S2

Problem Solving Homework

Problem 1:

1. **Equation Trees**
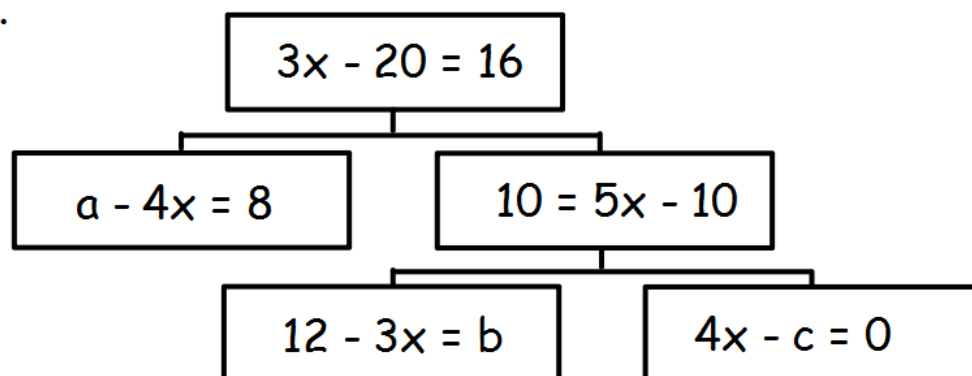
       i. Solve each equation

       ii. How are the solutions related?

$$2x - 40 = 20$$

$$4 - 2x = 0 \qquad 8 + 3x = 53$$
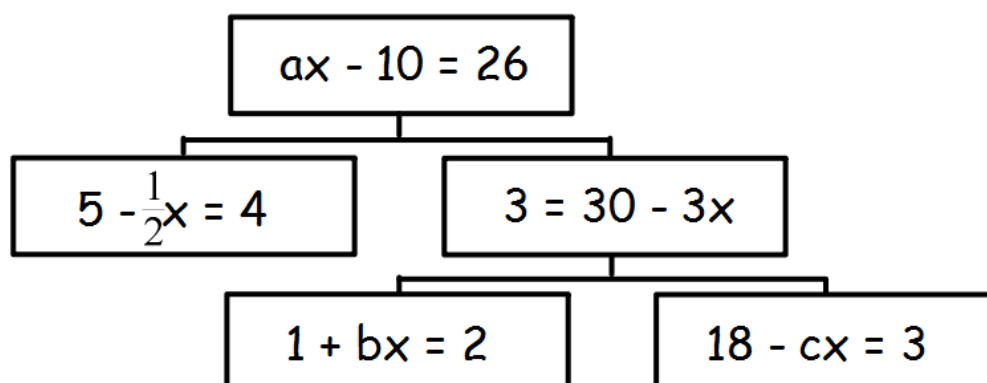
$$4x + 1 = 13 \qquad 12 - 2x = 2$$
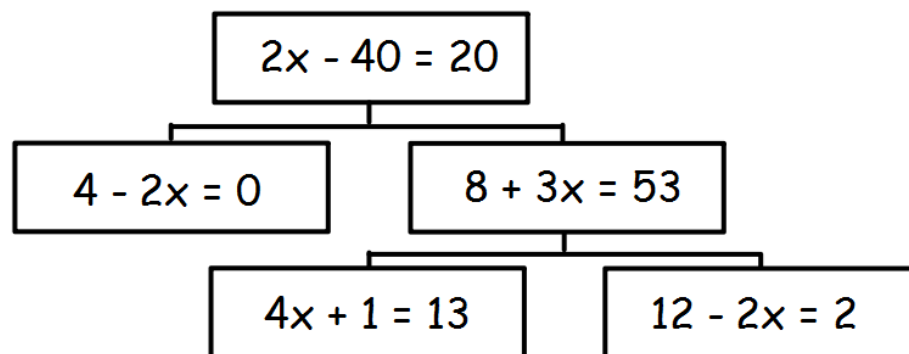
2. These equation trees have the same property. Some numbers have been replaced by the letters a, b and c.
   Find the value of a, b and c in each case.

   i.

$$3x - 20 = 16$$

$$a - 4x = 8 \qquad 10 = 5x - 10$$

$$12 - 3x = b \qquad 4x - c = 0$$

   ii.

$$ax - 10 = 26$$

$$5 - \tfrac{1}{2}x = 4 \qquad 3 = 30 - 3x$$

$$1 + bx = 2 \qquad 18 - cx = 3$$

Problem 2:

Problem 3:

1.

Problem 4:

1.

2.

Problem 5:

1.

Problem 6:

1.   i) In a 3 by 3 magic square, each number from 1 to 9 is used
      once, and the numbers in each row, column and diagonal have
      the same total.

   (b) (a)   Complete as many different magic squares as you can
             Which number is always in the same place?
   (b)   Take a = 5, b = 4, c = 1, and calculte the values of the
             entries in the following square. Is is a magic square?

a - b	a + b - c	a +c
a + b + c	a	a - b - c
a -c	a - b + c	a + b

Problem 7:

1. In a magic sqaure each row, column and diagonal adds up to the same magic number.

   (a) Complete this magic square where the magic number is 18. Sometimes a number is required, sometimes an expression.

   (b) For $x = 3$ check that your answer gives a magic sqaure.

   (c) No repeated numbers, zeros or negative numbers are allowed in a magic sqaure. Which other values of x produce a magic square?

Problem 8:

A	B	C	D	E	F	G	H	I	J	K	L	M
↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2

N	O	P	Q	R	S	T	U	V	W	X	Y	Z
↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
1.3	1.4	1.5	1.6	1.7	1.8	1.9	2	2.1	2.2	2.3	2.4	2.5

1. Use the code to read this message:
   || 2 × 0.9 | 0.1 + 0.1 | 4.2 ÷ 6 | 0.2 × 7 | 12.6 ÷ 9 | 1.21 ÷ 1.1||
   0.23 + 0.47 | 23.8 ÷ 17 | 9.9 - 8.8 | 2.32 ÷ 2.9 | 0.001 × 300|
   | 0 ÷ 9.99 | 1.2 × 1 × 0.2 | 14.3 -12.5 || 11.34 ÷ 6.3 | 0.7 |
   0.007 × 200 | 0.2 ÷ 0.12 | 6.69 - 5.59 | 0.11 + 0.19 ||
   77 ÷ 770 | 0.16 ÷ 0.4 || 0.83 + 0.27 | 2 × 0.7 | 7.15 ÷ 5.5 |
   | 2.9 - 2.3 | 400 ÷ 1000 | 8.5 ÷ 5 ||

2. A very fast growing sun-flower grows to a height of 12 feet in 12 weeks by doubling its height every week. If you only want your sunflower to be 6 feet tall, after how many weeks should you stop it growing?

Problem 9:

1. A collection of sheep and turkeys have a total of 99 heads and legs between them. There are twice as many turkeys as there are sheep. How many of each is there?

2. The Giffnock Maths Society are considering introducing a new operator symbol ¥

   Where ¥ means that you "double the first number then multiply your answer by the second number"

   For example: 4 ¥ 5 = (4 x 2) x 5
                                    = 40

   (a)   Calculate 5 ¥ 7
   (b)   If a ¥ 6 = 96   find a.

Problem 10:

1. The farmyard was full of cows and hens. All together there were 36 heads and 100 legs in the farmyard. How many cows were there?

2. Sharon has been using a faulty ballpoint pen. Can you work out the blotted digits?

Problem 11:

1. There are some sheep and geese in a field, and between them they possess 50 legs. If the sheep had had 2 legs each, and the geese had had 4 legs each there would have been 58 legs present. How many sheep and geese are there?

2. A wire frame is made for a model truck. Find an expression for the length of the wire needed. The lengths are in centimetres.
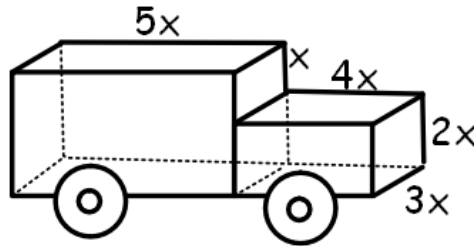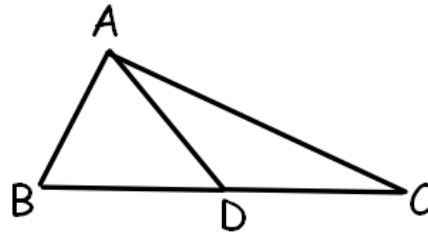

Problem 12:

1. Three times Euan's age in three years time, minus three times Euan's age three years ago, is equal to Euan's age now. How old is Euan?

2. Kim is twice as old as Sharon was when Kim was as old as Sharon is now. Kim is 24. How old is Sharon?

Problem 13:

In the Triangle ABC, AD = BD = CD
What is the size of the angle BAC?

A 60°   **B** 75°   **C** 90°   **D** 120°   **E** More information needed

Each Symbol represents a different number.
Work out the number for each symbol.

$$\text{✖} \times \text{✖} = 100$$

$$\text{✹} + \text{✹} + \blacksquare = \text{☺}$$

$$\blacksquare + \triangle = \text{☺}$$

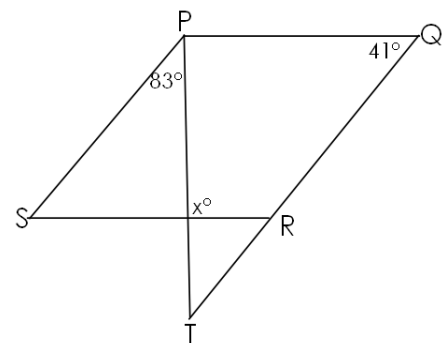$$\text{✖} + \text{✖} + \text{✖} = \text{✹} + \text{✹}$$

$$\triangle - \text{✖} = \blacksquare$$

$$\text{☺} = \qquad \text{✖} = \qquad \text{✹} = \qquad \blacksquare = \qquad \triangle =$$

Problem 14:

1. In the diagram the lines PQ and SR are parallel, as are the lines PS and QT.

   What is the value if x?

2. In a small group of people it was found that all of the following relationships could be claimed –

   Father, mother, son, daughter, brother, sister, cousin, nephew, niece, uncle and aunt.

   What is the smallest group of people that can manage this, and how must they be related?

Problem 15:

1. This sum is wrong.

$$
\begin{array}{r}
1 \quad 2 \quad 3 \\
+ \quad 2 \quad 9 \quad 7 \\
\hline
3 \quad 3 \quad 0 \\
\hline
\end{array}
$$

Change just one of the nine figure so as to make it correct.

2. When one particular number, written in figure, is turned upside down it increases in value by 21. Which number is it?

3. A many and his wife each weigh 50kg. The have two children each weighing 25kg. The whole family wish to cross a river in a rowing-boat which can only carry a maximum of 50kg. Every member of the family can row.

   How can they mange it?

Problem 16:

1. A man has to get a fox, a chicken, and a sack of corn across a river. He has a rowboat, and it can only carry him and one other thing. If the fox and the chicken are left together, the fox will eat the chicken. If the chicken and the corn are left together, the chicken will eat the corn. How does the man do it?

2. A frog is climbing a well 31 metres deep. It climbs 4 metres in 1 hour, but then slides back 1 foot as it rests for an hour.
   How long will it take to climb out of the well?

3. A 2cm cube is made from 8 wooden 1cm cubes.
   The outside faces are then painted white.

   How many of the smaller 1 cm cubes now have:
   (a) 3 white faces (b) 2 white faces (c) 1 white face (d) 0 white faces

Problem 17:

1.

On a dark night, four travellers arrive at a difficult river crossing. The crossing is made over a rickety wooden bridge which only takes the weight of two people.

Each crossing must be accompanied by their only torch.

Archie can cross in 3 minutes
Beth can cross in 5 minutes
Carol can cross in 9 minutes
David can cross in 10 minutes

What is the shortest time in which all four can compete the crossing?

2. Copy and complete this cross-number puzzle.
$x = 23$, $y = 7$, $z = 11$, $a = 120$, $b = 3$.

Across			Down	
1	$3x$		2	$a - x$
3	$3y$		4	$a + 2b$
5	yz		6	$6a$
7	$4x^2$		8	$z^2 + x$
9	$8a + 4z$		9	$3y^2$
10	y^2		11	$3x + 4y$
12	$a - 2x$			
13	$y^2 + 3b^2$			