St Ninian's High School

Higher Physics

Our Dynamic Universe
Pupil Notes

1.1 Equations of Motion

Vectors

Vectors and Scalars (Revision of National 5)

A **scalar** is a quantity that can be described by just a **magnitude (size)** and a unit. e.g. time 30 s, mass 20 kg.

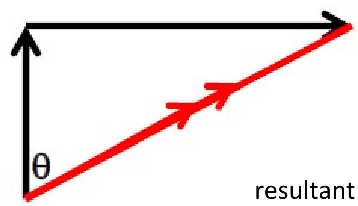
A **vector** is a quantity that is fully described with a **magnitude and *direction***.
e.g. a force of 50 N downwards, a velocity of 20 ms^{-1} East.

Adding Vectors (Revision of National 5)

This is more difficult than adding scalars as the direction of the vectors must be taken into account.
The addition of two vectors is called the ***resultant*** vector.
When you add vectors they have to be added ***tip-to-tail***.

- Each vector must be represented by a straight line of suitable scale.
- The straight line must have an arrow head to show its direction.
- The vectors must be joined one at a time so that the tip of the previous vector touches the tail of the next vector. i.e.

- A straight line is drawn from the starting point to the finishing point and the starting angle is marked.



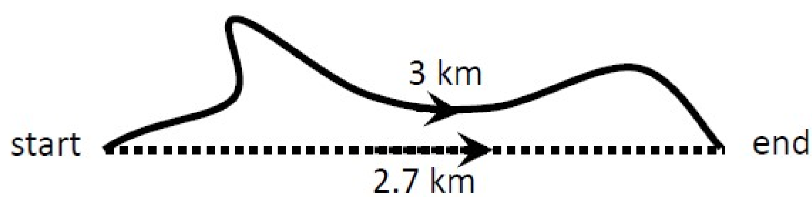
- The resultant should have 2 arrow heads to make it easy to recognise.
- If using a scale diagram the length and direction of this straight line gives the resultant vector.
- Alternatively you can use trigonometry and SOHCAHTOA or the sine or cosine rule to calculate the resultant.

Distance and Displacement (Revision of National 5)

The total **distance** travelled by an object is the **sum of the distances** of each stage of the journey. Since each stage has a different direction, the total distance has no single direction and therefore distance is a scalar.

The **displacement** of an object is the **shortest route between the start and finish point measured in a straight line**. Displacement (s) has a direction and is a vector.

Consider the journey below. A person walks along a path (solid line) from start to end.



They will have walked further following the path than if they had been able to walk directly from start to end in a straight line (dashed line).
The solid line denotes the **distance** = 3km. The dashed line denotes the **displacement** = 2.7 km East

Example:

A woman walks her dog 3 km due North (000) and then 4 km (030).
Find her

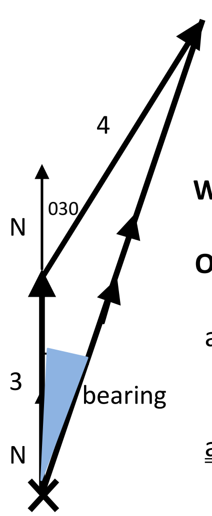
a) distance travelled
b) displacement.

Solution:

a) Distance travelled = 3km + 4km = 7km

b) **Use a ruler to measure the lengths of the vectors and a protractor to measure the bearing.**

- Choose an appropriate scale e.g. 1cm : 1km
- Mark the start point with an X, draw a North line and draw the first vector.
- Draw a North line at the tip of this vector and now draw the second vector (tip to tail) using a protractor to mark the correct angle.
- Draw the resultant vector from start to end using the double arrow.
- Measure the length of the line and the bearing.



When measuring bearings remember - from START – CLOCKWISE from NORTH

OR use the cosine rule to find the resultant and sine rule to find the angle

$a^2 = b^2 + c^2 - 2bc \cos \theta$
$= 3^2 + 4^2 - 2 \times 3 \times 4 \cos 150$
$= 25 + 20.78$
$\underline{\underline{a = 6.8\text{km}}}$

$\dfrac{\sin \theta}{4} = \dfrac{\sin 150}{6.8}$

$\sin \theta = \dfrac{4 \times \sin 150}{6.8} = 0.294$

$\underline{\underline{\theta = \sin^{-1} 0.294 = 17.1^o}}$

<u>Displacement = 6.8km (017)</u>

Speed and Velocity (Revision of National 5)

Speed is defined as the **distance travelled per second** and is measured in **metres per second**, **or ms^{-1}.**
Since distance and time are both scalar quantities then speed is also a scalar quantity.

$$\boxed{v = \frac{d}{t}}$$

The **velocity** of an object is defined as **the displacement per second** and has the symbol **v**.
Since displacement is a vector quantity that means that velocity is also a vector.
The equation for velocity is:

s- displacement in m

t- time in seconds

$$\boxed{v = \frac{s}{t}}$$

Note: The velocity calculated has the same direction as the displacement

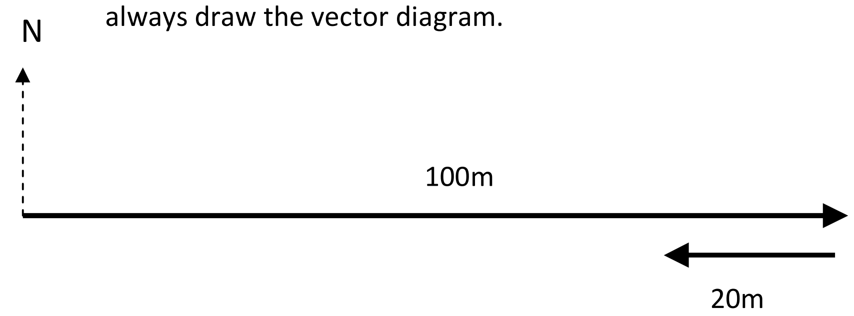
Example:

A runner sprints 100 m East along a straight track in 12 s and then takes a further 13 s to jog 20 m back towards the starting point.

(a) What distance does she run during the 25 s?
(b) What is her displacement from her starting point after the 25 s?
(c) What is her speed?
(d) What is her velocity?

Solution:

always draw the vector diagram.



(a) Distance
$d = 100 + 20$
$d = 120$ m

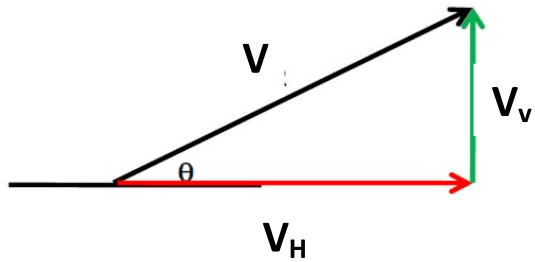
(b) Displacement
$s = 100 + (-20)$
$s = 80$ m (090)

(c) Speed
$v = d / t$
$v = 120/25$
$v = 4.8$ ms^{-1}

(d) Velocity
$v = s / t$
$v = 80/25$
$v = 3.2$ ms^{-1} (090)

Resolving Vectors

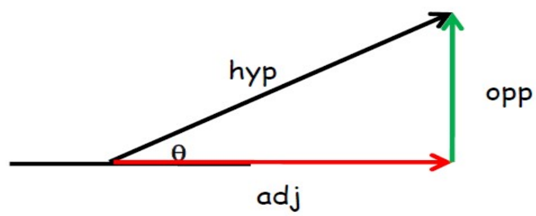
Two vectors can be added to give the resultant using vector addition.
A resultant vector can also be split into the two individual vectors that make it up.
Consider the following.



This shows a **resultant vector, V**, at an **angle ϴ** to the horizontal.
To travel to the end of the vector we could move in a straight line in the X direction and then a straight line in the Y direction as shown.

V_v is called the **vertical component** of the vector
V_H is called the **horizontal component** of the vector

Since we have a right angled triangle with a known angle we can use SOH-CAH-TOA to work out the unknown sides.



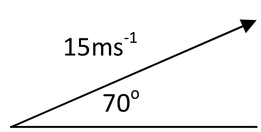
$\cos\theta = \frac{A}{H} = \frac{V_H}{V}$ and $\sin\theta = \frac{O}{H} = \frac{V_v}{V}$

horizontal component	vertical component
$V_H = V\cos\Theta$	$V_V = V\sin\Theta$

Example:

A football is kicked at an angle of 70° at 15 ms^{-1}.
Calculate:

a) the horizontal component of the velocity
b) the vertical component of the velocity.



Solution:

a) $V_H = V\cos\Theta$
$= 15\cos 70$
$= 5.2\ ms^{-1}$

b) $V_v = V\sin\Theta$
$= 15\sin 70$
$= 14.1\ ms^{-1}$

The Equations of Motion

Deriving Equations of Motion

The equations of motion can be applied to any object moving with constant acceleration in a straight line.
For all of these we will use quantities with symbols as follows:

s the displacement in metres (m)
u the initial velocity in metres per second (ms^{-1})
v the final velocity in metres per second (ms^{-1})
a the acceleration and is measured in metres per second per second (ms^{-2})
t the time in seconds (s)

(sometimes these equations are referred to as 'suvat' equations).

Equation of Motion 1:
Acceleration is the rate of change of velocity.
This can be written as

$$a = \frac{(v - u)}{t}$$

We can rearrange as $\boxed{\mathbf{v = u + at}}$

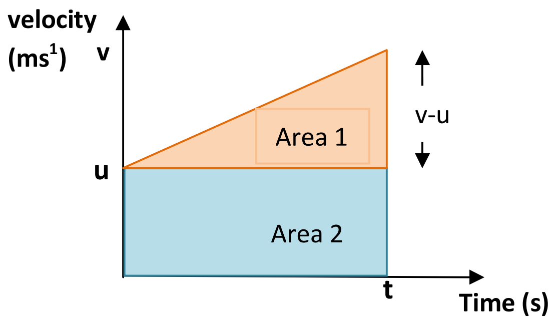
Example:
A racing car starts from rest and accelerates uniformly in a straight line at 12 ms^{-2} for 5.0 s.
Calculate the final velocity of the car.

Solution:

s	$v = u + at$
$u = 0\ ms^{-1}$ (rest)	$v = 0 + (12 \times 5.0)$
v	$v = 0 + 60$
$a = 12\ ms^{-2}$	$\underline{\underline{v = 60\ ms^{-1}}}$
$t = 5.0$ s	

Equation of Motion 2:

To find an expression for the displacement we use the fact that the displacement of an accelerating object is the area under the velocity-time graph.



Area 1 = ½ base x height
= ½ t x (v-u)
= ½(v-u)t

Substitute (v-u) = at from **Equation of Motion 1**

Area 1 = ½ at^2

Area 2 = base x height = t x u = ut

Total displacement (add area 1 and area 2)

$$\boxed{s = ut + \tfrac{1}{2}at^2}$$

Example:

A speedboat travels 400 m in a straight line when it accelerates uniformly from 2.5 ms^{-1} in 10 s. Calculate the acceleration of the speedboat.

Solution:

s = 400 m
u = 2.5 ms^{-1}
v
a = ?
t = 10 s

$s = ut + \frac{1}{2}at^2$
$400 = (2.5 \times 10) + (0.5 \times a \times 10^2)$
$400 = 25 + 50a$
$50a = 400 - 25 = 375$
$a = 375/50$
$\underline{\underline{a = 7.5\ ms^{-2}}}$

Equation of Motion 3:

To find the final equation we take **Equation of Motion 1** and square it

$v^2 = (u + at)^2$
$= u^2 + 2uat + a^2t^2$ | take out a common factor of 2a
$V^2 = u^2 + 2a(ut + \frac{1}{2}at^2)$ | substitute **Equation of Motion 2**
$v^2 = u^2 + 2as$

$$\boxed{v^2 = u^2 + 2as}$$

Equation of Motion 3 contd

Example:

A rocket is travelling through outer space with uniform velocity. It then accelerates at 2.5 ms^{-2} in a straight line in the original direction, reaching 100 ms^{-1} after travelling 1875 m.
Calculate the rocket's initial velocity.

Solution:

s = 1 875 m	$v^2 = u^2 + 2as$
u = ?	$100^2 = u^2 + (2 \times 2.5 \times 1875)$
v = 100 ms^{-1}	$10\,000 = u^2 + 9375$
a = 2.5 ms^{-2}	$u^2 = 10\,000 - 9375$
t	$u^2 = 625$
	$\underline{\underline{u = 25\ ms^{-1}}}$

The Equations of Motion with Decelerating Objects

When an object decelerates its velocity decreases. If the vector quantities in the equations of motion are positive, we represent the decreasing velocity by use of a **negative sign** in front of the acceleration value.

Example 1

A greyhound is running at 6.0 ms^{-1}. It decelerates uniformly in a straight line at 0.5 ms^{-2} for 4.0 s.
Calculate the displacement of the greyhound while it was decelerating.

Solution:

s = ?	$s = ut + \frac{1}{2}at^2$
u = 6.0 ms^{-1}	$s = (6.0 \times 4.0) + (0.5 \times \mathbf{-0.5} \times 4.0^2)$
v	$s = 24 + (-4.0)$
a = -0.5 ms^{-2}	$s = 20$ m
t = 4.0 s	

Example 2:

A curling stone leaves a player's hand at 5.0 ms^{-1} and decelerates uniformly at 0.75ms^{-2} in a straight line for 16.5 m until it strikes another stationary stone.
Calculate the velocity of the decelerating curling stone at the instant it strikes the stationary one.

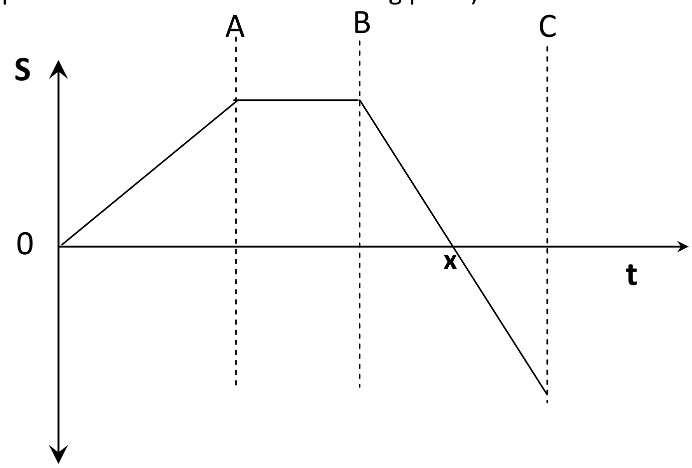
Solution:

s = 16.5 m	$v^2 = u^2 + 2as$
u = 5.0 ms^{-1}	$v^2 = 5.0^2 + (2 \times -0.75 \times 16.5)$
v = ?	$v^2 = 25 + (-24.75)$
a = -0.75 ms^{-2}	$v = \sqrt{0.25}$
t	$v = 0.5\ ms^{-1}$

Motion Graphs

Displacement – time graphs represent how far an object is from its starting point at some known time. Because displacement is a vector it can have positive and negative values.

(+ve and –ve will be opposite directions from the starting point).



Analyse the graph in sections:

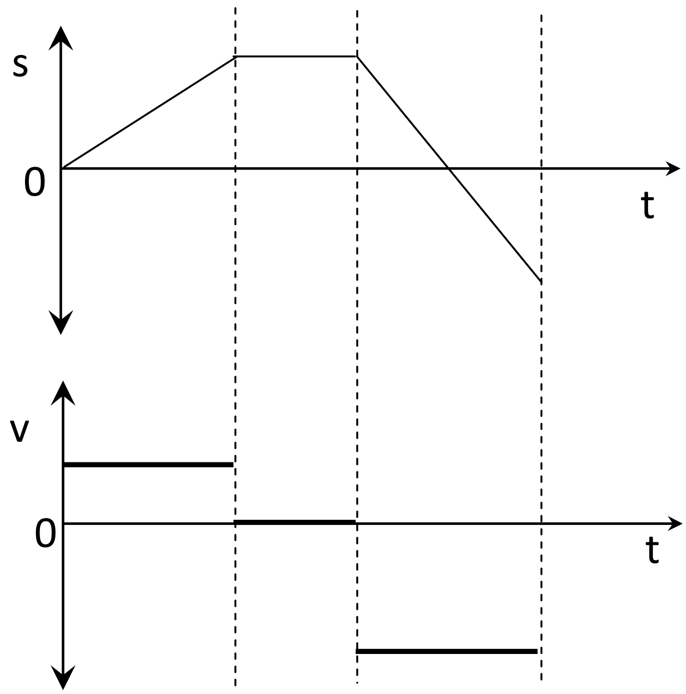
OA the object is moving away from the starting point. The displacement increases by the same amount each second, this is shown by the constant gradient. We can determine the velocity from the gradient of a displacement time graph.

Gradient = Displacement/time = velocity

AB the object has a constant displacement so is not changing its position, therefore it must be at rest. The gradient in this case is zero, which means the object has a velocity of 0 ms^{-1}

BC the object is now moving back towards the starting point, reaching it at time x (when the displacement is zero). It then continues to move away from the start, but in the opposite direction. The **gradient is constant**, so the object travels at **constant velocity** and in the opposite direction, because it's negative.

Converting Displacement – time Graphs to Velocity-time Graphs



We can sketch the velocity-time graph from the gradients of the displacement-time graph.
A positive gradient means a constant positive velocity, shown as a flat line on the velocity-time graph.
Zero gradient (a constant displacement) means the velocity is zero.
A negative gradient means a constant negative velocity.

There are no numerical values needed on the graphs above to describe the motion of the object.

<u>Reminder from National 5</u>

The area under a speed time graph is equal to the distance travelled by the object that makes the speed time graph.
In this course we are dealing with vectors so the statement above has to be changed to:
The area under a **velocity** time graph is equal to the **displacement of** the object that makes the speed time graph.
Any calculated areas that are below the time axis represent **negative** displacements.

Velocity – Time Graphs

All velocity time graphs that you encounter in this course will be of objects that have constant acceleration. You may be asked to identify or sketch the velocity-time graph for the motion of a falling object.

Example: The Bouncing Ball

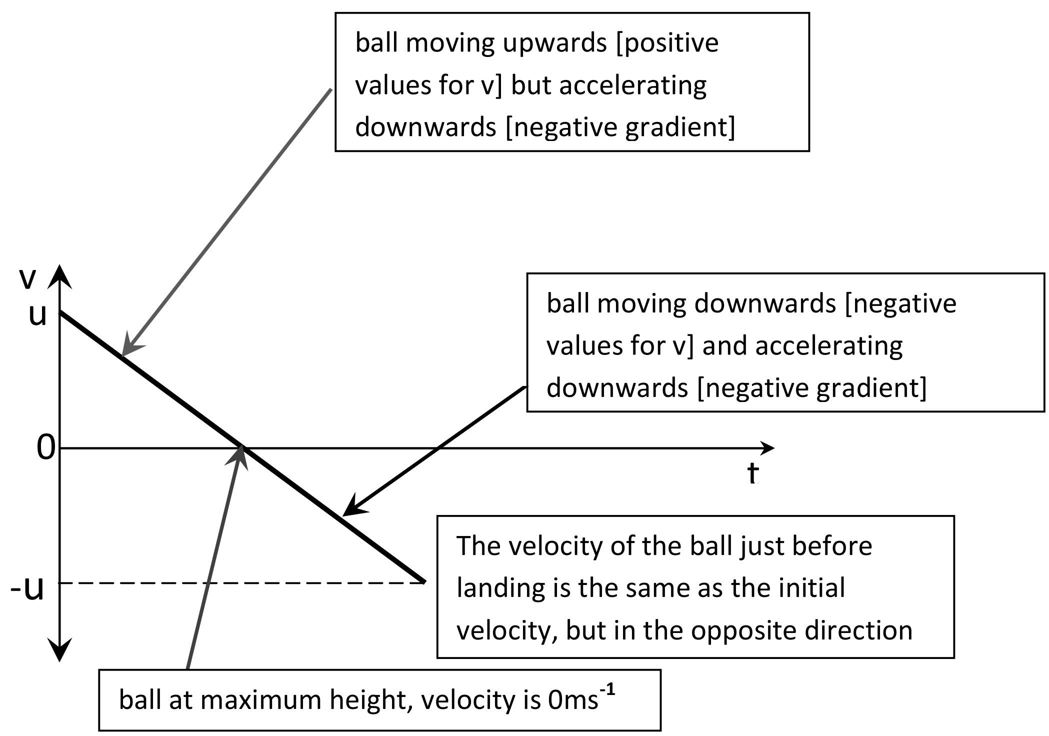
A ball is fired vertically into the air from the ground. The ball reaches its maximum height, falls, bounces and then rises to a new, lower, maximum height.

Solution:

The original direction of motion is up so upwards is the positive direction

Part One of Graph

The ball slows down as it moves upwards, having a velocity of zero at maximum height. The acceleration of the ball will be constant if we ignore air resistance. Once the ball reaches its maximum height it will fall and will accelerate at the same rate as when it was going up (the force acting is still the same), so the gradient of the line stays the same.

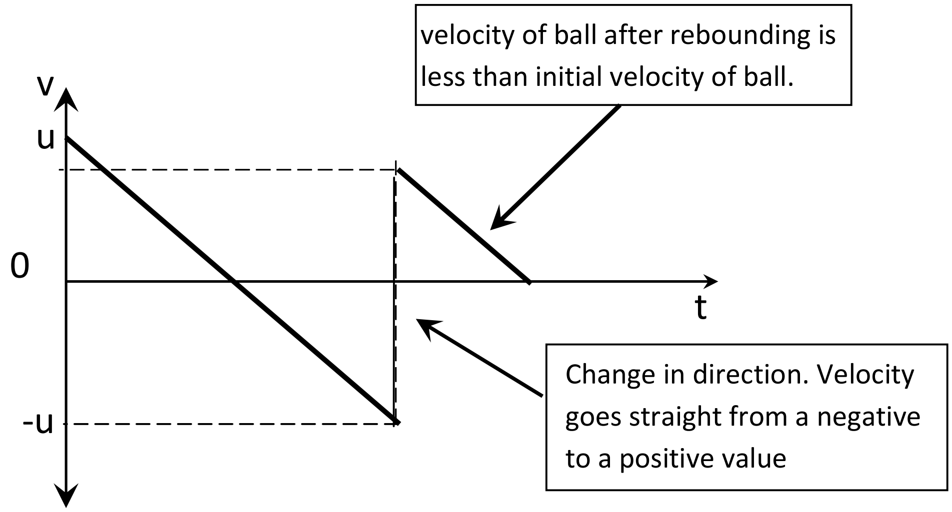


Velocity – Time Graphs (continued)

Part Two of Graph

The ball hits the ground. It will rebound and move upwards. We regard the time of contact with the ground as zero, although in reality there will be a finite time of contact with the ground.

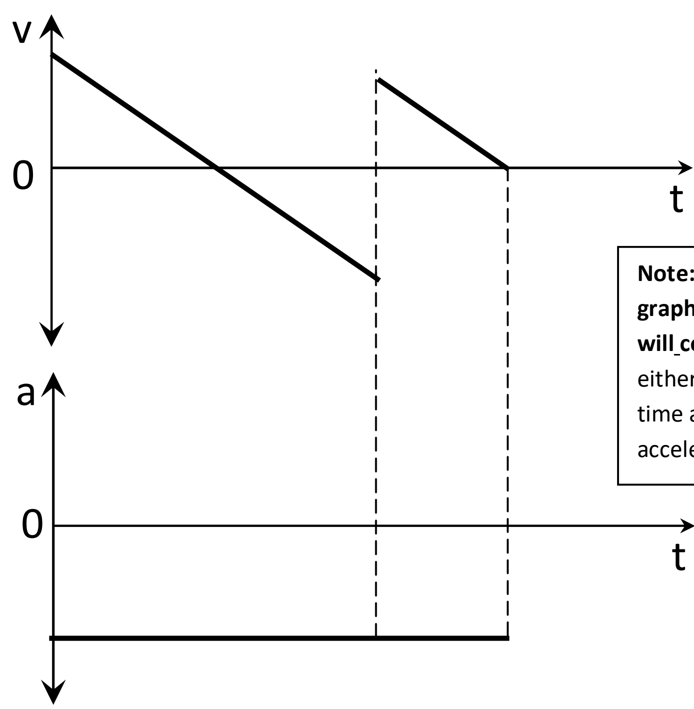
The acceleration of the ball after rebounding will be the same as the initial acceleration. The two lines will be parallel.



Note that the rebound velocity is less than u because energy is lost as heat when the ball bounces

Converting Velocity – Time Graphs to Acceleration – Time Graphs

What is important in this conversion is to consider the gradient of the velocity-time graph line.
In our example the gradient of the line is constant and has a **negative** value. This means the acceleration will have a single **negative** value.



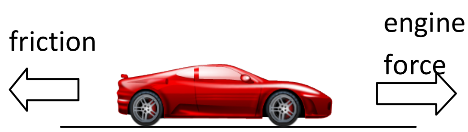
Note: All acceleration time graphs you are asked to draw will consist of horizontal lines, either above, below or on the time axis, because the accelerations will be constant.

1.2 Forces, Energy and Power

Forces

Newton's 1st Law of Motion (Revision of National 5)

An object will remain at rest or travel in a straight line at a constant velocity if the forces are balanced.



For a car moving in a straight line, if the engine force and friction force are equal, the car will continue to move at a constant velocity (in the same direction at the same speed).
If the same car is stationary (not moving) and all forces acting on it are balanced (same as no force at all) the car will not move.

Newton's 2nd Law of Motion (Revision of National 5)

When an **unbalanced force** acts, the velocity cannot remain constant and the acceleration produced will depend on:

- the mass (**m**) of the object **($a \propto 1/m$) - if mass increases, acceleration decreases and vice versa;**
- the unbalanced force **(F)** **($a \propto F$) - if Force increases acceleration increases and vice versa.**

This law can be summarised by the equation $\boxed{F = ma}$

Where F is the unbalanced force in Newtons (N)
m is the mass in kilograms (kg)
a is the acceleration in metres per second squared ms^{-2}

Resultant Forces

When several forces act on one object, they can be replaced by one force which has the same effect. This single force is called the **resultant** or **unbalanced** force. Remember that **friction** is a resistive force which always acts in the **opposite** direction to motion.

We use free body diagrams to show the forces acting on an object and to help determine the resultant force. **The direction and size of the force shown is important.**

Resultant Forces - Examples

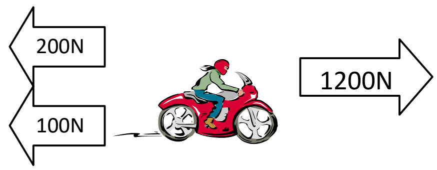
Example 1: Horizontal Forces

A motorcycle and rider of combined mass 650 kg provide an engine force of 1200 N. The friction between the road and motorcycle is 100N and the drag value = 200N.

Calculate:

a) the unbalanced force acting on the motorcycle
b) the acceleration of the motorcycle

Solution



a) Draw a free body diagram

$F_{un} = 1200 - (200 + 100)$
$\underline{\underline{F_{un} = 900\ N}}$ This 900 N force is the resultant of the 3 forces

b) To calculate acceleration we use the unbalanced force

$F_{un} = 900\ N$ $F_{un} = ma$
$a = ?$ $900 = 650 \times a$
$m = 650\ kg$ $\underline{\underline{a = 1.38\ ms^{-2}}}$

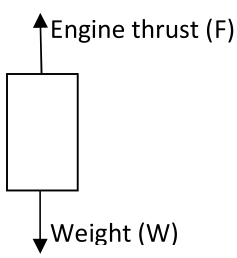
Example 2: Vertical (Rocket)

At launch, a rocket of mass 20 000 kg accelerates off the ground at 12 ms^{-2} (ignore air resistance)

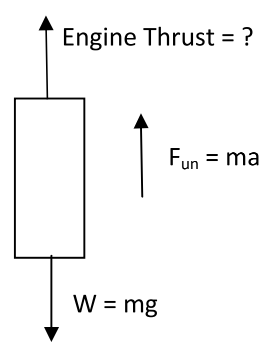
a) Draw a free body diagram to show all the vertical forces acting on the rocket as it accelerates upwards.
b) Calculate the engine thrust of the rocket which causes the acceleration of $12ms^{-2}$.

Solution:

a)



b) Calculate F_{un} and W (m= 20,000kg and g=9.8 Nkg^{-1} on Earth)



$F_{un} = m\,a$ $W = m\,g$
$F_{un} = 20\,000 \times 12$ $W = 20\,000 \times 9.8$
$F_{un} = 240\,000\ N$ $W = 196\,000\ N$

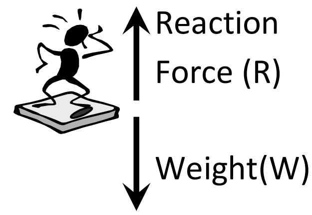
F_{un} = Engine Thrust – Weight (from the diagram)
240 000 = Engine thrust – 196 000
<u>Engine Thrust = 436 000 N</u>

Resultant Forces in Lifts

When you stand on a set of scales (a balance) the reading measures the upwards force they exert on you, called the **Reaction Force. This force will be different when you are stationary, moving at constant velocity or accelerating.**

From the free body diagram

$\mathbf{F_u = R - W}$ W is weight and
R is the Reaction force
or Reading on the balance



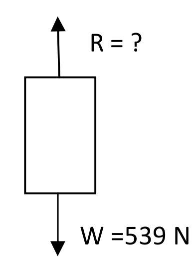
When forces are balanced, F_u is 0 and W = R. The reading on the balance is the same as your weight. This occurs when you are still, or travelling at constant velocity. You can't tell whether you are moving or not.

When there is an unbalanced force, W ≠ R. The reading on the balance will not be your weight. The reading will allow you to calculate the unbalanced force you can 'feel' acting on you. This unbalanced force could be acting up or down depending on the motion of the lift.

<u>Example:</u> A 55kg woman stands on a balance in a lift.

a) What is the reading on the balance when she accelerates upwards at $2ms^{-2}$?
b) What is the tension in the lift cable when the woman accelerates downwards at $2ms^{-2}$?

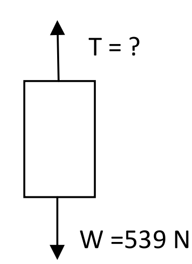
<u>Solution:</u>



a) We can calculate the unbalanced force from the acceleration.
$F_u = ma = 55 \times 2 = 110$ N

The weight of the woman $W = mg = 55 \times 9.8 = 539$ N

From the free body diagram
$F_u = R - W$
$110 = R - 539$
$\underline{\underline{R = 649 \text{ N}}}$ *she 'feels' heavier because she is accelerating up*.



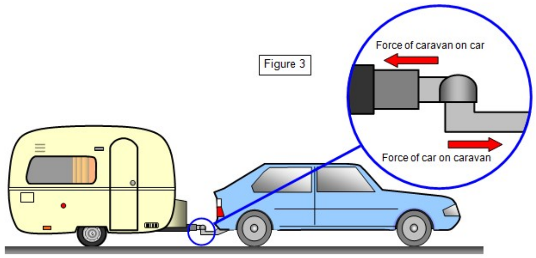
b) A **downward** direction gives a **negative** acceleration
(Note that the reaction force equals the tension in the lift cable).

$F_u = ma = 55 \times -2 = -110$ N $\qquad F_u = T - W$
$-110 = T - 539$
$\underline{\underline{T = 429 \text{ N}}}$ *the woman would 'feel' lighter.*

The reading on the scales would be higher when decelerating down (positive acceleration) and lower when decelerating up (negative acceleration).

Internal Forces

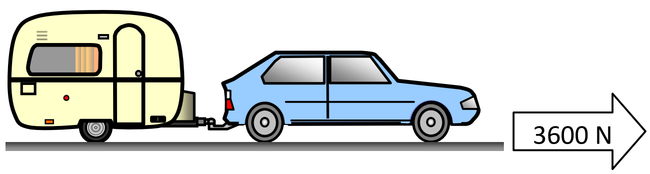
An example of an internal force is the tension in the tow bar (magnified below) when a car is pulling a caravan.



We can calculate the tension between the two objects if we know their acceleration.

Example 2:

A car of mass 700 kg pulls a 500 kg caravan with a constant engine thrust of 3.6 kN.
Calculate the tension in the tow bar during the journey (ignoring friction).



Solution:

Calculate the acceleration of the whole system using F = ma

F = 3600 N	a = F / m
m = 500 + 700 = 1200 kg	a = 3600 / 1200
a = ?	a = 3 ms^{-2}

The caravan is also accelerating at this rate. We can calculate the force needed to do this which is equal to the tension in the tow bar.

F = ?	F = ma
m = 500 kg	F = 500 x 3
a = 3ms^{-2}	<u>F = 1500 N = Tension in the towbar</u>

Newton's 3rd Law of Motion (Revision of National 5)

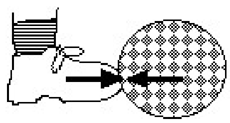
Newton noticed that forces occur in pairs. He called one force the **action** and the other the **reaction**. These two forces are always equal in size, but opposite in direction. They do not both act on the same object (do not confuse this with balanced forces).

Newton's Third Law can be stated as:

If an object A exerts a force (the action) on object B, then object B will exert an equal, but opposite force (the reaction) on object A.

For example:

a) Kicking a ball

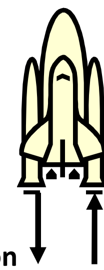


Action Reaction

Action: **The foot exerts a force on the ball to the right**

Reaction : **The ball exerts an equal Force on the left boot.**

b) Rocket flight



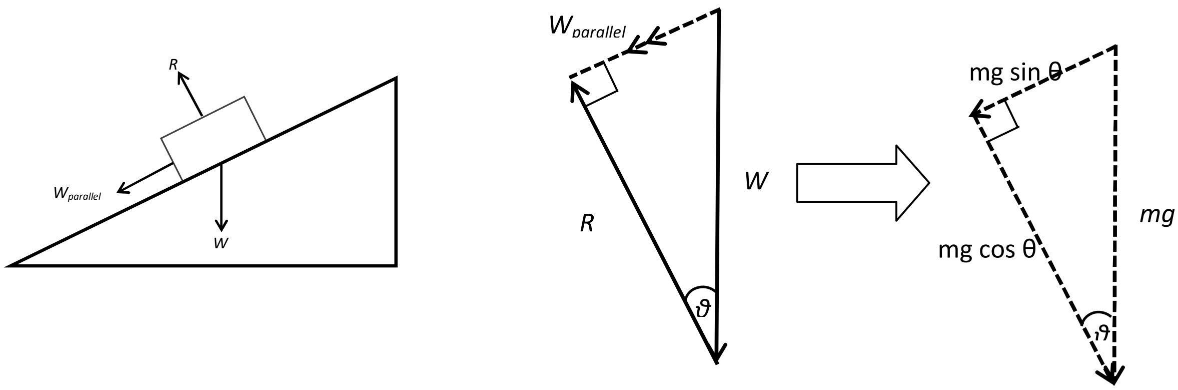
Action Reaction

Action: **The rocket pushes gases out of the back of the rocket**

Reaction: **The gases push the rocket in the opposite direction.**

Forces on a Slope – Component of Weight

The downward force acting on an object due to gravity is its weight, W = mg.
On a slope, we can show the weight as being made up of two components.
The component of Weight acting perpendicular to the slope is R, the Reaction force.



It is the **component of the weight** parallel to the slope that acts on the object, pulling it down the slope.

$$\boxed{W_{parallel} = mg \sin \theta}$$

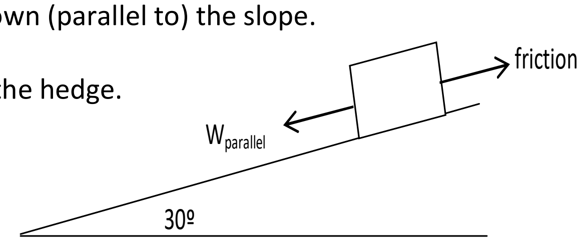
The reaction component $R = W_{perpendicular} = mg \cos \theta$ and is seldom required at higher level.

Component of Weight - Example

Example

A car of mass 1000 kg is parked on a hill. The slope of the hill is 20° to the horizontal. The brakes on the car fail. The car runs down the hill for a distance of 75 m until it crashes into a hedge. The average force of friction on the car as it runs down the hill is 250 N.

a) Calculate the component of the weight acting down (parallel to) the slope.
b) Find the acceleration of the car.
c) Calculate the speed of the car just before it hits the hedge.



Solution

a) $W_{parallel} = mg \sin \theta$
$= 1000 \times 9.8 \sin 30$
$\underline{\underline{W_{parallel} = 4900\text{ N}}}$

b) Calculate the unbalanced Force

$F_{un} = W_{parallel} – \text{Friction}$
$= 4900 – 250$
$F_{un} = 4650\text{ N}$

$a = F_{un} / m$
$a = 4650 / 1000$
$\underline{\underline{a = 4.65\text{ ms}^{-2}}}$

c) s = 75 m
u = 0 (parked at rest)
v = ?
$a = 4.65\text{ ms}^{-2}$
t =

$v^2 = u^2 + 2as$
$v^2 = 0 + 2 \times 4.65 \times 75$
$\underline{\underline{v = 26.4\text{ ms}^{-1}}}$

Energy & Power

Conservation of Energy

The principle of conservation of energy states that:

Energy cannot be created or destroyed, only converted from one form to another.

There are a number of equations for the different forms of energy:

Kinetic Energy	Potential Energy	Heat Energy
$E_k = ½ mv^2$	$E_p = mgh$	$E_H = c\,m\,\Delta T$

Work	Latent Heat Energy
$E_w = Fd$	$E_v = m L_v$ or $E_f = m L_f$

All forms of energy can be converted into any other form, so each of these equations can be equated to any other.

Power is a measure of the rate at which the energy is transferred and is measured in Watts.

$$P = \frac{E}{t}$$

Work Done against Friction

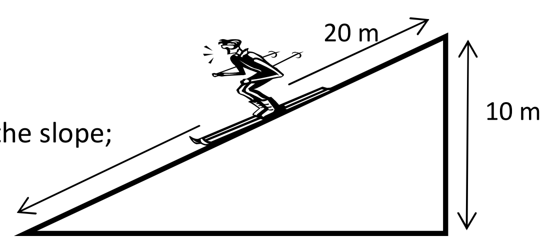
Work is done when Kinetic Energy is converted to Heat by Friction.

Example:

A skier of mass 60 kg slides from rest down a slope of length 20 m. The initial height of the skier was 10 m above the bottom and the final speed of the skier at the bottom of the ramp was 13 ms^{-1}.

Calculate:

a) The potential energy of the skier at the top of the hill.
b) The kinetic energy at the bottom of the slope
c) the work done against friction as the skier slides down the slope;
d) the average force of friction acting on the skier.



Solution:

a) $E_p = mgh$

$E_p = 60 \times 9.8 \times 10$

$E_p = 5880\ J$

b) $E_k = ½mv^2$

$= ½ \times 60 \times (13)^2$

$E_k = 5070\ J$

c) The work done against friction is the difference between E_p at the top and E_k at the bottom of the slope.

$E_p – E_k = 5880 - 5070 = 710\ J$

d) $E_w = F\,d$

$5880 = F \times 20$

$F = 5880/20$

$F = 294N$

1.3 Collisions, Explosions and Impulse

Momentum

Conservation of Momentum

Momentum, p, is the measure of an object's motion

$$\boxed{p = mv}$$

Where m is mass (kg)
v is velocity (ms^{-1})

Since velocity is a vector so is momentum. Its direction is shown by its sign. Usually to the right is positive and to the left negative.

In any collision of two objects, we say that momentum is conserved.
The **law of conservation of momentum** states that:

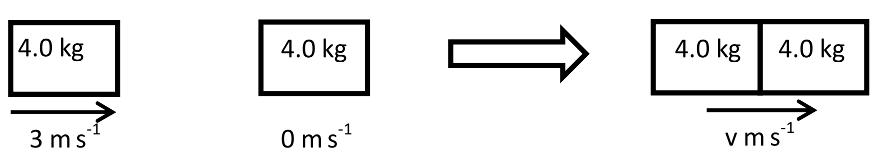
In the absence of net external forces, total momentum before = total momentum after

Collisions

The law of conservation of momentum can be used to analyse the motion of objects before and after a collision and an explosion. Let's deal with collisions first of all.
A collision is an event when two objects apply a force to each other for a relatively short time.

<u>Example:</u>
A trolley of mass 4.0 kg is travelling with a speed of 3 m s^{-1}. The trolley collides with a stationary trolley of equal mass and they move off together.
Calculate the velocity of the trolleys immediately after the collision.

<u>Solution</u>
Draw a diagram of the objects before and after the collision



Total momentum before = total momentum after

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$

$$(4.0 \times 3) + (4.0 \times 0) = (4 + 4) \times v$$

$$12 = 8v$$

$$v = 12 \div 8$$

$$\underline{\underline{v = 1.5 \text{ m s}^{-1}}}$$

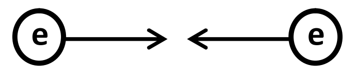
Kinetic Energy – Elastic and Inelastic Collisions

Elastic and Inelastic Collisions

When two objects collide their momentum is **always** conserved. Total energy is also conserved but, depending on the type of collision, kinetic energy may not be.

A collision is said to be **elastic if Kinetic Energy is conserved** (the same before the collision and after it) and **inelastic if Kinetic Energy is not conserved.**

In most collisions, like this one, the initial kinetic energy of the cars is converted to heat and sound during the collision.
The kinetic energy is **not conserved** so this is an **inelastic collision**.



When two electrons collide they will not actually come into contact with each other, as their electrostatic repulsion will keep them apart. There is no mechanism to convert their kinetic energy into another form and so it is **conserved** throughout the collision. This is an **elastic collision**.

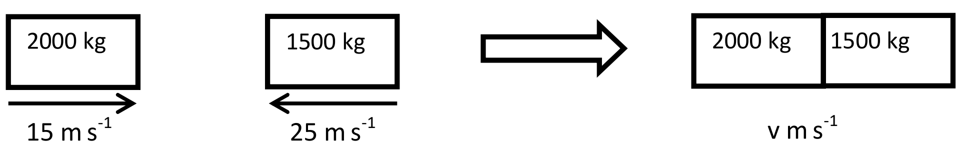
Example:

A car of mass 2000 kg is travelling at 15 m s^{-1}. Another car, of mass 1500kg and travelling at 25 m s^{-1} collides with it head on. They lock together on impact and move off together.

a) Determine the speed and direction of the cars after the impact.
b) Is the collision elastic or inelastic? Justify your answer.

Solution:

a) (remember objects moving to the left have a negative velocity)



total momentum before = total momentum after

$$m_1u_2 + m_2u_2 = (m_1 + m_2)\,v$$
$$(2000 \times 15) + (1500 \times (-25)) = (2000 + 1500) \times v$$
$$3 \times 10^4 + (-37.5 \times 10^4) = 3500\,v$$
$$-7.5 \times 10^3 = 3500\,v$$
$$V = -7.5 \times 10^3 \div 3500$$
$$= \underline{\underline{-2.1 \text{ m s}^{-1}}}$$

<u>The cars are travelling at 2.1 m s^{-1} to the left (from the negative sign).</u>

b) Calculate the Kinetic Energy of each car before the collision and add to get the total

$$E_k \text{ before} = \tfrac{1}{2}m_1u^2 + \tfrac{1}{2}m_2u^2$$
$$= (\tfrac{1}{2} \times 2000 \times 15^2) + (\tfrac{1}{2} \times 1500 \times 25^2)$$
$$= 2.25 \times 10^5 + 4.69 \times 10^5$$
$$= 6.94 \times 10^5 \text{ J}$$

$$E_k \text{ after} = \tfrac{1}{2}m_{total}v^2$$
$$= \tfrac{1}{2} \times 3500 \times 2.1^2$$
$$= 7.72 \times 10^3 \text{ J}$$

<u>E_k before is greater than E_k after so the collision is **inelastic**.</u>

Explosions

In a simple explosion two objects start together at rest then move off in opposite directions. Momentum must still be conserved. As the total momentum before is zero, the total momentum after must also be zero.

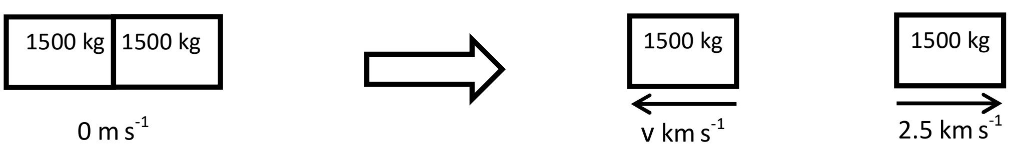
Example:

An early Stark Jericho missile is launched vertically and when it reaches its maximum height it explodes into two individual warheads.

Both warheads have a mass of 1500 kg and one moves off horizontally, with a velocity of 2.5 km s^{-1} (Mach 9) at a bearing of 090°.

Calculate the velocity of the other warhead.

Solution:



$$0 = m_1v_1 + m_2v_2$$
$$0 = 1500 \times v_1 + 1500 \times 2.5 \times 10^3$$
$$1500 \times v_1 = -1500 \times 2.5 \text{x} 10^3$$
$$v_1 = -\frac{3.75 \times 10^6}{1500}$$
$$v_1 = \underline{\underline{-2.5 \times 10^3 \text{ m s}^{-1}}}$$

(The negative sign in the answer indicates the direction of v_1 is opposite to that of v_2 i.e. 270° rather than 090°)

The Second warhead is travelling at 2.5 km s^{-1} on a bearing of 270°.

Newton's Third Law and Momentum

Collisions

p_{total} before = p_{total} after

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$m_1u_1 - m_1v_1 = m_2v_2 - m_2u_2$$
$$-m_1(v_1 - u_1) = m_2(v_2 - u_2)$$

$$\frac{-m_1(v_1 - u_1)}{t} = \frac{m_2(v_2 - u_2)}{t}$$

$$-m_1a_1 = m_2a_2$$
$$-F_1 = F_2$$

Collisions

p_{total} before = p_{total} after

$$0 = m_1v_1 + m_2v_2$$

$$m_1v_1 = -m_2v_2$$
$$\frac{m_1v_1}{t} = \frac{-m_2v_2}{t}$$
$$m_1a_1 = -m_2a_2$$
$$F_1 = -F_2$$

Impulse

A change in momentum is caused when a force is applied for a period of time and depends on:

- The size of the force
- The time the force acts for

Impulse is the product of force and time, measured in N s.

$$F = ma = \frac{m(v - u)}{t}$$

$$Ft = mv - mu$$

$$\boxed{Ft = \Delta p}$$

(impulse has no symbol of its own)

Impulse is equal to the change in momentum, and is also measured in kg m s^{-1}.

This means you can calculate the impulse from: $F \times t$ (N s), or $mv - mu$ (kg m s^{-1}).

Example:

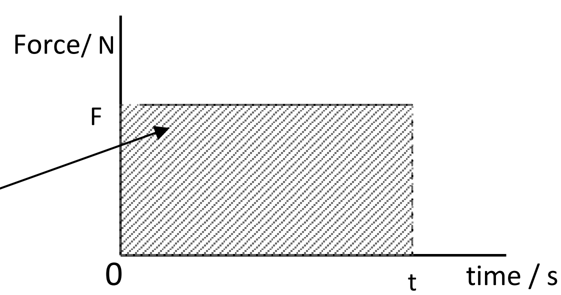
A force of 100 N is applied to a ball of mass 150 g for a time of 0.020 s. Calculate the final velocity of the ball.

Solution:

$F = 100\text{N}$
$m = 0.150 \text{ kg}$
$t = 0.020 \text{ s}$
$u = 0 \text{ m s}^{-1}$

$$Ft = mv - mu$$
$$100 \times 0.020 = 0.150 \times (v - 0)$$
$$2.0 = 0.150\, v$$
$$v = 2/0.15$$
$$\underline{\underline{v = 13.3 \text{ m s}^{-1}}}$$

Impulse Graphs



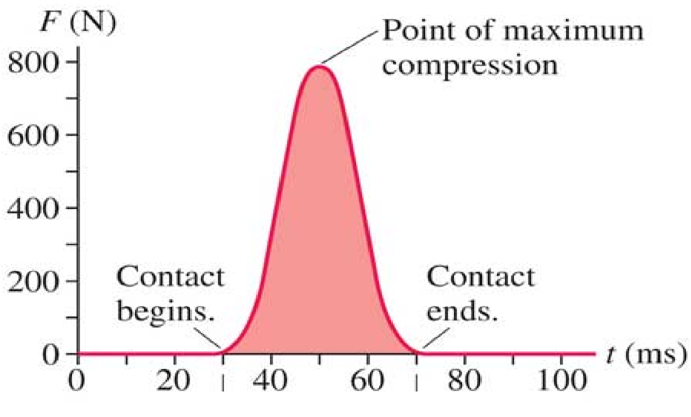
Ft = impulse = change in momentum

Ft = area under Force time graph

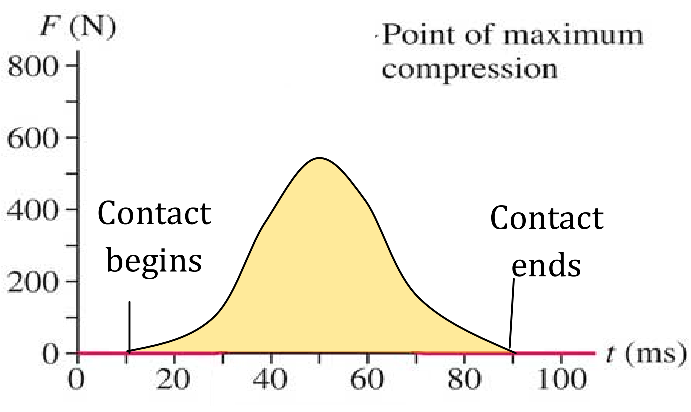
Impulse Graphs

In reality, the force applied is not usually constant.
Consider what happens when a ball is kicked.

Once the foot makes contact with the ball a force is applied, the ball will compress as the force increases. As the ball leaves the foot it will regain its original shape and the force applied will decrease. The graph below illustrates this



If a ball of the same mass that is softer is kicked and moves of with the same speed as that above, then a graph such as the one below will be produced.

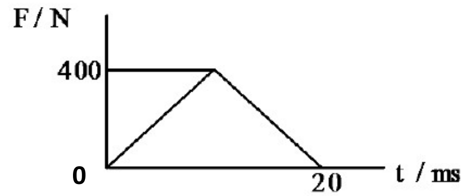


Since the ball has the same mass and moves off with the same speed its **change in momentum (impulse) will be the same** as the original. The maximum force applied is smaller but the contact time has increased. The graph is wider and has a smaller peak but **the area remains the same**.

Example:

A tennis ball of mass 100 g, initially at rest, is hit by a racquet.

The racquet is in contact with the ball for 20 ms and the force of contact varies over this period, as shown in the graph.



Determine the speed of the ball as it leaves the racquet

Solution:

(Notice that the graph is an ideal graph, without the rounded shape. This is to make it easy to calculate the area)

$$\begin{aligned} \text{Impulse} &= \text{area under graph} \\ &= \tfrac{1}{2} \times 20 \times 10^{-3} \times 400 \\ &= 4 \text{ N s} \end{aligned}$$

u = 0

m = 100 g = 0.1 kg

v = ?

$$\begin{aligned} Ft &= mv - mu \\ 4 &= 0.1v - (0.1 \times 0) \\ 4 &= 0.1v \\ \underline{\underline{v}} &\underline{\underline{= 40 \text{ ms}^{-1}}} \end{aligned}$$

Practical Applications – Car Safety

The occupants of a car will experience a stopping force during a collision which will depend on the time it is applied for. The greater the time you can take to decelerate an object, the smaller the force you need to apply. The goal of any restraint system is to help stop the passenger while doing as little damage to him or her as possible.

Airbags

The concept of the airbag has been around for many years. First patented for planes during WWII, commercial airbags appeared in cars during the 1980s.

Changing an object's momentum requires a force acting over a period of time. When a car crashes, the force required to stop any person inside it is very large because the car's momentum has changed instantly. The airbag is compressed as the stopping force is applied. It takes a longer time to slow the occupant’s speed to zero. The stopping force is reduced. The change in momentum remains the same.

Crumple Zones

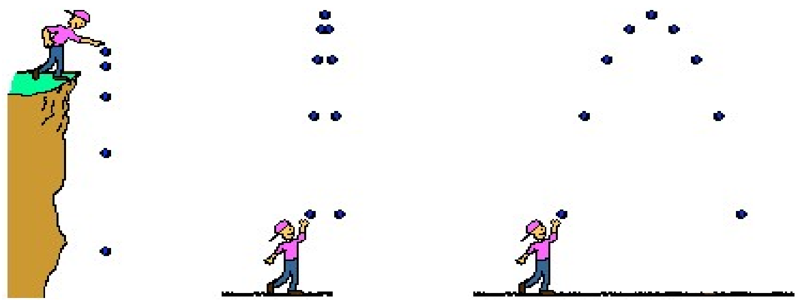
While certain parts of the car are designed to allow deformations (they crumple easily), the passenger cabin is strengthened by using high-strength steel and more beams. Crumple zones **increase the time** before the vehicle comes to a halt. This **reduces the force** experienced by the driver and occupants on impact. The change in momentum is the same with or without a crumple zone.

1.4 Gravitation

Projectile Motion

Projectiles

A projectile is any object, which, once projected, continues its motion by its own inertia and is influenced only by the downward force of gravity.



The motion of a projectile can be split into horizontal and vertical components. The horizontal and vertical components of the motion should be considered separately.

Vertical Motion Example

Objects moving vertically (up or down) experience an acceleration due to gravity of $9.8ms^{-2}$ downwards ($-9.8ms^{-2}$). We choose upwards as the positive direction so we have to be careful about the direction of displacement and velocity of the object.

Example:

A ball is thrown straight up with an initial velocity of $4ms^{-1}$.

a) How long will it take the ball to reach its maximum height?
b) What speed will it reach after 0.6s and what direction will it be travelling in?

Solution: This is just an equations of motion problem with a known acceleration

a) $u = 4ms^{-1}$ (up is positive)
we know $a = -9.8ms^{-2}$ *because this is vertical motion*
$t = ?$
$v = 0ms^{-1}$ at maximum height

$v = u + at$
$0 = 4 + (-9.8) \times t$
$-4 = -9.8t$
$\underline{\underline{t = 4/9.8 = 0.41s}}$

b) $u = 4ms^{-1}$ (up is positive)
$a = -9.8ms^{-}$
$t = 0.6$ s
$v = ?$

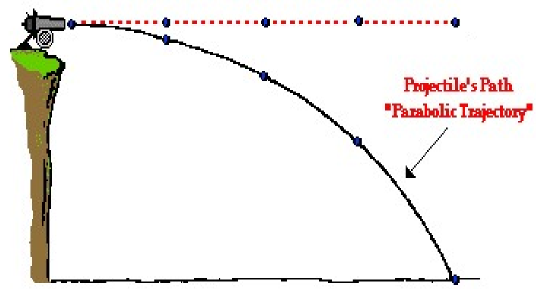
$v = u + at$
$v = 4 + (-9.8) \times 0.6$
$v = 4 - 5.88$
$\underline{\underline{v = -1.88ms^{-1}}}$

The negative sign tells us the direction is now downward

Horizontal Projection

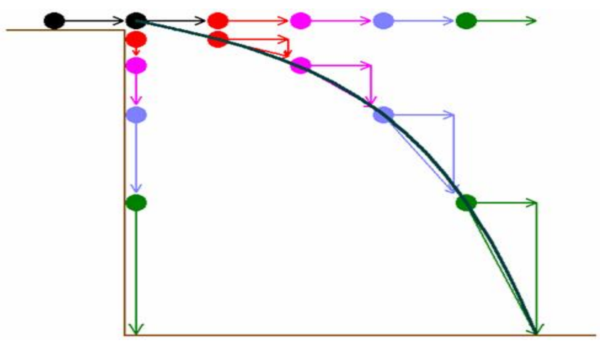
Here is a classic horizontal projectile scenario, from the time of Newton.

In projectile motion we **ignore all** air resistance, or any force other than gravity.



Analysis of this projectile shows the two different components of motion.

Horizontally: there are no forces acting on the cannonball and therefore *the horizontal velocity is constant.*



Vertically: The force due to gravity is constant and so the cannonball undergoes *constant vertical acceleration.*

The combination of these two motions is the reason why the projectile path is curved or *parabolic*.

Example:

The cannonball is projected horizontally from the cliff with a velocity of 100 m s^{-1}. The cliff is 20 m high. Determine:

a) the vertical speed of the cannonball, just before it hits the water;

b) if the cannonball will hit a ship that is 200 m from the base of the cliff.

Solution:

(Hint: time is the only quantity which can cross the horizontal and vertical barrier. Calculate t on one side and use it on the other)

Horizontal (use d = vt)	Vertical (use 3 equations of motion)
d = ?	s = 20 m
v = 100 m s^{-1}	u = 0
t = ?	v = ?
	a = 9·8 m s^{-2}
	t = ?

a) $v^2 = u^2 + 2as$

$v^2 = 0^2 + 2 \times 9{\cdot}8 \times 20$

$v^2 = 392$

$v = 19{\cdot}8$ m s^{-1}

b) $v = u + at$

$19{\cdot}8 = 0 + (9{\cdot}8 \times t)$

$t = \frac{19{\cdot}8}{9{\cdot}8}$

$t = 2{\cdot}02$ s

now use t = 2.02 s on the horizontal side

$d = vt$

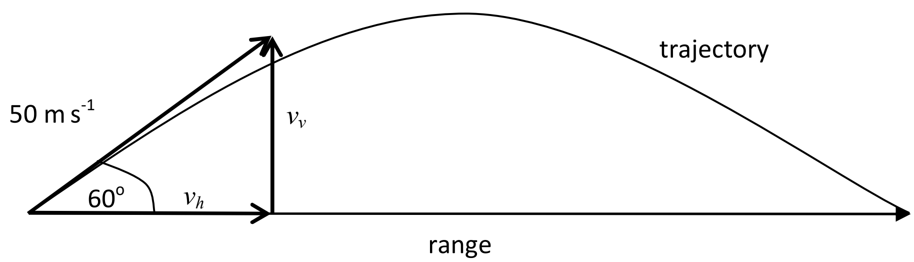
$d = 100 \times 2{\cdot}02$

$d = 202$ m **The cannonball will hit the ship.**

Projection at an Angle

Projectiles at an angle are an application for our knowledge of splitting vectors into their horizontal and vertical components.

An example is when an athlete throws the javelin at a velocity of 50 ms^{-1} at an angle of 60^o to the horizontal.



Horizontal and vertical motions must still be treated separately. This means that the velocity at an angle **must** be split into its **vertical and horizontal** components before any further consideration of the projectile.

> You will never use the velocity at an angle (here 50ms^{-1}) directly in any calculation!

Points to remember!!!
For projectiles fired at an angle above a horizontal surface:

- The path of the projectile is symmetrical, in the horizontal plane, about the highest point. This means that:

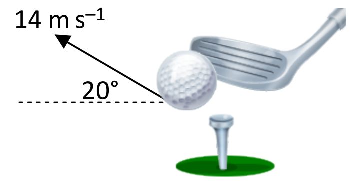
$$\text{initial vertical velocity} = -\text{final vertical velocity}$$
$$u_v = -v_v$$

- The time of flight = 2 × the time to the highest point.
- The vertical velocity at the highest point is zero.
- The horizontal distance is also known as the range.

Projection at an Angle Calculation

Example:

A golfer hits a stationary ball and it leaves his club with a velocity of 14 ms^{-1} at an angle of 20° to the horizontal.



a) Calculate:
   i. the horizontal component of the velocity of the ball;
   ii. the vertical component of the velocity of the ball.

b) Calculate the time for the ball to reach its maximum height.

c) Calculate the total time of flight of the ball

d) How far down the fairway does the ball land?

Solution:

Horizontal	Vertical
a) i. $v_H = v \cos \Theta$ $v_H = 14 \cos 20$ $v_H = 13.1\ ms^{-1}$	a) ii. $v_v = v \sin \Theta$ $v_v = 14 \sin 20$ $v_v = 4.8\ ms^{-1}$

b)

$s = ?$

$u = v_v = 4.8 ms^{-1}$

$v = 0$ (at top)

$a = -9{\cdot}8\ m\ s^{-2}$

$t = ?$

$v = u + at$

$0 = 4{\cdot}8 + (-9{\cdot}8 \times t)$

$9{\cdot}8 \times t = 4{\cdot}8$

$t = \frac{4{\cdot}8}{9{\cdot}8}$

$t = 0{\cdot}49$ s

time to max height is 0.49 s

c)

$t = 2 \times 0.49$ s

time of flight = 0.96 s

d)

$d = ?$

$v = v_H = 13.1\ ms^{-1}$

$t = 0.96$ s

$d = v\,t$

$d = 13.1 \times 0.96$

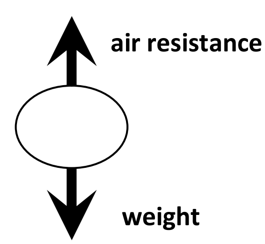
$d = 12.8$ m

the ball lands 12.8m down the fairway

Free Fall

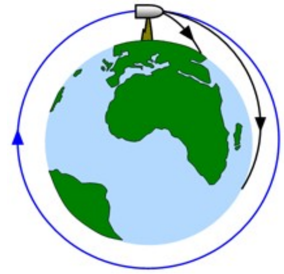
An object is in **free fall** if it is falling due to its Weight (the Force of Gravity). This downward unbalanced force causes it to accelerate (at 9.8ms^{-2} on Earth). However, the force of friction is also acting on the object in the opposite direction to its motion and increases as it speeds up.

If the object falls through a large enough distance then the air resistance may increase in magnitude until it equals the weight of the object. The **forces become balanced** and the object falls with a constant velocity, known as **terminal velocity**.



Newton and Gravity

Newton's Thought Experiment – Satellite's orbit as an Application of Projectiles



Newton suggested that if a cannon fired a cannonball it would fall towards the Earth. At higher speeds the cannonball would travel further. Eventually, if fired fast enough, the ball would then fall towards the Earth but never land since the curvature of the Earth would be the same as the flight path of the cannonball.
This would be a satellite.
You would need a high mountain and an enormous cannon, but it would work.

Mass, Weight and the Force of Gravity

Mass & Weight
Mass is a measure of how much matter an object contains and is measured in kilograms (kg). This will only change if matter is added to or taken from the object.

Weight is the force of gravity acting on an object and is measured in Newtons (N).
Weight can change depending on where we are in the universe.

Force of Gravity

Gravity is the force of attraction between objects that have mass. Any object that has mass will have its own gravitational field. The magnitude of the field depends on the mass of the object. The larger the mass, the larger the force. Everything on Earth has its own gravitational field but the gravitational forces are so small that we don't notice them.

Newton's Universal Law of Gravitation

Newton's Law of gravitation states that the gravitational attraction between two objects (m_1 and m_2) is directly proportional to the mass of each object and is inversely proportional to the square of their distance (r) apart.

$$\boxed{F = \frac{Gm_1 m_2}{r^2}}$$

G is the universal constant of gravitation = $6{\cdot}67 \times 10^{-11}$ m^3 kg^{-1} s^{-2}.

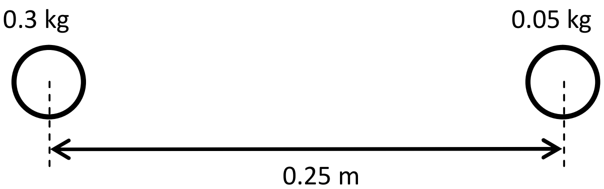
Gravitational force is always attractive, unlike electrostatic or magnetic forces.

The distance r between the two objects is the distance between their **centres of mass**. This is especially important when considering planetary bodies. For example, the radius of the orbit of the moon is only the distance from the surface of the Earth to the surface of the Moon, not the distance between their centres of mass.

Newton's Universal Law of Gravitation

Example

Consider a folder, of mass 0.3 kg and a pen, of mass 0.05 kg, sitting on a desk, 0.25 m apart. Calculate the magnitude of the gravitational force between the two masses. (Assume they can be approximated to spherical objects).



Solution

$F = ?$

$m_1 = 0.3$ kg

$m_2 = 0.05$ kg

$r = 0.25$ m

$G = 6{\cdot}67 \times 10^{-11}\ m^3\ kg^{-1}\ s^{-2}$

$$F = \frac{G\, m_1\, m_2}{r^2}$$

$$F = \frac{6.67 \times 10^{-11} \times 0.3 \times 0.05}{0.25^2}$$

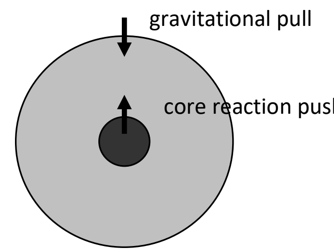
$$F = \underline{\underline{1.6 \times 10^{-11}\ N}}$$

Note that the mass of the Earth is 5.97×10^{24} kg and it would exert just less than 0.5N on the pen. The effect of the force the folder exerts on the pen is negligible (can be ignored).

Application of Gravitational Force

Formation of the Universe

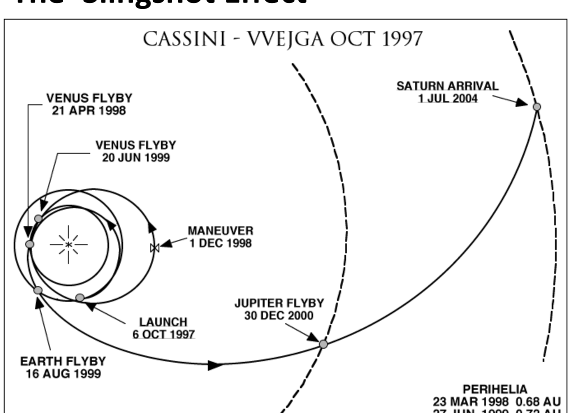
Gravity is a force that permeates the entire universe; scientists believe that stars were formed by the gravitational attraction between hydrogen molecules in space. Over time, the mass became large enough for enormous forces at the centre to cause the hydrogen molecules to fuse together, generating energy. This is what is happening in the centre of the sun. The energy radiating outwards from the centre of the sun counteracts the gravitational force trying to compress the sun inwards.



Star

In time (about 4 billion years) the hydrogen will be used up, the reaction will stop and the sun will collapse under its own gravity.

The 'Slingshot Effect'



Another application of the gravitational force is the use made of the 'slingshot effect' to get some 'free' energy to accelerate spacecraft. The craft travel very close to a planet, where they accelerate due to the gravitational field of the planet. If the trajectory is correct the craft will then pass the planet with the increased speed. Too close and the craft will orbit the planet and eventually crash.

1.5 Special Relativity

Reference Frames

Relativity is all about observing events and measuring physical quantities, such as distance and time, from different reference frames. Here is an example of the same event seen by three different observers, each in their own frame of reference:

Event 1: You are reading on the train. The train is travelling at 60 mph.

Observer	**Observer Location**	**Observation**
1	Passenger sitting next to you	You are stationary
2	Person standing on the platform	You are travelling towards them at 60 mph.
3	Passenger on train travelling at 60 mph in opposite direction	You are travelling towards them at 120 mph.

This example works well as it only involves objects travelling at relatively low speeds. The comparison between reference frames does not work in quite the same way, however, if objects are moving close to the speed of light.

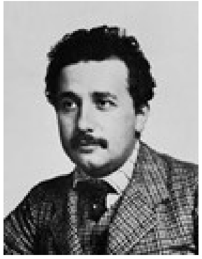
Event 2: You are reading on an interstellar train. The train is travelling at 2×10^8 m s^{-1}.

Observer	**Observer Location**	**Observation**
1	Passenger sitting next to you	You are stationary
2	Person standing on the platform	You are travelling towards them at 2×10^8 ms^{-1}
3	Passenger on train travelling at 2×10^8 ms $^{-1}$ in the opposite direction	You are travelling towards them at **4×10^8 ms^{-1} – impossible!**

The observation made by observer 3 is impossible as an object cannot travel faster than the speed of light in any reference frame and it would certainly be impossible to watch something travel faster than light, so this scenario is impossible.

An inertial reference frame is one moving at a constant speed, where Newton's laws hold. Galileo was one of the first scientists to consider the idea of relativity. He stated that the laws of physics should be the same in all inertial frames of reference. If you are on a bus moving at constant speed you experience the laws in the same way as when you are at rest.

Introduction to Relativity



If we are to explain the motion of objects in fast moving reference frames we use Einstein's Theory of Special Relativity, which he proposed in 1905. This was one of four world changing theories published by Einstein that year, known as the Annus Mirabilis (miracle year) papers. Einstein was 26.

Relativity has allowed us to examine the mechanics of the universe far beyond that of Newtonian mechanics, especially the more extreme phenomena such as black holes, dark matter and the expansion of the universe, where the usual laws of motion and gravity appear to break down.
Special Relativity was the first theory of relativity Einstein proposed. It was termed as 'special' as it only considers the 'special' case of reference frames moving at **constant** speed. Later he developed the theory of general relativity which considers accelerating frames of reference.

The Principles of Relativity

Using his imagination and performing thought (gedanken) experiments like those on the previous page, Einstein came up with two principles (postulates) to explain the problem of fast moving reference frames. Experimental evidence came later.

1. **When two observers are moving at constant speeds relative to one another (in inertial reference frames), they will observe the same laws of physics.**
2. **The speed of light (in a vacuum) is the same for all observers.**

This means that no matter how fast you go, you can never catch up with a beam of light, since it always travels at $3{\cdot}0 \times 10^8$ ms^{-1} **relative** *to you*.

Example**:**
A car ship travelling through space at 90% of the speed of light switches on its headlights.
What is the speed of the light bean as observed by
   a) A passenger on the ship
   b) An observer on Earth

Solution:

   a) The passenger is stationary with regard to the ship. The light beam travels away from them at 3×10^8 ms^{-1}.

   b) The speed of light does not vary for observers in any reference frame. An observer on Earth will also observe light of the beams travelling at 3×10^8 ms^{-1}.

Time Dilation Equation

A consequence of the speed of light remaining constant is that measured time will change for a moving system depending on who is observing the motion.

A person travelling at speed v will observe an event taking time t, while a stationary observer will see the same event taking more time, t'. We say the observer time is dilated.

$$\mathbf{t' = \frac{t}{\sqrt{1 - \left(\frac{v^2}{c^2}\right)}}}$$

t' is always observed by the stationary observer, observing the object moving at speed e.g. the person on a train platform watching the train go by, or an observer on Earth watching a fast moving ship.

Example:

A rocket is travelling past Earth at a constant speed of 2.7×10^8 ms^{-1}.
The pilot measures the journey as taking 240 minutes.
How long did the journey take when measured by an observer on Earth?

Solution:

t = 240 minutes
c = 3×10^8 ms^{-1}
v = 2.7×10^8 ms^{-1}
t' = ?

$$t' = \frac{t}{\sqrt{1 - \left(\frac{v^2}{c^2}\right)}}$$

$$t' = \frac{240}{\sqrt{1 - \left(\frac{2.7 \times 10^8}{3.0 \times 10^8}\right)^2}}$$

t' = 550.5977 minutes

t' = 550 minutes (to 2 significant figures)

An observer on Earth would measure the journey as taking 550 minutes.

Time Dilation Experimental Evidence

Further evidence in support of special relativity comes from the field of particle physics. Muons are produced in the upper layers of the atmosphere by cosmic rays (high-energy photons from space) and can be detected at the Earth's surface. The speed of muons high in the atmosphere is 99·9653% of the speed of light.

The half-life of muons when measured in a laboratory is about 2·2 μs.

<u>Example:</u>
Show by calculation, why time dilation is necessary to explain the observation of muons at the surface of the Earth.

<u>Solution:</u>
Without considering time dilation, the distance the muons cover before they decay is small

$t = 2.2\ \mu s = 2.2 \times 10^{-6}$ s
$v = 0.999653 \times 3.00 \times 10^{8} = 2.998956 \times 10^{8}\ ms^{-1}$
$d = ?$

$d = vt$
$d = 2.998956 \times 10^{8} \times 2.2 \times 10^{-6}$
$d = 660$ m

but, from the point of view of an observer on Earth the time taken for the muons to fall is

$$t' = \frac{t}{\sqrt{1-\left(\frac{v^2}{c^2}\right)}}$$

$$t' = \frac{2.2 \times 10^{-6}}{\sqrt{1-(0.999653)^2}}$$

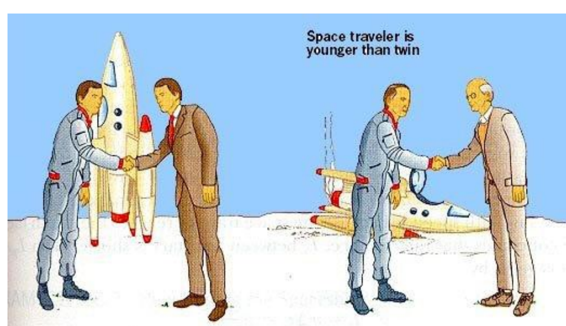
$$t' = 84 \times 10^{-6} s$$

$d' = vt$
$d' = 2.998956 \times 10^{8} \times 84 \times 10^{-6}$
$d' = 2.52 \times 10^{4}$ m

In the reference frame of an observer on Earth the half-life of the muon is recorded as 84 μs and from this perspective, the muon has enough time to travel the many kilometres to the Earth's surface, before it decays.

A Twin Paradox

You leave Earth and your twin to go on a mission in a spaceship travelling at 90% the speed of light on a return journey that lasts 20 years. When you get back you find that 46 years will have elapsed on Earth. Your clock will have run slowly compared to one on Earth, however as far as you were concerned the clock would have been working correctly on your spaceship.
You will look 26 years younger than your twin.



The paradox lies in the thought that the clock on Earth is *also* slow *relative* to the one on the spacecraft. This would mean the Earth twin could be younger or that there may be no difference in age. In fact, the space twin will agree that the journey took a shorter time, because of **length contraction**. An analysis of the time observed by each will show that **the space twin will actually be younger on return.**

Length Contraction

Another implication of Einstein's theory is the shortening of length when an object is moving. Consider the muons discussed above. Their large speed means they experience a longer half-life due to time dilation. An equivalent way of thinking about this is that the fast moving muons observe a much shorter (or contracted) distance travelled. The distance is contracted by the same amount that the time has increased (or dilated). A symmetrical formula for length contraction can be derived.

$$l' = l\sqrt{1-\frac{v^2}{c^2}}$$

Where **l** is the distance measured by an observer who is stationary and **l'** is the contracted distance observed by someone who is moving at speed.

Example:

A space ship is flying away from Earth towards Proxima Centauri, our nearest star, at 0.8c. Compare the distance to Proxima Centauri, which is 4.2 ly, with the length measured by the pilot of the spacecraft.

Solution:

Length contraction only takes place in the direction that the object is travelling.

l = 4.2 ly (light years)
l' = ?
v = 0.8 c

$$l' = l\sqrt{1-\frac{v^2}{c^2}}$$

$$l' = 4.2\sqrt{1-0.8^2}$$

$$l' = 2.52$$

$$l' = 2.5 \text{ ly}$$

So the Pilot of the ship measures their journey as 2.5 ly.
The pilot of the space ship will measure the distance in front of him, between Earth and Proxima as **less than the distance measured by a stationary observer**.

Length Contraction – A Paradox

Another apparent paradox thrown up by special relativity: consider a train that is just longer than a tunnel. If the train travels at high speed through the tunnel does length contraction mean that, from our stationary perspective, it fits inside the tunnel? How can this be reconciled with the fact that from the train's reference frame the tunnel appears even shorter as it moves towards the train? The key to this question is simultaneity, i.e. whether different reference frames can agree on the exact time of particular events. In order for the train to fit in the tunnel the front of the train must be inside at the same time as the back of the train. Due to time dilation, the stationary observer (you) and a moving observer on the train cannot agree on when the front of the train reaches the far end of the tunnel or the rear of the train reaches the entrance of the tunnel. If you work out the equations carefully then you can show that even when the train is contracted, the front of the train and the back of the train will not both be inside the tunnel at the same time!

The Lorentz Factor

The time dilation equation can be written as: $t' = t\gamma$

where **γ (gamma)** is known as the **Lorentz Factor**.

The Lorentz factor is the figure we use to adjust the observed time due to the effects of relativity.

It is used in the study of special relativity and is given by:

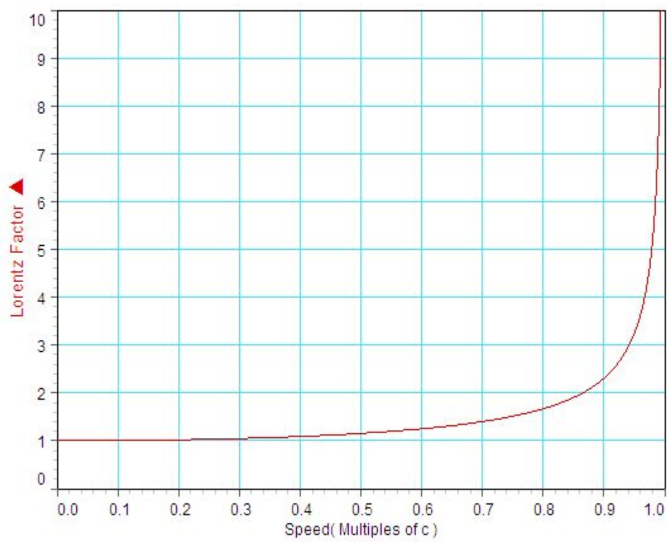
$$\gamma = \frac{1}{\sqrt{1-\left(\frac{v^2}{c^2}\right)}}$$

Why don't we notice relativistic time differences in everyday life?

A graph of the Lorentz factor versus speed (measured as a multiple of the speed of light) is shown below. For small speeds (i.e. less than 0.1 times – or 10% of - the speed of light) the Lorentz factor is close to 1 and relativistic effects are negligibly small. It is unlikely that we would observe these effects as speeds we observe are much lower than this.

In our calculations, we won't consider relativistic effects for speeds below 10% of the speed of light.

However, the speed of satellites is fast enough that, small differences in time will accumulate and seriously affect the synchronisation of global positioning systems (GPS) and television satellites with users on Earth. They have to be specially programmed to allow for the effects of special relativity (and also general relativity, which is not covered here). Very precise measurements of these small changes in time have been performed on fast-flying aircraft and agree with predicted results within experimental error.



Can we travel faster than light speed?

What would happen if the speed of an object was the same as the speed of light?

v = c so $\gamma = \frac{1}{\sqrt{1-\left(\frac{c^2}{c^2}\right)}}$ $\gamma = \frac{1}{\sqrt{1-1}}$ $\gamma = \frac{1}{0}$

The effects of relativity mean that as we get close to the speed of light a journey would seem to take an infinite amount of time to an outside observer at rest.

1.6 The Expanding Universe

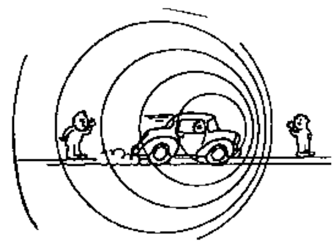
The Doppler Effect and Redshift

The Doppler Effect

The Doppler Effect is the change in the observed frequency of a wave, when the source or observer is moving.

In this course we will concentrate on a wave source moving at **constant speed** relative to a stationary observer.

We notice the Doppler effect when a police car, ambulance or fire engine passes. We hear the pitch of their siren increase as they come towards us and then decrease as they move away.

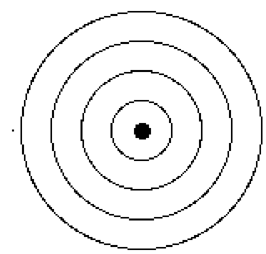


The Doppler Effect applies to all waves, including light if we allow for relativistic effects.

Uses of the Doppler Effect

- Police radar guns use the Doppler effect to measure the speed of motorists.
- Doppler is used to measure the speed of blood flow in veins to check for deep vein thrombosis [DVT] in medicine.

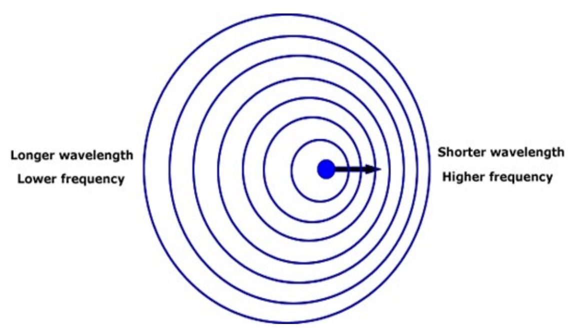
Stationary Source



A stationary source produces sound waves at a constant frequency **f**. The wave fronts travel away from the source at a constant speed, in all directions. The distance between wave fronts is the wavelength. All observers hear the same frequency, which is equal to the actual frequency of the source: $\mathbf{f = f_0}$.

Moving Source

The sound source now moves to the right with a speed $\mathbf{v_s}$. The wave fronts are produced with the same frequency as before. However, in the time taken for the production of each new wave the source has moved to the right. This means that the wave fronts on the left are created further apart and the on the right, they are created closer together. This leads to the spreading out and bunching up of waves as shown here and hence the change in frequency.



The frequency of the source remains constant, it is the observed frequency that changes.

The Doppler Effect Equations

For a stationary observer with a wave source **moving towards them**, the relationship between the frequency, $\mathbf{f_s}$, of the source and the observed frequency, $\mathbf{f_o}$, is:

$$\boxed{f_o = f_s\left(\frac{v}{v - v_s}\right)}$$

v = speed of the wave
v_s = speed of source
f_s = frequency source
f_o = observed frequency

(sounds seem higher moving towards the observer)

For a stationary observer with a wave source **moving away from them**, the relationship between the frequency, $\mathbf{f_s}$, of the source and the observed frequency, $\mathbf{f_o}$, is:

$$\boxed{f_o = f_s\left(\frac{v}{v + v_s}\right)}$$

(sounds seem lower moving away from the observer)

Example:

A woman is standing at the side of a road. A lorry, moving at 14 $m\,s^{-1}$, sounds its horn as it is passing her. The horn has a frequency of 300 Hz.

a) Calculate the frequency heard by the woman when the lorry is approaching her.
b) Calculate the wavelength heard by the woman when the lorry is moving away.

Solution:

f_s = 300Hz

V = 340ms^{-1}

V_s =340ms^{-1}

a) The frequency seems higher on approach

$$f_o - f_s\left(\frac{v}{v - v_s}\right)$$

$$f_o = 300\left(\frac{340}{340 - 14}\right)$$

$$f_o = 313 \text{ Hz}$$

b) The frequency would seem lower moving away

$$f_o = f_s\left(\frac{v}{v + v_s}\right)$$

$$f_o = 300\left(\frac{340}{340 + 14}\right)$$

$$f_o = 288 \text{ Hz}$$

Use the wave equation to find the wavelength.

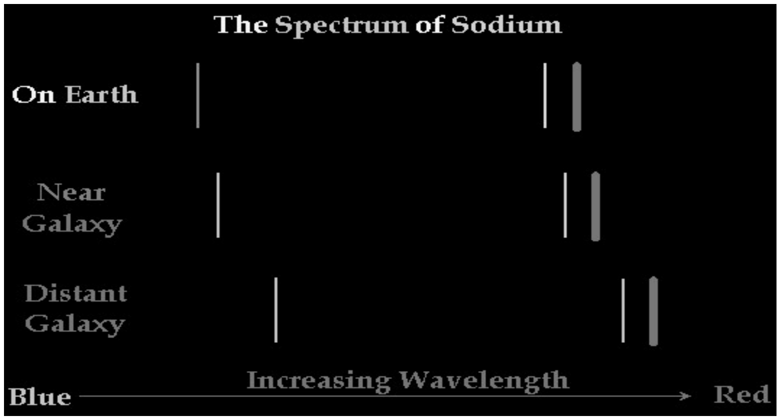
$$\lambda = \frac{v}{f}$$

$$\lambda = \frac{340}{288}$$

= <u>1.18 m</u>

RedShift

Moving light sources also appear to shift their frequency (or wavelength) although the explanation is more complex because of relativistic effects.
The light emitted by a star is made up of the line spectra emitted by the different elements present in that star. Each of these line spectra is unique to the element and these spectra are constant throughout the universe.
Since these line spectra are so recognisable, we can compare the wavelength of spectra produced by these elements, on Earth, with the wavelength of spectra emitted by a distant star or galaxy.



Redshift is the observed increase in wavelength of light emitted from distant galaxies compared with the wavelength of the same light sources on Earth, which occurs because they are moving away from us.
Galaxies moving towards us would emit light that would appear to be reduced in wavelength or blueshifted.
In the 1920's astronomer Edwin Hubble examined the spectral lines from various elements and found that the wavelengths of the spectra emitted by each galaxy were shifted **towards the red** by a specific amount. Hubble found that all distant galaxies appeared to be moving away from us, evidence that the Universe is expanding.

Calculation of Redshift

Redshift, **z**, of a galaxy is given by:

$$z = \frac{\lambda_{observed} - \lambda_{rest}}{\lambda_{rest}} = \frac{\Delta\lambda}{\lambda_{rest}}$$

Redshift of galaxies can also be shown to be the ratio of the velocity of the galaxy to the velocity of light:

$$z = \frac{v_{galaxy}}{c}$$

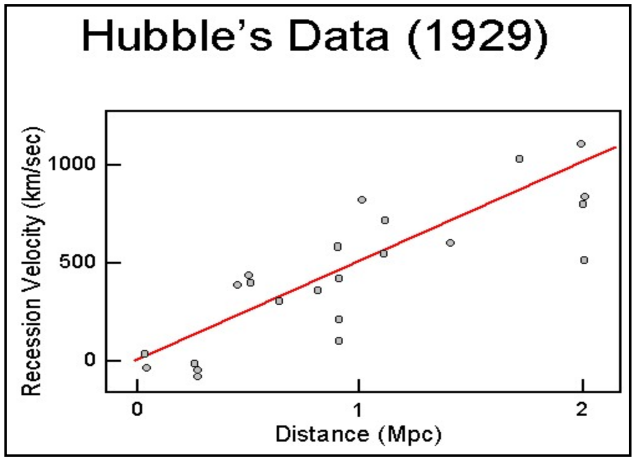
Redshift has no unit of its own as it is always calculated from the ratio of quantities with the same unit.
These relationships also allow us to calculate the speed at which an exoplanet is orbiting its parent star, or the velocity of stars orbiting a galactic core

Hubble's Law

Hubble examined the redshift of galaxies at varying distances from the Earth. The size of the redshift depends on the velocity of the galaxy. He found that the further away a galaxy was, the faster it was travelling away from us. The relationship between the velocity of a galaxy **v**, as it recedes from us, and its distance **d**, is known as **Hubble's Law.** The graph shows the data collected by Hubble.

The graph is a straight line.
The gradient of the line is

$$H_o = \frac{v}{d}$$



H_o is known as Hubble's constant and has the unit s^{-1}.
It's stated value is $2.3 \times 10^{-18}\ s^{-1}$.
The value of the Hubble constant is not known exactly, however as more accurate measurements are made, especially for the distance to a galaxy, the range of possible values has reduced.

Using Hubble's Law

Example:
Light from a distant galaxy is found to contain the spectral lines of hydrogen. The light causing one of these lines has a measured wavelength of 466 nm. When the same line is observed from a hydrogen source on Earth it has a wavelength of 434 nm.
(Hubble's constant is taken as $H_o = 2.3 \times 10^{-18}\ s^{-1}$).

a) Calculate the Doppler shift, *z*, for this galaxy.
b) Calculate the speed at which the galaxy is moving relative to the Earth.
c) How far away is this Galaxy from earth?

Solution:

a)

$$z = \frac{\lambda_{observed} - \lambda_{rest}}{\lambda_{rest}}$$

$$z = \frac{466 - 434}{434}$$

$$= 0.07\ \text{nm}$$

b)

$$z = \frac{v_{galaxy}}{c}$$

$$0.07 = \frac{v_{galaxy}}{3 \times 10^8}$$

$$V_{galaxy} = 21000\text{kms}^{-1}$$

c)

$$H_o = v / d$$

$$2.3 \times 10^{-18} = 2.1 \times 10^{9} / d$$

$$\underline{d = 9.1 \times 10^{26}\text{m}}$$

Calculating the Age of the Universe

Hubble's observations show that the universe is expanding. This means that in the past the galaxies were closer to each other than they are today. By working back in time it is possible to calculate a time when all the galaxies were at the same point in space. This allows the age of the universe to be calculated.

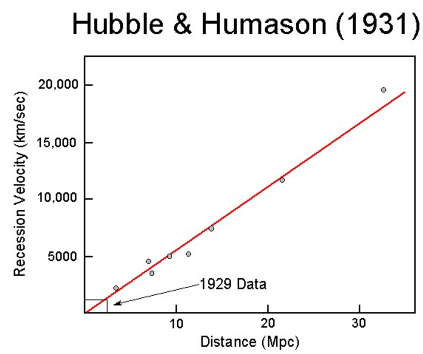
v = speed of galaxy receding from us
d = distance of galaxy from us
H_0 = Hubble's constant
t = time taken for galaxy to travel that distance, i.e. the age of the universe

$$t = \frac{d}{v} \qquad \text{and since } v = H_o d$$

The age of the Universe $\boxed{t = \dfrac{1}{H_0}}$

Currently, using this method, NASA estimate the age of the universe to be **13·7 billion years**.

Since Hubble's time, there have been other major breakthroughs in astronomy and our ability to make accurate observations of much more distant objects. These support the findings of Hubble, but allow the age of the universe to be calculated even more accurately.



The Origins of the Universe

There were two theories regarding the origins of the universe

1. **The Steady State Universe: where the universe had always been and would always continue to be in existence.**

2. **The Created Universe: where at some time in the past the universe was created (now referred to as the Big Bang Theory).**

Ironically the term ‘Big Bang’ was coined by Fred Hoyle, a British astronomer who was the leading supporter of the Steady State theory and who was vehemently opposed to the, currently named, Big Bang theory.

Evidence suggests that the universe started as a singularity, with a sudden appearance of energy which consequently became matter after rapid expansion began around 13.8 billion years ago.

Investigating the Origins of the Universe

Measuring the Temperature of Stars

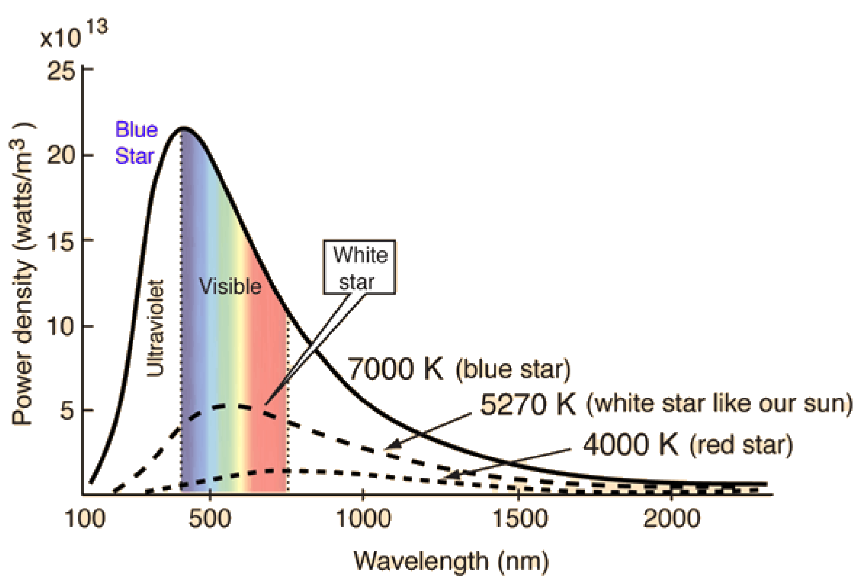
To determine the temperature of distant stars and galaxies we look at the intensity and frequency of the light they emit.

In the same way that iron glows when heated (changing from dull red, to red, orange then yellow) the temperature of star determines the frequency or wavelength of light it emits.

This idea has been with us for a long time; Jožef Stefan proposed in 1879 that the power irradiated from an object was proportional to its temperature in Kelvin to the fourth power: $P = \sigma T^4$

The graph of light emitted shows that the energy of radiation emitted increases to a peak value and then decreases at longer wavelengths.

The peak wavelength is shorter for hotter objects i.e. hotter stars give out more blue light.



Measuring the Luminosity of Stars

To measure the luminosity (brightness) of a star we need to know how distant it is.

1. Parallax

Closer stars appear to move their position as the Earth moves around the Sun, compared with more distant objects. We can use this to work out their distance from Earth in Parsecs.

The definition of the parsec is the distance between the sun and an astronomical object (star,galaxy etc) when the parallax angle is 1 arcsecond. There are 3600 arcseconds in 1 degree. *(1 Mparsec = 3.2 x* 10^6 *light years = 3.1 x* 10^{22}*m)*

2. Standard candles

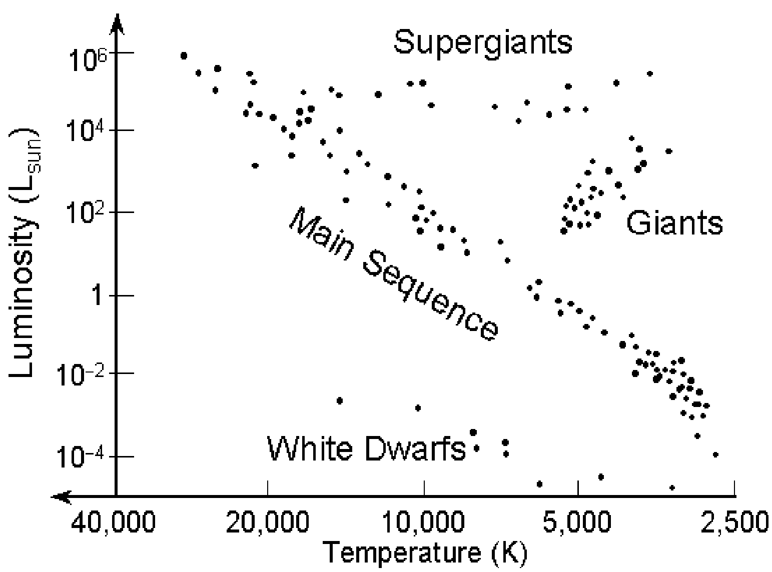
Cepheid variables are stars which periodically change their brightness. The period of variation is linked to their brightness and so we can determine how bright they are.

Supernovae can also be used because of their exceptional brightness – but they can't be observed for long.

Life Cycle of Stars

A **Hertzsprung-Russell diagram** plots the brightness or luminosity of stars against their temperature. The graph can be used to tell the life cycle of a star and to predict whether it will become a red giant or a black hole and can be used to estimate the age of stars, by looking at their position in the main sequence.

(The Sun is found on the main sequence with a luminosity of 1 and a temperature of around 5,400 Kelvin).

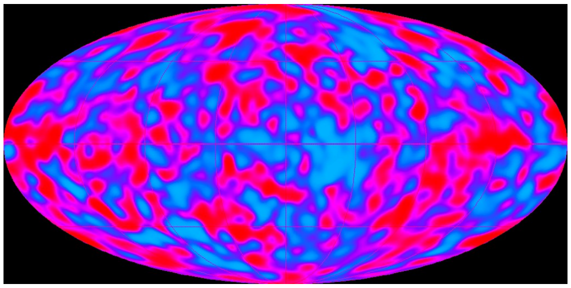


Evidence for the Big Bang Theory

1. Redshift and Expanding Universe

The redshift of light from distant galaxies is evidence that they are moving away from us and that the universe is expanding. If we reverse the expansion, it indicates that the universe has moved outward from a single point.

2. Cosmic Microwave Background Radiation



A map of the Cosmic Microwave Background

Cosmic Microwave Background Radiation is the residual background radiation from the cooling of the hot, dense plasma, first formed after the Big Bang. It is detected in the microwave region, representing a temperature of around 2.7K. It is largely uniform in every direction throughout space and is referred to as the 'afterglow' of the Big Bang. Its existence was predicted in 1948 and it was detected (largely by accident) in 1965 by Penzias & Wilson. The COBE (Cosmic background explorer) satellite confirmed this discovery in 1992.

3. Abundance of Hydrogen & Helium

As the universe cooled the first element to be formed was Hydrogen. Scientists predicted that there should be a significantly greater proportion of hydrogen in the universe because of this and the next most abundant should be Helium.

The elements present in the universe can be determined by spectroscopy. The latest proportions are given in the table shown.

These observations confirm that Hydrogen and Helium are indeed, the most abundant elements.

Element	Relative Abundance
Hydrogen	10 000
Helium	1 000
Oxygen	6
Carbon	1
All others	1

4. Olber's Paradox

His paradox was in answer to the question, "why is the sky dark at night?"

If the universe followed the Steady State model then there should be an even distribution of stars in all directions. All the stars in the universe should be visible. This means the light from the stars should reach Earth and the sky should be bright at night.

The Big Bang theory gives a finite age to the universe, and only stars within the observable universe can be seen. This means that only stars within the distance of 15 000 light years will be observed. Not all stars will be within that range and so the dark sky can be explained.

The Future of the Universe

The rate at which distant galaxies are moving away from us is evidence for an expanding Universe. What is not known however is, what will happen to the universe in the future.

There are essentially two scenarios.

1. Closed universe: the universe will slow its expansion and eventually begin to contract.
2. Open universe: the universe will continue to expand forever.

The Force of Gravity would be responsible for the contraction of the Universe, but this can only happen if the mass of the Universe is large enough

To estimate the mass of the Universe, we begin by finding the mass of galaxies. We can use the orbital speed of stars within those galaxies to find their masses. We can also estimate the number of stars in the galaxy and use this to estimate it's mass. However, when we compare the two figures they are very different. It would seem that up to 68% of the mass of these galaxies cannot be detected by us and has been called **Dark Matter**.

As the calculated mass of the universe seems to be large enough to cause contraction of the Universe, we should find that the rate of expansion of the Universe is decreasing. However, by comparing the recession speed of stars which are very far away, we can tell that the rate of expansion is in fact increasing. The existence of **Dark Energy** has been proposed to explain this effect, which is overcoming the gravitational force, pushing the mass of the universe further apart.