BT 1 Answers

1) $m_{PR} = \frac{2-(-1)}{-6-(-1)} = \frac{3}{-5}$

$\therefore m_\perp = \frac{5}{3}$ $(m_1 m_2 = -1)$

$Q(2,1)$ $m = 5/3$

$y - b = m(x-a)$
$y - 1 = \frac{5}{3}(x-2)$
$3y - 3 = 5x - 10$
$3y = 5x - 7$

2) a) mid pt AB $(-2, 2)$
C $(5,2)$ * Taken from diagram
$\therefore y = 2$

b) Mid pt BC $(1, 0)$
$m_{BC} = \frac{2-(-2)}{5-(-3)} = \frac{4}{8} = \frac{1}{2}$
$\therefore m_\perp = -2$ $(m_1 m_2 = -1)$

$y - b = m(x-a)$
$y - 0 = -2(x-1)$
$y = -2x + 2$

c) $-2x + 2 = 2$
$-2x = 0$
$x = 0$ $\therefore (0,2)$

3) $y = \frac{7}{3}x^{-2}$

a) $dy/dx = -\frac{14}{3}x^{-3}$
$= \frac{-14}{3x^3}$

b) $y = 3x + \frac{1}{3}x^{-1}$
$dy/dx = 3 - \frac{1}{3}x^{-2}$
$= 3 - \frac{1}{3x^2}$

c) $y = 2x^{-2} + 7x^{-3}$
$dy/dx = -4x^{-3} - 21x^{-4}$
$= -\frac{4}{x^3} - \frac{21}{x^4}$

d) $y = x^{1/2}(x - x^3)$
$y = x^{3/2} - x^{7/2}$
$dy/dx = 3/2\, x^{1/2} - 7/2\, x^{5/2}$

4) $f(x) = \frac{x^2 + 3x + 2}{x^{1/2}}$

$f(x) = x^{3/2} + 3x^{1/2} + 2x^{-1/2}$
$f'(x) = \frac{3}{2}x^{1/2} + 3/2\, x^{-1/2} - x^{-3/2}$
$= \frac{3}{2}\sqrt{x} + \frac{3}{2\sqrt{x}} - \frac{1}{\sqrt{x^3}}$

$f'(4) = \frac{3}{2}\sqrt{4} + \frac{3}{2\sqrt{4}} - \frac{1}{\sqrt{4^3}}$
$= 3 + \frac{3}{4} - \frac{1}{8}$
$= \frac{24}{8} - \frac{6}{8} - \frac{1}{8}$
$= \frac{17}{8} = 2\frac{1}{8}$

5) $y = x^3 - 4x - 5$

$m = dy/dx$

$dy/dx = 3x^2 - 4$

$m = 3(1)^2 - 4$

$= 3 - 4$

$= -1$

$x = 1 \quad y = 1^3 - 4(1) - 5$

$= 1 - 4 - 5$

$= -8$

$(1, -8)$

$y - b = m(x - a)$

$y + 8 = -1(x - 1)$

$y + 8 = -x + 1$

$y = -x - 7$

6) Cuts x-axis at $y = 0$

a) $4x^2(x - 3) = 0$

$x = 0, \; x = 3$

$(0,0) \quad (3,0)$

Cuts y-axis at $x = 0$

$(0,0)$

b) Stationary points occur when $dy/dx = 0$

$f(x) = 4x^3 - 12x^2$

$f'(x) = 12x^2 - 24x$

$12x^2 - 24x = 0$

$12x(x - 2) = 0$

$x = 0 \quad x = 2$

$x = 0 \quad y = 0 \quad (0,0)$

$x = 2 \quad y = 4(2)^3 - 12(2)^2$

$= 32 - 48$

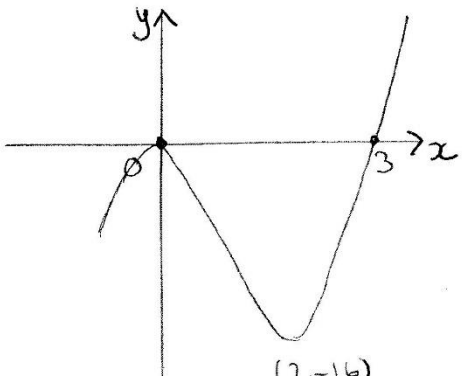
$= -16 \quad (2, -16)$

x	$\rightarrow$	0	$\rightarrow$
	$+$	0	$-$
dy/dx	/	—	\

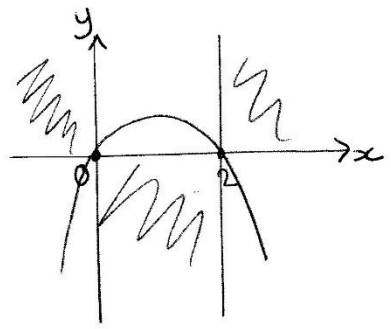
max at $(0,0)$

x	$\rightarrow$	2	$\rightarrow$
	$-$	0	$+$
dy/dx	\	—	/

min $(2, -16)$



7)



8) i) $u_{n+1} = 0.65u_n + 8$ $\quad u_0 = 25$

$u_1 = 0.65 \times 25 + 8$ $\quad u_2 = 0.65 \times 24.25 + 8$ $\quad u_3 = 23.4456$

$= 24.25$ $\quad = 23.7625$ $\quad u_4 = 23.24$

ii) Limit exists because $-1 < 0.65 < 1$

$L = 0.65L + 8$

$0.35L = 8$

$L = \frac{8}{0.35}$

$= \frac{160}{7}$ $\left(22.86\right)$

In the long term, the level of sewage in the loch will settle at 22.86 tonnes.

9) $u_n = 0.9u_{n-1} + 2$ $\quad u_1 = 3$

a) $u_2 = 0.9 \times 3 + 2$ $\quad$ b) $u_3 = 0.9 \times 4.7 + 2$ $\quad \therefore n = 3$

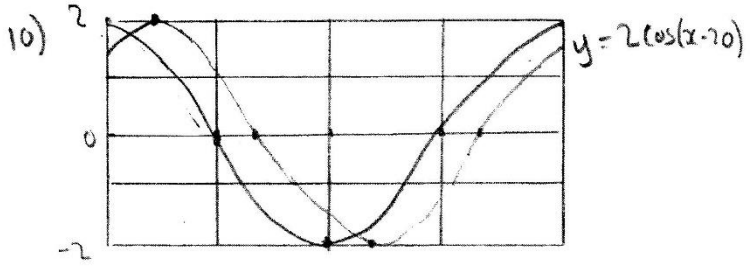
$= 4.7$ $\quad = 6.23$

c) $L = 0.9L + 2$

$0.1L = 2$

$L = 20$

10)



11) $\sin 2x = \frac{1}{2}$

$\pi - \alpha$	α
S ✓	A ✓
T	C

$\alpha = \frac{\pi}{6}$

$2x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$

$x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$