Fundamental Principles

(1) a) $1.0\ ms^{-1}$ N
b) $2.0\ ms^{-1}$ S

(2) a) $0.8\ ms^{-1}$ E
b) $2.8\ ms^{-1}$ E
c) $2.2\ ms^{-1}$ W

(3) a) $3.0\ ms^{-1}$
b) $2.0\ ms^{-1}$

(4) A – Einstein's B – same C – zero
D – velocity E – speed of light
F – slow G – shortened

(5) $2000\ km\ h^{-1}$

(6) a) No
b) Yes

(7) $3 \times 10^{8}\ ms^{-1}$

(8) a) $t = \frac{d}{v}$

$= \frac{3 \times 10^{10}}{3 \times 10^{8}}$

$= 100s$

b) $t = \frac{d}{v}$

$= \frac{3 \times 10^{10}}{3 \times 10^{8}}$

$= 100\ s$

(9) a) $3 \times 10^{8}\ ms^{-1}$ b) $3 \times 10^{8}\ ms^{-1}$

(10) a) $3 \times 10^7 ms^{-1}$ b) $1.5 \times 10^8 ms^{-1}$

c) $1.8 \times 10^8 ms^{-1}$ d) $2.4 \times 10^8 ms^{-1}$

(11) a) c b) $0.67c$

c) $0.5c$ d) $0.33c$

Time Dilation

(12) $$t' = \frac{t}{\sqrt{1 - \left(\frac{v^2}{c^2}\right)}}$$

(13) a) $$t' = \frac{20}{\sqrt{1 - \left(\frac{1 \times 10^8}{3 \times 10^8}\right)^2}}$$

$$= \frac{20}{\sqrt{1 - \frac{1}{9}}}$$

$$= 21.2 \text{ hours}$$

b) $$t' = \frac{10}{\sqrt{1 - \left(\frac{2.25 \times 10^8}{3 \times 10^8}\right)^2}}$$

$$= \frac{10}{\sqrt{1 - 0.5625}}$$

$$= 15.1 \text{ years}$$

(13) c) $$t' = \frac{t}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

$$1400 = \frac{t}{\sqrt{1 - \frac{4}{9}}}$$

$$1400 = \frac{t}{0.745}$$

$$t = 1043\,s$$

d) $$t' = \frac{t}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

$$1.4 \times 10^{-4} = \frac{t}{\sqrt{1 - \frac{1}{9}}}$$

$$1.4 \times 10^{-4} = \frac{t}{0.942}$$

$$t = 1.32 \times 10^{-4}\,s$$

(13) e) $t' = \dfrac{t}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$

$84 = \dfrac{60}{\sqrt{1-\left(\frac{v}{3\times10^8}\right)^2}}$

$\sqrt{1-\left(\frac{v}{3\times10^8}\right)^2} = \dfrac{60}{84}$

$= 0.714$

$1-\left(\frac{v}{3\times10^8}\right)^2 = 0.51$

$-\left(\frac{v}{3\times10^8}\right)^2 = -0.49$

$\dfrac{v}{3\times10^8} = 0.7$

$v = 2.1\times10^8 \text{ ms}^{-1}$

(13) f) $t' = \dfrac{t}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$

$21 = \dfrac{20}{\sqrt{1-\left(\frac{v}{3\times10^8}\right)^2}}$

$\sqrt{1-\left(\frac{v}{3\times10^8}\right)^2} = 0.952$

$1-\left(\frac{v}{3\times10^8}\right)^2 = 0.907$

$-\left(\frac{v}{3\times10^8}\right)^2 = -0.093$

$\frac{v}{3\times10^8} = 0.305$

$v = 9.15\times10^7 \text{ ms}^{-1}$

(14) a) $t' = \dfrac{t}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$

$= \dfrac{15}{\sqrt{1-\left(\frac{2}{3}\right)^2}}$

$= \dfrac{15}{0.75}$

$= 20 \text{ mins}$ 11.20 am

b) $t' = \dfrac{t}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$

$= \dfrac{15}{\sqrt{1 - \left(\frac{2}{3}\right)^2}}$

$= \dfrac{15}{0.75}$

$= 20$ mins. 11.20 am

(15) $t' = \dfrac{t}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$

$= \dfrac{10}{\sqrt{1 - \left(\frac{0.81}{1}\right)^2}}$

$= \dfrac{10}{0.586}$

$= 17.1$ billion years

(16) $$t' = \frac{t}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

$$14.65 = \frac{t}{\sqrt{1 - \left(\frac{0.75}{1}\right)^2}}$$

$$= \frac{t}{\sqrt{1 - \frac{9}{16}}}$$

$$14.65 = \frac{t}{0.661}$$

$$t = 0.661 \times 14.65$$

$$t = 9.69s$$

(17) $$t' = \frac{t}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

$$4 \times 10^{-4} = \frac{t}{\sqrt{1 - \left(\frac{8 \times 10^7}{3 \times 10^8}\right)^2}}$$

$$4 \times 10^{-4} = \frac{t}{0.964}$$

$$t = 0.964 \times 4 \times 10^{-4}$$

$$t = 3.9 \times 10^{-4} s$$

(18)

$$t' = \frac{t}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

$$14 = \frac{10}{\sqrt{1 - \frac{v^2}{(3\times10^8)^2}}}$$

$$\sqrt{1 - \frac{v^2}{9\times10^{16}}} = 0.71$$

$$1 - \frac{v^2}{9\times10^{16}} = 0.51$$

$$0.49 = \frac{v^2}{9\times10^{16}}$$

$$v^2 = 4.41\times10^{16}$$

$$v = 2.1\times10^8 \text{ ms}^{-1}$$

(19)

$$t' = \frac{t}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

$$40 = \frac{10}{\sqrt{1 - \frac{v^2}{(3\times10^8)^2}}}$$

$$\sqrt{1 - \frac{v^2}{9\times10^{16}}} = 0.25$$

$$1 - \frac{v^2}{9\times10^{16}} = 0.0625$$

$$0.9375 = \frac{v^2}{9\times10^{16}}$$

$$v^2 = 8.4\times10^{16}$$

$$v = 2.9\times10^8 \text{ ms}^{-1}$$

(20)

$$t' = \frac{t}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

$$2 = \frac{1}{\sqrt{1 - \frac{v^2}{(3\times10^8)^2}}}$$

$$\sqrt{1 - \frac{v^2}{9\times10^{16}}} = 0.5$$

$$1 - \frac{v^2}{9\times10^{16}} = 0.25$$

$$0.75 = \frac{v^2}{9\times10^{16}}$$

$$v^2 = 6.75\times10^{16}$$

$$v = 2.6\times10^8 \, ms^{-1}$$

Length Contraction

(21)

$$l' = l\sqrt{1 - \frac{v^2}{c^2}}$$

(22) a) $l' = l\sqrt{1 - \frac{v^2}{c^2}}$

$l' = 5\sqrt{1 - \frac{(1\times10^8)^2}{(3\times10^8)^2}}$

$= 5\sqrt{1 - 1/9}$

$= 5 \times 0.943$

$= 4.71\ m$

b) $l' = l\sqrt{1 - \frac{v^2}{c^2}}$

$= 15\sqrt{1 - \frac{(2\times10^8)^2}{(3\times10^8)^2}}$

$= 15\sqrt{1 - 4/9}$

$= 15 \times 0.745$

$= 11.2\ m$

c) $l' = l\sqrt{1 - \frac{v^2}{c^2}}$

$0.15 = l\sqrt{1 - \frac{(2.25\times10^8)^2}{(3\times10^8)^2}}$

$0.15 = l\sqrt{1 - 0.5625}$

$0.15 = l \times 0.66$

$l = 0.227\ km$

d) $l' = l\sqrt{1 - v^2/c^2}$

$150 = l\sqrt{1 - \frac{(1.04\times10^8)^2}{(3\times10^8)^2}}$

$150 = l\sqrt{1 - 0.120}$

$150 = l \times 0.937$

$l = 160\,mm$

e) $l' = l\sqrt{1 - v^2/c^2}$

$30 = 35\sqrt{1 - \frac{v^2}{(3\times10^8)^2}}$

$0.857 = \sqrt{1 - \frac{v^2}{9\times10^{16}}}$

$0.735 = 1 - \frac{v^2}{9\times10^{16}}$

$\frac{v^2}{9\times10^{16}} = 0.265$

$v^2 = 2.39\times10^{16}$

$v = 1.54\times10^8\,ms^{-1}$

f) $l' = l\sqrt{1 - v^2/c^2}$

$10 = 11\sqrt{1 - \frac{v^2}{c^2}}$

$0.91 = \sqrt{1 - \frac{v^2}{9\times10^{16}}}$

$0.826 = 1 - \frac{v^2}{9\times10^{16}}$

$\frac{v^2}{9\times10^{16}} = 0.174$

$v^2 = 1.56 \times 10^{16}$

$v = 1.25 \times 10^8 \, ms^{-1}$

(23) $l' = l\sqrt{1 - \frac{v^2}{c^2}}$

$= 20\sqrt{1 - \frac{(1.8\times10^8)^2}{(3\times10^8)^2}}$

$= 20\sqrt{1 - 0.36}$

$= 20 \times 0.8$

$= 16m$

(24)

$$l' = l\sqrt{1 - v^2/c^2}$$

$$= 2\sqrt{1 - \frac{0.9^2}{1^2}}$$

$$= 2\sqrt{1 - 0.81}$$

$$= 2 \times 0.436$$

$$= 0.872\ m$$

(25)

$$l' = l\sqrt{1 - \frac{v^2}{c^2}}$$

$$160 = l\sqrt{1 - \frac{0.8^2}{1^2}}$$

$$160 = l\sqrt{1 - 0.64}$$

$$160 = l \times 0.6$$

$$l = 267\ m$$

(26)

$$l' = l\sqrt{1 - v^2/c^2}$$

$$0.8 = l\sqrt{1 - \frac{0.5^2}{1^2}}$$

$$0.8 = l\sqrt{1 - 0.25}$$

$$0.8 = l \times 0.866$$

$$l = 0.92\ km$$

(27)

$$l' = l\sqrt{1 - v^2/c^2}$$

$$0.5 = 1\sqrt{1 - \frac{v^2}{(3\times10^8)^2}}$$

$$0.5 = \sqrt{1 - \frac{v^2}{9\times10^{16}}}$$

$$0.25 = 1 - \frac{v^2}{9\times10^{16}}$$

$$\frac{v^2}{9\times10^{16}} = 0.75$$

$$v^2 = 6.75\times10^{16}$$

$$v = 2.6\times10^8 ms^{-1}$$

(28)

$$l' = l\sqrt{1 - v^2/c^2}$$

$$150 = 220\sqrt{1 - \frac{v^2}{(3\times10^8)^2}}$$

$$0.682 = \sqrt{1 - \frac{v^2}{9\times10^{16}}}$$

$$0.465 = 1 - \frac{v^2}{9\times10^{16}}$$

$$\frac{v^2}{9\times10^{16}} = 0.535$$

$$v^2 = 4.82\times10^{16}$$

$$v = 2.19\times10^8 ms^{-1}$$

(29) $L' = L\sqrt{1 - v^2/c^2}$

$99 = 100\sqrt{1 - \frac{v^2}{(3\times10^8)^2}}$

$0.99 = \sqrt{1 - \frac{v^2}{9\times10^{16}}}$

$0.98 = 1 - \frac{v^2}{9\times10^{16}}$

$\frac{v^2}{9\times10^{16}} = 0.02$

$v^2 = 1.8\times10^{15}$

$v = 4.2\times10^7 \text{ ms}^{-1}$

Relativity

(30) a) $v = \frac{d}{t}$

$= \frac{240}{1\times10^{-6}}$

$= 2.4\times10^8 \text{ ms}^{-1}$

b) $t' = \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}}$

$= \frac{1\times10^{-6}}{\sqrt{1 - \frac{(2.4\times10^8)^2}{(3\times10^8)^2}}}$

$= \frac{1\times10^{-6}}{\sqrt{1 - 0.64}}$

$= 1.67\times10^{-6} \text{ s}$

c) $l' = l\sqrt{1 - \frac{v^2}{c^2}}$

$= 240\sqrt{1 - \frac{(2.4\times10^8)^2}{(3\times10^8)^2}}$

$= 240\sqrt{1 - 0.64}$

$= 144\ m$

(81) a) $t' = \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}}$

$1 = \frac{t}{\sqrt{1 - \frac{0.95^2}{1^2}}}$

$\sqrt{1 - 0.9025} = t$

$t = 0.31$ years

b) $d = v\ t$

$= 2.85\times10^8 \times (365\times24\times60\times60)0.31$

$= 2.8\times10^{15}\ m$

c) $d = v\ t$

$= 2.85\times10^8 \times (365\times24\times60\times60)$

$= 8.99\times10^{15}\ m$

(32) a) $$t' = \frac{t}{\sqrt{1 - v^2/c^2}}$$

$$= \frac{2.6 \times 10^{-8}}{\sqrt{1 - \frac{0.99^2}{1^2}}}$$

$$= \frac{2.6 \times 10^{-8}}{\sqrt{0.0199}}$$

$$= 1.84 \times 10^{-7} s$$

b) $$d = v\ t$$
$$= (0.99 \times 3 \times 10^8) \times 1.84 \times 10^{-7}$$
$$= 54.7 m$$

(33) a) $$d = v\ t$$
$$= 2.4 \times 10^8 \times 5 \times 10^{-7}$$
$$= 120 m$$

b) $$l' = l\sqrt{1 - v^2/c^2}$$

$$= 120\sqrt{1 - \frac{2.4^2}{3^2}}$$

$$= 120\sqrt{1 - 0.64}$$

$$= 72 m$$

(34) a) $T = \frac{1}{f}$

$= \frac{1}{0.2}$

$= 5s$

b) $t' = \frac{t}{\sqrt{1 - v^2/c^2}}$

$= \frac{5}{\sqrt{1 - \frac{0.84^2}{1^2}}}$

$= \frac{5}{\sqrt{1 - 0.7056}}$

$= 9.2s$

35

$$t' = \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$0.25 = \frac{0.15}{\sqrt{1 - \frac{v^2}{(3\times10^8)^2}}}$$

$$\sqrt{1 - \frac{v^2}{9\times10^{16}}} = \frac{0.15}{0.25}$$

$$1 - \frac{v^2}{9\times10^{16}} = 0.36$$

$$\frac{v^2}{9\times10^{16}} = 0.64$$

$$v^2 = 5.76\times10^{16}$$

$$v = 2.4\times10^8 \text{ ms}^{-1}$$

36 a)

$$l' = l\sqrt{1 - \frac{v^2}{c^2}}$$

$$= 10\sqrt{1 - \frac{0.999^2}{1^2}}$$

$$= 10\sqrt{1.999\times10^{-3}}$$

$$= 0.447 \text{ km}$$

$$= 447 \text{ m}$$

(36) b) $t = \frac{d}{v}$

$= \frac{447}{2.997 \times 10^8}$

$= 1.49 \times 10^{-6} s$

(37) a) $l' = l\sqrt{1 - v^2/c^2}$

$3.6 = 4.2\sqrt{1 - \frac{v^2}{(3\times10^8)^2}}$

$0.857 = \sqrt{1 - \frac{v^2}{9\times10^{16}}}$

$0.735 = 1 - \frac{v^2}{9\times10^{16}}$

$\frac{v^2}{9\times10^{16}} = 0.265$

$v = 1.54 \times 10^8 ms^{-1}$

b) $t = \frac{d}{v}$

$= \frac{4.2\times(365\times24\times60\times60\times3\times10^8)}{1.56\times10^8}$

$= 2.55 \times 10^8 s$

(37) c) $t = \frac{d}{v}$

$= \frac{3.6 \times (365 \times 24 \times 60 \times 60 \times 3 \times 10^8)}{1.56 \times 10^8}$

$= 2.18 \times 10^8 s$

(38) a) $t' = \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}}$

$= \frac{2.6 \times 10^{-8}}{\sqrt{1 - \frac{0.995^2}{1^2}}}$

$= \frac{2.6 \times 10^{-8}}{\sqrt{0.009975}}$

$= 2.6 \times 10^{-7} s$

b) $d = v \quad t$
$= (0.995 \times 3 \times 10^8) \times 2.6 \times 10^{-7}$
$= 77.6 m$

c) $d = v \quad t$
$= (0.995 \times 3 \times 10^8) \times 2.6 \times 10^{-8}$
$= 7.76 m$

Section 5: Special relativity

Relativity – Fundamental principles

1. (a) $1{\cdot}0\ ms^{-1}$ north
   (b) $2{\cdot}0\ ms^{-1}$ south

2. (a) $0{\cdot}8\ ms^{-1}$ east
   (b) $2{\cdot}8\ ms^{-1}$ east
   (c) $2{\cdot}2\ ms^{-1}$ west

3. (a) $3{\cdot}0\ ms^{-1}$
   (b) $2{\cdot}0\ ms^{-1}$

4. A = Einstein's; B = same; C = zero; D = velocity; E = speed of light; F = slow; G = shortened

5. $2000\ km\ h^{-1}$

6. (a) No
   (b) Yes

7. $3 \times 10^8\ ms^{-1}$

8. (a) 100 s
   (b) 100 s

9. (a) $3 \times 10^8\ ms^{-1}$
   (b) $3 \times 10^8\ ms^{-1}$

10. (a) $0{\cdot}3 \times 10^8\ ms^{-1}$
    (b) $1{\cdot}5 \times 10^8\ ms^{-1}$
    (c) $1{\cdot}8 \times 10^8\ ms^{-1}$
    (d) $2{\cdot}4 \times 10^8\ ms^{-1}$

11. (a) c
    (b) 0·67 c
    (c) 0·5 c
    (d) 0·33 c

SOLUTIONS

Relativity – Time dilation

1. $t' = \dfrac{t}{\sqrt{1-(\frac{v^2}{c^2})}}$

2. (a) 21·2 h
   (b) 15·1 year
   (c) 1043 s
   (d) $1{\cdot}32 \times 10^{-4}$ s
   (e) $2{\cdot}10 \times 10^{8}$ ms^{-1}
   (f) $9{\cdot}15 \times 10^{7}$ ms^{-1}

3. (a) 11.20 am
   (b) 11.20 am

4. 17·1 billion years

5. 9·69 s

6. $3{\cdot}9 \times 10^{-4}$ s

7. $2{\cdot}1 \times 10^{8}$ ms^{-1} or 0·70 c

8. 2·90 10^{8} ms^{-1} or 0·97 c

9. $2{\cdot}60 \times 10^{8}$ ms^{-1}

Relativity – Length contraction

1. $l' = l\sqrt{(1 - v^2/c^2)}$

2. (a) 4·71 m
   (b) 11·2 m
   (c) 0·227 km
   (d) 160 mm
   (e) $1{\cdot}55 \times 10^{8}$ ms^{-1}
   (f) $1{\cdot}25 \times 10^{8}$ ms^{-1}

3. 16 m

4. 0·872 m

5. 267 m

6. 0·92 km

7. $2{\cdot}60 \times 10^{8}$ ms^{-1}

8. $2{\cdot}19 \times 10^{8}$ ms^{-1}

9. $4{\cdot}23 \times 10^{7}$ ms^{-1} or 0.14 c

Relativity questions

1. (b) $1{\cdot}67 \times 10^{-6}$ s
   (c) 144 m

2. (a) 0·31 of a year
   (c) $8{\cdot}97 \times 10^{15}$ m

3. (a) $1{\cdot}84 \times 10^{-7}$ s
   (b) 54·6 m or 54·7 m

4. (a) 120 m
   (b) 72 m

5. (a) 5 s
   (b) 9·22 s

6. 0·8 c

7. (a) 447 m
   (b) $1{\cdot}49 \times 10^{-6}$ s

8. (a) 0·52 c
   (b) $2{\cdot}55 \times 10^{8}$ s
   (c) $2{\cdot}18 \times 10^{8}$ s

9. (a) $2{\cdot}60 \times 10^{-7}$ s
   (b) 77·6 m
   (c) 7·75 m or 7·76 m

0.099a