Constructing Distance Time Graphs

1. Brian and his wife set off for a day's outing to Barnsby-on-Sea.

They set out at 0900 along the motorway and stopped for coffee, before finishing the rest of their journey along the A25 road.

(a) How long was the first part of their journey along the motorway ?

(b) How long did they stop for coffee ?

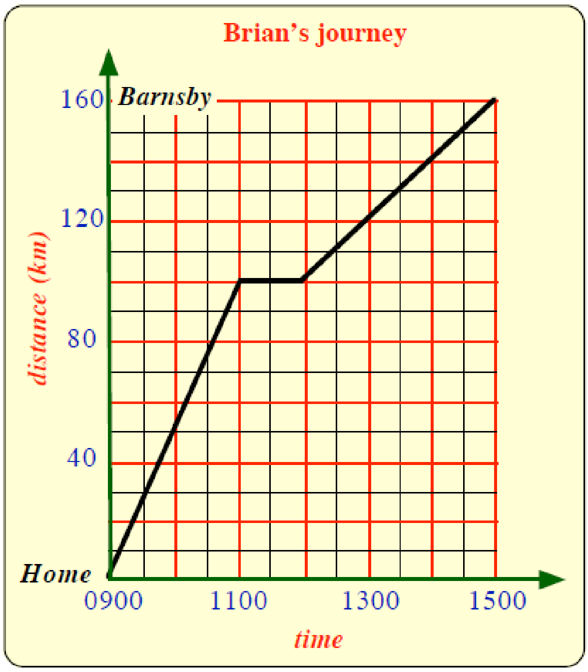
(c) When did they arrive in Barnsby ?

(d) Calculate their speed :–

(i) on the motorway.

(ii) between 1100 and 1200.

(iii) along the A25.



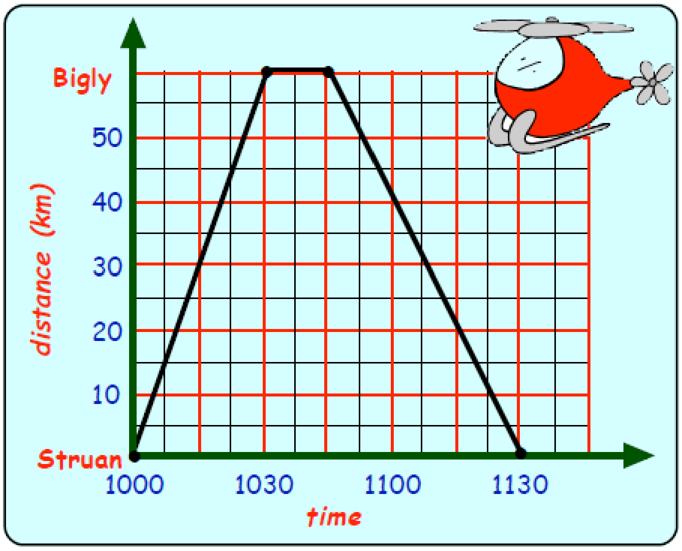
2. A helicopter flew from Struan to Bigly, dropped off supplies and returned to Struan.

(a) For how long was the helicopter on the ground at Bigly ?

(b) Calculate its speed for the outward flight to Bigly.

(c) It hit a "head wind" on the way back. Calculate the return speed.

(d) From your answers to (b) and (c), say whether the "head wind" slowed it down or helped it go faster.



3. Gemma left Brioch Harbour in her dinghy at 10·00 am and set sail for Toule.

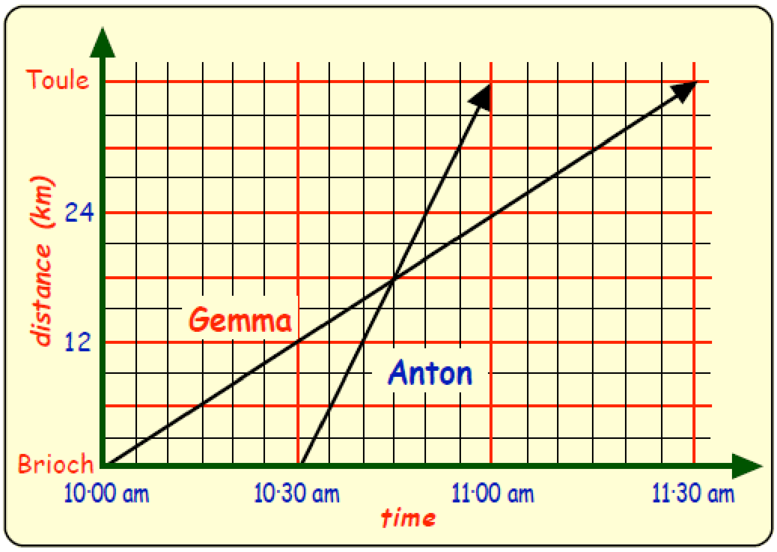
Anton left Brioch at 10·30 am in his small motor launch.

(a) Calculate Gemma's speed.

(b) Calculate Anton's speed.

(c) When did Anton's launch overtake Gemma's dinghy ?

(d) How far away from **Toule** were they when Anton overtook Gemma ?



4. A goods train and a passenger train left 2 stations heading towards each other, one from London and one from Edinburgh.

The goods train was the slower of the two.

(a) Which line, A or B, represents the goods train's journey ? (*Explain why*).

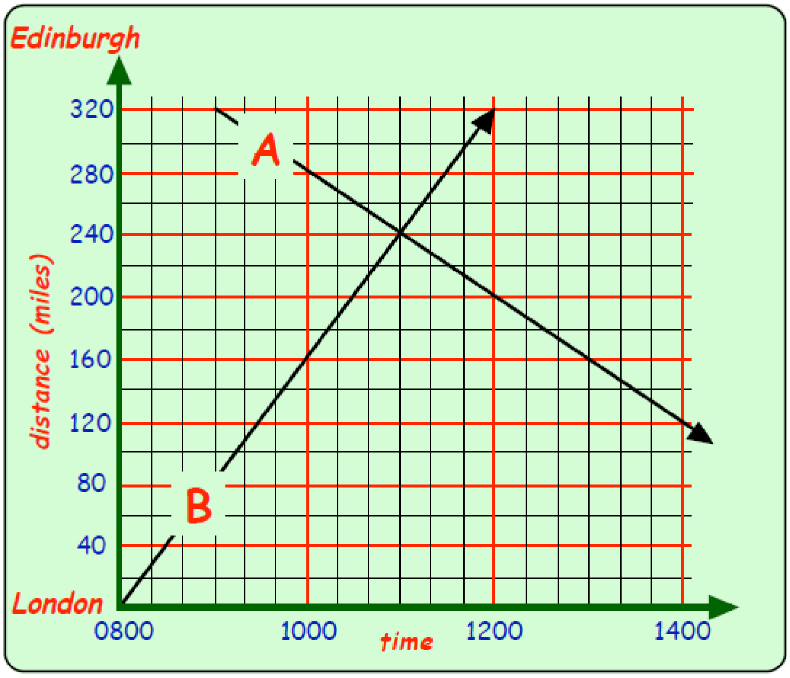
(b) Calculate the :-

(i) goods train's speed.

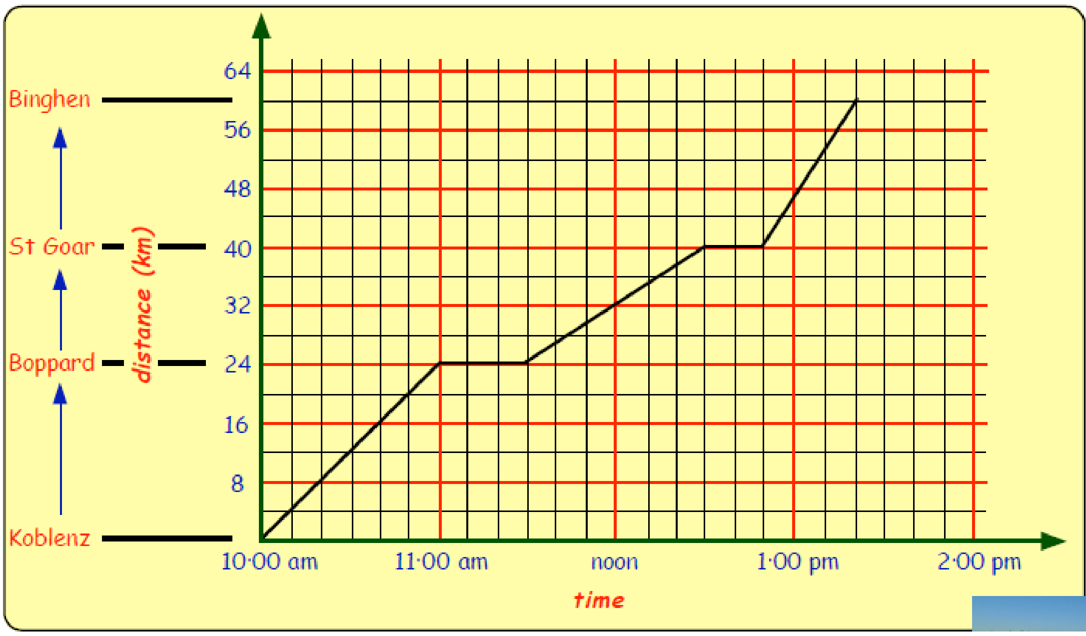
(ii) passenger train's speed.

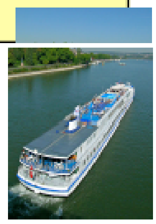
(c) At what time did the two trains pass ?

(d) At what time should train A reach London ?



5. This diagram shows the journey of a pleasure boat on a Rhine cruise.





(a) Make a neat copy of this timetable and complete it for the pleasure boat's trip.

Koblenz	Boppard		St Goar		Binghen
depart	arrive	leave	arrive	leave	arrive
10·00 am →	?	?	?	?	?

(b) How many km is it from :– (i) Koblenz to Boppard (ii) St Goar to Binghen ?

(c) Calculate the average speed of the boat :–

(i) from Koblenz to Boppard (ii) from Boppard to St Goar

(iii) from St Goar to Binghen (iv) from Koblenz to Binghen.

6. Andrea left at 8·00 am, and drove her coach from Inverness to Stirling, 150 miles away.

She drove at an average speed of 40 miles per hour for the first 60 miles.

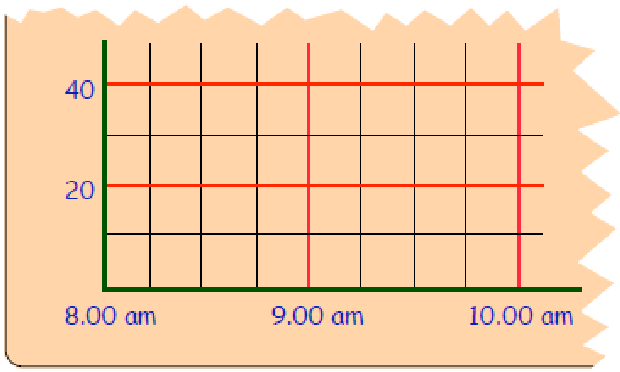
Andrea stopped for 15 minutes for petrol and a break.

She then set off again and reached Stirling at 11·15 am.

(a) For how long was she driving before she stopped for her break ?

(b) What was her average speed after her break ?

(c) Copy and complete the graph for Andrea's journey.



Applications of Speed, Distance and Time

Part 1

1. A car travels a distance of 1800 metres in a time of 90 seconds. Calculate the average speed of the car in metres per second.

2. Jane jogs to work every day at an average speed of 4 m/s. Most days it takes her 800 seconds to reach work. Calculate how far she jogs.

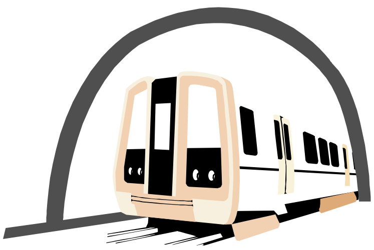
3. A model train travels round 20 m of track at an average speed of 2·5 m/s. How long does this take?

4. Christopher takes 15 seconds to swim one length of a swimming pool. If the pool is 90 metres long calculate his average speed.

5. How far will a cyclist travel in 60 seconds if he is travelling at an average speed of 13 m/s?

6. Calculate a hurdler's time if she completes the 400 m hurdle race at an average speed of 7 m/s.

7. How far will a jet aircraft travel in 5 minutes if it flies at 400 metres per second?

8. 

The Channel Tunnel is approximately 50 km long. How long will it take a train travelling at 80 m/s to travel from one end of the tunnel to the other?

9. A hill walker walks at an average speed of 1·6 m/s. How long will it take her to cover a distance of 33 km?

10. A lorry takes 4 hours to travel 150 km. Calculate the average speed of the lorry in m/s.

11. Richard Noble captured the world land speed record in 1983 in his vehicle Thrust 2. The car travelled 5km in 3·5 seconds. Calculate the average speed of the car in m/s.

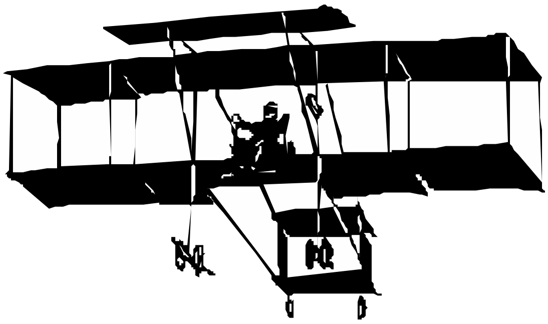
12. The table below shows part of a timetable for the Glasgow to Aberdeen train

Station	*Departure time*	*Distance* (km)
Glasgow	1025	0
Perth	1125	100
Dundee	1148	142
Aberdeen	1324	250

(*a*) Calculate the average speed of the train in m/s over the whole journey.

(*b*) Between which stations is the train's average speed greatest?

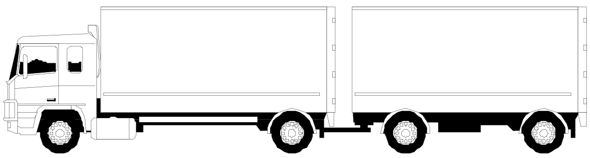
13. The Wright brothers were the first people to fly an aeroplane. Their first flight in 1903 lasted only 12 seconds and covered just 36 metres.



(a) Calculate the average speed of the plane during that first journey.

(b) Today a Tornado fighter jet can fly at Mach 2 (twice the speed of sound). How long would it take the jet to travel 36 metres? (Speed of sound in air = 340 m/s)

14. A long distance lorry driver has 3 hours to travel 210 km to catch the Northlink ferry.



a) Calculate the average speed at which the lorry must travel in order to reach the ferry on time. Give your answer in km/h.

b) Due to heavy traffic the lorry has an average speed of 60 km/h for the first 100 km. Calculate how long this leg of the journey takes.

c) At what speed must the lorry travel for the rest of the journey if the driver is to catch the ferry? Give your answer in km/h.

15. The cheetah is the fastest mammal on earth. It can run at an average speed of 40 m/s but can only maintain this speed for short periods of time. Cheetahs prey on antelopes. The average speed of an antelope is 35 m/s. The antelope can maintain this speed for several minutes.

a) Calculate how far a cheetah could run in 12 seconds if it maintained an average speed of 40 m/s.
b) How long would it take an antelope to run 480 m?
c) A cheetah is 80 m away from an antelope when it begins to chase it. The antelope sees the cheetah and starts to run at the same instant than the cheetah begins its chase. Both animals run at their average speeds and the cheetah is able to run for 15 s. Show by calculation whether or not the cheetah catches the antelope.

16. Before a major motor race the competitors complete practice circuits in their cars. These practice runs are timed and used to determine the position of each car at the starting grid for the race. The race circuit is 3·6 km long.

In a particular race each driver completed four practice laps. The practice lap times for the top three drivers are shown in the table.

Driver Name	Lap Times (s)			
	1	2	3	4
Mickey	45·8	43·4	46·4	48·2
Donald	44·7	46·2	44·6	49·5
Goofy	46·3	44·8	45·1	43·8

a) Which driver had the fastest average speed during lap 1?
b) Calculate the average speed during lap 2.
c) For each driver calculate their average speed in metres per second for the complete practice run.
d) Which driver is most likely to win the race?

Part 2

1. Calculate how far a car travels in 300 seconds when it is travelling at a top speed of 30 m/s.

2. How long does it take to walk to school if you walk at an average speed of 3 m/s and you live 900 metres away?

3. Find the average speed of a motor boat which takes 350 seconds to cover a 10 000 m course.

4. A runner takes 35 seconds to run round 250 metres of a track. What is his average speed?

5. A train travelling at 35 m/s takes 15 seconds to pass through a tunnel. How long is the tunnel?

6. 

   Find the average speed of Sammy Snail who slithers 0·005 m in 4 seconds.

7. How long does the TGV take to travel 60 000 m given that it goes at an average speed of 30 m/s.

8. A school bus takes 20 minutes to travel 15 km. What is it's average speed? (Give your answer in m/s)

9. A bird maintains an average speed of 11·2 m/s for 5 minutes. How far does it travel?

10. How long does a roller blader take to travel 2km if his average speed is 7 m/s?

The Area of a Rhombus and a Kite

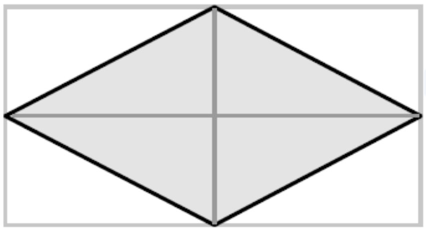
Exercise 3

1. (a) Make an accurate drawing of a rhombus with diagonals measuring 8 cm and 6 cm.
   (*Draw the 2 diagonals 8 cm by 6 cm meeting at right angles in the middle.*)

   (b) On your diagram, draw a rectangle round the rhombus.

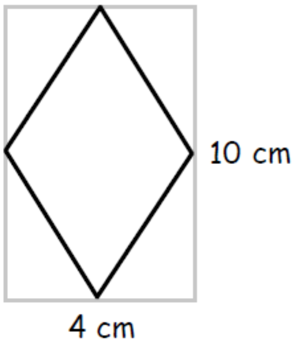
   (c) Calculate the area of the rectangle.

   (d) Now calculate the area of the rhombus.



2. For each rhombus below :- (i) sketch it.
   (ii) surround it with a rectangle.
   (iii) calculate the area of the rectangle.
   (iv) calculate the area of the rhombus.

(a)



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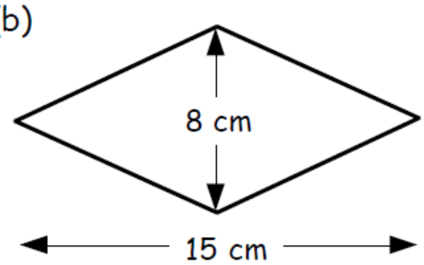

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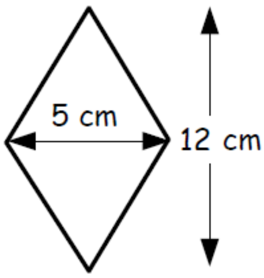

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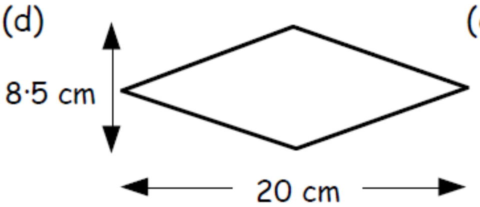

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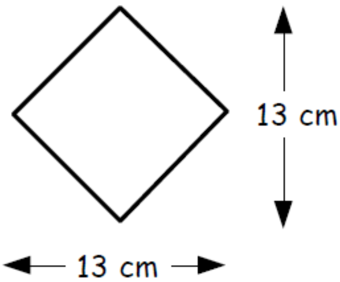

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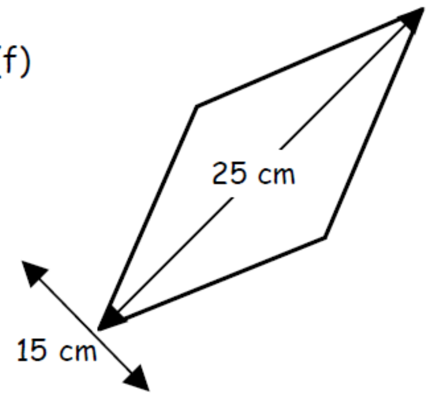
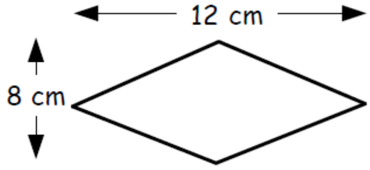
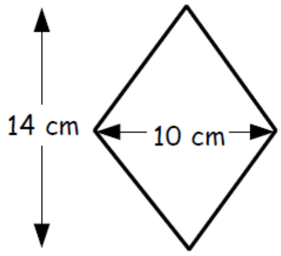


3. Use the formula "Area of Rhombus = $\frac{1}{2}(D \times d)$" to find the areas of these rhombi :-

(a)



(b)



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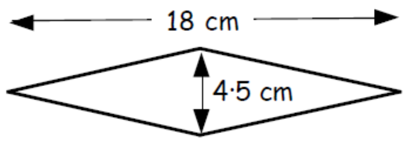

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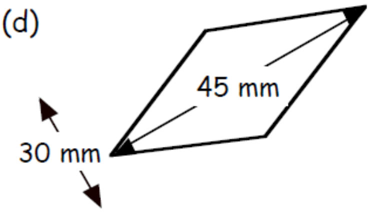

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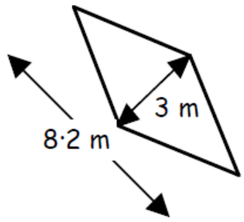

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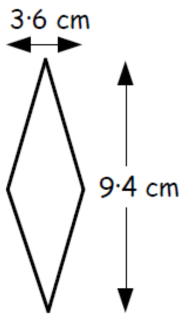
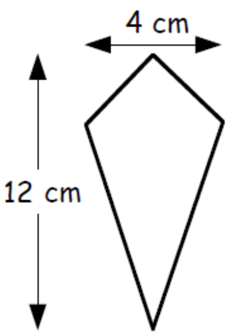


4. Use the formula "Area of Kite = $\frac{1}{2}(D \times d)$" to find the areas of these kites :-

(a)



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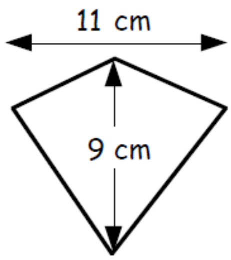

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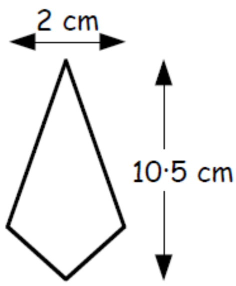

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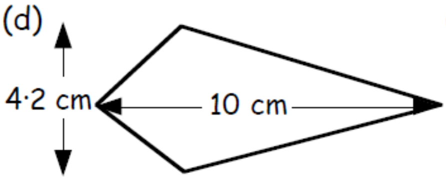

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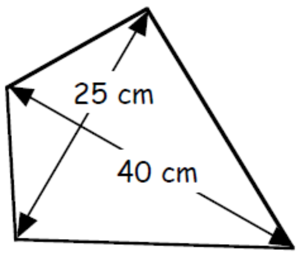

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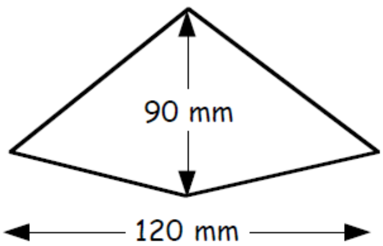
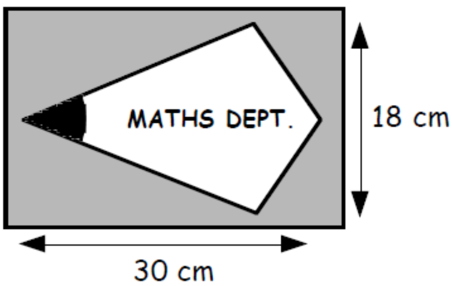


5. On parents' evenings, the maths department put up this wooden sign on the first floor of the school to direct parents to their rooms.

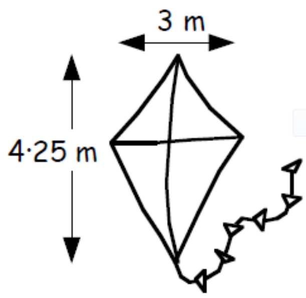
Calculate the area of the wooden kite-shape.

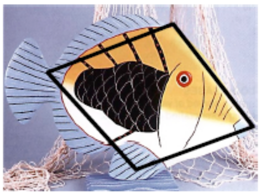


6. A giant polythene kite flew above the marquee at the wedding reception of the managing director of "Kites-R-4-U".

The kite was strengthened by 2 plastic poles measuring 4·2 metres and 3 metres which were fitted as diagonals of the feature.

Calculate the area of the giant kite.



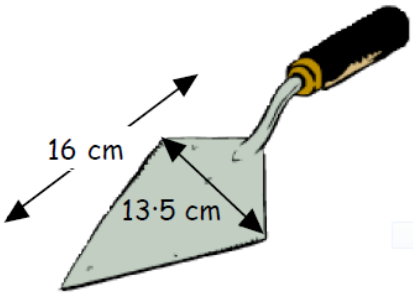
7. 

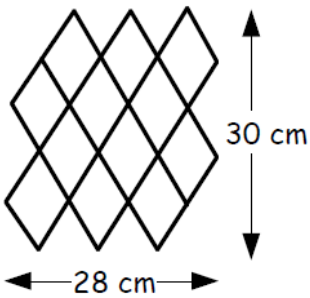
Local fishermen used to nickname this fish "The Rhombus".

Find the approximate area of its body if its measurements are 25 cm long and 9 cm in height.

8. The base of the trowel shown is in the shape of a kite.

Find its area.



9. 

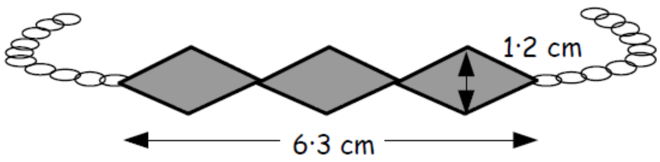
A tiling company glued 12 similar rhombus-shaped tiles onto a plywood board and used this to illustrate how their tiles gelled together to make ideal designs.

Calculate the area covered by ALL the tiles.
(*Hint - calculate the dimensions of one of the rhombi first*)

10. The main design on the pair of cufflinks shown is in the shape of rhombus.
The diagonals of each rhombus are 0·8 centimetres and 1·2 centimetres.

Calculate the total area taken up by the faces of rhombi.

11. Marjorie's necklace was made up with 3 identical golden rhombi on a chain.



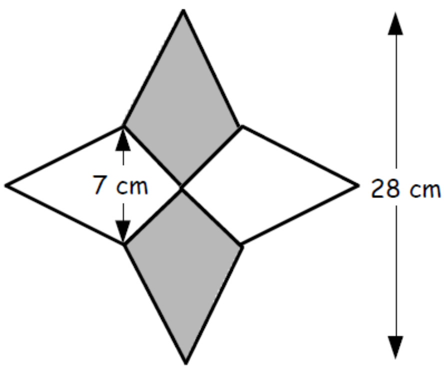
The 3 rhombi together measure 6·3 centimetres long and each has a height of 1·2 centimetres.

Calculate :-

(a) the length of the diagonal of one of the rhombi.

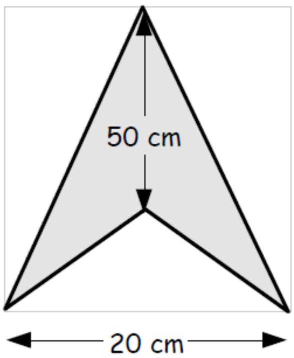
(b) the total area of the 3 golden rhombi.

12. Calculate the area of the star-shape, constructed from 4 identical kites.

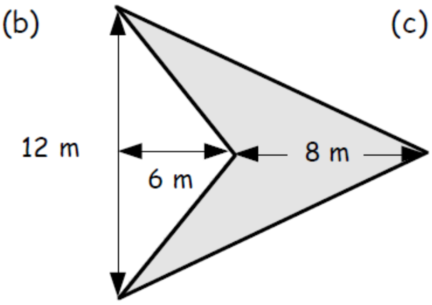


13. Calculate the area of each V-kite.

(a)



(b)



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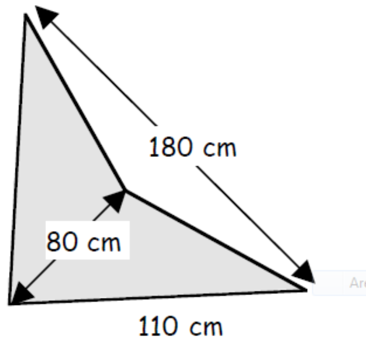
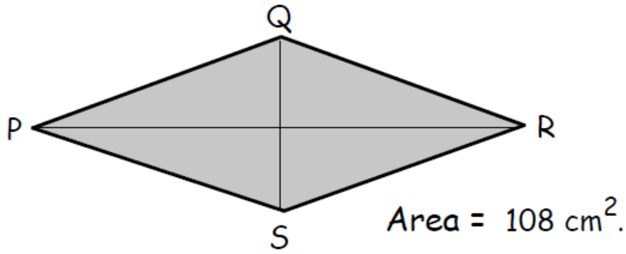


14. The area of rhombus PQRS is 108 cm^2.
The length of diagonal PR is 24 cm.

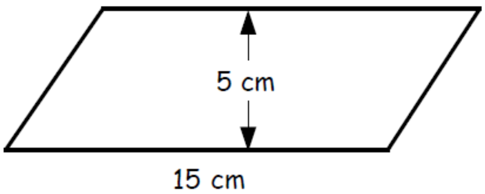
Find the length of diagonal QS.



The Area of a Parallelogram

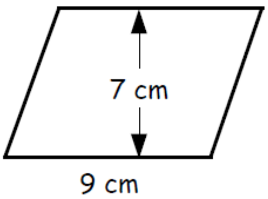
Exercise 4

1. This is a sketch of a parallelogram.
Use the formula $A = B \times H$ to find its area.



2. Calculate the areas of these parallelograms :-

(a)



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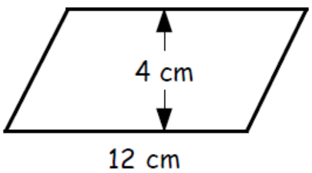

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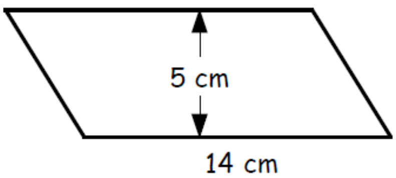

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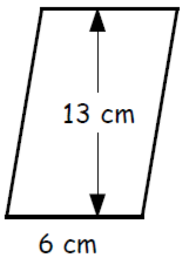

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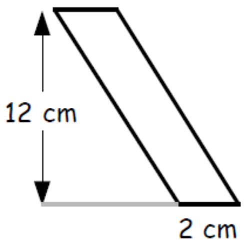

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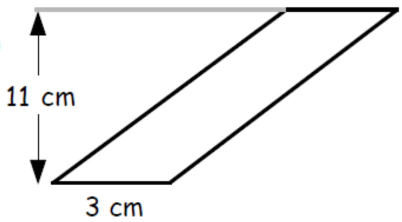

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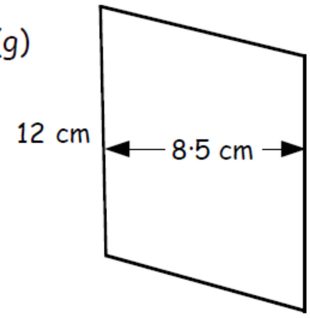
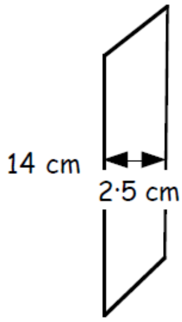
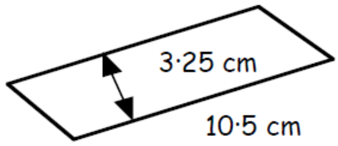


(h)

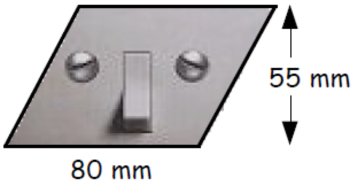


(i)



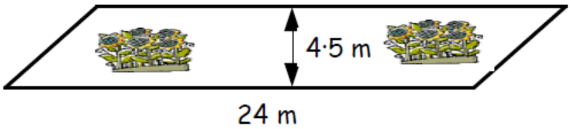
3. This light switch is in the shape of a parallelogram.

   Calculate its area.

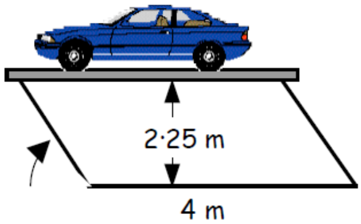


4. Mrs Galbraith made her front garden into a parallelogram shape.

   Calculate the area of her garden.



5.

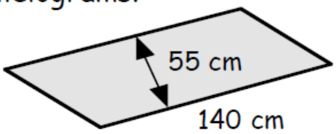


The ramp in the garage is the form of a parallelogram.

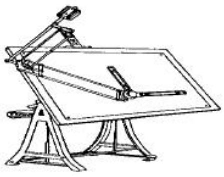
Calculate the area of the gap shown.

6. Council workers use this machine when mending roads. Many of its moving parts are parallelograms.

   Find the area of the one shown.

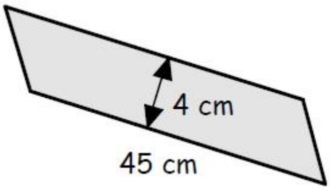


7.



Fraser, an architect, often uses parts of parallelograms when drawing up plans.

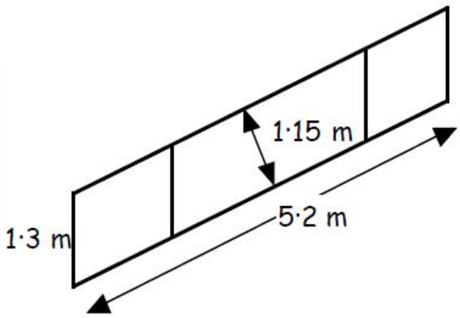
Calculate the area of this part.



8. The movable stairway is used at many older airports to allow passengers to disembark from aircraft.

   Again, a parallelogram shape is noticeable.

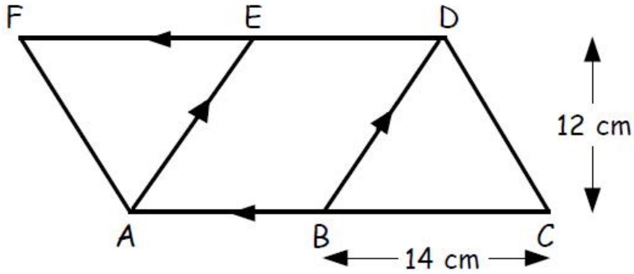
   Find the area of the large parallelogram.

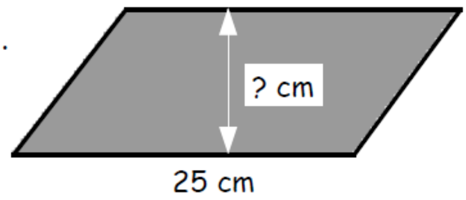


9. Look at the diagram shown and :-

   (a) name 2 parallelograms.

   (b) calculate the area of each one.



10. 

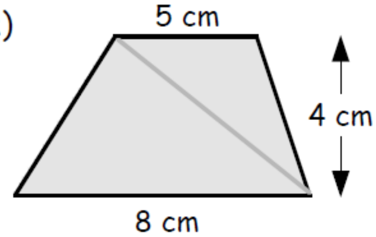
The area of the shaded parallelogram is 350 cm^2.

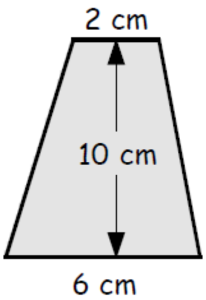
What is its height ?

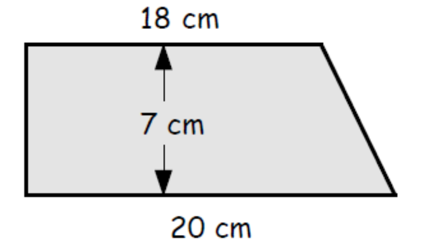
The Area of a Trapezium

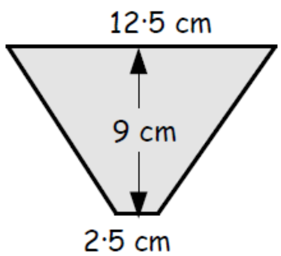
Exercise 5

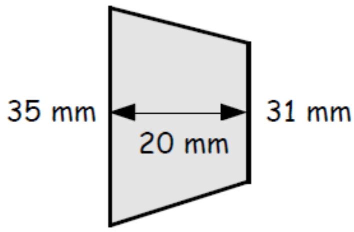
1. For each of the following, sketch and split each trapezium into 2 triangles, and calculate the area :-

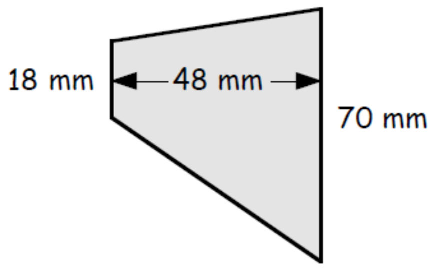
(a) 

(b) 

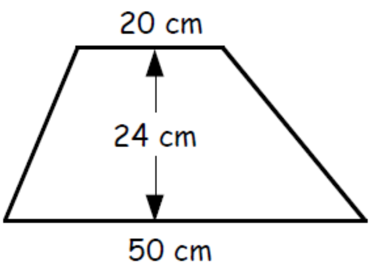
(c) 

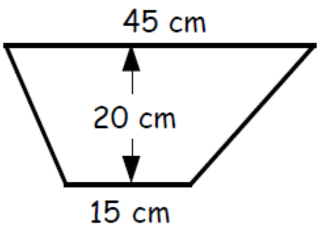
(d) 

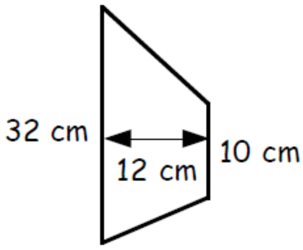
(e) 

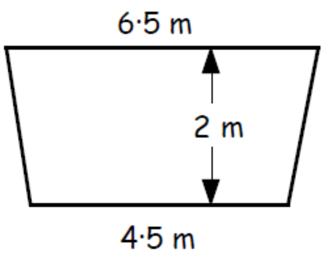
(f) 

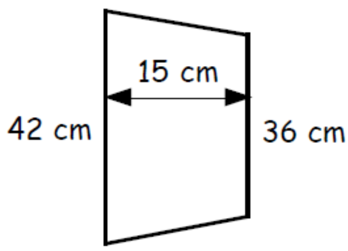
2. Use the formula $Area = \frac{1}{2}(a + b)h$ to calculate the area of each of the trapezia :-

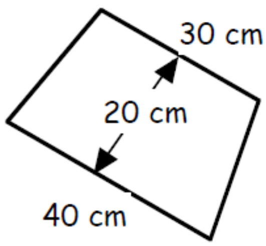
(a) 

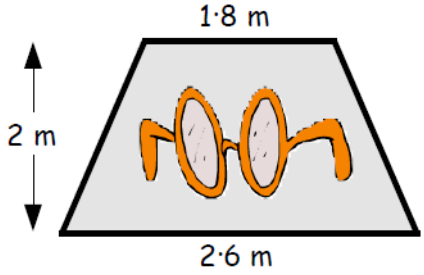
(b) 

(c) 

(d) 

(e) 

(f) 

3. 

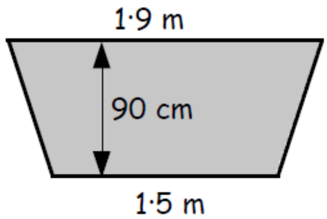
The sign outside "Trapezia Spectacles" is similar to the one shown.

Calculate the area of the trapezium.

4. Calculate out the area of this Malaysian stamp.

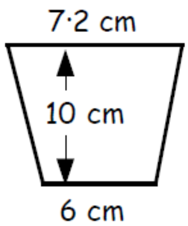
5. The top of this office table is in the shape of a trapezium.

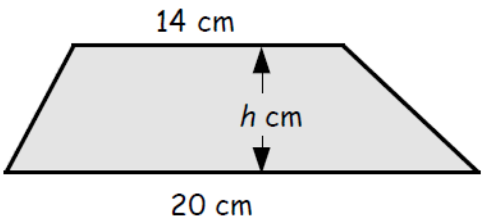
Find its area in cm^2.



6. The gaps in the alloy wheels are trapezium shaped.

Calculate the total area taken up by the 8 gaps in this wheel.



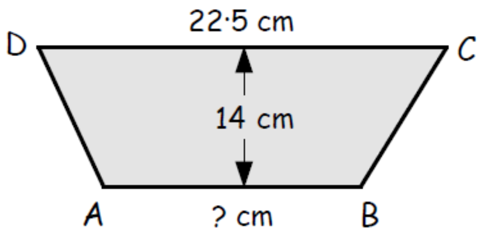
7. 

The area of this trapezium is 136 cm^2.

Calculate its height (h cm).

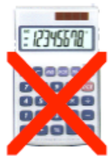
8. The area of the trapezium ABCD is 273 cm^2.

Calculate the length of the line AB.



Division of Fractions

Exercise 5



1. Copy each of the following and complete :-

(a) $\frac{3}{4} \div \frac{3}{5}$

$= \frac{3}{4} \times \frac{5}{3}$

$= \frac{?}{12} = \frac{?}{4} = 1\frac{?}{4}$

(b) $\frac{5}{6} \div \frac{2}{3}$

$= \frac{5}{6} \times \frac{3}{2}$

$= \frac{?}{12} = 1\frac{?}{?}$

(c) $\frac{3}{4} \div \frac{5}{6}$

$= \frac{3}{4} \times \frac{6}{5}$

$= \frac{?}{20} = \frac{?}{?}$

2. Divide the following fractions and simplify (where possible) :-

(a) $\frac{2}{5} \div \frac{2}{3}$ (b) $\frac{5}{6} \div \frac{7}{12}$ (c) $\frac{3}{7} \div \frac{6}{7}$ (d) $\frac{3}{10} \div \frac{4}{5}$

(e) $\frac{3}{8} \div \frac{5}{6}$ (f) $\frac{7}{12} \div \frac{7}{8}$ (g) $\frac{11}{16} \div \frac{5}{8}$ (h) $\frac{2}{9} \div \frac{1}{6}$

(i) $\frac{7}{10} \div \frac{3}{5}$ (j) $\frac{7}{16} \div \frac{3}{10}$ (k) $\frac{8}{9} \div \frac{3}{4}$ (l) $\frac{1}{5} \div \frac{1}{7}$

3. How many $\frac{2}{5}$'s are there in $\frac{3}{10}$'s ?

4. How many pieces of cloth $\frac{1}{8}$ metre long, can I cut from a piece $\frac{2}{3}$ metre long ?

5. Copy and complete the following :-

(a) $2\frac{1}{4} \div 1\frac{1}{5}$

$= \frac{9}{4} \div \frac{6}{5}$

$= \frac{9}{4} \times \frac{?}{6}$

= ..... = ...

(b) $4\frac{2}{3} \div 1\frac{2}{5}$

$= \frac{14}{3} \div \frac{7}{5}$

$= \frac{14}{3} \times \frac{?}{?}$

= ..... = ...

(c) $2\frac{2}{3} \div 3\frac{1}{5}$

$= \frac{?}{3} \div \frac{?}{5}$

= ....

= ..... = ...

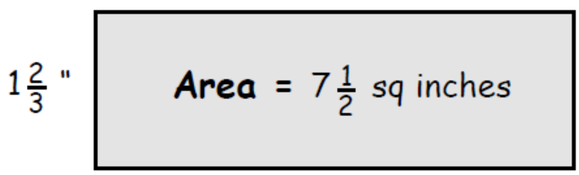
6. Divide the following fractions in the same way (simplify if possible) :-

(a) $3\frac{1}{3} \div 1\frac{1}{2}$ (b) $2\frac{1}{5} \div 1\frac{1}{2}$ (c) $4\frac{1}{3} \div 2\frac{3}{4}$ (d) $1\frac{2}{7} \div 2\frac{2}{3}$

(e) $4\frac{1}{4} \div 3\frac{3}{5}$ (f) $6\frac{1}{2} \div 2\frac{1}{4}$ (g) $1\frac{3}{5} \div 4\frac{2}{3}$ (h) $7\frac{1}{2} \div 1\frac{3}{7}$

(i) $5\frac{1}{3} \div 1\frac{3}{5}$ (j) $4\frac{1}{2} \div 5\frac{1}{4}$ (k) $6 \div 1\frac{1}{2}$ (l) $8 \div 2\frac{2}{3}$

7. The area of this rectangular piece of card is $7\frac{1}{2}$ square inches.
It is $1\frac{2}{3}$ inches wide. Calculate its length.



8. A $4\frac{1}{2}$ metre length of plank weighs $10\frac{1}{8}$ kilograms.

(a) What does 1 metre of the plank weigh ?

(b) What is the weight of a $3\frac{1}{4}$ metre plank of the same type of wood ?

9. Danny's mum found that she was $1\frac{1}{5}$ times as tall as Danny was.

If his mum was $1\frac{3}{4}$ metres tall, how tall was Danny ?

10. 

$2\frac{1}{4}$ laps of the park took Tommy Muir, walking his dog, $12\frac{1}{2}$ minutes.

How long, on average, did each lap take ?

Division of Decimals by a Single Decimal Digit

Exercise 6

1. Find :-

(a) 8 ÷ 0·2 (b) 16 ÷ 0·4 (c) 25 ÷ 0·5 (d) 48 ÷ 0·6

(e) 56 ÷ 0·7 (f) 81 ÷ 0·9 (g) 100 ÷ 0·1 (h) 99 ÷ 0·9

2. Find :-

(a) 1·4 ÷ 0·7 (b) 2·6 ÷ 0·2 (c) 5·6 ÷ 0·8 (d) 5·4 ÷ 0·6

(e) 2·55 ÷ 0·5 (f) 9·24 ÷ 0·6 (g) 22·26 ÷ 0·7 (h) 37·36 ÷ 0·8

3. Calculate :-

(a) 8 ÷ 0·02 (b) 40 ÷ 0·08 (c) 4·2 ÷ 0·03 (d) 6·3 ÷ 0·07

(e) 0·024 ÷ 0·08 (f) 0·081 ÷ 0·09 (g) 0·005 ÷ 0·01 (h) 0·015 ÷ 0·05

4. Calculate :-

(a) 0·27 ÷ 0·003 (b) 0·64 ÷ 0·004 (c) 0·48 ÷ 0·006 (d) 0·035 ÷ 0·007

(e) 0·065 ÷ 0·005 (f) 0·008 ÷ 0·002 (g) 0·0153 ÷ 0·003 (h) 0·906 ÷ 0·006

5. Calculate :-

(a) 42 ÷ 60 (b) 18 ÷ 20 (c) 15 ÷ 500 (d) 12 ÷ 400

(e) 54 ÷ 900 (f) 32 ÷ 8000 (g) 210 ÷ 7000 (h) 350 ÷ 5000

6. 4000 floppy disks can store 6160 megabytes.

How many megabytes can be stored on one such disk ?

7. A small paint pen for colour testing holds 0·08 litres of paint.

How many pens can be filled from a drum which contains :-

(a) 1·6 litres (b) 40 litres (c) 100 litres (d) 0·72 litres ?

8. A box of 2000 large envelopes weighs 1·4 kg, not including the weight of the box itself.

Work out the weight of one envelope, (a) in kg's. (b) in grams.

9. Try these :-

(a) 10 ÷ 0·0002 (b) 50 ÷ 0·0005 (c) 3·33 ÷ 0·0003 (d) (-0·42) ÷ 0·7

(e) 0·18 ÷ (-0·6) (f) (-0·24) ÷ (-0·4) (g) 0·0001 ÷ 0·001 (h) 0·0005 ÷ 0·005

Removing Brackets

Remove these brackets :-

1. (a) $-2(3x + 5)$ (b) $-3(4x - 1)$ (c) $-6(5x + 7)$ (d) $-4(4x - 8)$

(e) $-7(4-2x)$ (f) $-8(3 + 9x)$ (g) $-(5x+9)$ (h) $-(3x - 6)$

(i) $-(x + 1)$

2. (a) $3 - 2(4x + 1)$ (b) $8 - 4(3x - 1)$ (c) $10 - 5(5x - 3)$ (d) $9 - 4(6x + 2)$

3. (a) $-2(a + 1)$ (b) $-3(x - 2)$ (c) $-5(3 + d)$ (d) $-4(5 - c)$

(e) $-(p + q)$ (f) $-(p - q)$ (g) $-6(d + e)$ (h) $-5(d - e)$

(i) $-p(p + 4)$ (j) $-h(h - 1)$ (k) $-x(1 + x)$ (l) $-2m(m + 3)$

(m) $-a(4a - 1)$ (n) $-h(5h + 4k)$ (o) $-x(5y - 4x)$ (p) $-2x(x - 3k)$

4. (a) $4(x + 1) - 2(x + 2)$ (b) $5(a + 2) - 4(a + 2)$ (c) $3(b + 5) - 2(b + 7)$
   (d) $3(2c + 4) - 2(c + 5)$ (e) $6(3p + 2) - 4(p + 3)$ (f) $4(x + 3) - 2(x - 3)$
   (g) $5(x + 1) - 3(x - 2)$ (h) $6(1 + 2e) - 2(1 - e)$ (i) $10(2 - v) - 12(1 - v)$
   (j) $x(x + 1) + 2(x - 1)$ (k) $n(n + 6) - 4(n + 1)$ (l) $w(3w - 1) - 2(3w - 8)$

5. (a) $7 - 2(y + 3)$ (b) $5 - 2(p - 1)$ (c) $3 - 3(d - 1)$
   (d) $4 + 3(h + 1)$ (e) $2 + 8(2 - c)$ (f) $4 - 2(1 - u)$
   (g) $9(b - 2) - 8$ (h) $-2(n - 1) + 3$ (i) $m + 3(m - 4)$
   (j) $x - (3 - x)$ (k) $9k - 3(k + 6)$ (l) $3w - 2(2 - 3w)$

Equations with Brackets

1 $5(2x + 3) - 3(2x + 4) = 15$

2 $2(4x + 2\frac{1}{2}) - 2(2x + 1) = 11$

3 $3(3x + 4) - 2(2x + 1) = 30$

4 $5(6x + 3) - 4(5x + 1) = 41$

5 $3(6x + 1) - 2(7x - 4) = 43$

6 $5(4x + 1) - 9(2x - 3) = 38$

7 $9(2x - 1) - 3(5x - 3) = 18$

8 $3(4x - 2) - 2(5x - 7) = 20$

9 $5(x - 3) - 2(x - 3) = 9$

10 $3(2 + 4x) + 3(5 - x) = 30$

11 $3x + 4 + 2(x - 3) = 8$

12 $3x + 4 - 2(x - 3) = 8$

13 $2(x - 1) - 4 + x = 12$

14 $4(x + 1) - 2x + 7 = 11$

15 $^{3}3(2x + 7) - 2(x - 1) = 19$

16 $6(x + 1) - 4(x + 1) = 0$

17 $3(2x + 5) - 2(2x - 3) = 27$

18 $5(8x - 4) - 7(5x - 2) = 14$

19 $7(5x + 2) + 4(8 - 7x) = 60$

20 $3(9x - 2) + 2(7 - 12x) = 20$

21 $8(x - 5) + 2(9 - 2x) = 2$

22 $7(3x - 4) - 4(3 - x) = 10$

23 $4x - 3 - 7(6 - x) = 10$

24 $6x + 8 - 2(2x - 9) = 31$

25 $13x - 2(4 + 5x) - 2 = 2$

26 $8 - 3(7 - 2x) + 4x = 12$

27 $5(4x + 1) - 3(6x + 1\frac{1}{2}) = 0$

28 $6x + 3 + 2(4 - x) = 3$

29 $5(3 - 2x) - 4(1 - 2x) = 12$

30 $0.3(4 + 2x) - 0.1(x + 2) = 0$

Ratio

Exercise 1 (Oral Exercise)

1. Look at the picture.
   Write down the ratio of : -

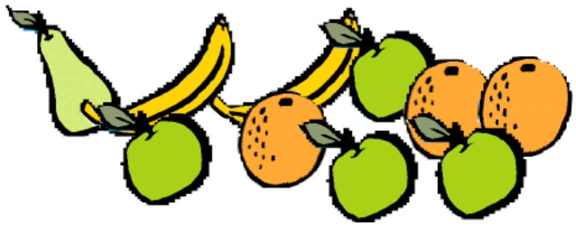
   (a) cars : buses (b) buses : cars

2. Look at this picture.
   Write down the ratio of : -

   (a) apples : oranges (b) apples : pears

   (c) oranges : pears (d) bananas : apples

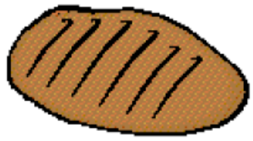
   (e) pears : bananas (f) bananas : pears.



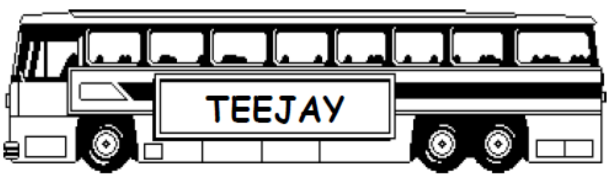
3. In a baker shop there are 122 loaves, 169 rolls and 59 baguettes.
   Write down the ratio of : -

   (a) loaves : baguettes (b) baguettes : rolls

   (c) rolls : baguettes (d) rolls : loaves.



4.



On a school trip there are 21 girls, 19 boys and 11 adults.

Write down the ratio of : -

(a) boys : girls (b) adults : girls

(c) children : adults (d) adults : people.

5. An accurately drawn rectangle has its dimension as shown. Write down the ratio of :-

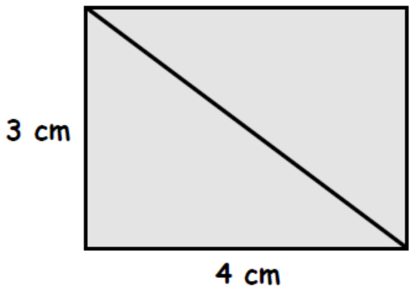
   (a) length : breadth

   (b) length : perimeter

   (c) length : area (*ignore units*)

   (d) area : perimeter (*ignore units*)

   (e) length : diagonal length



Exercise 2 (no calculator)

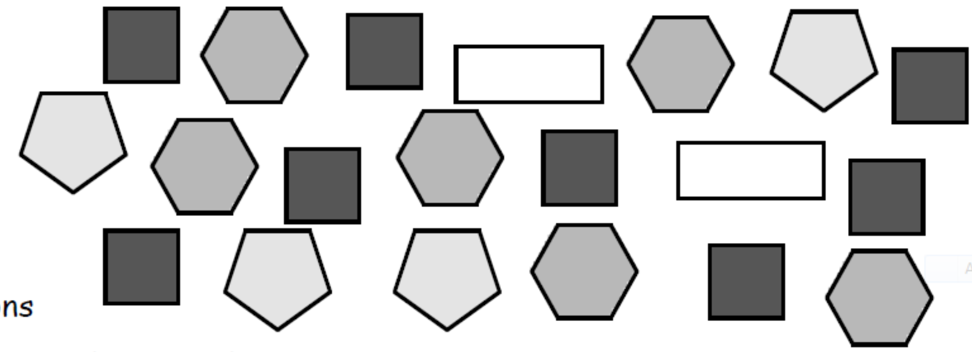
1. Simplify each ratio by dividing each value by 3 : -

   (a) 6 : 9 (b) 15 : 21 (c) 30 : 33 (d) 3 : 27 (e) 300 : 663

2. Copy the ratios and simplify each as far as possible : -

   (a) 3 : 36 (b) 12 : 48 (c) 30 : 180 (d) 7 : 56
   (e) 11 : 121 (f) 33 : 12 (g) 22 : 99 (h) 17 : 51
   (i) 26 : 130 (j) 57 : 171 (k) 33 : 242 (l) 15 : 615
   (m) 25 : 90 (n) 3 : 27 (o) 25 : 1250 (p) 24 : 144
   (q) 10 000 : 200 (r) 30 000 : 6000 (s) 2 : 4 : 10 (t) 14 : 84 : 21

3. Write down each ratio in its simplest from : -



   (a) pentagons : hexagons
   (b) squares : pentagons
   (c) rectangles : squares
   (d) quadrilaterals : hexagons
   (e) quadrilaterals : (pentagons + hexagons).

4. (a) A farmer has 18 sheep and 32 cows in a field.
   Write down the ratio of cows : sheep in its simplest form.

   (b) The farmer's field measures 20 metres by 35 metres.
   Write down the ratio of area : perimeter in its simplest form. (*ignore units*).

5. A large container has dimensions 4 by 3 by 2 metres.
   A small container has dimensions 2 by 2 by 1 metres.

   Write down the ratio of **volumes** (small : large) in its simplest form.

6. In a week Barry earns £250, Sharon earns £300 and Del earns £450.

   Write down the following ratios of wages in their simplest forms : -

   (a) Del : Barry (b) Sharon : total wages (c) Del : Sharon : Barry.

7. Simplify the following to a unitary ratio each time : -

(a) $\frac{1}{3} : 2$ (b) $\frac{1}{3} : 5$ (c) $\frac{1}{2} : 6$ (d) $\frac{1}{2} : 2$ (e) $\frac{1}{4} : 9$ (f) $\frac{1}{4} : 12$

(g) $\frac{1}{5} : 15$ (h) $\frac{1}{8} : 8$ (i) $\frac{1}{7} : 13$ (j) $\frac{1}{15} : 20$ (k) $\frac{1}{4} : \frac{1}{2}$ (l) $\frac{1}{2} : \frac{1}{8}$

8. Write each of the following in its simplest form (not all give unitary ratios) : -

(a) $\frac{2}{3} : 4$ (b) $\frac{2}{3} : 5$ (c) $\frac{3}{4} : 15$ (d) $\frac{2}{5} : 10$ (e) $\frac{4}{7} : 2$ (f) $\frac{9}{10} : \frac{1}{2}$

(g) $\frac{7}{10} : 0{\cdot}6$ (h) $\frac{5}{6} : 5{\cdot}4$ (i) $\frac{3}{5} : 50$ (j) $\frac{3}{4} : 11$ (k) $\frac{9}{10} : 180$ (l) $\frac{2}{5} : \frac{1}{2}$

9. A recipe needs $\frac{1}{2}$ kilogram of butter, $\frac{1}{4}$ kilogram of flour and $\frac{1}{10}$ kilogram of sugar.

Write in its simplest form the ratio of : -

(a) butter : flour (b) flour : butter

(c) sugar : butter (d) flour : sugar.

When working with ratios, the two units **must** be the same.

10. Write down each ratio in its simplest form : -

(a) $\frac{1}{4}$ of an hour : 30 minutes (*hint : - change both to minutes*)

(b) $\frac{1}{4}$ kg : 150 g (c) $\frac{1}{4}$ litres : 25 ml (d) $\frac{1}{2}$ metre : 200 cm

(e) 20 kg : 200 g (f) 10 litres : 100 ml (g) 3 kilometres : 200 m

(h) 1 km : 10 cm (i) 2 tonnes : 100 g (j) 30 minutes : 1 day

(k) 1 week : days in April (l) one million millimetres : one kilometre.

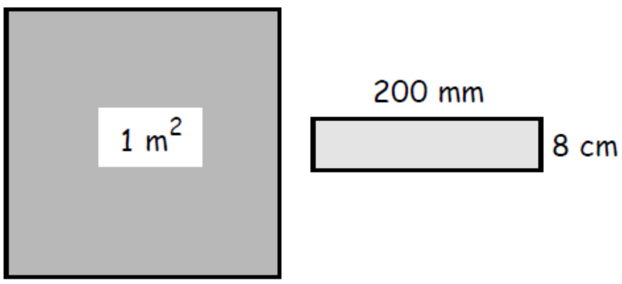
11. A rectangle has length 200 mm and breadth 8 cm.

A square has an area of 1 m^2.

Write in its simplest form the ratio of : -

(a) area of rectangle : area of square.

(b) length of rectangle : length of square.

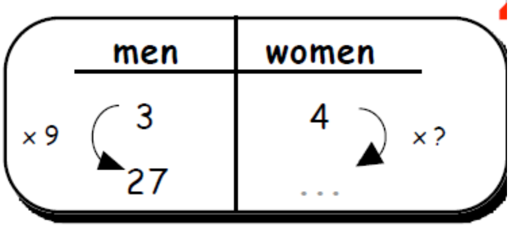


Ratio Calculations

Exercise 3 (no calculator)

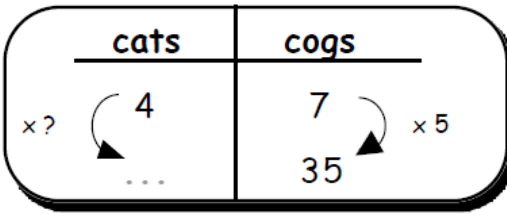
1. (a) On a train the ratio of men to women is 3 : 4.

If there are 27 men on the train, how many women are there ?



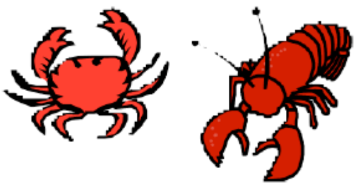
(b) In a Cat & Dog home the ratio of cats to dogs is 4 : 7.

If there are 35 dogs in the home , how many cats are there ?



(c) In an orchard the ratio of apple trees to pear trees is 9 : 11.

If there are 27 apple trees, how many pear trees are there ?

2. 

In a large aquarium the ratio of crabs to lobsters is 3 : 5.

(a) If there are 12 crabs, how many lobsters are there ?

(b) If there are 30 lobsters, how many crabs are there ?

3. The ratio of Bob's weekly wage to Janet's weekly wage is 5 : 7.

(a) If Bob earns £250, how much would Janet earn ?

(b) If Janet earns £210, how much would Bob earn ?

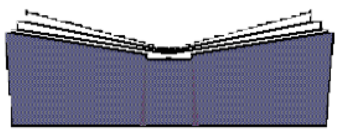
4. 

In a school the ratio of girls to boys is 8 : 7.

(a) If there are 400 girls, how many boys are there ?

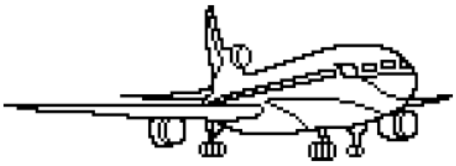
(b) If there are 651 boys, how many girls are there ?

5. The ratio of vowels to consonants in a book was 11 : 23.



(a) If there are 13 200 vowels, how many consonants are there ?

(b) If there are 690 000 consonants, how many vowels are there ?

6. 

A model aeroplane has a scale of 1 : 40.

(a) If the wing span on the model is 25 centimetres, what would be the wingspan of the real aeroplane ?

(b) If the real aeroplane has length 8 metres, what is the length of the model aeroplane ?

7. The table shows the ratios of blue and red paint for making different shades of purple.

Which shade of purple will I get if I mix : -

(a) 300 ml of blue and 500 ml of red ?

(b) 1·8 litres of blue and 200 ml of red ?

(c) 900 ml of blue and 1·5 litres of red ?

(d) 1 litre of blue and 300 ml of red ?

(e) 500 ml of red and 0·7 litres of blue ?

(f) 2·25 litres of red and 1·35 litres of blue ?

Mix in the ratio

Colour	***Blue***	**:**	***Red***
Very dark purple	9	:	1
Dark purple	10	:	3
Purple	7	:	5
Light purple	3	:	5
Very light purple	2	:	9

Class Intervals

The test scores of a group are to be entered into a frequency table. (The first 6 have been done)

~~12~~ ~~23~~ ~~41~~ ~~55~~ ~~77~~ ~~15~~ 32 40
51 69 21 12 16 43 56 71
32 75 34 42 55 76 21 73
22 56 41 19 20 47 78 17

Class Intervals	Tally	Frequency
10 - 19	II	
20 - 29	I	
30 - 39		
40 - 49	I	
50 - 59	I	
60 - 69		
70 - 79	I	

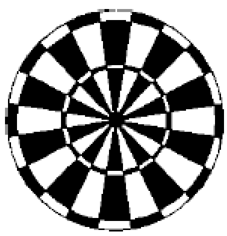
1. (a) Copy and complete the frequency table above.

(b) How many students scored over 49 ?

(c) Draw a neat labelled **bar graph** to show this information.

2. Each number below shows the score of 3 darts thrown by each member of class $1A_3$.

15 13 31 42 64 34 32 20 11 8 21
55 19 51 45 64 35 75 50 46 55 67
21 33 12 6 40 79 76 47 29 10 15



(a) How many numbers are in each interval ?

(b) How many intervals will there be in the table ?

(c) Copy and complete the table.

(d) How many pupils are in class $1A_3$?

(e) How many pupils scored under 30 ?

(f) Draw a neat labelled bar graph showing this information.

Class Intervals	Tally	Frequency
0 - 9		
10 - 19		
20 - 29		
30 - 39		
40 -		

3. The number of pets in each class in a school is shown below.

1	14	8	27	16	7	12	15	21	20	17	0	11	15	10
12	14	4	5	10	14	11	9	19	15	21	13	4	11	16

Show this information on a frequency table. (*Use class intervals of 0 - 4, 5 - 9, 10 - 14, etc*)

4. A class were asked to tidy their bedrooms and say how many coins they found !

The number of coins found by each pupil is shown.

(a) Find the range.

(b) Which of these would be the best class interval to start with : -

(0 - 9) or (0 - 3) or (0 - 4) or (0 - 2) ?

(c) Construct a frequency table using your chosen class interval.

(d) Draw a neat labelled bar graph to show this information.

4	3	18	15	31	9	0	2
11	6	27	15	12	11	15	4
22	15	16	26	25	17	13	3
9	7	1	9	16	7	21	10
12	20	1	14	19	3	0	12

5. A list of waiting times (in minutes) in a doctors surgery are shown.

(a) Find the range.

(b) Which of these would be the best class interval to use : -

(0 - 9) or (0 - 1) or (0 - 4) or (0 - 3) ?

(c) Construct a frequency table showing this information.

0	4	22	11	11	19	10	12
5	8	26	25	15	17	18	2
20	13	19	21	22	13	23	13
8	9	1	6	26	8	18	10
14	10	3	24	17	5	3	22

6. For each table below, construct a frequency table using an appropriate class interval.

(a)

14	13	18	15	11	9	4	1
15	34	32	25	12	16	15	14
9	15	18	25	25	19	14	3
9	8	2	7	16	27	23	20
22	20	11	13	16	30	4	22

(b)

10	35	28	45	71	69	50	42
11	36	27	15	62	72	65	54
42	35	26	16	25	37	43	53
69	52	47	31	29	19	47	31
20	12	60	51	24	49	43	40

(c)

127	152	163	174	101	133	167	155	171	110	117	129
111	134	125	164	115	122	150	160	129	144	141	153
130	128	166	154	122	169	140	151	163	162	100	174

(d)

3·6	2·3	4·6	1·7	5·6	4·2	1·1	4·0	5·2	6·3	6·9	4·1
2·5	2·8	1·3	2·5	6·6	5·1	1·4	4·6	2·2	3·3	5·1	0·4
5·0	2·9	4·3	2·1	5·4	4·6	5·3	6·1	2·2	5·7	5·8	1·3

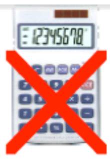
Stem-and-leaf Graphs

Age in Years

stem	leaves
2	4 6 8
3	0 1 3
4	4 4 5 6 7 9
5	0 3 4 9
6	1 4 5 6

Key :-
2 | 4 means 24

Exercise 6



1. The 2nd line of the above graph reads 30, 31 and 33 years of age.

   (a) Write the ages given by each line in the graph above.

   (b) (i) What age was the youngest person in the queue ?

   (ii) What age was the oldest person in the queue ?

   (c) How many people were in the queue ?

2. The ages of a group of people waiting in a queue at a bank were recorded and put into the stem and leaf graph shown.

   (a) The first line (level 2) reads 21 years, 22 years, 24 years and 27 years.

   Write out the ages in level 3.

   (b) Write out the ages of level 4.

   (c) What age was the :-

   (i) youngest person (ii) oldest person ?

   (d) Were most of the people in their 20's, 30's or 40's ?

Age in years

2	1 2 4 7
3	1 7 9
4	2 2 3 6 8 8
5	3 3 3 5
6	0 8

Key :
2 | 4 means 24

3. Some pupils were asked how much money they had. The results are shown in the stem and leaf graph.

   (a) List the amount of money each pupil had.

   (b) Which level has the most data ?

   (c) Which amount of money appears the most often (mode) ?

   (d) How many pupils were asked in the survey ?

Pupil's money

1	1 2 5 7 8
2	0 1 2 6
3	4 4 4 5 8 8
4	9
5	0 3

Key :
1 | 2 means £1·20

4. The table shows the time it took in seconds for a puzzle to be solved by some students.

(a) Write a key for this stem and leaf graph.

(b) State what was the :-

(i) fastest time (ii) slowest time,

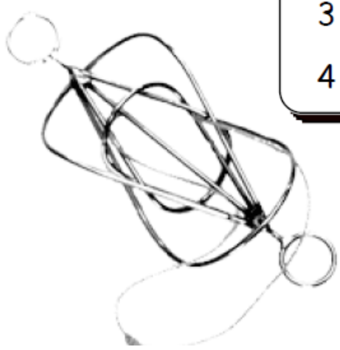
taken for the puzzle.

(c) How many pupils tried the puzzle.

(d) How many pupils took more than 22 seconds to complete the puzzle ?

(e) Find the modal time (mode).

(f) Work out the median (middle) time.



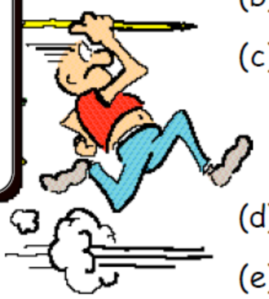
Puzzle Time

0	9					
1	6	8	9			
2	0	1	1	1	1	4
3						
4	1	7				

5. This stem and leaf graph has not been put in order.

Javelin throw

0	9					
1						
2	7	2	6	8	2	
3	9	6	1	9	2	9
4	2	1	5	0		



The graph shows the lengths (in metres) thrown in a javelin competition.

(a) Copy the graph, but this time show the distances in order.

(b) Write a key for this graph.

(c) What was the :-

(i) greatest distance thrown ?
(ii) least distance thrown ?

(d) What does the empty space at "1" mean ?

(e) Find the :- (i) mode
(ii) median.

6. For each set of data :- (i) Construct an ordered stem and leaf graph with a key.
(ii) Find the mode and median.

(a)

14	13	18	15	11	9	4	1
15	34	32	25	12	16	15	14
9	15	18	25	25	19	14	3
9	8	2	7	16	27	23	20
22	20	11	13	16	30	4	22

(b)

11	22	27	49	61	68	60	52
45	34	47	25	52	62	65	45
24	52	62	61	52	31	63	33
59	42	37	21	29	19	47	34
30	22	60	41	34	59	53	10

(c)

137	142	153	164	111	123	157	165	161	104	107	119
101	124	135	154	125	132	140	160	139	154	151	123
140	138	156	164	132	159	160	111	143	152	110	164

Back to back stem and leaf diagrams

1. Shown below is a back-to-back stem and leaf diagram, giving the age and gender of people at a 50th birthday party.

Age of People

Male		Female
9 8 7 1	1	5
5 1 0	2	0 2 3
9 5 3 3 2	3	6 7 7 8 9
8 6 6 6 6	4	0 2 4 4 4
3 1 0	5	2 4 9

(a) How many **males** at the party are aged :–

(i) 33 (ii) 46 (iii) 54 ?

(b) Find the **modal** age and **median** age of

(i) the males. (ii) the females.

2. The information below gives the ages of men and women who took part in a competition.

Men	23	35	45	32	19	23	33	37
Women	22	18	19	23	27	27	30	29

(a) Draw an **ordered back to back** stem and leaf diagram to represent this information.

(b) Find the **modal** and **median** ages of :–

(i) the men. (ii) the women.

3. (a) Draw an **ordered back to back** stem and leaf diagram showing information about the weights (*in kilograms*) of a group of children given below.

Boys	24	19	18	26	34	26	25	30
Girls	40	23	34	21	25	29	30	25

(b) Find the **modal** and **median** ages of :–

(i) the girls. (ii) the boys.

4. Ten senior citizens aged in their **60's** and ten senior citizens aged in their **70's**, were asked how many times per year they went to the hairdresser's.

Number of Hair-do's

sixties		seventies
3	0	1
3 2 2 1	1	3 5
0	2	2 2 4
7 1	3	0
9 4	4	8
	5	1 2

Key :- 2 | 1 means 12 times, and 4 | 8 means 48 times.

a One person gets her hair done once per year. In which age group is she (60's or 70's) ?

b For the 10 senior citizens in their 60's, add **all** the visits together.
How many times did they visit the hairdresser altogether ?

c Do the same for the 10 senior citizens in their 70's.

d Who went more often – the 10 in their sixties or the 10 in their seventies ?

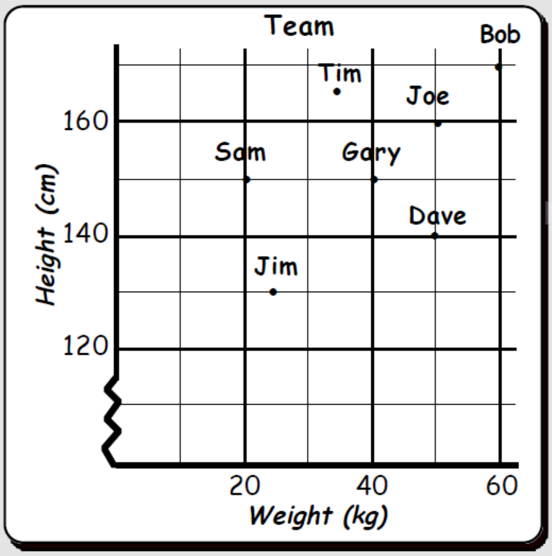
Scattergraphs

This scattergraph displays the **heights** and **weights** of a sevens football team.

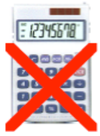
Gary weighs 40 kg.

Joe is 160 cm tall.

Jim is 130 cm tall and weighs approximately 25 kg.



Exercise 7



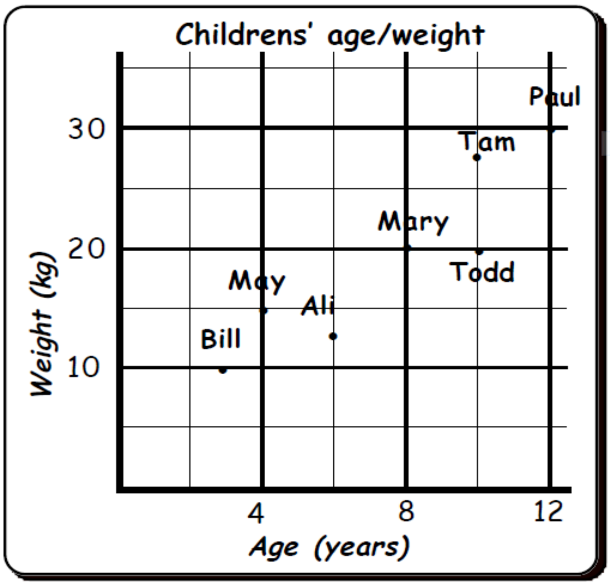
1. For the scattergraph above, write down the height and weight of each player.

2. This scattergraph shows the ages and weights of several children.

   (a) Who is :- (i) the youngest
   (ii) the lightest
   (iii) the oldest
   (iv) the heaviest child ?

   (b) Write down the age and weight of each child.

When two quantities are strongly connected, we say there is a strong **correlation** between them.



3. Say whether you think there will be a **correlation** between :-

   (a) the temperature and the sales of ice-cream.

   (b) the temperature and the amount of people on a beach.

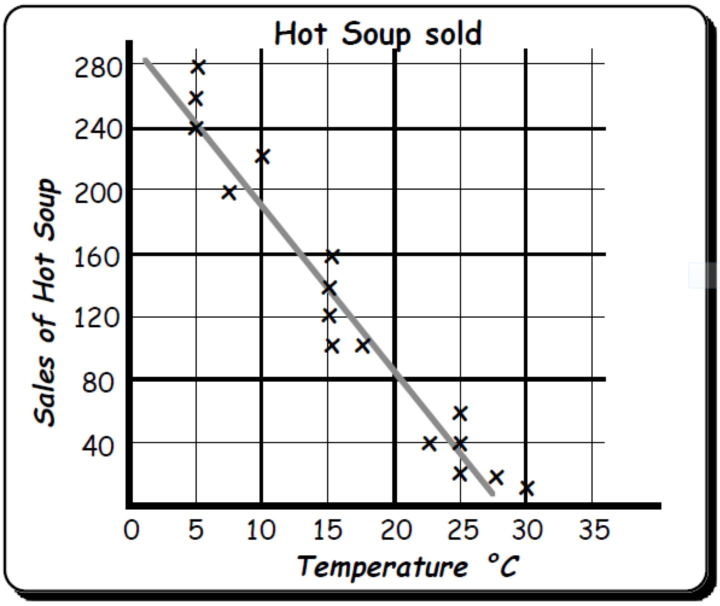
   (c) the amount of rain and the sales of umbrellas.

   (d) the distance a taxi travels and the fare.

   (e) the temperature and the sales of gloves.

   (f) the number of workmen and the time taken to build a wall.

4. This scattergraph shows the sales of cups of hot soup at a football ground.



This would be called a strong **negative correlation** since all the points lie roughly on a straight line going downwards from left to right.

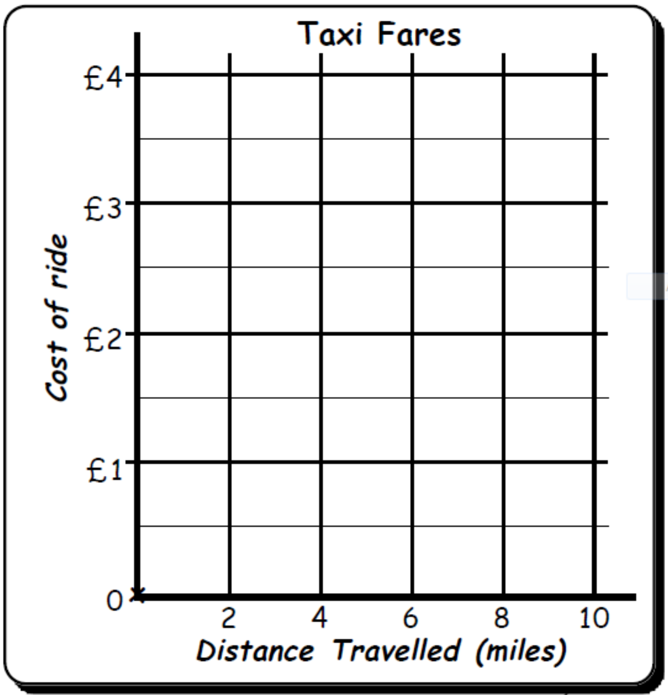
The line is called a **line of best fit**.

Use the line of best fit to estimate :-

(a) the sales at 20°C.

(b) the temperature when the sales were approximately 240 cups.

5. This graph represents the cost of different taxi fares and the distances travelled.



(a) Copy the graph.

(b) Use the table below to plot the points on the graph.

Distance (km)	Cost (£)
2	1·50
3	2·50
2	1·75
5	3·25
5	3·50
6	4·00

(c) Does this graph show a strong negative or positive correlation ?

(d) Draw a best line of fit on your graph.

(e) Estimate how much a 4 kilometre journey would cost.

6. For each data set below, construct a scattergraph and show a best line of fit.

(a)

Age (years)	0	1	1	2	3	3	3	4	4	5	6	7	8	8	9	9	10	10
Car price (£1000)	10	9	8	8	7	6	5	5	4	2	3	3	3	2	2	1	2	1

(b)

Temp. (°C)	0	5	5	5	10	15	20	20	20	25	25	30	30	25	20
No. of People in the park	1	3	5	5	10	15	25	35	20	40	50	60	55	35	30

Adding and Subtracting Integers

Write down each question first, then the answer :-

1. (a) 6 + 9 (b) 2 + 11 (c) 0 + 23 (d) 10 + (-7)
   (e) 8 + (-2) (f) 7 + (-7) (g) 2 + (-6) (h) 3 + (-13)
   (i) 0 + (-20) (j) (-5) + 11 (k) (-6) + 6 (l) (-3) + 15
   (m) (-9) + 5 (n) (-11) + 4 (o) 1 + (-17) (p) (-8) + (-5)
   (q) (-9) + (-9) (r) (-13) + (-17) (s) (-15) + 7 (t) (-21) + (-19)
   (u) (-80) + 60 (v) (-35) + (-55) (w) 10 + (-45) (x) (-3·6) + (-2·4)

2. (a) 9 - 3 (b) 10 - 10 (c) 4 - 1 (d) 3 - 5
   (e) 5 - 10 (f) 2 - 12 (g) 0 - 15 (h) (-1) - 4
   (i) (-7) - 3 (j) (-11) - 5 (k) (-1) - 21 (l) 0 - 35
   (m) 19 - 39 (n) (-15) - 25 (o) 100 - 300 (p) (-71) - 29
   (q) 0 - 22 (r) (-10) - 10 (s) 6 - 22 (t) (-25) - 35
   (u) (-1) - 1 (v) (-63) - 27 (w) (-13) - 13 (x) (-2·5) - 3·5

3. (a) 2 + 8 (b) 3 + (-10) (c) 1 - 11 (d) (-5) + 15
   (e) -7 + (-8) (f) 6 - 14 (g) (-5) - 7 (h) (-40) + (-60)
   (i) (-20) + 35 (j) 0 - 27 (k) 0 + (-27) (l) (-18) + (-12)
   (m) 22 + (-15) (n) (-10) + 3 (o) (-41) + 41 (p) 45 - 75
   (q) (-27) + 14 (r) 0 + (-35) (s) (-101) + 99 (t) 19 + (-21)

Exercise 4

1. Copy and complete the following :-

   (a) 8 - (-3) = 8 + 3 = ...
   (b) 11 - (-9) = 11 + 9 = ...
   (c) 6 - (-5) = 6 + ... = ...
   (d) 30 - (-20) = 30 + ... = ...
   (e) 13 - (-6) = ... + ... = ...
   (f) 4 - (-4) = ... + ... = ...

2. Show your steps in finding the following :-

(a) 6 - (-9) (b) 12 - (-13) (c) 0 - (-11) (d) 4 - (-16)

(e) 15 - (-7) (f) 35 - (-15) (g) 7 - (-7) (h) 600 - (-400)

(i) 23 - (-37) (j) 6·5 - (-3·5) (k) 2·1 - (-3·2) (l) $\frac{1}{2} - (-\frac{1}{2})$

3. Copy and complete :- (*Remember to use a thermometer scale if it helps*)

(a) - 4 - (-6) = - 4 + 6 = ...

(b) (-2) - (-7) = -2 + 7 = ...

(c) (-10) - (-15) = -10 + ... = ...

(d) (-8) - (-12) = -8 + ... = ...

(e) (-40) - (-30) = ... + ... = ...

(f) (-5) - (-5) = ... + ... = ...

4. Show your steps in finding the following :-

(a) (-2) - (-6) (b) (-3) - (-9) (c) (-8) - (-11) (d) (-9) - (-6)

(e) (-1) - (-2) (f) (-13) - (-7) (g) (-14) - (-14) (h) (-50) - (-120)

(i) (-24) - (-4) (j) (-2·5) - (-4·5) (k) (-0·9) - (-0·4) (l) $(-\frac{1}{2}) - (-\frac{1}{2})$

5. The same idea works with algebraic expressions. Find :-

(a) $4x - (-3x)$ (b) $8x - (-10x)$ (c) $0 - (-5x)$ (d) $4a - (-9a)$

(e) $5p - (-8p)$ (f) $7w - (-13w)$ (g) $8g - (-12g)$ (h) $60f - (-20f)$

(i) $(-3m) - (-7m)$ (j) $(-9k) - (-4k)$ (k) $(-5n) - (-5n)$ (l) $(-b) - (-2b)$

(m) $(-6q) - (q)$ (n) $(-11z) -(-15z)$ (o) $(-6c) - (-12c)$ (p) $(-23g) - (-23g)$

6. A great big **MIXTURE**.

Find :-

(a) (-3) + 8 (b) (-4) - 6 (c) 2 - (-9) (d) (-11) + 15

(e) (-17) + 17 (f) 8 - 22 (g) 0 - 13 (h) (-7) + 17

(i) 8 - (-12) (j) (-3) - (-4) (k) 7 - (-7) (l) (-22) + 42

(m) $3x - (-4x)$ (n) $(-5p) + 11p$ (o) $10a - (-2a)$ (p) $(-3g) - 12g$

(q) $a - (-a)$ (r) $0 - (-5p)$ (s) 101 - (-99) (t) $65f - 95f$

(u) $2a^2 - 5a^2$ (v) $(-7t^2) + 15t^2$ (w) (-1000) + 3000 (x) $(-2\frac{1}{2}) - 3\frac{1}{2}$

Multiplication and Division of Integers

Exercise 5 (no calculator)

1. Write down each of the following and find the answers :-

(a) $4 \times (-5)$ (b) $6 \times (-7)$ (c) $2 \times (-9)$ (d) $5 \times (-5)$

(e) $(-8) \times 3$ (f) $(-9) \times 4$ (g) $(-11) \times 2$ (h) $(-10) \times 7$

(i) $6 \times (-8)$ (j) $8 \times (-3)$ (k) $4 \times (-12)$ (l) $7 \times (-7)$

(m) $9 \times (-1)$ (n) $(-9) \times 3$ (o) $(-2) \times 10$ (p) $(-9) \times 5$

2. Write down each of the following and find the answers :-

(a) $(-30) \div 6$ (b) $(-20) \div 5$ (c) $(-56) \div 7$ (d) $(-63) \div 9$

(e) $(-40) \div 2$ (f) $(-90) \div 10$ (g) $(-33) \div 3$ (h) $(-32) \div 4$

(i) $(-8) \div 8$ (j) $(-5) \div 1$ (k) $(-54) \div 6$ (l) $(-100) \div 5$

3. Find the answers to the following :-

(a) $(4 \times 9) \div 6$ (b) $(2 \times (-10)) \div 5$ (c) $3 \times (-2) \times 4$ (d) $5 \times (-1) \times 6$

(e) $3 \times (-8) \div 6$ (f) $(-6) \times 6 \div 4$ (g) $6 \times (-4) \div 2$ (h) $10 \times (-10) \div 5$

4. Find the following :- (*hint : find the bit in brackets first*)

(a) $(8 + (-5)) \times 7$ (b) $6 \times (4 - 7)$ (c) $((-10) + 2) \times 2$

(d) $((-4) - 8) \div 2$ (e) $10 \times (12 - 14)$ (f) $(8 - 3) \times (-5)$

(g) $((-3) - 4) \times 5$ (h) $(6 + (-12)) \div 3$ (i) $((-9) - 11) \div 5$

5. (a) What do you think the answer to $10 \div (-2)$ will be ? 5 or -5 ?

(b) If you think 5, check if $5 \times (-2)$ really takes you back to the original 10.

(c) If it doesn't, then the answer must be **-5** !

6. Write down each of the following and find the answers :-

(a) $20 \div (-5)$ (b) $24 \div (-6)$ (c) $18 \div (-9)$ (d) $25 \div (-5)$

(e) $36 \div (-4)$ (f) $40 \div (-8)$ (g) $7 \div (-1)$ (h) $42 \div (-3)$

(i) $96 \div (-8)$ (j) $100 \div (-5)$ (k) $120 \div (-6)$ (l) $49 \div (-7)$

(m) $1 \div (-1)$ (n) $7 \div (-2)$ (o) $30 \div (-4)$ (p) $3 \div (-6)$

7. Write down each of the following and find the answers :-

(a) $(-4) \times (-3)$ (b) $(-5) \times (-2)$ (c) $(-7) \times (-9)$ (d) $(-8) \times (-4)$

(e) $(-7) \times (-8)$ (f) $(-8) \times (-8)$ (g) $(-1) \times (-14)$ (h) $(-10) \times (-9)$

(i) $(-5) \times (-5)$ (j) $(-20) \times (-3)$ (k) $(-4) \times (-50)$ (l) $(-400) \times (-10)$

8. Find the answers to the following :-

(a) $(-20) \div (-5)$ (b) $(-18) \div (-3)$ (c) $(-32) \div (-4)$ (d) $(-22) \div (-2)$

(e) $(-36) \div (-9)$ (f) $(-40) \div (-8)$ (g) $(-54) \div (-6)$ (h) $(-80) \div (-4)$

(i) $(-84) \div (-7)$ (j) $(-120) \div (-6)$ (k) $(-200) \div (-10)$ (l) $(-168) \div (-3)$

9. (a) $(4 \times (-9)) \div 6$ (b) $((-2) \times (-10)) \div 5$ (c) $3 \times (-2) \times (-4)$

(d) $3 \times (-8) \div (-6)$ (e) $(-8) \times (-3) \div (-4)$ (f) $(-5) \times 6 \div (-2)$

(g) $(5 + (-8)) \times (-6)$ (h) $(-7) \times (3 - 9)$ (i) $((-10) + (-2)) \div (-3)$

10. (a) $(-2) \times (-3) \times (-4)$ (b) $(-3) \times (-4) \times (-5)$ (c) $(-4) \times (-5) \times (-6)$

(d) $(-3)^2$ (e) $(-5)^2$ (f) $(-10)^2$

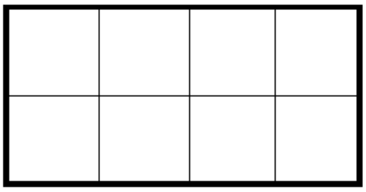
(g) $(-1)^2$ (h) $(-1)^3$ (i) $(-1)^4$

Enlargement and Reduction

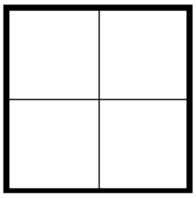
Exercise 1

1. Make a neat "two-times" enlargement of each of these shapes :- (each box = 1 cm long)

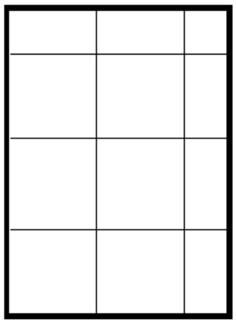
(a)



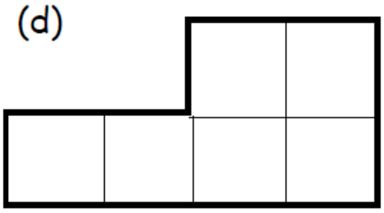
(b)



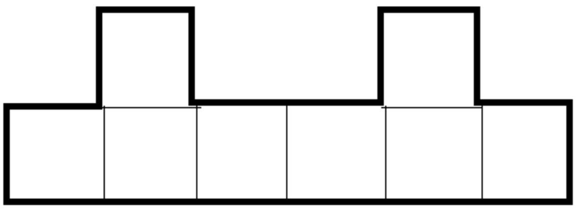
(c)



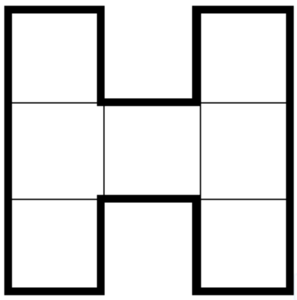
(d)



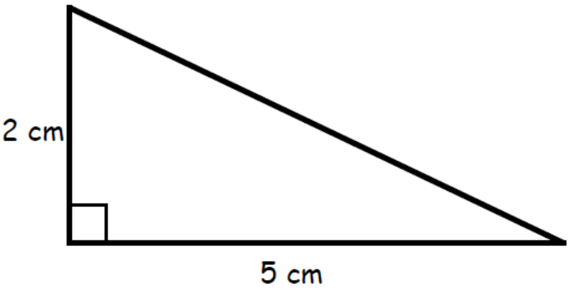
(e)



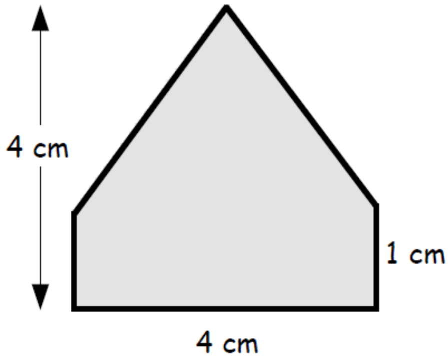
(f)



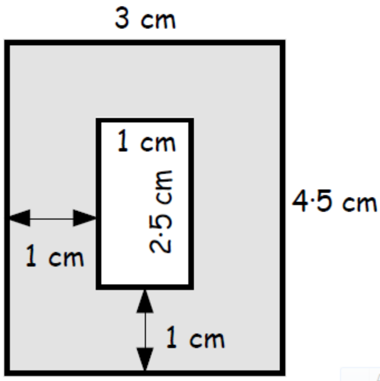
(g)



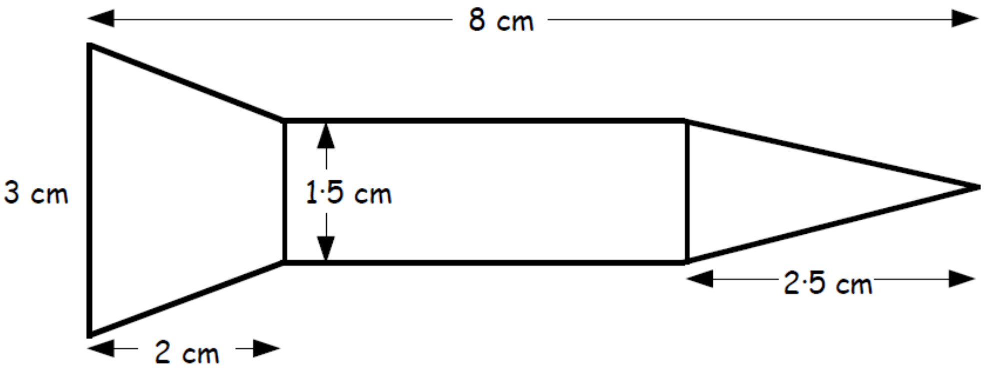
(h)



(i)

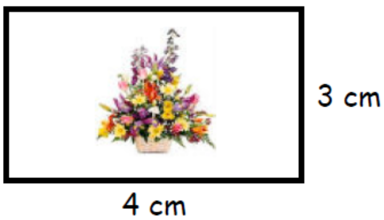


(j)



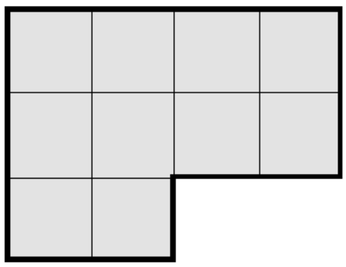
2. Make enlargements OR reductions of the following shapes using the given scales :-

(a)



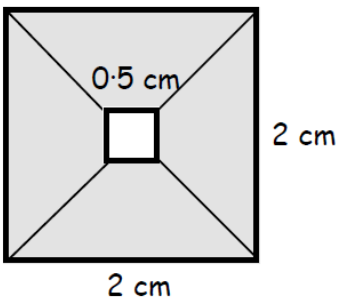
make a **three times** enlargement.

(b)



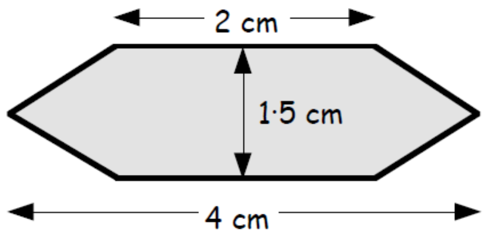
make a **four times** enlargement.

(c)



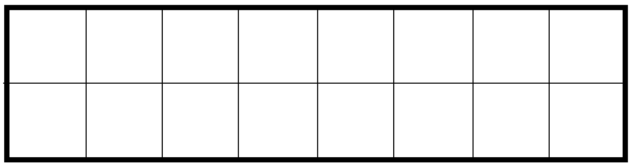
make a **six times** enlargement.

(d)



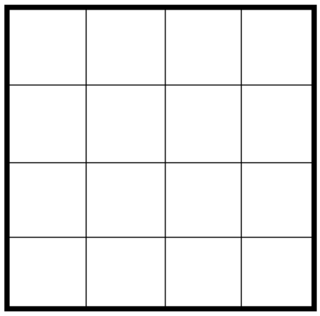
make a **four times** enlargement.

(e)



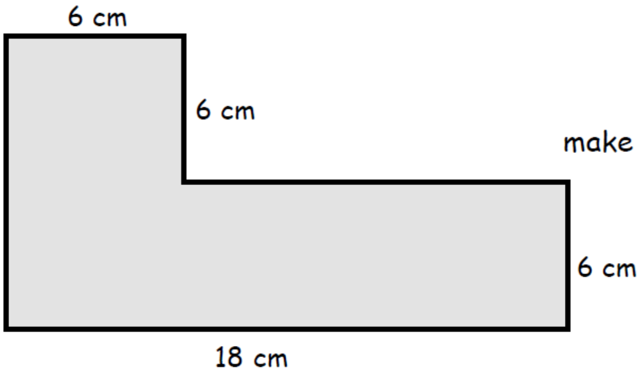
reduce this shape to **half** its size.

(f)



make this a **quarter** of its size here

(g)

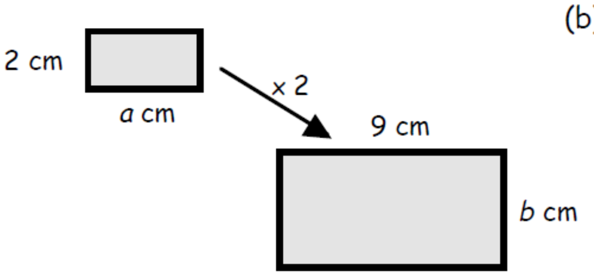


make this **one third** of its size.

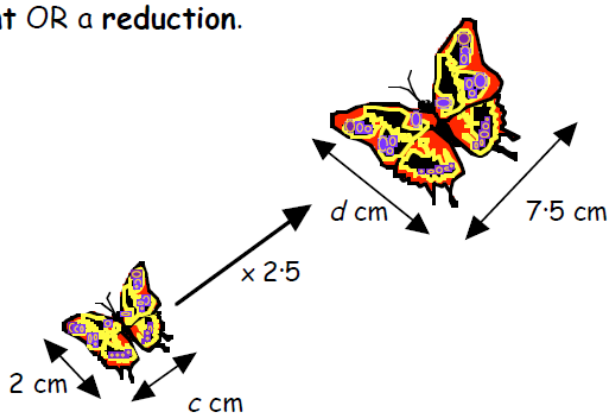
3. Each pair of pictures shows either an **enlargement** OR a **reduction**.

Calculate the unknown sizes. (Do not measure).

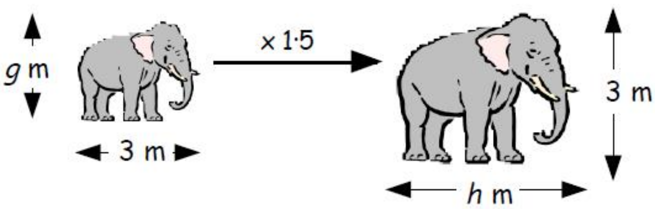
(a)



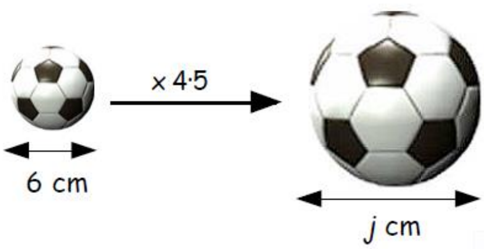
(b)



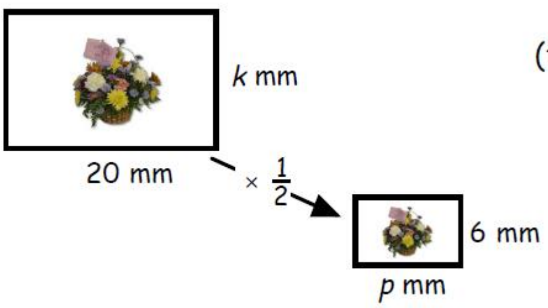
(c)



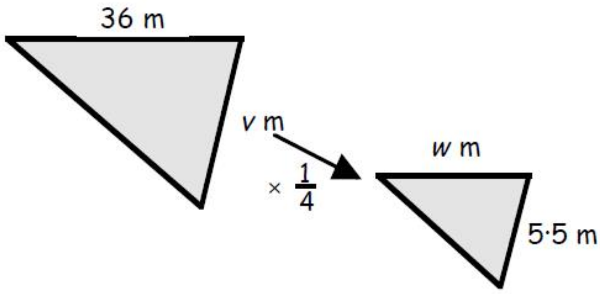
(d)



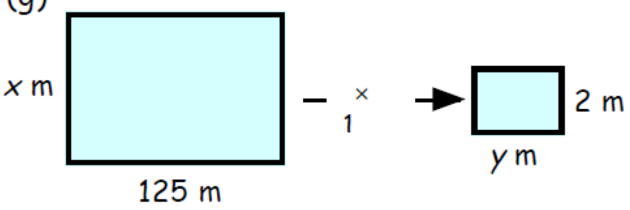
(e)



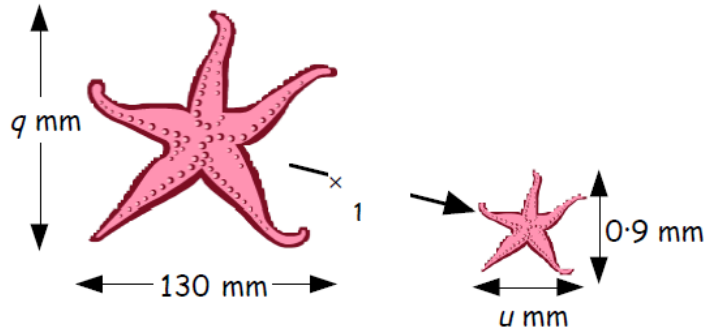
(f)



(g)



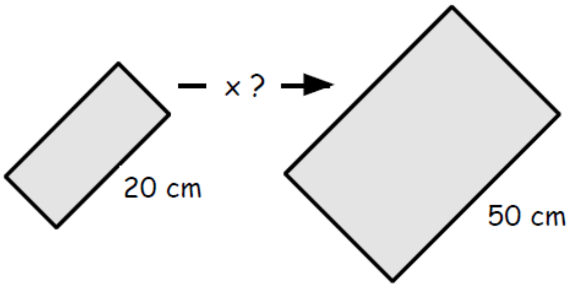
(h)



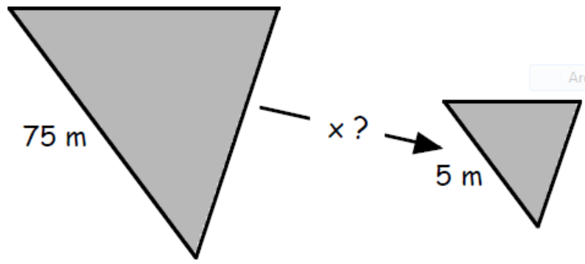
4. Each pair of pictures below shows either an **enlargement** OR a **reduction**.

In each case below, find the **enlargement factor** or the **reduction factor**.

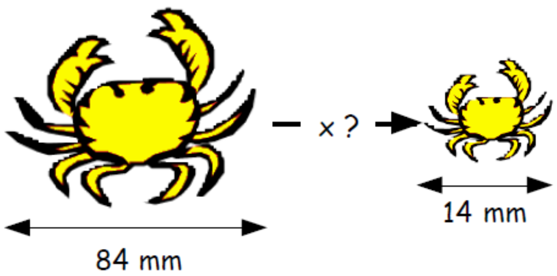
(a)



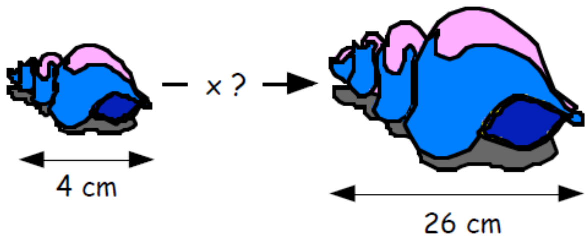
(b)



(c)



(d)



Scale, Reading Maps and Interpreting Distances

Exercise 4

1. Measure the length of each line (in cm) and calculate the length it really represents.

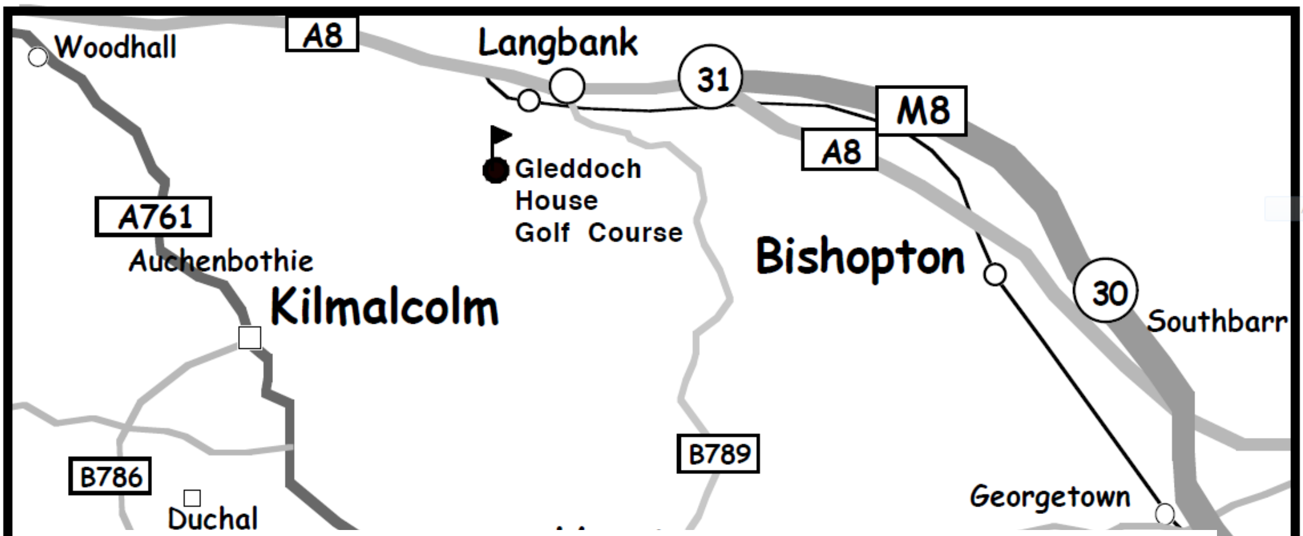
(a) scale 1 : 10

(b) scale 1 : 50

(c) scale 1 : 1000

(d) scale 1 : 50 000

2. The map below shows an area in the West of Scotland. Its scale is 1 : 50 000.



Use a ruler to measure and then write down the direct distance (in cm) from :-

(a) Kilmacolm to Woodhall.

(b) Houston to Gleddoch House Golf Course.

(c) Quarrier's Village to Bishopton.

(d) Woodhall to Georgetown.

3. Now calculate the real direct distance (in km) from :-

(a) Kilmacolm to Woodhall.

(b) Houston to Gleddoch House Golf Course.

(c) Quarrier's Village to Bishopton.

(d) Woodhall to Georgetown.

4. Jack is planning a skiing trip. On his map he measures that the distance from his home to the best ski slope is 18·3 cm.

If the scale of his map is 1 : 25 000, find how far away Jack lives from this ski slope.

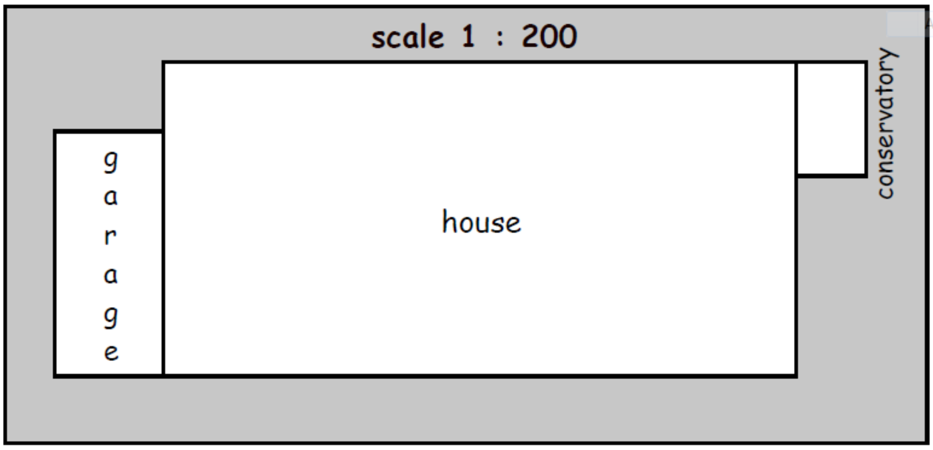
5. The distance from Golding to Beachhead is 10·7 cm on a map which has a scale of 1 : 20 000. Calculate the real distance between the two towns.

6. 

On an architect's plan the height of the lighthouse is measured as 8·5 cm.

If the scale of his plan is 1 : 500, find the real height of the lighthouse, in metres.

7. This is a plan of Jamie's house and garden.



(a) Measure and write down the dimensions (length and breadth) of :-

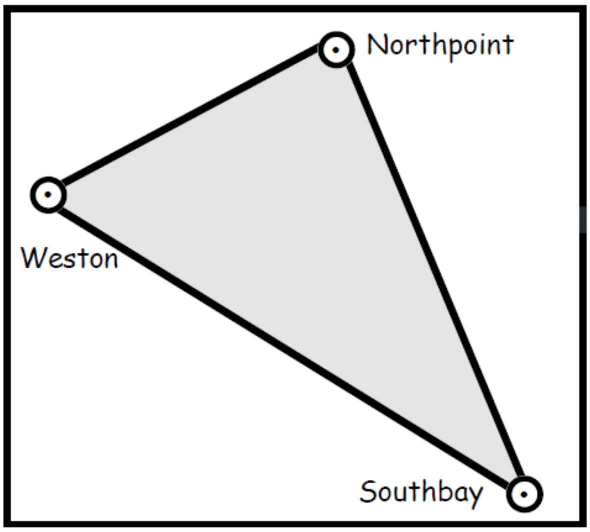
(i) the house,

(ii) the garage,

(iii) the conservatory.

(b) Calculate the real dimensions of the three buildings in metres.

8. This map shows the railway lines which link the 3 busiest towns on a holiday island. The actual distance from Northpoint to Southbay is 12 km.



(a) Measure and write down the distance from Northpoint to Southbay.

(b) Calculate the scale of the map.

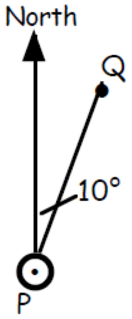
(c) Calculate the real distance from Southbay to Weston.

Bearings (Problems)

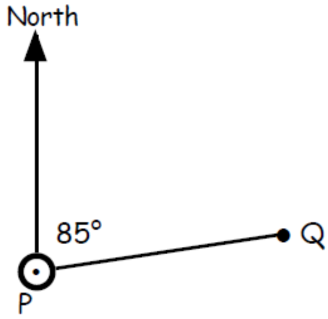
Exercise 5

1. Calculate (do not measure) the bearing of P **from** Q in the diagrams below

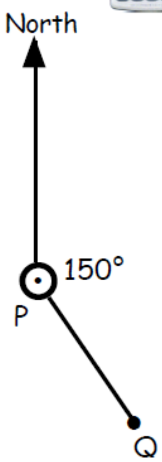
(a)



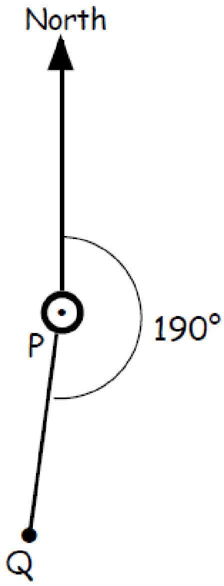
(b)



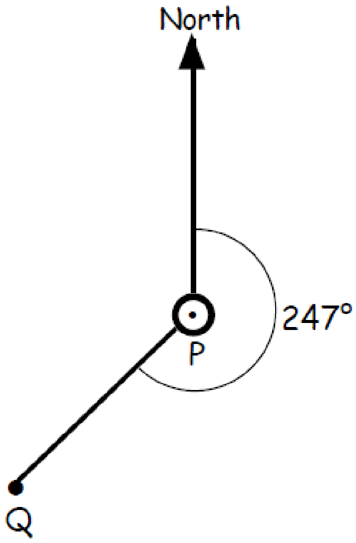
(c)



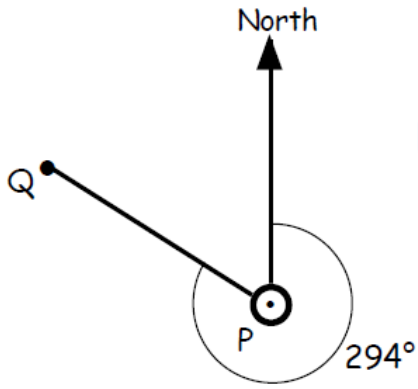
(d)



(e)



(f)



2. The scaled plan shows the position of three towns in central Dreamland.

The scale of the plan is 1 : 50 000.

(a) **Calculate** the **real** distances between the towns in kilometres.

(b) Use a protractor to measure the bearing of :-

(i) Elfton from Fairytown.

(ii) Twinkleton from Elfton.

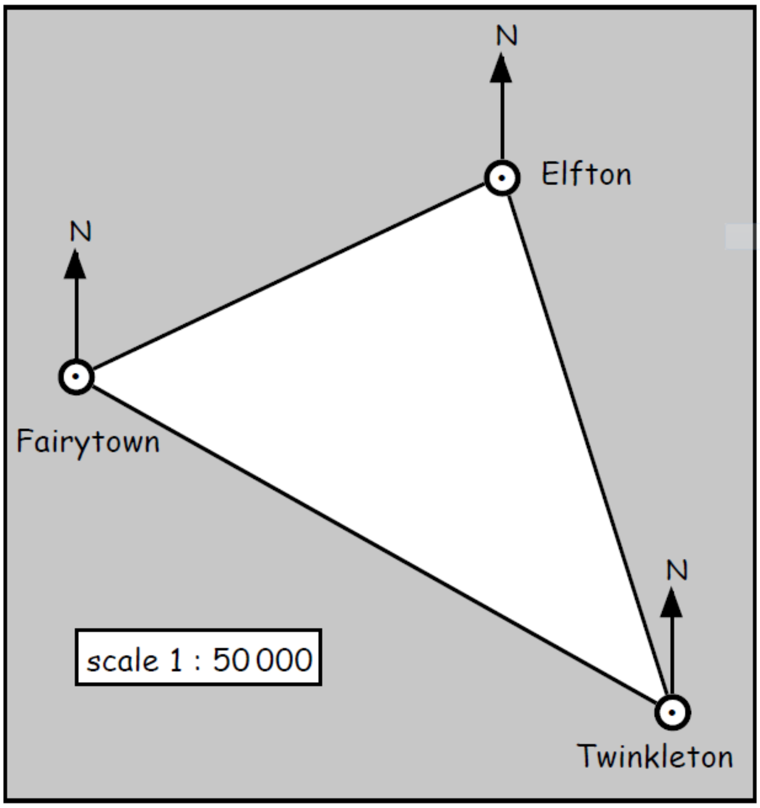
(iii) Twinkleton from Fairytown.

(c) **Calculate** the bearing of :-

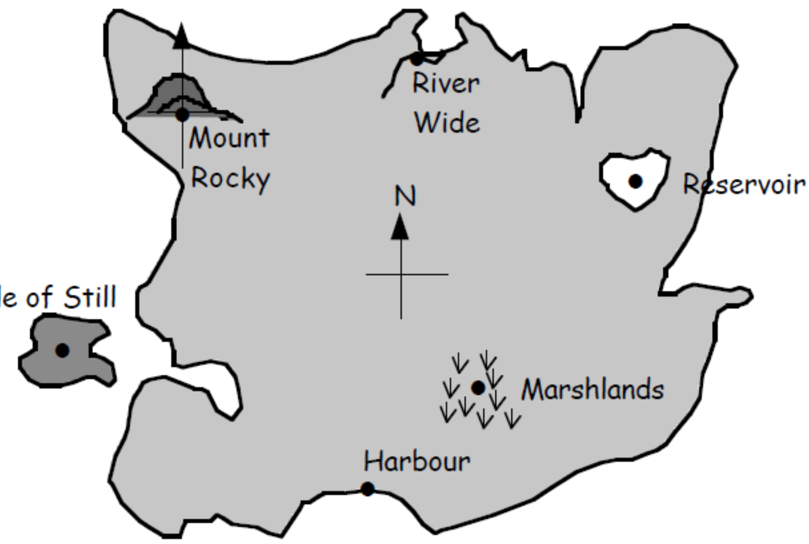
(i) Fairytown from Elfton.

(ii) Elfton from Twinkleton.

(iii) Fairytown from Twinkleton.



3.



scale 1 : 15 000

(a) Find the **real** distance in km from :-

(i) River Wide to the Harbour.

(ii) Isle of Still to the Reservoir.

(b) How far is it from Mount Rocky to the Marshlands ?

(c) Now measure the bearing from Mount Rocky to the Marshlands.

(d) **Calculate** the bearing from the Marshlands to Mount Rocky.

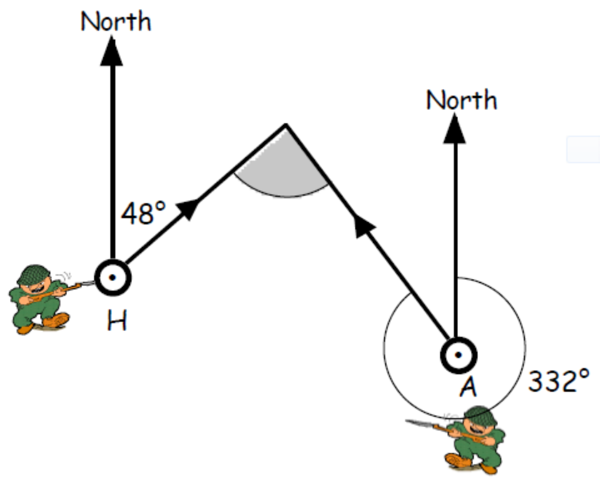
4. As part of a military training exercise, two teams of cadets are marching to a rendezvous point.

The Highlanders (H) are travelling on a bearing of 048°.

The Argyllans (A) are on a course of 332°.

At what angle (shaded) will their courses meet ?

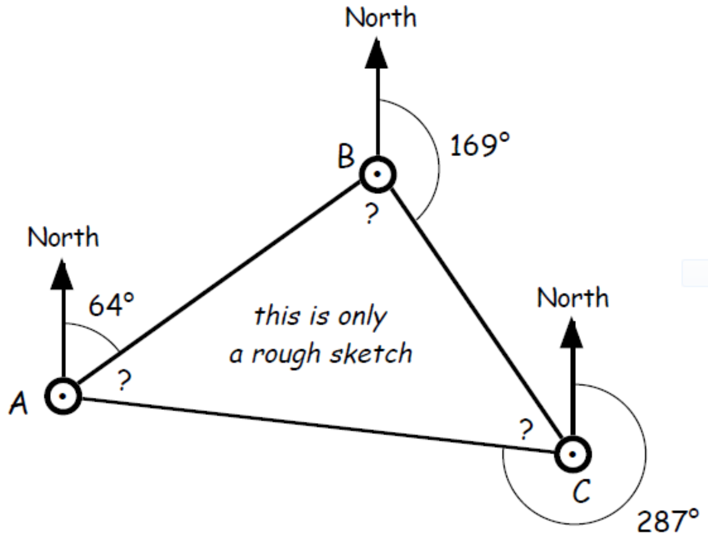
Calculate - do NOT measure.



5. The vertices of triangle ABC are shown, together with bearings from A to B, B to C and C back to A.

Calculate the sizes of :-

$\angle ABC$, $\angle ACB$ and $\angle BAC$.

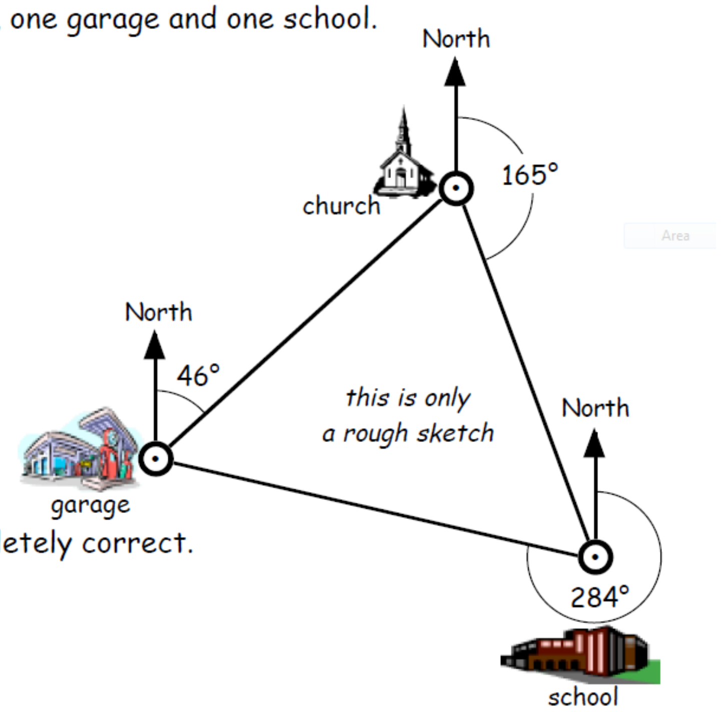


6. The small village of Adensport has one church, one garage and one school.

On a map of the village, the bearing of :-

- the church from the garage is 046°.
- the school from the church is 165°.
- the garage from the school is 284°.

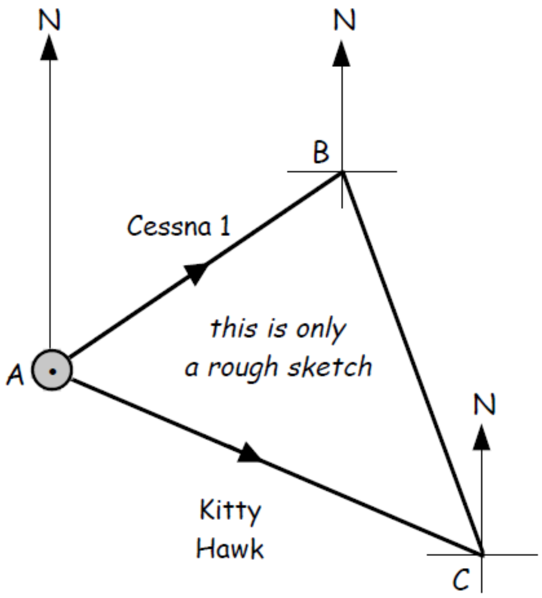
A visitor to the village looks at the map and thinks that the distance from the garage to the church looks the same as the distance from the garage to the school.



Prove that in fact, the visitor's idea is completely correct.

Calculations, no measuring !

7.



Two aeroplanes leave an airport (A) at the same time.

The Cessna 1 flies on a bearing 068° to B.

The Kitty Hawk flies on a bearing 115° to C.

From B, the bearing of the Kitty Hawk is 160°.

Make a neat **sketch** of the journeys.

Calculate, and mark on your sketch :-

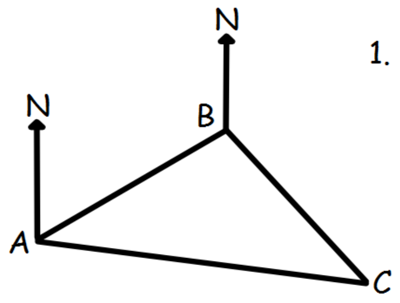
(a) the bearing of A from B.

(b) the bearing of B from C.

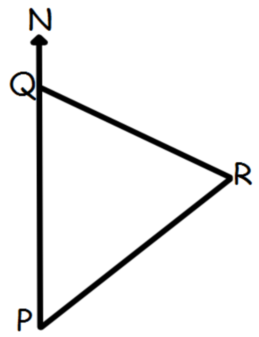
(c) the bearing of A from C.

Bearings - Worksheet 1 – Annotating Diagrams

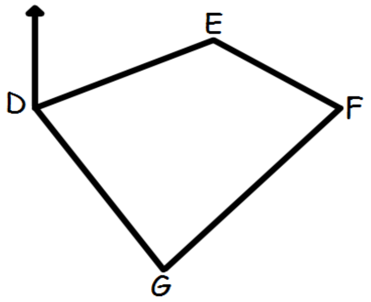
Copy the diagrams below and annotate them with the information given



1. B is on a bearing of 55° from A
   C is on a bearing of 98° from A
   A in on a bearing of 240° from B
   A is on a bearing of 280° from C

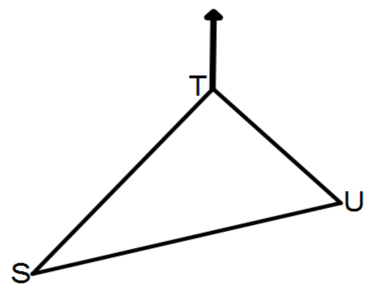


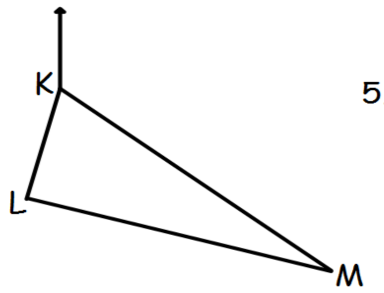
2. From Q the bearing of R is 120°
   From R the bearing of Q is 295°
   From P the bearing of R is 50°



3. E is on a bearing of 65° from D
   G is on a bearing of 225° from F
   G in on a bearing of 145° from D

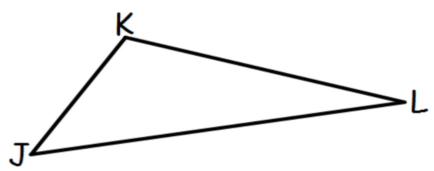
4. From S the bearing of U is 75°
   From T the bearing of S is 220°
   From U the bearing of S is 255°



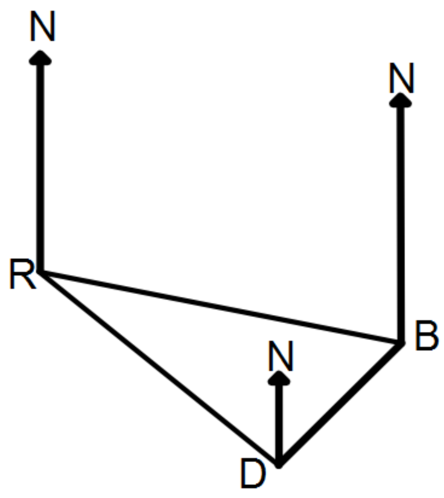


5. M is on a bearing of 130° from K
   K is on a bearing of 310° from M
   K in on a bearing of 15° from L

6. From J the bearing of K is 35°
   From J the bearing of L is 80°
   From K the bearing of L is 105°
   From L the bearing of K is 285°
   From L the bearing of J is 260°



Bearings - Worksheet 2 – Calculating Bearings



Example 1

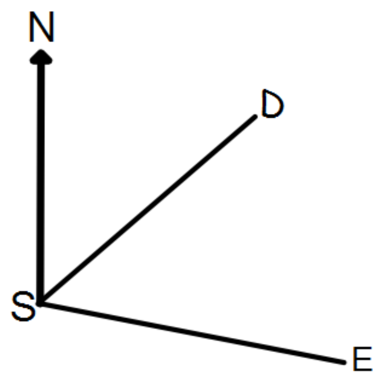
B is on a bearing of 100° from R
D is on a bearing of 225° from B
D is on a bearing of 130° from R

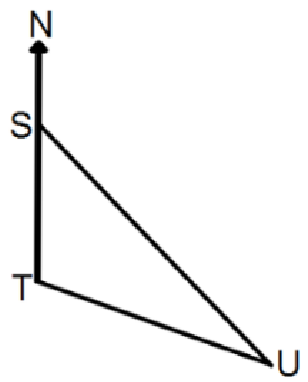
(a) Calculate the size of angle BDR
(b) Write down the bearing of R from D

Example 2

D is on a bearing of 50° from S
E is on a bearing of 102° from S
D is on a bearing of 340° from E

(a) Calculate the bearing of E from D
(b) Calculate the size of angle SDE





Example 3

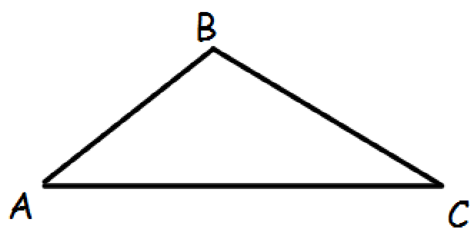
U is on a bearing of 110° from T
U is on a bearing of 136° from S

(a) Calculate the bearing of S from U
(b) Calculate the size of angle SUT

Example 4

C is due East from A
C is on a bearing of 138° from B
A is on a bearing of 230° from B

(a) Calculate the bearing of B from A
(b) Calculate the size of angle ACB



Tolerance

Metric Measure - Revision of Units

Express:

1.	in cm:	a) 1m 52cm	b) 1m 47cm	c) 5m 9cm
		d) 6cm 8mm	e) 25cm 2mm	f) 19cm 9mm
2.	in m:	a) 6m 48cm	b) 40m 50cm	c) 17m 8cm
3.	in km:	a) 5km 283m	b) 10km 35m	c) 1km 1m
4.	in kg	a) 2kg 486g	b) 5kg 48g	c) 5kg 5g
5.	in litres	a) 3L 673ml	b) 2L 30ml	c) 20L 2ml
6.	in g	a) 2.675kg	b) 0.3kg	c) 0.06kg

7. 3 flasks contain 2L 38ml, 12L 107ml, and 9L 8ml of liquid.

   a) How many ml of liquid are there altogether?

   b) Express your answer in litres.

8. A railway truck can carry 15 500kg.

a) How many tonnes is this? (1 tonne = 1000kg)
b) How many tonnes can a train with 30 trucks take?

9. The distance a pupil has to walk to school is 1km 86m.

How many metres will the pupil walk going to and from school for a school week?

10. Find the area of a rectangle (in mm^2) with a length 5cm 4mm and breadth 3cm 2mm.

11. A rectangular field measures 260m by 180m.

Calculate its area in hectares. (1 hectare = $10000m^2$)

Tolerance

1. Give the greatest and least acceptable measurements for the following:

a) (12 ±1)g b) (76 ±2)m

c) (4·3 ±0·1)cm d) (6·3 ±0·5)s

e) (48 ±0·35)kg f) (1·4 ±0.05)cm

2. A box of drawing pins contains 50 ±5 pins.

What is the minimum number of pins acceptable in any box?

3. A set of fitness weights are manufactured with a tolerance of ±80g.

Write down the maximum and minimum possible weights of a 5kg dumbbell. (**Answer in g)**

4. A company manufactures PVC windows to a tolerance of ±2mm.

A window is ordered measuring 90cm by 60cm.

Write down the dimensions of the largest possible window that would be acceptable. (**Answer in mm)**

5. The actual weight of corn flakes in a box is claimed to be ±5% of what is labelled on the box.

A 500g box has its contents weighed and the actual weight of corn flakes is 469g.

Is this acceptable according to the tolerance given? **Explain your answer.**

6. Pieces of tubing are required with lengths given by (6 ±0·2)cm.

Which of the following will be accepted, and which rejected?

a) 6·3cm	b) 5·6cm	c) 6·09cm
d) 5·82cm	e) 5·98cm	f) 6·18cm

7. A GPS sports watch measures the distance covered by a runner. It is accurate to ±50m. According to the watch, a runner covers 1·75km in 6 minutes.

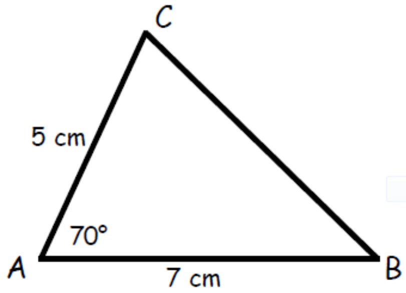
What is the fastest possible speed of the runner?

Drawing Triangles

Exercise 2

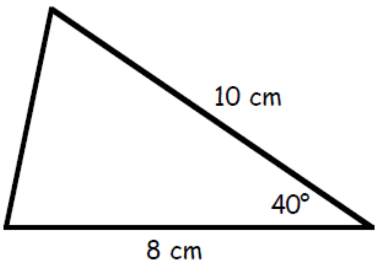
1. On the right is a sketch of ΔABC.
Follow the instructions to draw it accurately :-

Step 1 :-	Draw line AB = 7 cm
Step 2 :-	Put your protractor at A and mark (with an X) an angle of 70°.
Step 3 :-	Draw line AC, from A through the X, to point C. (*Make sure it is 5 centimetres long*).
Step 4 :-	Join C to B to complete the triangle.

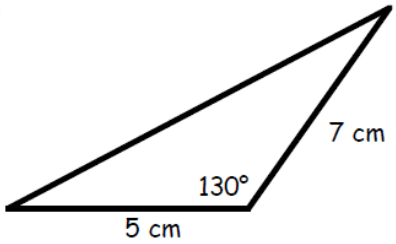


2. Make accurate drawings of the following triangles :-

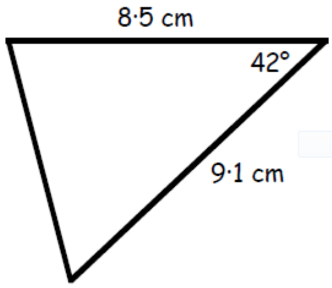
(a)



(b)



(c)



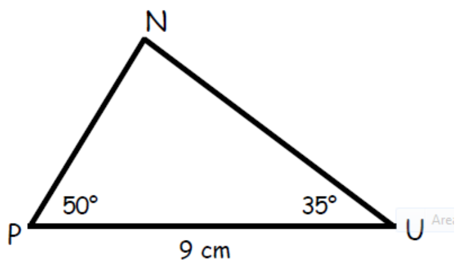
3. Make accurate drawings of the following triangles :-
(*Make rough sketches of the triangles first before drawing them accurately*).

(a) Draw ΔPQR where PQ = 11 cm, QR = 9 cm and $\angle PQR$ = 60°.

(b) Draw ΔTAN where AN = 12 cm, AT = 7·5 cm and $\angle TAN$ = 110°.

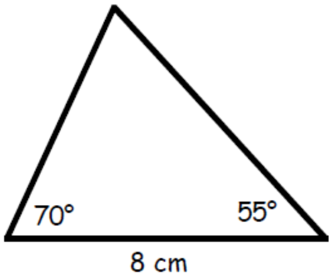
4. Shown is a rough sketch of ΔPUN.
Follow the instructions to draw it accurately :-

Step 1 :-	Draw line PU = 9 cm
Step 2 :-	Put your protractor at P and mark (with an X) an angle of 50°.
Step 3 :-	Draw a line from P through the X.
Step 4 :-	Put your protractor at U and mark (with an X) an angle of 35°.
Step 5 :-	Draw a line from U through the X, to meet your first line at point N.

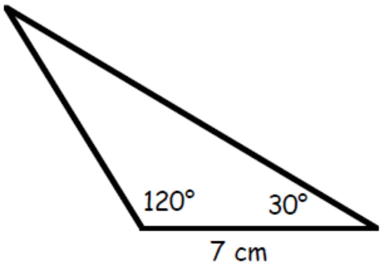


5. Make accurate drawings of the following triangles :-

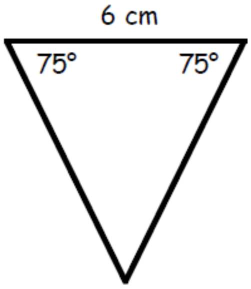
(a)



(b)



(c)

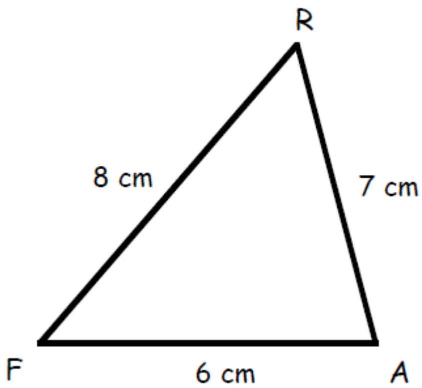


6. Make accurate drawings of the following triangles :-
(*Make rough sketches of the triangles first before drawing them accurately*).

(a) Draw ΔABC where AB = 10 cm, ∠CAB = 50° and ∠CBA = 65°.

(b) Draw ΔRYT where RY = 5 cm, ∠TRY = 35° and ∠TYR = 125°.

7.

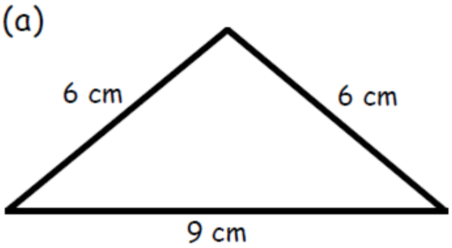


Shown is a sketch of ΔFAR.
Draw it accurately using the following instructions :-

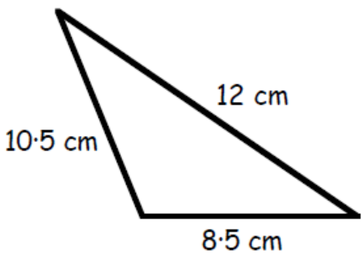
Step 1 :-	Draw line FA = 6 cm
Step 2 :-	Set your compasses to 8 cm, place the compass point on F and draw a light arc.
Step 3 :-	Now set your compasses to 7 cm, place the compass point on A and draw a 2nd arc.
Step 4 :-	Call this point where the arcs meet R and join R to F and to A.

8. Make accurate drawings of the following triangles :-

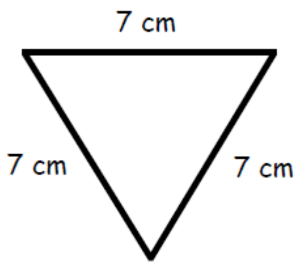
(a)



(b)



(c)

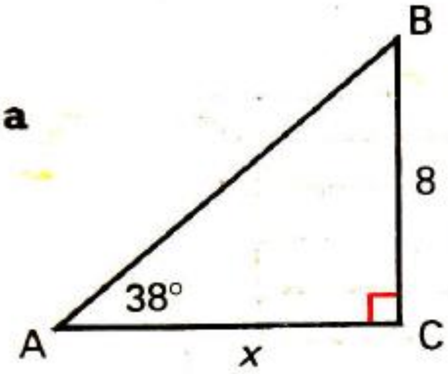


TRIGONOMETRY Finding the Adjacent

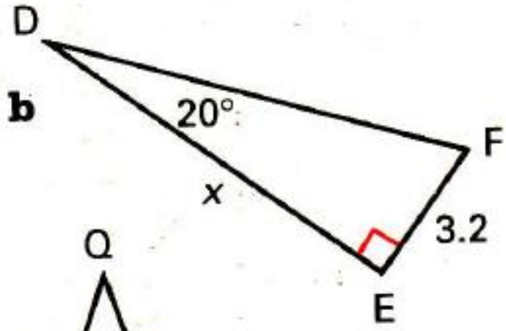
Give all your answers to 1 decimal place

1 Calculate x in each triangle.

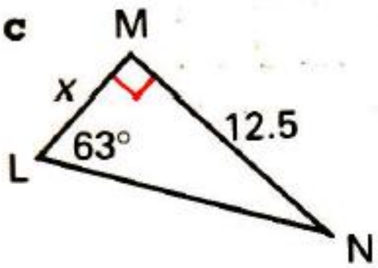
a



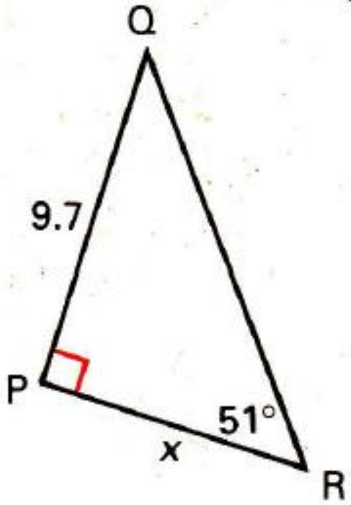
b



c

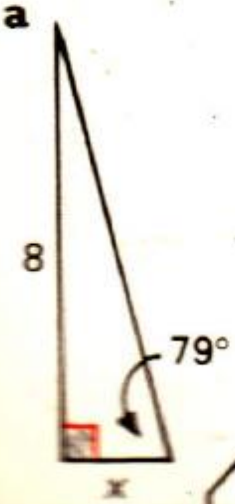


d

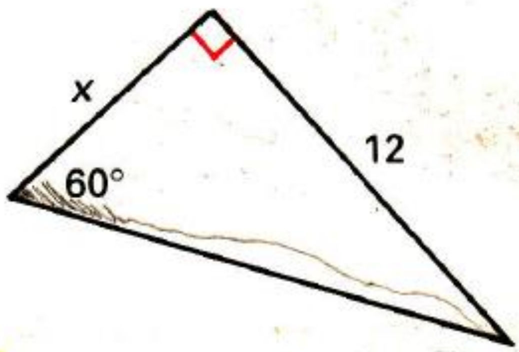


2 Calculate x in each diagram.

a

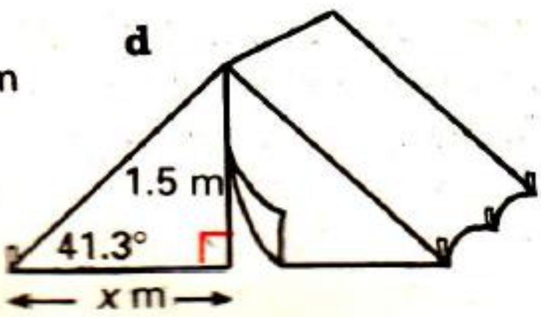


b



c

d



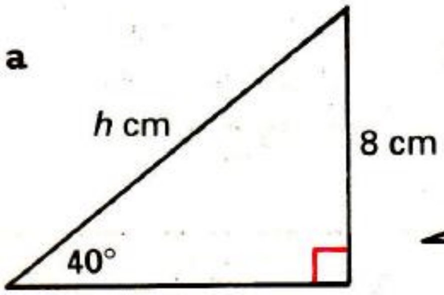
TRIGONOMETRY

Finding the Hypotenuse

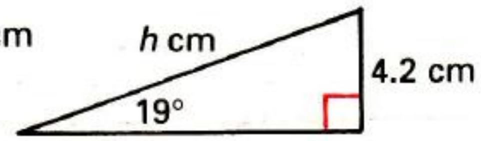
EXERCISE 6B

Calculate h, correct to 1 decimal place, in each triangle.

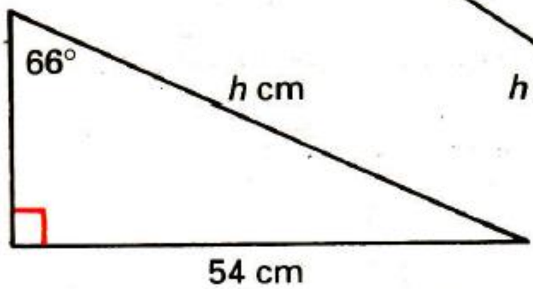
a



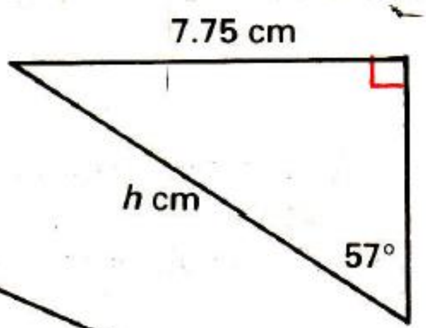
b



c

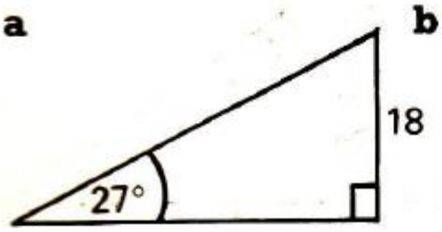


d

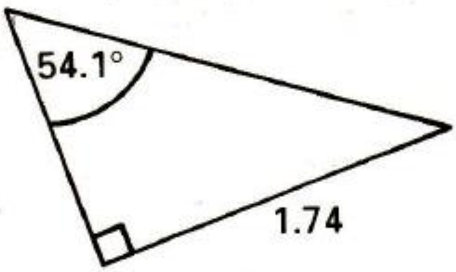


2 Calculate the length of the hypotenuse of each right-angled triangle.

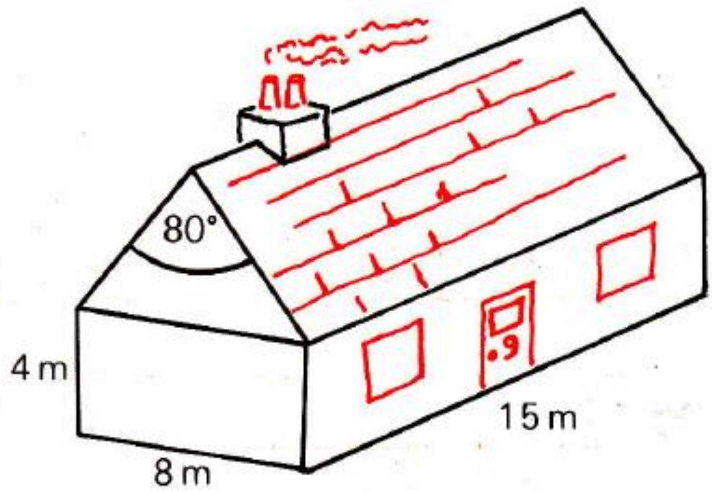
a



b



3 Number 9 Tay View is a small bungalow.

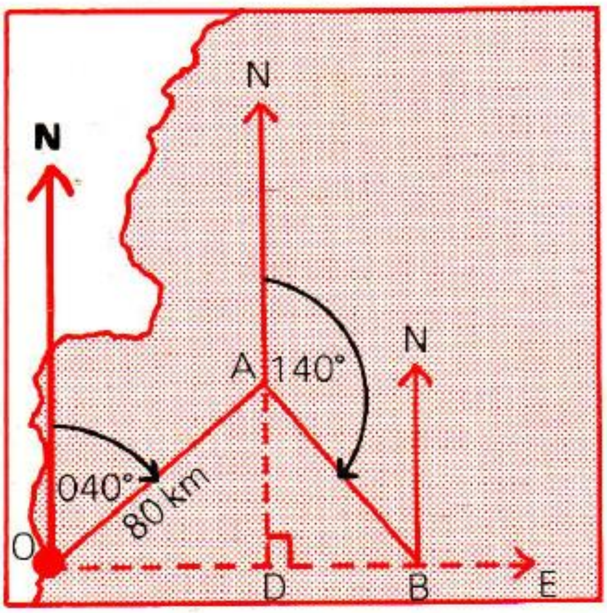


Calculate, to the nearest centimetre:

a the length of the sloping edge of the roof

b the height of the roof ridge above the ground.

4 The *Mary Anne* sets course from her harbour, O, on a bearing of 040°, and sails for 80 km. She then changes course to 140° and sails until she is due east of the harbour.



a What is the furthest north the *Mary Anne* goes from the harbour?

b What is the length of her journey from A to B?

c How far is she from the harbour at B?

The Golden Ratio

Introduction

What is the Golden Ratio?

Well, before we answer that question let's examine an interesting sequence (or list) of numbers. We'll start with the numbers 1 and 1. To get the next number we add the previous two numbers together. So now our sequence becomes 1, 1, 2. The next number will be 3. What do you think the next number in the sequence will be?

If you said 4, then unfortunately you are incorrect. Remember, we add the previous two numbers to get the next. So the next number should be 2+3, or 5.

Here is what our sequence should look like if we continue on in this fashion for a while:

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, . . .

Now, I know what you might be thinking: "What does this have to do with the Golden Ratio?" The answer is forthcoming. This sequence of numbers was first discovered by a man named Leonardo Fibonacci, and hence is known as Fibonacci's sequence. The relationship of this sequence to the Golden Ratio lies not in the actual numbers of the sequence, but in the ratio of the consecutive numbers. Let's look at some of these ratios:

$$2/1 = 2\cdot0$$

$$3/2 = 1\cdot5$$

$$5/3 = 1\cdot67$$

$$8/5 = 1\cdot6$$

$$13/8 = 1\cdot625$$

$$21/13 = 1\cdot615$$

$$34/21 = 1\cdot619$$

$$55/34 = 1\cdot618$$

$$89/55 = 1\cdot618$$

Aha! Notice that as we continue down the sequence, the ratios seem to be converging upon one number (from both sides of the number)! Notice that I have rounded my ratios to the third decimal place. If we examine 55/34 and 89/55 more closely, we will see that their decimal values are actually not the same. But what do you think will happen if we continue to look at the ratios as the numbers in the sequence get larger and larger? That's right: the ratio will eventually become the same number, and that number is the Golden Ratio! The Golden Ratio is what we call an irrational number: it has an infinite number of decimal places and it never repeats itself! Generally, we round the Golden Ratio to 1·618. We work with another important irrational number in Geometry: pi, which is approximately 3·14. Since we don't want to make the Golden Ratio feel left out, we will give it its own Greek letter: phi. One more interesting thing about Phi is its reciprocal. If you take the ratio of any number in the Fibonacci sequence to the next number (this is the reverse of what we did before), the ratio will approach the approximation 0·618. This is the reciprocal of Phi: 1 / 1·618 = 0·618. It is highly unusual for the decimal integers of a number and its reciprocal to be exactly the same. In fact, I cannot name another number that has this property! This only adds to the mystique of the Golden Ratio and leads us to ask: What makes it so special?

NOTE: WHEN MEASURING, A DEGREE OF ERROR MUST BE AGREED. TIME PERMITTING, AN EXPLANATION OF WHY ERRORS OCCUR COULD BE DISCUSSED

1·5< phi < 1·8 IS A SUGGESTION.

ACTIVITIES

Activity 1

The Golden Ratio in Everyday Objects

You will need some measuring tools in this activity. Rulers, tape measures etc.

The Golden Ratio is not just some number that maths teachers think is cool. The interesting thing is that it keeps popping up in strange places - places that we may not ordinarily have thought to look for it. It is important to note that Fibonacci did not "invent" the Golden Ratio; he just discovered one instance of where it appeared naturally. In fact civilizations as far back and as far apart as the Ancient Egyptians, the Mayans, as well as the Greeks discovered the Golden Ratio and incorporated it into their own art, architecture, and designs. They discovered that the Golden Ratio seems to be Nature's perfect number. For some reason, it just seems to appeal to our natural instincts. The most basic example is in rectangular objects. Look at the following rectangles:

Now ask yourself, which of them seems to be the most naturally attractive rectangle? If you said the first one, then you are probably the type of person who likes everything to be symmetrical. Most people tend to think that the third rectangle is the most appealing. Measure each rectangle's length and width, and compare the ratio of length to width for each rectangle.

Have you figured out why the third rectangle is the most appealing? That's right - because the ratio of its length to its width is the Golden Ratio! For centuries, designers of art and architecture have recognized the significance of the Golden Ratio in their work. We will learn more about that later. For right now, let's see if we can discover where the Golden Ratio appears in everyday objects. Use your measuring tool to compare the length and the width of rectangular objects in the classroom or in your house (depending on where you are right now). Try to choose objects that are meant to be visually appealing. Some suggestions are listed below, but by all means add to the list.

Object	Length	Width	Ratio
Picture			
Doorway			
TV Screen			
Textbook			

Activity 2/3

The Golden Ratio in Architecture/Art

Pupils should research this at home on the internet. The following is only a guide.

Spend some time the following period discussing the pupils' findings.

As you have probably guessed by now, the Golden Ratio appears in architecture as well. This practice goes back for centuries and into many countries. Unfortunately, we cannot travel all over the world to search for the Golden Ratio in architecture.

The Golden Ratio and Art

Now let's go back and try to discover the Golden Ratio in art. We will concentrate on the works of Leonardo da Vinci, as he was not only a great artist but also a genius when it came to mathematics and invention. Your task is to find at least one of the following da Vinci paintings on the Internet. Make sure that you find the entire painting and not just part of it. The best way to do this is to use a search engine.

List of paintings to look for:

The Annunciation

Madonna with Child and Saints

The Mona Lisa

St. Jerome

If you are having difficulty finding the images, try a search using the words "da Vinci" and "art gallery" together.

Directions for finding evidence of the Golden Ratio in each painting:

(This has proven to be quite a difficult task. Perhaps it is something to discuss with the pupils.)

***The Annunciation* -** Using the left side of the painting as a side, create a square on the left of the painting by inserting a vertical line. Notice that you have created a square and a rectangle. The rectangle turns out to be a Golden

Rectangle, of course. Also, draw in a horizontal line that is 61.8% of the way down the painting (.618 - the inverse of the Golden Ratio). Draw another line that is 61·8% of the way up the painting. Try again with vertical lines that are 61·8% of the way across both from left to right and from right to left. You should now have four lines drawn across the painting. Notice that these lines intersect important parts of the painting, such as the angel, the woman, etc. Coincidence? I think not!

***Madonna with Child and Saints* -** Draw in the four lines that are 61·8% of the way from each edge of the painting. These lines should mark off important parts of the painting, such as the angels and the baby Jesus in the center.

***The Mona Lisa* -** Measure the length and the width of the painting itself. The ratio is, of course, Golden. Draw a rectangle around Mona's face (from the top of the forehead to the base of the chin, and from left cheek to right cheek) and notice that this, too, is a Golden rectangle.

***St. Jerome* -** Draw a rectangle around St. Jerome. Conveniently, he just fits inside a Golden rectangle.

Conclusions - Leonardo da Vinci's talent as an artist may well have been outweighed by his talents as a mathematician. He incorporated geometry into many of his paintings, with the Golden Ratio being just one of his many mathematical tools. Why do you think he used it so much? Experts agree that he probably thought that Golden measurements made his paintings more attractive. Maybe he was just a little too obsessed with perfection. However, he was not the only one to use Golden properties in his work.

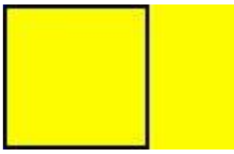
Activity 4

Constructing a Golden Rectangle – Method 1

Isn't it strange that the Golden Ratio came up in such unexpected places? Well let's see if we can find out why. The Greeks were the first to call phi the Golden Ratio. They associated the number with perfection. It seems to be part of human nature or instinct for us to find things that contain the Golden Ratio naturally attractive - such as the "perfect" rectangle. Realizing this, designers have tried to incorporate the Golden Ratio into their designs so as to make them more pleasing to the eye. Doors, notebook paper, textbooks, etc. all seem more attractive if their sides have a ratio close to phi. Now, let's see if we can construct our own "perfect" rectangle.

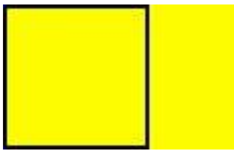
You will need a piece of paper, a pencil, and a protractor to complete this activity.

We'll start by making a square, any square (just remember that all sides have to have the same length, and all angles have to measure 90 degrees!):



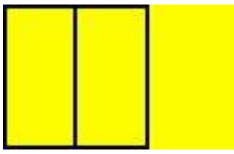
Please note that everyone will have different size squares to begin with.

Now, let's divide the square in half (bisect it). Be sure to use your protractor to divide the base and to form another 90 degree angle:



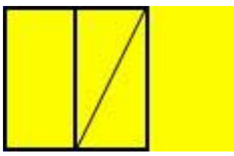
Notice that we have made two rectangles.

Now, draw in one of the diagonals of one of the rectangles:

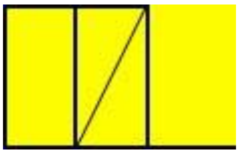


Measure the length of the diagonal and make a note of it.

Now extend the base of the square from the midpoint of the base by a distance equal to the length of the diagonal (the length of the diagonal should be equal to the distance from the midpoint of the OLD base to the edge of your NEW base):



Construct a new line perpendicular to the base at the end of our new line, and then connect to form a rectangle:



Measure the length and the width of your rectangle.

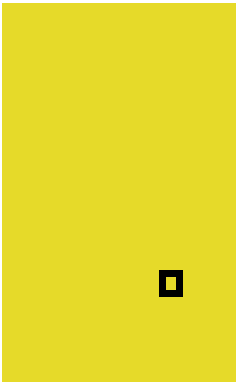
Now, find the ratio of the length to the width.

Are you surprised by the result? The rectangle you have made is called a Golden Rectangle because it is "perfectly" proportional.

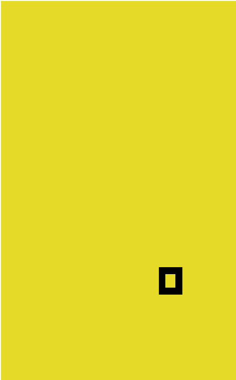
Activity 5

Constructing a Golden Rectangle – Method 2

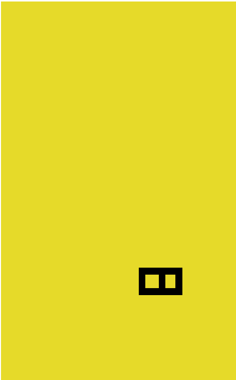
Now, let's try a different method that will relate the rectangle to the Fibonacci series we looked at. We'll start with a square. The size does not matter, as long as all sides are congruent. We'll use a small square to conserve space, because we are going to build our golden rectangle around this square. Again, please note that the golden area is what your rectangle will eventually look like.



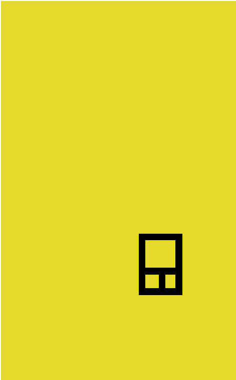
Let's call this square a unit square, and say that it has a side of length 1. Now, let's build another, congruent square right next to the first one:



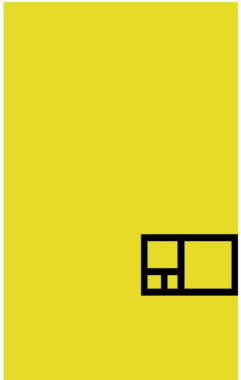
Now we have a rectangle with width 1 and length 2 units. Let's build a square on top of this rectangle, so that the new square will have a side of 2 units:



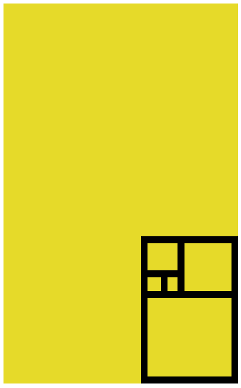
Notice that we have a new rectangle with width 2 and length 3. Let's continue the process, building another square on the right of our rectangle. This square will have a side of 3:



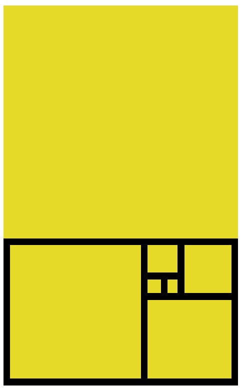
Now we have a rectangle of width 3 and length 5. Again, let's build upon this rectangle and construct a square underneath, with a side of 5:



The new rectangle has a width of 5 and a length of 8. Let's continue to the left with a square with side 8:



Have you noticed the pattern yet? The new rectangle has a width of 8 and a length of 13. Let's continue with one final square on top, with a side of 13:

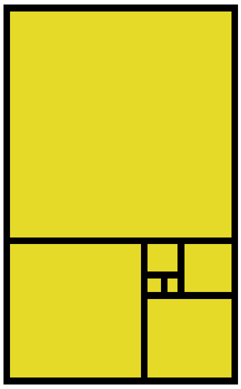


Our final rectangle has a width of 13 and a length of 21. Notice that we have constructed our golden rectangle using square that had successive side lengths from the Fibonacci sequence (1, 1, 2, 3, 5, 8, 13, ...)! No wonder our rectangle is golden! Each successive rectangle that we constructed had a width and length that were consecutive terms in the Fibonacci sequence. So if we divide the length by the width, we will arrive at the Golden Ratio! Of course, our rectangle is not "perfectly" golden. We could keep the process going until the sides approximated the ratio better, but for our purposes a length of 21 and a width of 13 are sufficient. Let's take one last look at our rectangle:

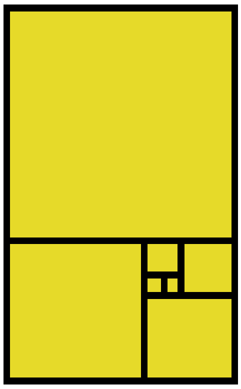
Activity 6

Constructing a Golden Spiral

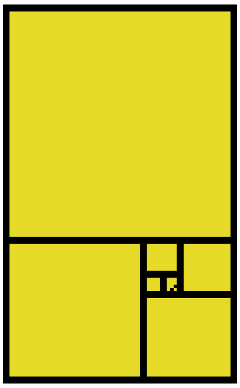
Notice how we built our rectangle in a counterclockwise direction. This leads us into another interesting characteristic of the Golden Ratio. Let's look at the rectangle with all of our construction lines drawn in:



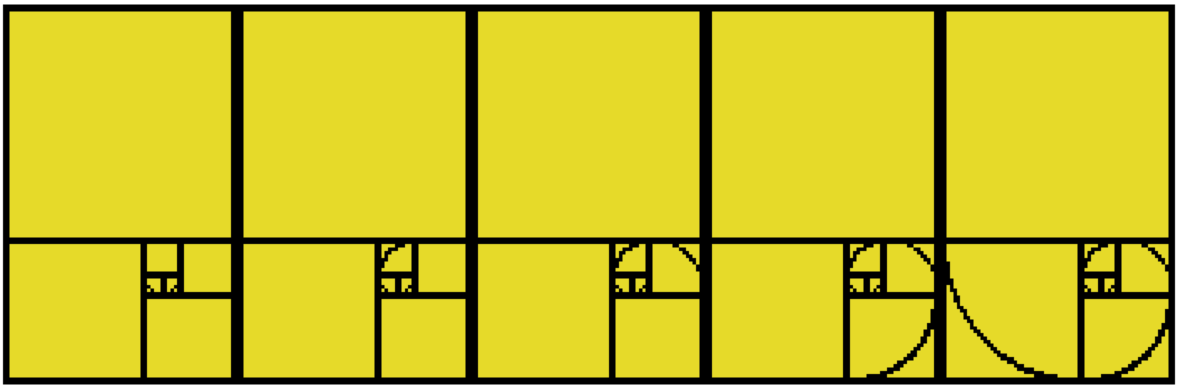
We are going to concentrate on the squares that we drew, starting with the two smallest ones. Let's start with the one on the right. Connect the upper right corner to the lower left corner with an arc that is one fourth of a circle:



Then continue your line into the second square on the left, again with an arc that is one fourth of a circle:



We will continue this process until each square has an arc inside of it, with all of them connected as a continuous line. The line should look like a spiral when we are done. Here is an example of what your spiral should look like (move your cursor over the images to see them change):



Now what was the point of that? The point is that this "golden spiral" occurs frequently in nature. If you look closely enough, you might find a golden spiral in the head of a daisy, in a pinecone, in sunflowers, or in a nautilus shell that you might find on a beach. Here are some examples:

This rectangle should seem very well proportioned to you, i.e. it should be pleasing to the eye. If it isn't, maybe you need your eyes checked! Go ahead and decorate or color in your rectangle to make it even more attractive if you wish.

Activity 7

The Golden Ratio in Nature

Pupils should research this at home prior to the lesson. Again, the following is only a guide for your discussion.

So, why do shapes that exhibit the Golden Ratio seem more appealing to the human eye? No one really knows for sure. But we do have evidence that the Golden Ratio seems to be Nature's perfect number. Take, for example, the head of a daisy (see sheet):

Somebody with a lot of time on their hands discovered that the individual florets of the daisy (and of a sunflower as well) grow in two spirals extending out from the center. The first spiral has 21 arms, while the other has 34. Do these numbers sound familiar? They should - they are Fibonacci numbers! And their ratio, of course, is the Golden Ratio. We can say the same thing about the spirals of a pinecone, where spirals from the center have 5 and 8 arms, respectively (or of 8 and 13, depending on the size)- again, two Fibonacci numbers:

A pineapple has three arms of 5, 8, and 13 - even more evidence that this is not a coincidence. Now is Nature playing some kind of cruel game with us? No one knows for sure, but scientists speculate that plants that grow in spiral formation do so in Fibonacci numbers because this arrangement makes for the perfect spacing for growth. So for some reason, these numbers provide the perfect arrangement for maximum growth potential and survival of the plant.

Activity 8 The Perfect Face

Do some faces seem attractive to you? Many people seem to think so. But why? Is there something specific in each of their faces that attracts us to them, or is our attraction governed by one of Nature's rules? Does this have anything to do with the Golden Ratio? I think you already know the answer to that question. Let's try to analyze faces to see if the Golden Ratio is present or not.

Here's how we are going to conduct our search for the Golden Ratio: we will measure certain aspects of each person's face. Then we will compare their ratios. Let's begin. We will need the following measurements, to the nearest tenth of a centimeter:

a = Top-of-head to chin = ☐ cm

b = Top-of-head to pupil = ☐ cm

c = Pupil to nosetip = ☐ cm

d = Pupil to lip = ☐ cm

e = Width of nose = ☐ cm

f = Outside distance between eyes = ☐ cm

g = Width of head = ☐ cm

h = Hairline to pupil = ☐ cm

i = Nosetip to chin = ☐ cm

j = Lips to chin = ☐ cm

k = Length of lips = ☐ cm

l = Nosetip to lips = ☐ cm

Now, find the following ratios:

a/g = ☐ cm

b/d = ☐ cm

i/j = ☐ cm

i/c = ☐ cm

e/l = ☐ cm

f/h = ☐ cm

k/e = ☐ cm

Did any of these ratios come close to being Golden? If not, then maybe this face isn't so perfect after all. Of the face above, who has the most "Golden" one? Try finding a face that you find attractive and see how Golden it is.

Alternate activity: For those of you who are artistically inclined, see if you can draw the perfect face. Keep the ratios above in mind when designing your face. After you have completed your sketch, prove that it is Golden by computing the ratios above. Turn in your sketch and analysis to your instructor.

In the Human Body

Pupils can research this at home and discuss their findings in class. Here are some links to good website

http://www.goldennumber.net/body.htm

http://milan.milanovic.org/math/english/golden/golden2.html

Conclusions

So is Nature playing some kind of cruel game with us mathematicians? We don't know. But something tells me that if there is a grand design for the "Matrix" that we live in, then the Golden Ratio plays a large role in that design in one way or another. Why else would this specific number appear at completely different times, in completely different places, in places as varied as plants to artwork to shapes and to architecture? Who knows where else the Golden Ratio lies undiscovered? Only time will tell. All we can say for now is that math is definitely a part of Nature, and there is still a lot more for us to learn.