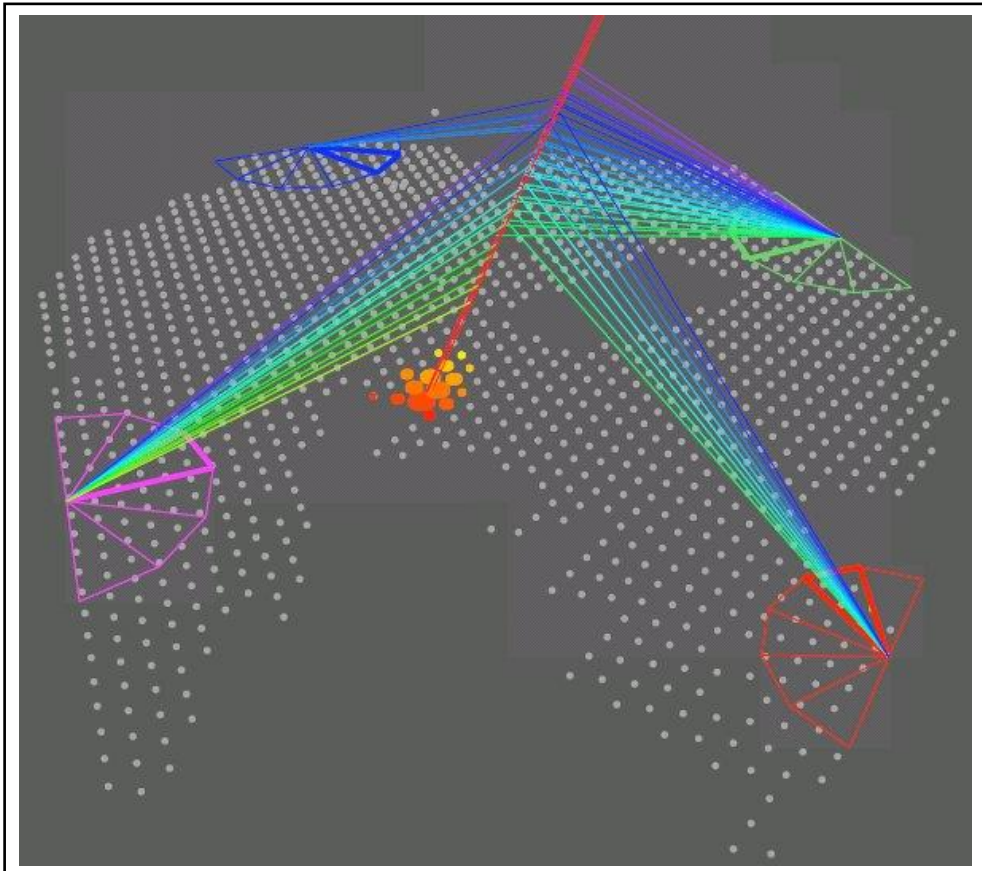




St Ninian's High School

Advanced Higher Physics

Quanta



Cover image: Cosmic ray event detected at Auger Observatory, Argentina

Quantum Theory

An Introduction to Quantum Theory

Quantum mechanics was developed in the early twentieth century to explain experimental observations that could not be explained by classical physics. In many cases various quantisation 'rules' were proposed to explain these experimental observations but these did not have any classical justification. This results in physics that is difficult to reconcile with everyday experience or with normal intuition.

This led to two of the most famous physicists of the twentieth century to make the following statements:

If quantum mechanics hasn't profoundly shocked you, you haven't understood it yet.

Neils Bohr

I think it safe to say that no one understands quantum mechanics.

Richard Feynman

However, quantum mechanics has been one of the most successful theories in physics explaining many experimental observations such as blackbody radiation, the photoelectric effect, spectra, atomic structure and electron diffraction.

Blackbody Radiation

Towards the end of the nineteenth century there was interest in the frequencies (or wavelengths) emitted by a 'black body' when the temperature is increased. When an object is heated it can radiate large amounts of energy as infrared radiation. We can feel this if we place a hand near, but not touching, a hot object. As an object becomes hotter it starts to glow a dull red, followed by bright red, then orange, yellow and finally white (white hot). At extremely high temperatures it becomes a bright blue-white colour.

Measurements were made of the intensity of the light emitted at different frequencies (or wavelengths) by such objects. In addition measurements were made at different temperatures. In order to improve the experiment and avoid any reflections of the radiation, a cavity was used with a small hole, which emits the radiation: a black body.

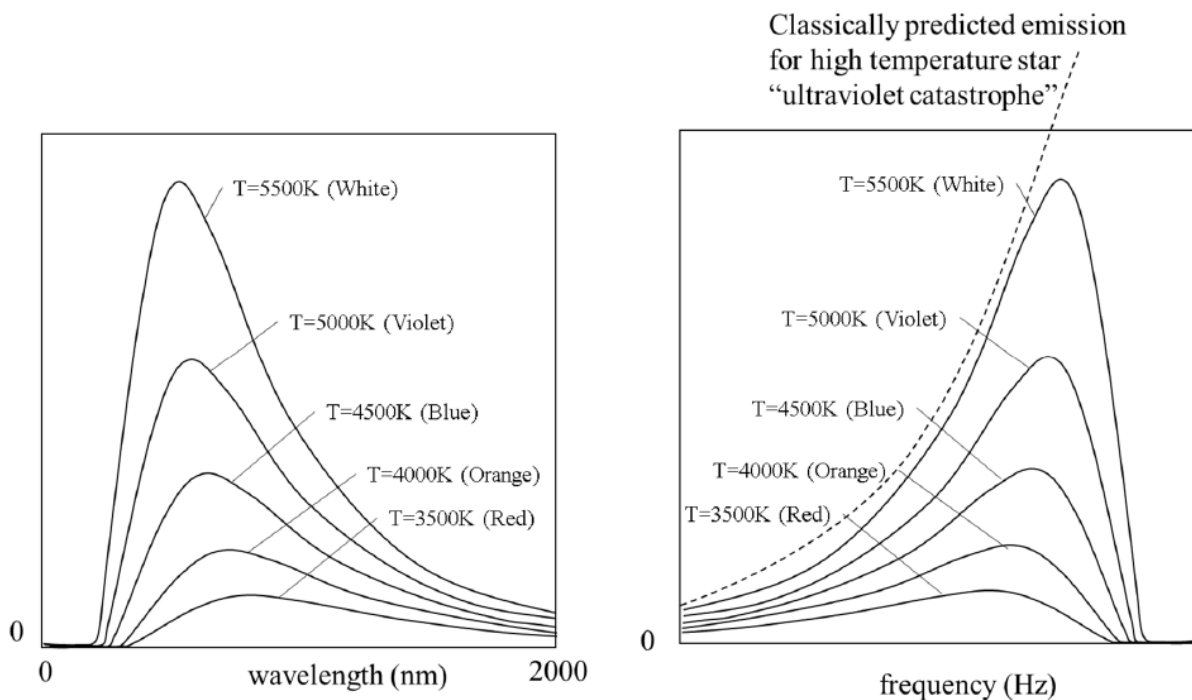
A surface that absorbs all wavelengths of electromagnetic radiation is also the best emitter of electromagnetic radiation at any wavelength. Such an ideal surface is called a **black body**. The continuous spectrum of radiation it emits is called black-body radiation.

It was found that the amount of black-body radiation emitted at any frequency depends **only** on the temperature, not the actual material.

Specific intensity is a measure of the radiation *emitted* by a body. Irradiance is a measure of the radiation *received* by a surface.

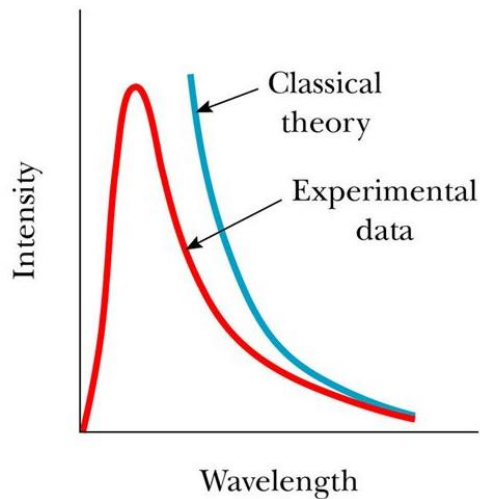
Graphs of specific intensity against wavelength (or frequency) are shown in Figure 1. As the temperature increases, each maximum shifts towards the higher frequency (shorter wavelength).

Black Body Radiation Curves



Graphs of specific intensity against wavelength and against frequency.

Attempts to obtain theoretically the correct black-body graph using classical mechanics failed. Wien obtained an equation that 'fitted' observations at low wavelengths (high frequencies).



Later Lord Rayleigh obtained an equation that 'fitted' at high wavelengths (low frequencies) but tended off to infinity at low wavelengths (high frequencies). This divergence was called the **ultraviolet catastrophe** and puzzled many leading scientists of the day.

In 1900 Planck looked at the two equations and produced a 'combined' relationship, which gave excellent agreement with the experimental curve. However, initially this relationship could not be derived from first principles. It was a good mathematical 'fudge'!

Planck studied his relationship and the theory involved and noticed that he could resolve the problem by making the assumption that the absorption and emission of radiation by the oscillators could only take place in 'jumps' given by:

$$E = nhf \quad (1)$$

where E is energy, f is frequency, h is a constant and $n = 0, 1, 2, 3, \dots$

Using this assumption he could derive his equation from first principles. The constant of proportionality h was termed Planck's constant. (The word quantum, plural quanta, comes from the Latin 'quantus', meaning 'how much'.)

It must be emphasised that Planck did this in a mathematical way with no justification as to why the energy should be quantised – but it worked! To Planck the oscillators were purely theoretical and radiation was not actually emitted in 'bundles', it was just a 'calculation convenience'. It was some years before Planck accepted that radiation was really in energy packets.

The Photoelectric Effect

In 1887 Hertz observed that a spark passed between two plates more often if the plates were illuminated with ultraviolet light. Later experiments by Hallwachs and Lenard gave the unexpected results we are familiar with, namely:

- (a) the non-emission of electrons with very bright but low frequency radiation on a metal surface, e.g. very bright red light, and
- (b) the increase in the speed of the emitted electron with frequency but not with intensity. Increasing the intensity only produced more emitted electrons.

These results were unexpected because energy should be able to be absorbed continuously from a wave. An increase in the intensity of a wave also means an increase in amplitude and hence a larger energy.

In 1905 Einstein published a paper on the photoelectric effect entitled *On a Heuristic Viewpoint Concerning the Production and Transformation of Light*. He received the Nobel Prize for Physics for this work in 1921. The puzzle was why energy is not absorbed from a continuous wave, e.g. any electromagnetic radiation, in a cumulative manner. It should just take more time for energy to be absorbed and an electron emitted but this does not happen. Einstein proposed that electromagnetic radiation is emitted and absorbed in small packets. (The word 'photon' was introduced by Gilbert in 1926.) The energy of each packet is given by:

$$E = hf \quad (2)$$

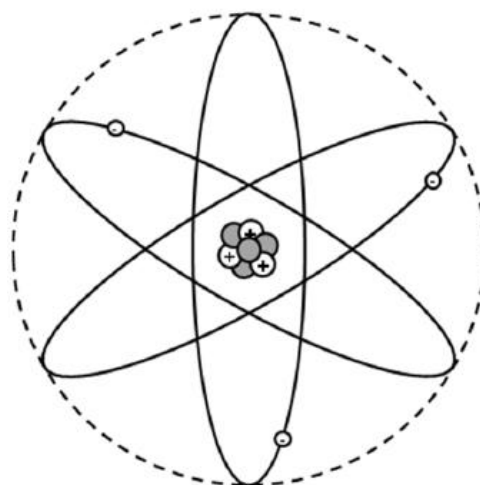
where E is the energy of a 'packet' of radiation of frequency f .

This proposal also explained why the number of electrons emitted depended on the irradiance of the electromagnetic radiation and why the velocity of the emitted electrons depended on the frequency. It did not explain the 'packets' or why they should have this physical 'reality'.

Models of the Atom

Rutherford's scattering experiment indicated that the majority of the mass of the atom was in a small nucleus, with the electrons 'somewhere' in the atomic space. He and his assistants could not 'see' the electrons. A picturesque model of the atom, similar to a small solar system, came into fashion. This model had some features to commend it. Using classical mechanics, an electron in an orbit could stay in that orbit, the central force being balanced by electrostatic attraction. However, the electron has a negative charge and hence it should emit radiation, lose energy and spiral into the nucleus.

The current theory was insufficient. Why do the electrons 'remain in orbit'? Do they in fact 'orbit'?



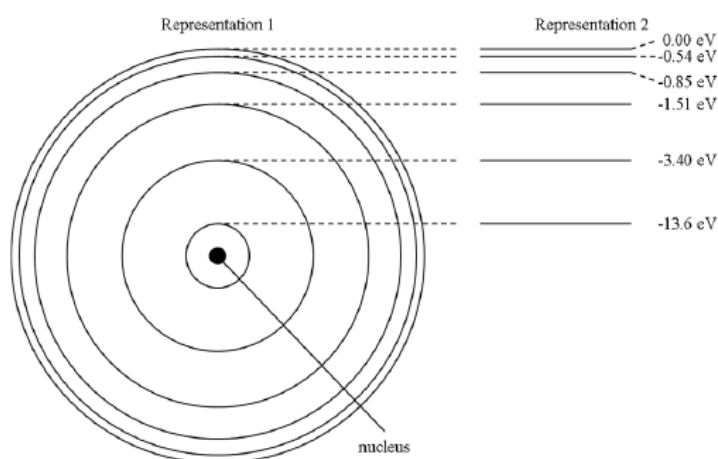
In the late nineteenth century attempts were made to introduce some 'order' to the specific frequencies emitted by atoms. Balmer found, by trial and error, a simple formula for a group of lines in the hydrogen spectra in 1885.

$$\frac{1}{\lambda} = R \left(\frac{1}{2^2} - \frac{1}{n^2} \right) \quad (3)$$

where λ is the wavelength, R the Rydberg constant, n is an integer 2, 3, 4,...

Other series were then discovered, e.g. Lyman with the first fraction $1/1^2$ and Paschen with the first fraction $1/3^2$. However, this only worked for hydrogen and atoms with one electron, e.g. ionised helium, and moreover did not provide any theoretical reason why the formula should work.

In 1913 Bohr introduced the idea of **energy levels**. Each atom has some internal energy due to its structure and internal motion but this energy cannot change by any variable amount, only by specific discrete amounts. Any particular atom, e.g. an atom of gold say, has a specific set of energy levels. Different elements each have their own set of levels. Experimental evidence of the day provided agreement with this idea and energy level values were obtained from experimental results. Transitions between energy levels give the characteristic line spectra for elements.



In order to solve the problem that an electron moving in a circular orbit should continuously emit radiation and spiral into the nucleus, Bohr postulated that an electron can circulate in certain permitted, stable orbits without emitting radiation. He made the assumption that the normal electromagnetic phenomena did not apply at the atomic scale! Furthermore he made an intuitive guess that angular momentum is quantised. It is said that he noticed that the units of Planck's constant (J s^{-1}) are the same as those of angular momentum ($\text{kg m}^2 \text{s}^{-1}$). The allowed orbit, of radius r , of an electron must have angular momentum of an integral multiple of $h/2\pi$:

$$mvr = \frac{nh}{2\pi} \quad (4)$$

where n is an integer 1, 2, 3,...

The angular momentum of a particle, of mass m , moving with tangential speed v , is mvr .

Thus for any specific orbit n we can calculate the radius of that orbit given the tangential speed or vice versa.

Theoretical aside

For the hydrogen atom with a single electron, mass m_e revolving around a proton (or more correctly around the centre of mass of the system), we can assume the proton is stationary since it is ~2000 times bigger. Hence, equating the electrostatic force and centripetal force $m_e v^2/r$.

$$\frac{1}{4\pi\epsilon_0} \frac{e^2}{r_n^2} = \frac{m_e v_n^2}{r_n} \text{ where } \epsilon_0 \text{ is the permittivity of free space} \quad (5)$$

for the n th orbit.

Equations (4) and (5) can be solved simultaneously to give:

$$r_n = \epsilon_0 \frac{n^2 h^2}{\pi m_e e^2} \text{ and } v_n = \frac{1}{\epsilon_0} \frac{e^2}{2\pi h} \quad (6)$$

for the n th orbit.

Calculating r_1 for the radius of the first Bohr orbit uses data given in assessments, namely h , m_e , e , ϵ_0 , and gives $r_1 = 5.3 \times 10^{-11} \text{ m}$.

These equations give the values of the radii for the **non-radiating** orbits for hydrogen and the value of n was called the **quantum number** of that orbit.

Example problem

For the hydrogen atom, calculate the velocity of an electron in the first Bohr orbit of radius $5.3 \times 10^{-11} \text{ m}$.

Using $mvr = \frac{nh}{2\pi}$ with $n = 1$ we obtain $v = 2.2 \times 10^6 \text{ m s}^{-1}$.

Note: Bohr's theory only applies to an atom with *one* electron, e.g. the hydrogen atom or ionised helium atom.

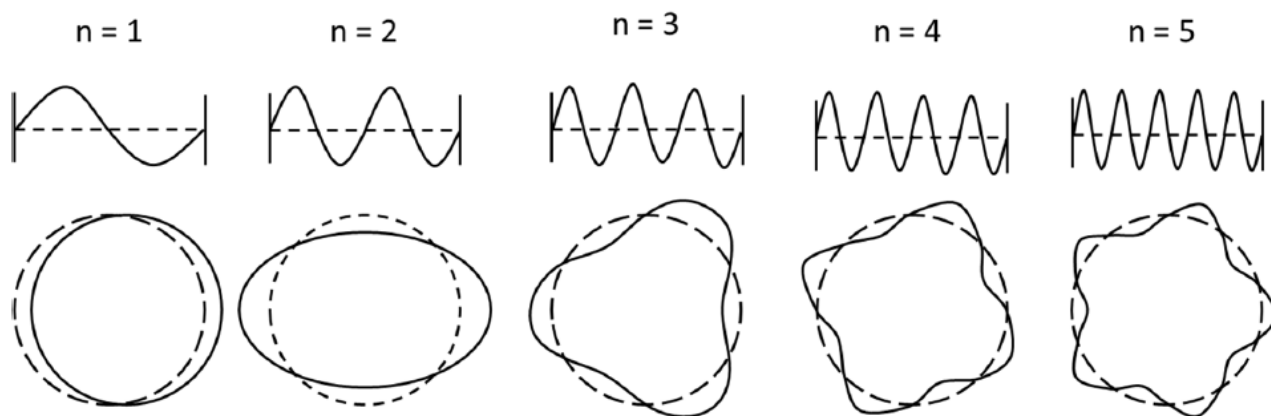
However, as mentioned above, the idea of energy levels can be extended to all atoms, not just hydrogen.

So this theory is not complete since it did not allow any **prediction** of energy levels for any specific element nor did it explain why angular momentum should be quantised or why electrons in these orbits did not radiate electromagnetic energy! Another question was, 'what happens during a transition?'

Wave-Particle Duality

Electrons as “Standing Waves”

An extension by Louis de Broglie suggested that electron orbits are standing waves. The electron, now behaving like a wave, forms a standing wave of an integral number of wavelengths that just ‘fits’ into the circumference of an orbit. This is represented here:



The dashed lines represent the circumference, while the solid line represents the hypothesised wave-like path of the electron.

This is a picturesque idea but somewhat old-fashioned since an electron does not take a ‘wiggly’ path around the nucleus. This is an example of a model that should not be taken too seriously and could lead to poor understanding. Quantum mechanics goes on to show that we cannot describe the motion of an electron in an atom in this way.

De Broglie Wavelength

We use the word ‘particle’ to describe localised phenomena that transport mass and energy, and the word ‘wave’ to describe delocalised (spread out) phenomena that carry energy but no mass.

Experimental observations seem to suggest that both electromagnetic radiation and electrons can behave like particles *and* like waves. They exhibit both wave phenomena, such as interference and diffraction, and particle phenomena, for example photons causing electron emission in the photoelectric effect or electron ‘billiard ball type’ collisions.

An electron can show wave-like phenomena. In the mid-1920s G P Thomson, in Aberdeen, bombarded a thin metal with an electron beam and obtained a diffraction ring. In 1927 Davisson and Germer directed a beam of electrons onto the surface of a nickel crystal and observed the reflected beam. They had expected to see diffuse reflection since even this smooth surface would look ‘rough’ to the tiny electrons. To their surprise, they observed a similar pattern to X-ray diffraction from a surface. Thomson and Davisson were awarded the Nobel Prize in 1937 for demonstrating the wave-like properties of electrons. (Thomson’s father, J J Thomson, won the Nobel Prize in 1906 for discovering the electron as a particle.)

In 1923 de Broglie suggested that since light had particle-like properties, perhaps nature was dualistic and particles had wave-like properties.

From relativity theory, the energy of a particle with zero rest mass, e.g. a photon, is given by $E = pc$ and we know that $E = hf$, hence $p = h/\lambda$.

Thus the wave and particle are related through its momentum.

For a particle $p = mv$ and for a wave $p = h/\lambda$, giving a relationship $mv = h/\lambda$ or:

$$\lambda = \frac{h}{p} \text{ where } p \text{ is the momentum and } h \text{ Planck's constant} \quad (7)$$

Thus we can calculate the **de Broglie wavelength** of a particle of velocity v .

Example problems

1. A neutron and an electron have the same speed.
Which has the longer de Broglie wavelength?

The electron, since the neutron has the larger mass. (The mass is in the denominator.)

2. An electron microscope uses electrons of wavelength of 0.01 nm.

What is the required speed of the electrons?

Using $\lambda = \frac{h}{p}$ for electrons and $p = mv$ gives:

$$0.04 \times 10^{-9} = (6.63 \times 10^{-34}) / (9.11 \times 10^{-31} \times v)$$

and

$$v = 1.8 \times 10^{-7} \text{ m s}^{-1}$$

Notice that this wavelength of 0.04 nm is very much smaller than that of blue light. Hence the use of electrons in microscopes can improve the resolution of the image compared to optical microscopes.

Quantum Mechanics

The more you see how strangely Nature behaves, the harder it is to make a model that explains how even the simplest phenomena actually works.

Richard Feynman

Matter was thought to be 'atomistic' with 'particles' making basic interactions and the properties of the particles continually changing smoothly from place to place. Waves moved continuously from place to place.

Classical mechanics could not explain the various 'quantisation rules', which attempted to give some limited agreement between observation and theory. The apparent dual wave particle nature of matter could not be explained. With quantum theory these ideas needed to be revised.

There are various forms of quantum mechanics: Heisenberg's matrix mechanics, Erwin Schrödinger's wave mechanics, Dirac's relativistic field theory and Feynman's sum over histories or amplitude mechanics.

In essence quantum mechanics provides us with the means to calculate **probabilities** for physical quantities. Exact physical quantities, e.g. position or velocity, do not have unique values at each and every instant.

Balls in quantum mechanics do not behave like balls in classical mechanics ... an electron between release and detection does not have a definite value for its position. This does not mean that the electron has a definite position and we don't know it. It means the electron just does not have a position just as love does not have a colour.

Strange World of Quantum Mechanics, D Styer

Quantum theories incorporate the following concepts:

- (i) Transitions between stationary states are **discrete**. There is no meaning to any comment on a system in an intermediate state.

Depending on the experiment, matter or waves may behave as a wave or a particle. However, in a certain way they act like both together. It is just not a sensible question in quantum mechanics to ask if matter is a wave or a particle.

- (ii) Every physical situation can be characterised by a wavefunction (or other mathematical formalism). This wavefunction is not directly related to any actual property of the system but is a description of the potentialities or possibilities within that situation. The wavefunction provides a statistical ensemble of similar observations carried out under the specified conditions. It does not give the detail of what will happen in any particular individual observation. The probability of a specific observation is obtained from the square of the wavefunction. This is an important and non-intuitive idea.

The quantum probability aspect is very different from classical physics, where we consider there is an actual state and any 'probability' comes from our inadequate measuring or statistical average.

In the quantum domain we can only calculate **probabilities**. For example, we cannot state when a particular nucleus will decay (although we can measure a half-life) but we could calculate the probability of a particular nucleus decaying after a certain time. This is typical of the rules of quantum mechanics – the ability to calculate probabilities.

Quantum mechanics has enjoyed unprecedented practical success. Theoretical calculations agree with experimental observations to very high precision.

Quantum mechanics also reminds us that there is discreteness in nature and there are only probabilities.

Double-slit experiment

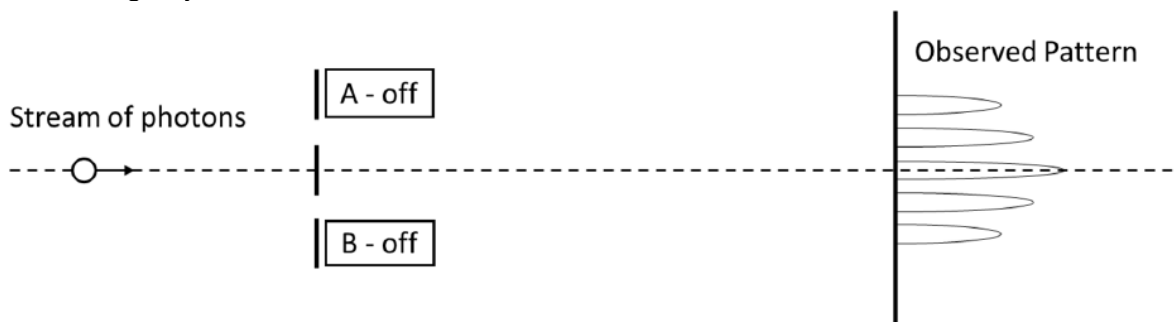
The double-slit experiment with light shows an interference pattern. This is a standard experiment to demonstrate that light is a wave motion.

There is a central maximum opposite the central axis between the two slits as shown below.

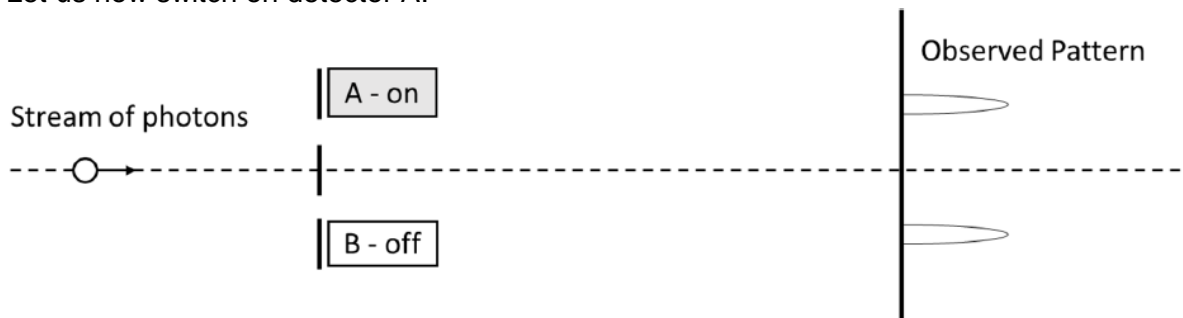


In more recent years this experiment has been performed with **single** photons and a detector screen. Each photon reaches the screen and the usual interference pattern is gradually built up.

The question is how does a single photon 'know about' the slit it does *not* pass through? Let us place a detector near each slit as shown below. In this diagram the detectors are switched off and *not* making any measurements.



Let us now switch on detector A.



We lose the interference effect and simply obtain a pattern for particles passing through two slits. We would get the same pattern if we switched on detector B instead of detector A or if we switched on both detectors.

It seems if we ask the question 'Where is the photon?' or 'Which slit does the photon pass through?' and set up an experiment to make a measurement to answer that question, e.g. determine which slit the photon passed through, we do observe a 'particle' with a position but lose the interference effect.

It appears that the single photon in some way does 'know about' both slits. This is one of the non-intuitive aspects of quantum mechanics, which suggests that a single particle can pass through *both* slits.

A very similar double-slit experiment can be performed with electrons. Again we can arrange for only one electron to 'pass through' the slits at any one time. The position of the electrons hitting the 'screen' agrees with our familiar interference pattern. However, as soon as we attempt to find out which slit the electron passes through we lose the interference effect. For electron interference the spacing of the 'slits' must be small. Planes of atoms in a crystal can be used to form the slits since electrons have a very small associated wavelength, the de Broglie wavelength.

Conclusions

These observations are in agreement with quantum mechanics. We cannot measure wave and particle properties at the same time. What we find is that we cannot predict the exact path that either an electron or photon will follow, and therefore predict where it lands on the screen, in this experiment.

We also find that we cannot make a direct measurement of the particles locations, as this will act in itself will interfere with the outcome of the experiment!

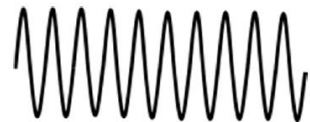
What we can do is say what probability the electron or photon has of landing at a particular point on the screen. The maxima are areas where there is a high probability of arriving, hence many electrons/photons arrive here and we see a bright patch. The minima represent areas of low probability, fewer electrons/photons arrive and it is dimmer/dark. The sum of all of these probabilities is 1. This ties in with the idea of Feynman's sum over histories.

The Uncertainty Principle

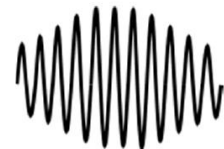
A theoretical introduction

Using the wave theory of quantum mechanics outlined above we can produce a wavefunction describing the 'state' of the system, e.g. an electron. However, we find that it is not possible to determine with accuracy *all* the observables for the system. For example, we can compute the likelihood of finding an electron at a certain position, e.g. in a box. The wavefunction may then be effectively zero everywhere else and the uncertainty in its position may be very small inside the box. If we then consider its momentum wavefunction we discover that this is very spread out, and there is nothing we can do about it. This implies that *in principle*, if we 'know' the position, the momentum has a very large uncertainty.

Consider a wave with a single frequency. Its position can be thought of as anywhere along the wave but its frequency is uniquely specified.



Now consider a wave composed of a mixture of slightly different frequencies, which when added together produces a small 'wave packet'. The position of this wave can be quite specific but its frequency is conversely non-unique



Heisenberg's Uncertainty Principle should more appropriately be called Heisenberg's Indeterminacy Principle since we can measure either x or p_x with very low 'uncertainty' but we cannot measure both. If one is certain, the other is indeterminate.

Theoretical considerations also shows that the energy E and time t have this dual indeterminacy.

A thought experiment to illustrate Heisenberg's Uncertainty Principle

In classical physics it was assumed that all the attributes, such as position, momentum, energy etc, could be measured with a precision limited only by the experiment.

In the atomic domain is this still true?

Let us consider an accurate method to determine the position of an electron in a particular direction, for example in the x direction. The simplest method is to use a 'light gate', namely to allow a beam of electromagnetic radiation to hit the electron and be interrupted in its path to a detector. To increase the accuracy we can use radiation of a small wavelength, e.g. gamma rays. However, we note that by hitting the electron with the gamma rays the velocity of the electron will alter (a photon-electron collision). Now the velocity or momentum of the electron in the x direction will have changed. Whatever experiment we use to subsequently measure the velocity or momentum cannot determine the velocity before the electron was 'hit'. To reduce the effect of the 'hit' we can decrease the frequency of the radiation, and lose some of the precision in the electron's position. We just cannot 'win'!

Heisenberg's Uncertainty Principle is stated as

$$\Delta x \Delta p_x \geq \frac{h}{4\pi} \quad (8)$$

where Δx is the uncertainty in the position, Δp_x is the uncertainty in the component of the momentum in the x direction and h is Planck's constant.

Quantum mechanics can show that there are other pairs of quantities that have this indeterminacy, for example energy and time:

$$\Delta E \Delta t \geq \frac{h}{4\pi} \quad (9)$$

where ΔE is the uncertainty in energy and Δt is the uncertainty in time.

We notice that the pairs of quantities in these relationships (termed conjugate variables) have units that are the same as those of h , namely J s. For energy and time this is obvious. For position and momentum we have m kg m s^{-1} , which we can adjust as $\text{kg m}^2 \text{s}^{-2} \text{s}$, multiplying by s^{-1} and s. The $\text{kg m}^2 \text{s}^{-2}$ is J, giving the required total unit of J s.

The question 'Does the electron have a position and momentum before we look for it?' can be debated. Physicists do not have a definitive answer and it depends on the interpretation of quantum mechanics that one adopts. However, this is not a useful question since the wavefunctions give us our information and there is a limit on what we can predict about the quantum state. We just have to accept this. It is worth reiterating that quantum mechanics gives superb agreement with experimental observations.

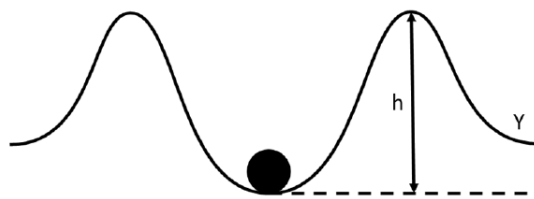
Using quantum mechanics the spectral lines for helium and other elements can be calculated and give excellent agreement with experimental observations.

More importantly, quantum mechanics provides a justification for the ad hoc quantisation 'rules' introduced earlier and gives us a very useful tool to explain theoretically observed phenomena and make quantitative and accurate predictions about the outcomes of experiments.

Potential wells and quantum tunnelling

Imagine a ball in a 'dip'. The shape of the ups and downs is irrelevant.

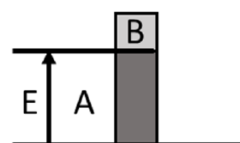
The ball cannot get to position Y unless it receives energy $E = mgh$.



The ball is in a 'potential well' of 'height' mgh . This means that the ball needs energy E equal to or greater than mgh in order to 'escape' and get to position Y.

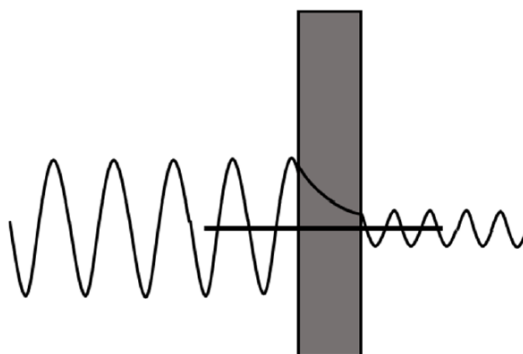
In the quantum world things are a touch different, although the concept of a 'potential well' or 'potential barrier' is useful.

Now let us consider an electron with some energy E on the left-hand side of a barrier of energy greater than E . The electron is thus confined to side A. It does not have enough energy to 'get over' the barrier and 'escape'



Not so according to quantum theory.

The wavefunction is continuous across a barrier. The amplitude is greater in region A but it is finite, although much smaller, outside region A to the right of the barrier. Although the probability of finding the electron in region A is very high, there is a **finite** probability of finding the electron beyond the barrier.



The probability depends on the square of the amplitude. Hence it appears that the electron can 'tunnel out'. This is called **quantum tunnelling** and has some interesting applications.

Examples of quantum tunnelling

Alpha decay

For some radioactive elements, e.g. polonium 212, the alpha particles are held in the nucleus by the residual strong force and do not have enough energy to escape. However, because of quantum tunnelling they do escape and quantum mechanics can calculate the half-life. In 1928 George Gamow used quantum mechanics (Schrödinger wave equation) and the idea of quantum tunnelling to obtain a relationship between the half-life of the alpha particle and the energy of emission. Classically the alpha particle should not escape.

Scanning tunnelling microscope

A particular type of electron microscope, the scanning tunnelling microscope, has a small stylus that scans the surface of the specimen. The distance of the stylus from the surface is only about the diameter of an atom. Electrons 'tunnel' across the sample. In this way the profile of the sample can be determined. Heinrich Rohrer and Gerd Binnig were awarded the Nobel Prize for their work in this field in 1986.

Virtual particles

Another interesting effect of the Uncertainty Principle is the 'sea' of virtual particles in a vacuum. We might expect a vacuum to be 'empty'. Not so with quantum theory.

A particle can 'appear' with an energy ΔE for a time less than Δt where

$$\Delta E \Delta t \geq \frac{h}{4\pi}.$$

Do these virtual particles 'exist'? This is not really a sensible question for quantum mechanics. We cannot observe them in the short time of their existence. However, they are important as 'intermediate' particles in nuclear decays and high energy particle collisions and if they are omitted theoretical agreement with observations may not be obtained. Virtual particles are important when using Feynman diagrams to solve problems.

Particles from Space - Cosmic Rays

Austrian physicist Victor Hess made measurements of radiation at high altitudes from a balloon, to try and get away from possible sources on Earth. He was surprised to find the measurements actually **increased** with altitude.

It was thought this radiation was coming from the Sun, but Hess obtained the same results after repeating his experiments during a nearly complete solar eclipse (1912), thus ruling out the Sun as the (main) source of radiation. Tracks produced by cosmic rays can be observed using a cloud chamber. Robert Millikan later coined the phrase 'cosmic rays', believing them to be electromagnetic in nature.

By measuring the intensity of cosmic rays at different latitudes (they were found to be more intense in Panama than in California), Compton showed that they were being deflected by the Earth's magnetic field and so must consist of electrically charged particles, i.e. electrons or protons rather than photons in the form of gamma radiation.

Origin and composition

Cosmic rays originate from outer space, they consist of high energy charged particles, travelling close to the speed of light, mainly protons and alpha particles (helium nuclei).

Table 1 Composition of cosmic rays

Nature	Approximate percentage of all cosmic rays
Protons	89
Alpha particles	9
Carbon, nitrogen and oxygen nuclei	1
Electrons	less than 1
Gamma radiation	less than 0.1

The energies of cosmic rays cover an enormous range, with the most energetic having energies much greater than those capable of being produced in current particle accelerators.

The highest energies produced in particle accelerators are of the order of 1 tera electronvolt (10^{12} eV). Cosmic rays have been observed with energies ranging from 10^9 to 10^{20} eV. Those with energies above 10^{18} eV are referred to as ultra-high-energy cosmic rays (UHECRs).

The 'Oh-my-God' (OMG) particle with energy of 3×10^{20} eV was recorded in Utah in 1991.

Converting to joules (J), 3×10^{20} eV = $3 \times 10^{20} \times 1.6 \times 10^{-19}$ J = 48 J, ie ~50 J.

That is enough energy to throw a 25 kg mass (e.g. a bag of cement) 2 m vertically upwards. It is also approximately equal to the kinetic energy of a tennis ball served at about 100 mph by Andy Murray.

Such UHECRs are thought to originate from fairly local (in cosmological terms) distances, i.e. within a few hundred million light years. This is because were they to originate from further away it would be hard to understand how they get all the way here at all, since the chances are they would have interacted with Cosmic Microwave Background Radiation (CMBR) photons along the way, producing pions.

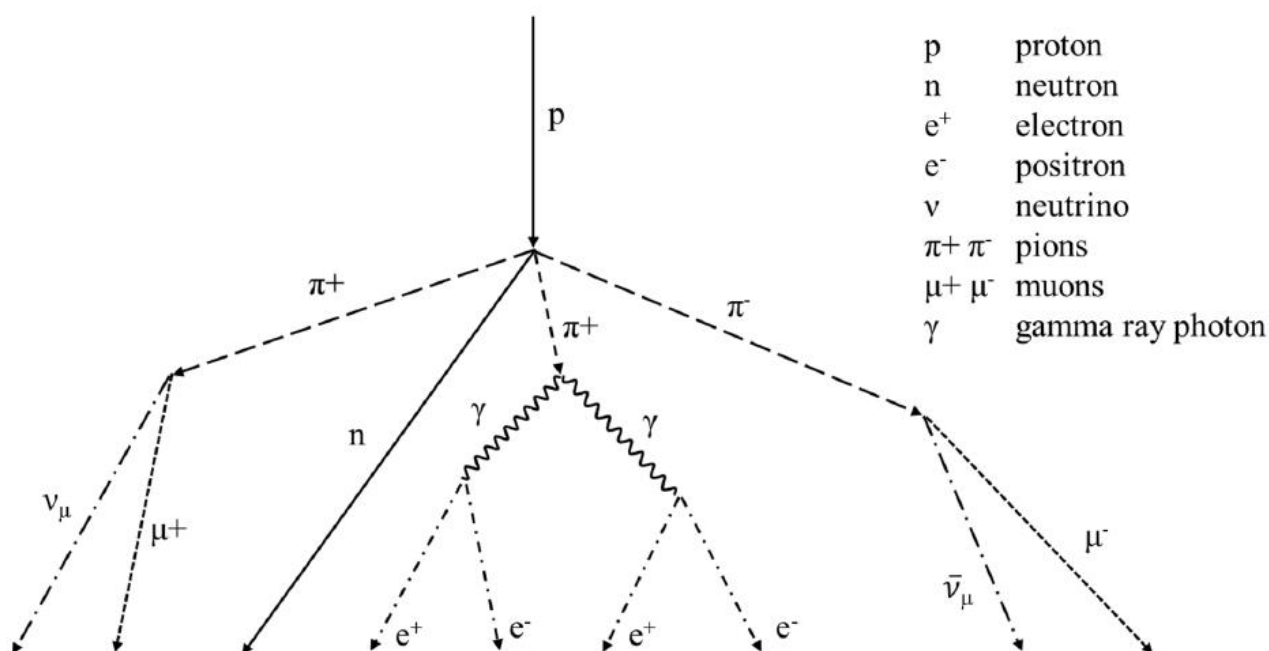
Origin

Intermediate energy cosmic rays are presumed to be created within our galaxy, often in connection with supernovae. Supernovae since they are believed to be the main source of cosmic rays.

Interaction with the Earth's atmosphere - Cosmic Showers

When cosmic rays reach the Earth, they interact with the Earth's atmosphere, producing a chain of reactions resulting in the production of a large number of particles known as a **cosmic air shower** (Figure 1). Analysing these showers allows the initial composition and energies of the original (primary) cosmic rays to be deduced.

When cosmic rays from space (primary cosmic rays) strike particles in the atmosphere they produce secondary particles, which go on to produce more collisions and particles, resulting in a shower of particles that is detected at ground level. The primary cosmic rays can usually only be detected directly in space, for example by detectors on satellites, although very high energy cosmic rays, which occur on rare occasions, can penetrate directly to ground level.



Cherenkov radiation - Air shower particles can travel at relativistic speeds. Although relativity requires that nothing can travel faster than the speed of light in a vacuum, particles may exceed the speed of light in a particular medium, for example water. Such particles then emit a beam of Cherenkov radiation – the radiation that causes the characteristic blue colour in nuclear reactors. (This is a bit like the optical equivalent of a sonic boom.)

Atmospheric fluorescence - When charged particles pass close to atoms in the atmosphere, they may temporarily excite electrons to higher energy levels. The photons emitted when the electrons return to their previous energy levels can then be detected.

Magnetosphere – Background information

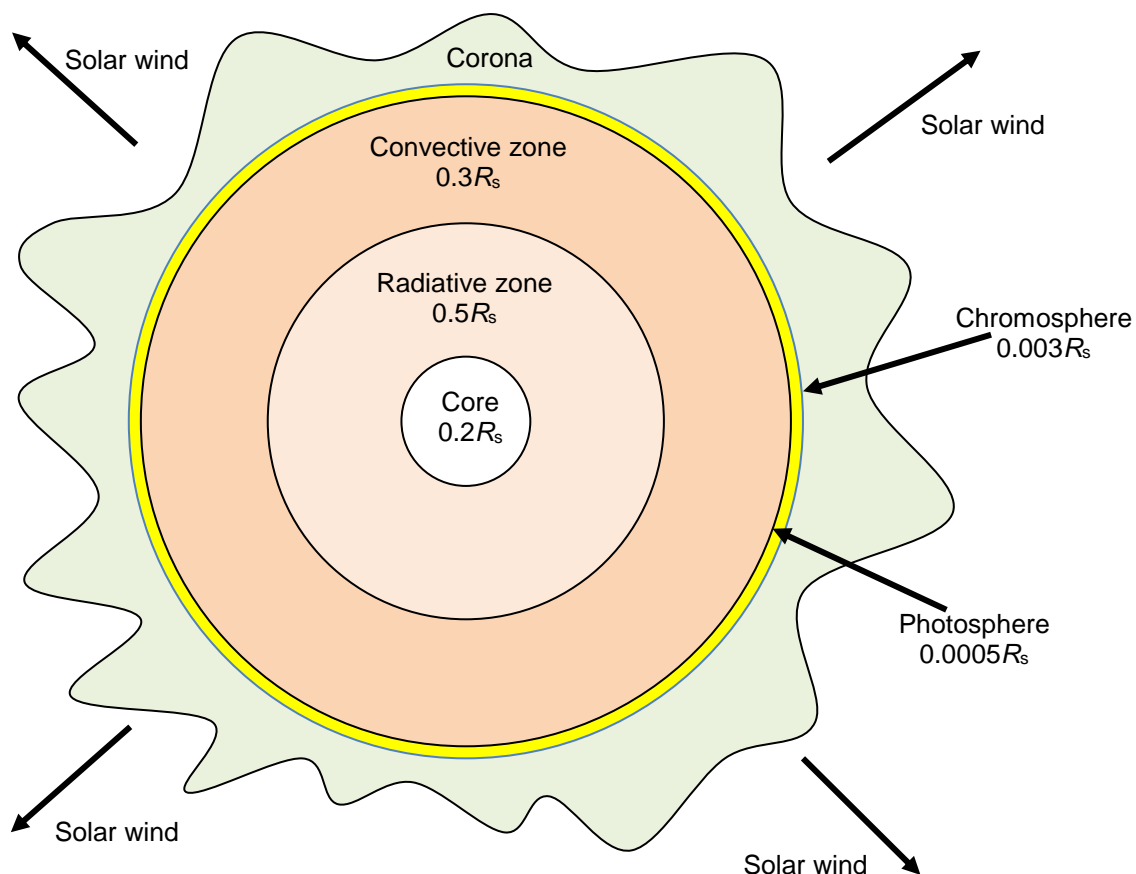
Structure of the Sun

The interior of the Sun consists of three main regions:

1. the **core**, within which nuclear fusion takes place
2. the **radiative zone**, through which energy is transported by photons
3. the **convective zone**, where energy is transported by convection.

The extended and complex solar atmosphere begins at the top of the convective zone, with the **photosphere**.

The photosphere is the visible surface of the Sun and appears smooth and featureless, marked by occasional relatively dark spots, called sunspots.



The solar wind

There is a continual flow of charged particles emanating from the Sun because of the high temperature of the corona. This gives some particles sufficient kinetic energy to escape from the Sun's gravity.

Solar winds originate from the sun, they are charged particles in the form of a plasma – roughly the same number of protons and electrons.

It can be thought of as an extension of the corona itself and as such reflects its composition. The solar wind also contains about 8% alpha particles (i.e. helium nuclei) and trace amounts of heavy ions and nuclei (C, N, O, Ne, Mg, Si, S and Fe).

The solar wind travels at speeds of between 300 and 800 kms^{-1} , with gusts recorded as high as 1000 km^{-1} (2.2 million miles per hour).

Comet tails

Although it was known that solar eruptions ejected material that could reach the Earth, no-one suspected that the Sun was continually losing material regardless of its apparent activity. It had been known for a long time that comet tails always pointed away from the Sun, although the reason was unknown. It was realised this could be explained if the comet was moving in flow of gas streaming away from the Sun. The comet's tail was acting like a wind-sock. In the early 1950s Biermann concluded that even when the Sun was quiet, with no eruptions or sunspots, there was still a continuous flow of gas from it.

Coronal holes

The magnetic field lines from coronal holes don't loop back onto the surface of the corona. Instead they project out into space like broken rubber bands, allowing charged particles to spiral along them and escape from the Sun. There is a marked increase in the solar wind when a coronal hole faces the Earth.

Solar flares

Solar flares are explosive releases of energy that radiate energy over virtually the entire electromagnetic spectrum, from gamma rays to long wavelength radio waves. They also emit high-energy particles called **solar cosmic rays**. These are composed of protons, electrons and atomic nuclei that have been accelerated to high energies in the flares. Protons (hydrogen nuclei) are the most abundant particles followed by alpha particles (helium nuclei). The electrons lose much of their energy in exciting radio bursts in the corona. These generally occur near sunspots, which leads to the suggestion they are magnetic phenomena. It is thought that magnetic field lines become so distorted and twisted that they suddenly snap like rubber bands. This releases a huge amount of energy, which can heat nearby plasma to 100 million kelvin in a few minutes or hours. This generates X-rays and can accelerate some charged particles in the vicinity to almost the speed of light.

The energies of solar cosmic ray particles range from millielectronvolts (10^{-3} eV) to tens of gigaelectronvolts (10^{10} eV).

The highest energy particles arrive at the Earth within half an hour of the flare maximum, followed by the peak number of particles 1 hour later. Particles streaming from the Sun after solar flares or other major solar events can disrupt communications and power delivery on Earth.

A major solar flare in 1989:

- caused the US Air Force to temporarily lose communication with over 2000 satellites
- induced currents in underground circuits of the Quebec hydroelectric system, causing it to be shut down for more than 8 hours.

The solar cycle

All solar activities show a cyclic variation with a period of about 11 years.

Sunspots

When an image of the sun is focussed on a screen dark spots called sunspots are often visible. By observing over a number of days they will be seen to move (due to the rotation of the Sun) and also change in size, growing or shrinking. The sunspots look dark because they are cooler than the surrounding photosphere. A large group of sunspots is called an **active region** and may contain up to 100 sunspots.

The general pattern of the movement of sunspots (individual sunspots may appear and disappear – short-lived ones only lasting a few hours whereas others may last for several months) shows that the Sun is rotating with an average period of about 27 days with its axis of rotation tilted slightly to the plane of the Earth's orbit.

Unlike the Earth, the Sun does not have a single rotation period. The period is 25 days at the Sun's equator and lengthens to 36 days near the poles. Sections at different latitudes rotate at different rates and so this is called **differential rotation**.

The three main features of the solar cycle are:

1. the number of sunspots
2. the mean latitude of sunspots
3. the magnetic polarity pattern of sunspot groups.

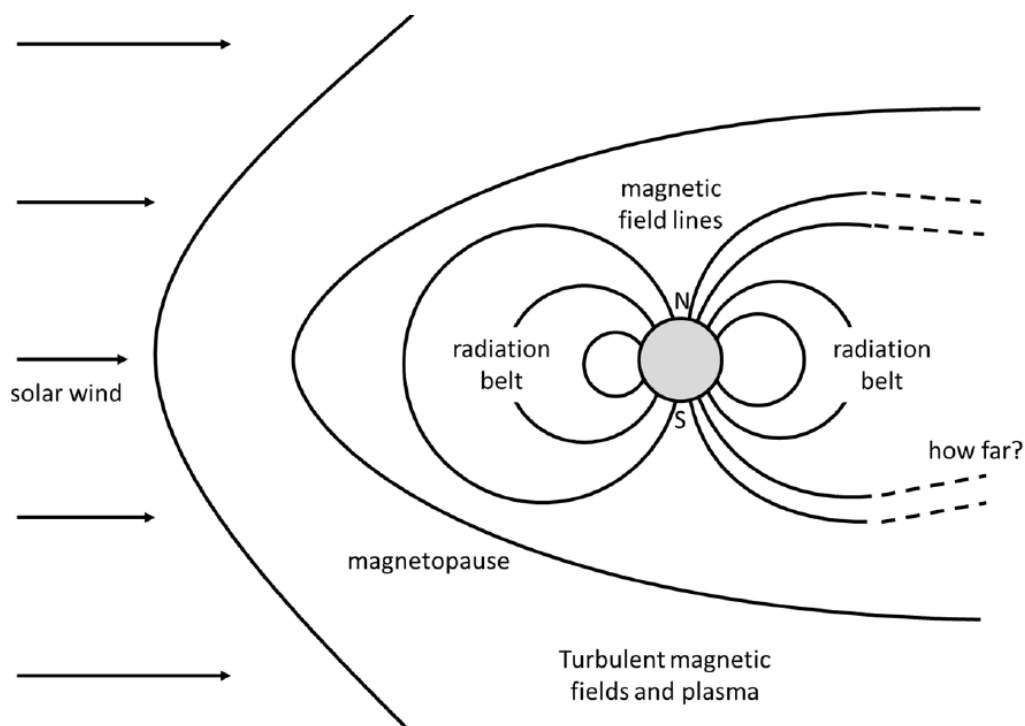
The number of sunspots increases and decreases with an 11-year cycle, the mean solar latitude at which the sunspots appear progresses towards the solar equator as the cycle advances and the magnetic polarity pattern of sunspot groups reverses around the end of each 11-year cycle (making the full cycle in effect 22 years).

The magnetosphere

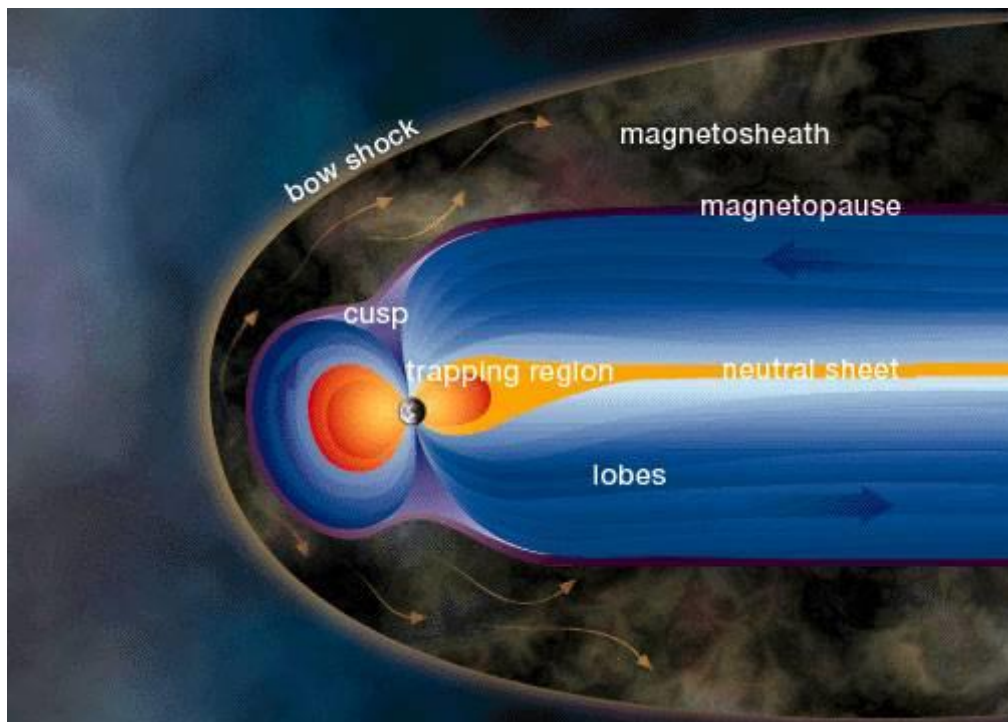
The magnetosphere is the part of the Earth's atmosphere dominated by the Earth's magnetic field. This region also contains a diffuse plasma of protons and electrons. The magnetic field resembles that of a bar magnet, tipped at about 11° to the Earth's rotational axis. However, the magnetic field is believed to be generated by electric currents in conducting material inside the Earth, like a giant dynamo. Geological evidence shows that the direction of the Earth's magnetic field has reversed on several occasions, the most recent being about 30,000 years ago. This lends evidence for the 'dynamo' model as the reversal can be explained in terms of changes in the flow of conducting fluids inside the Earth.

Interaction of the solar wind with the Earth's magnetic field

The solar wind interacts with the magnetosphere and distorts its pattern from the simple bar magnet model outlined above. The Earth's magnetic field also protects it from the solar wind, deflecting it a bit like a rock deflecting the flow of water in a river. The boundary where the solar wind is first deflected is called the bow shock. The cavity dominated by the Earth's magnetic field is the **magnetosphere**, see Figures 4 and 5.

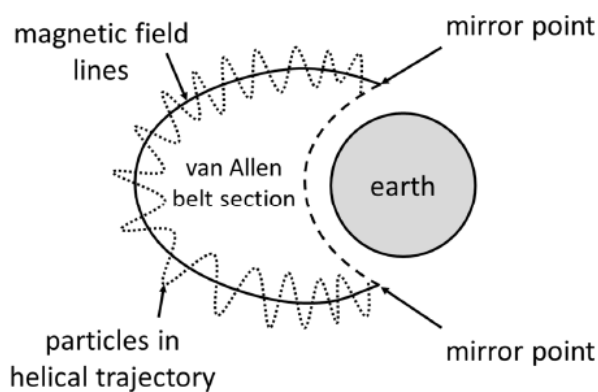


High-energy particles from the solar wind that leak into the magnetosphere and become trapped to form the Van Allen belts of radiation. These are toroidal in shape and concentric with the Earth's magnetic axis. There are two such belts: the inner and the outer.



The inner Van Allen belt lies between one and two Earth radii from its axis, ($R_E < \text{inner belt} < 2R_E$) then there is a distinct gap followed by the outer belt lying between three and four Earth radii ($3R_E < \text{outer belt} < 4R_E$). The inner belt traps protons with energies of between 10 and 50 MeV and electrons with energies greater than 30 MeV. The outer belt contains fewer energetic protons and electrons.

The charged particles trapped in the belts spiral along magnetic field lines and oscillate back and forth between the northern and southern mirror points with periods between 0.1 and 3 seconds as shown below.



Particles in the inner belt may interact with the thin upper atmosphere to produce the aurorae. These result from the excitation of different atoms in the atmosphere, each of which produces light with a characteristic colour due to the different energy associated with that atomic transition.

Charged particles in a magnetic field

The force acting on a charge q , moving with velocity v through a magnetic field B is given by:

$$F = qvB$$

where F , v and B are all mutually at right angles to each other.

Circular motion

As F_B is always at right angles to v , the particle will move with uniform motion in a circle, where F_c is the central force (assuming any other forces are negligible), so:

$$F = \frac{mv^2}{r}$$

Equating the magnetic force to the central force we get: $F_c = F_B$

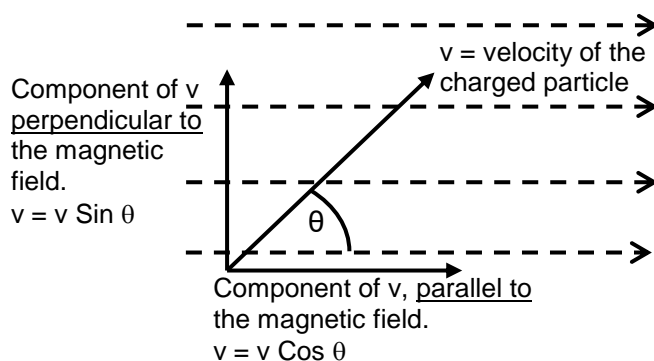
$$\frac{mv^2}{r} = qvB$$

so

$$r = \frac{mv}{qB}$$

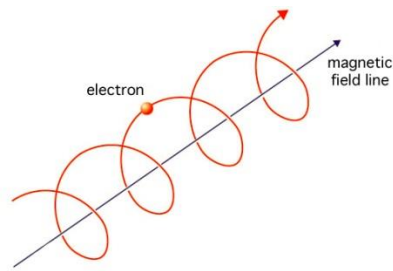
Hence when a charged particle enters a uniform magnetic field at 90° to the field lines it will follow a circular path as described above.

Helical motion



If a charged particle crosses the magnetic field lines at an angle θ , then its velocity can be resolved a component perpendicular to the field and the other parallel to it.

The perpendicular component provides the central force, which produces uniform circular motion as shown above. The component parallel to the magnetic field does not cause the charge to experience a magnetic force so it continues to move with constant velocity in that direction, resulting in a helical path.



Aurorae

Solar winds ie charged particles, travel towards the earth will spiral along the Earth's magnetic field lines to the North or South poles. When they reach the Earth's upper atmosphere they collide with air atoms excite them which then emit a photon to produce the aurora.

Collisions with atomic oxygen produce green light while collision with Nitrogen produces pink and violet light.

Aurora borealis in the Northern hemisphere –is known as the northern lights; aurora australis in the southern hemisphere – southern lights. Usually within 20° of the north or south poles.



Data

Common Physical Quantities

QUANTITY	SYMBOL	VALUE
Gravitational acceleration	g	9.8 m s^{-2}
Radius of Earth	R_E	$6.4 \times 10^6 \text{ m}$
Mass of Earth	M_E	$6.0 \times 10^{24} \text{ kg}$
Mass of Moon	M_M	$7.3 \times 10^{22} \text{ kg}$
Mean radius of Moon orbit		$3.84 \times 10^8 \text{ m}$
Universal constant of gravitation	G	$6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$
Speed of light in vacuum	c	$3.0 \times 10^8 \text{ m s}^{-1}$
Speed of sound in air	v	$3.4 \times 10^2 \text{ m s}^{-1}$
Mass of electron	m_e	$9.11 \times 10^{-31} \text{ kg}$
Charge on electron	e	$-1.60 \times 10^{-19} \text{ C}$
Mass of neutron	m_n	$1.675 \times 10^{-27} \text{ kg}$
Mass of proton	m_p	$1.673 \times 10^{-27} \text{ kg}$
Planck's constant	h	$6.63 \times 10^{-34} \text{ J s}$
Permittivity of free space	ϵ_0	$8.85 \times 10^{-12} \text{ F m}^{-1}$
Permeability of free space	μ_0	$4\pi \times 10^{-7} \text{ H m}^{-1}$

Astronomical Data

Planet or satellite	Mass/ kg	Density/ kg m ⁻³	Radius/ m	Grav. accel./ m s ⁻²	Escape velocity/ m s ⁻¹	Mean dist from Sun/ m	Mean dist from Earth/ m
Sun	1.99×10^{30}	1.41×10^3	7.0×10^8	274	6.2×10^5	--	1.5×10^{11}
Earth	6.0×10^{24}	5.5×10^3	6.4×10^6	9.8	11.3×10^3	1.5×10^{11}	--
Moon	7.3×10^{22}	3.3×10^3	1.7×10^6	1.6	2.4×10^3	--	3.84×10^8
Mars	6.4×10^{23}	3.9×10^3	3.4×10^6	3.7	5.0×10^3	2.3×10^{11}	--
Venus	4.9×10^{24}	5.3×10^3	6.05×10^6	8.9	10.4×10^3	1.1×10^{11}	--