\tan x + 1)^2}$$

$\tan^2 x - \sec^2 x = -1$

$$= \frac{\sec x(\tan x - 1)}{(\tan x + 1)^2}$$

$$= \frac{\sec x}{\tan x + 1} \cdot \frac{\tan x - 1}{\tan x + 1}$$

$$= f(x)\,\frac{\tan x - 1}{\tan x + 1} \quad \text{as required}$$

b) $\dfrac{\tan x - 1}{\tan x + 1} = \dfrac{f'(x)}{f(x)}$

$\Rightarrow \displaystyle\int \frac{\tan x - 1}{\tan x + 1}\,dx = \int \frac{f'(x)}{f(x)}\,dx$

$= \ln\left[f(x)\right] + C$

$= \ln\left[\dfrac{\sec x}{\tan x + 1}\right] + C$

14 a) $a = 15 + x - 2x^2 \quad ms^{-2}$

max V occurs $a = 0$

$\Rightarrow 15 + x - 2x^2 = 0$

$2x^2 - x - 15 = 0$

$2x^2 - 6x + 5x - 15 = 0$

$2x(x-3) + 5(x-3) = 0$

$(2x+5)(x-3) = 0$

$\cancel{x = -\frac{5}{2}} \quad x = 3\text{ m}$

b) (i) $WD = \displaystyle\int_0^3 F\,dx$

$= \displaystyle\int_0^3 5(15 + x - 2x^2)\,dx$

$= \displaystyle\int_0^3 75 + 5x - 10x^2\,dx$

$= \left[75x + \frac{5}{2}x^2 - \frac{10}{3}x^3\right]_0^3$

$= \dfrac{315}{2} - 0 = 157.5\text{ J.}$

(ii) $WD = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$157.5 = \frac{1}{2} \times 5 \times v^2 - 0$

$v = \sqrt{\frac{157.5}{\frac{1}{2}\times 5}}$

$= \sqrt{63} = 7.937\ldots = \underline{7.94 \text{ ms}^{-1}}$

15

a) i) $u_J = \begin{pmatrix} 240 \\ 0 \\ 50 \end{pmatrix}$

$a_J = \begin{pmatrix} 3000 \\ 0 \\ 80 \end{pmatrix}$

$t = 12\text{min} = 0.2\text{hr}.$

$v = u + at$

$v = \begin{pmatrix} 240 \\ 0 \\ 50 \end{pmatrix} + \begin{pmatrix} 3000 \\ 0 \\ 80 \end{pmatrix} \times 0.2$

$= \begin{pmatrix} 840 \\ 0 \\ 66 \end{pmatrix}$ km/hr.

speed $= \sqrt{840^2 + 66^2}$

$= 842.58$

$= 843$ km/hr.

(ii) $v = \int a \, dt$

$v = \int (3000\underline{i} + 80\underline{j}) \, dt$

$v = 3000t\underline{i} + 80t\underline{k} + C$

at $t=0$ $\quad 240\underline{i} + 50\underline{k} = 3000t\,\underline{i} + 80t\,\underline{k} + C$

$C = 240\underline{i} + 50\underline{k}$

$v = (3000t + 240)\underline{i} + (80t + 50)\underline{k}$ ms^{-1}

$$s = \int v \, dt$$

$$= \int (3000t + 240)\underline{i} + (80t + 50)\underline{k} \, dt$$

$$s = (1500t^2 + 240t)\underline{i} + (40t^2 + 50t)\underline{k} + C$$

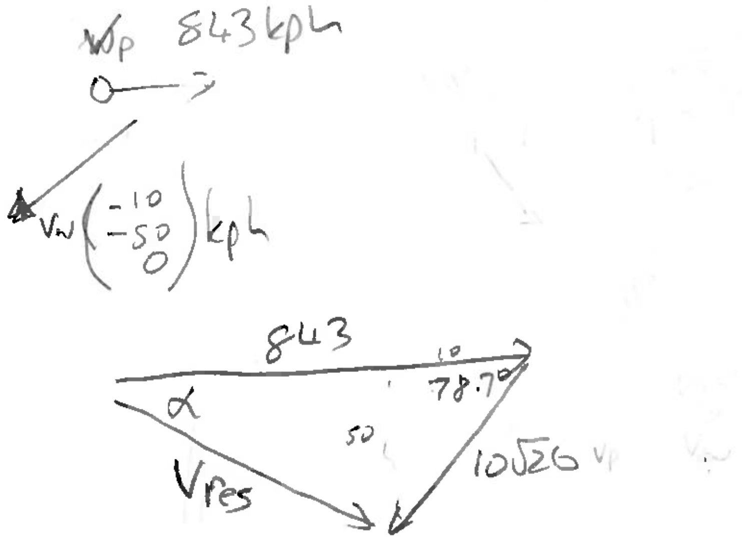
$t=0$ $\quad 0 = C$

$$S = (1500t^2 + 240t)\underline{i} + (40t^2 + 50t)\underline{k}$$

$t=0.2$ $\quad S = (1500(0.2)^2 + 240(0.2))\underline{i} + (40(0.2)^2 + 50(0.2))\underline{k}$

$$= 108\underline{i} + 11.6\underline{k} \text{ m}$$

b) (i)



$\theta = \tan^{-1}\left(\frac{50}{10}\right)$

$= 78.7°$

$$V_{res}^2 = 843^2 + (10\sqrt{26})^2 - 2 \times 843 \times 10\sqrt{26} \times \cos 78.7°$$

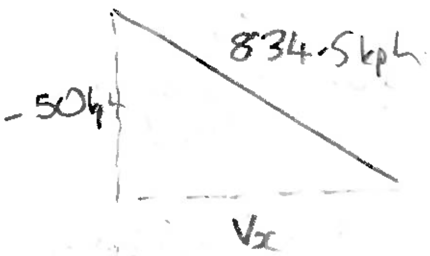
$$= 696403.614\ldots$$

$$V_{res} = 834.5 \text{ kph}.$$

$$\frac{\sin\alpha}{10\sqrt{26}} = \frac{\sin 78.7}{834.5} \qquad \alpha = \sin^{-1}\left(\frac{10\sqrt{26}\sin 78.7}{834.5}\right)$$

$$\alpha = 3.4°$$

b)(ii) $t = 90 - 12 = 78 \text{ min} = 1.3 \text{ hrs}$



$V_x = \sqrt{834.5^2 - 50^2}$

$= 833 \text{ kph}$

$S_x(1.3) = 108 + (833 \times 1.3) = 1190.9 \text{ km}$

$S_y(1.3) = 50 \times 1.3 = 65 \text{ km}$

$S_z = 11.6 \text{ km}$

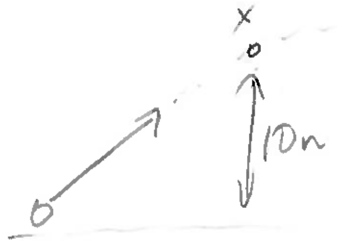
$S = \sqrt{1190.9^2 + 11.6^2 + 65^2}$

$\underline{\underline{S = 1192.7 \text{ km}}}$

16 $m = 0.1kg$

a) $E_k = 20J$

$h = 10m$



$t=0$ $E_T = 20J$

$E_T = E_k + E_p$

$20 = \frac{1}{2}mv^2 + mg \times 10$

$\frac{20 - g}{\frac{1}{2} \times 0.1} = v^2$

$v = \sqrt{\frac{20 - g}{0.05}} = 2\sqrt{51} = \underline{\underline{14.3 ms^{-1}}}$

b) $v = \begin{pmatrix} 4 \\ 5 \end{pmatrix} ms^{-1}$ $|v| = \sqrt{4^2 + 5^2} = \sqrt{41} = 6.40 ms^{-1}$

$E_T = E_k + E_p$

$20 = \frac{1}{2} \times 0.1 \times 6.4^2 + 0.1 \times g \times h$

$h = \frac{20 - (\frac{1}{2} \times 0.1 \times 6.4^2)}{0.1g}$

$= 18.318...$

$= \underline{\underline{18.3 m}}$

c) At max height $V_V = 0$ $\therefore E_{kV} = 0J$.

$V_H = 4ms^{-1}$ $E_{kH} = \frac{1}{2} \times 0.1 \times 4^2 = \underline{\underline{0.8J}}$.