WELLINGTON
COLLEGE

13+ SCHOLARSHIP EXAMINATION 2015

MATHEMATICS 11

TIME ALLOWED: 1 hour 30 minutes

The marks available for each question are printed in square brackets.

You may not use a calculator in this examination.

- **This exam has seven questions. Each is worth twelve marks.**
- **Start each question on a fresh sheet of paper and put the pages in question order at the end of the exam.**
- **You should aim for complete answers rather than partially-answered questions; answer as many questions as you can.**
- **Credit will be given for the clarity of your work and your explanations.**

1. (a) In 10 years' time I will be twice as old as I was 5 years ago.

   (i) Express this statement as an equation. [2]

   (ii) Find my age. [1]

   (b) Two years ago, Emmet was ten years older than twice Marty's age.

   (i) Show that in eight years' time, Emmet will be exactly twice Marty's age. [6]

   (ii) Given that the sum of their ages is 59 years, find Marty's age. [3]

2. A new operation, $\otimes$, is defined so that $a \otimes b = ab - a - b$.

   (a) Show that $3 \otimes 5 = 7$. [2]

   (b) Find the value of $(4 \otimes 3) \otimes 5$. [3]

   (c) Show that $x \otimes 1 = -1$, regardless of the value of x. [2]

   (d) Simplify the expression $x \otimes 2$. [2]

   (e) Solve the equation $(x \otimes 4) \otimes 2 = 3$ [3]

3. In music, a crotchet lasts for a whole beat and a quaver lasts for half a beat.

   A crotchet is notated ♩ and a quaver is notated ♪.

   There are two ways of combining these notes to make a rhythm which lasts one whole beat:

   ♩ or ♪♪.

   (a) Show clearly that there are five ways of making a 2-beat rhythm using only these notes. [3]

   The above tells us that there are five possible 2-beat rhythms.

   (b) Determine the number of possible 1½-beat rhythms.
   Explain clearly how you know there are no others. [3]

   (c) Determine the number of possible 3-beat rhythms.
   Explain clearly how you know there are no others. [6]

4. (a) My hot tap, running alone and on fully, will completely fill my bath in 20 minutes.
   My cold tap, running alone and on fully, will completely fill my bath in 15 minutes.

   Determine how long it would take to fill the bath if I run both together, fully on.

   Give your answer in minutes; use fractions if you need to. [6]

   (b) The hot water in my house is supplied from a tank in the loft.

   If I run the bath tap on full, it will drain the tank entirely in 40 minutes.
   If I run the tap in the bathroom sink on full, it will drain the tank entirely in 80 minutes.
   If I run the kitchen tap on full, it will drain the tank entirely in 60 minutes.

   I set all three taps running at half their maximum rate.
   Determine how long it will take to drain the entire tank.

   Give your answer in minutes; use fractions if you need to. [6]

5. You may already know that $\pi \approx \frac{22}{7}$.

In fact, $\pi < \frac{22}{7}$, although they are so close that $\frac{22}{7} - \frac{1}{320} < \pi < \frac{22}{7}$.
You may wish to use this fact when answering the following questions.

(a) Determine which is larger, $\frac{13}{7}$ or $\frac{24}{13}$.
Show enough working to justify your answer. [3]

(b) Determine which is larger, the area of a circle with radius 9 or the area of a square with side length 16.
Show enough working to justify your answer. [4]

(c) Determine which is larger, the circumference of a circle with radius 16 cm, or the perimeter of a square with side length 25.
Show enough working to justify your answer. [5]

6. This question is about **Euclid's Algorithm**, which is a way of finding the Highest Common Factor (HCF) of two numbers.
You do not need to have learnt about Euclid's Algorithm before.

An algorithm is a set of rules which you can apply repeatedly to generate a desired result.
Euclid was an ancient Greek mathematician, whose most famous achievements are in the field of geometry.

To find the HCF of *a* and *b*, start with a table containing two columns, with the numbers *a* and *b* at the top, then apply the following rules:

- If the smaller number is a factor of the larger number, then stop: the smaller number is HCF(*a*, *b*).
- If not, subtract the smaller number from the larger number and write the new numbers on the next row of the table.

For example, to find HCF(969, 391):

969	391	*Write out a and b.*
578	391	*Subtract 391 from 969 to give 578*
187	391	*Subtract 391 from 578 to give 187*
187	204	*Subtract 187 from 391 to give 204*
187	17	*Subtract 187 from 204 to give 17*

Since 17 is a factor of 187, we can stop: HCF(969, 391) = 17.
If you hadn't noticed this, but had carried on going, the next line would have read:

170	17	*Subtract 17 from 187 to give 170*

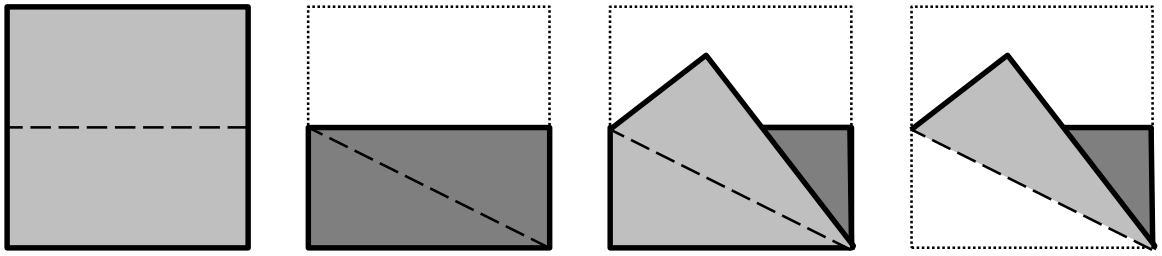
So we still would have stopped with 17, because it is easy to see that 17 is a factor of 170.

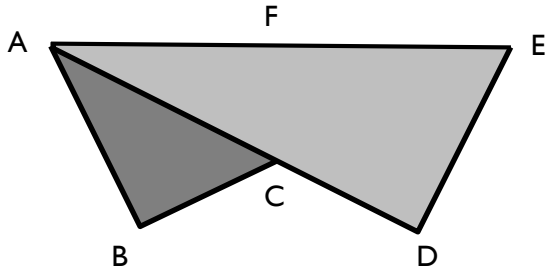
(a) Use Euclid's Algorithm to show that HCF(81, 24) = 3. [6]
You do not need to replicate what is shown in italics above, but you must show the full table of values.

(b) Use Euclid's Algorithm to find HCF(1071, 462). [4]

(c) Use Euclid's Algorithm to find HCF(96, 25). [2]

7. I take a square piece of paper (dark grey on one side, and light grey on the other).
I fold it in half to form a rectangle, as illustrated.
I then lift one corner and fold diagonally from the centre line to the opposite corner.
Finally, I repeat the diagonal fold for the back half of the paper.





Given that the original square had side length 2, and that F is the midpoint of AE,

(a) Write down the length of DE. [1]

(b) Find the length of AE, giving your answer in an exact form. [3]

(c) Hence show that $FC = \frac{1}{4}\sqrt{5}$.
Explain your reasoning. [3]

(d) Hence or otherwise show that $AC = \frac{5}{4}$. [3]

(e) Show that triangle ABC has area $\frac{3}{8}$. [2]