Homework 1

MAC - Partial Fractions

1. Express as a single fraction a) $\frac{3}{w+2}+\frac{5}{w}$ b) $\frac{2}{b+1}+\frac{3}{b+2}$ c) $\frac{2}{t-7}-\frac{5}{t+9}$

2. Express each of the following as partial factions:

(a) $\frac{37x-81}{(x-3)(x+7)(2x-3)}$ (b) $\frac{2x+1}{(x-3)^2}$

(c) $\frac{8x-1}{(x-2)(x^2+1)}$ (d) $\frac{2x^3+11}{(x^2-4)(x-3)}$ (Careful here!)

3. Simplify: (a) $\frac{x+2}{x+5}$ (b) $\frac{x^3-5x^2+9x-7}{x^2-2x+3}$

4. Express as a single logarithm:

(a) $7\ln 2-3\ln 12+5\ln 3$ (b) $\ln 12-(\tfrac{1}{2}\ln 9+\tfrac{1}{3}\ln 8)$

Homework 2

MAC – Differentiation 1

1. Differentiate the following with respect to *x*, simplifying your answers where possible:

(a) $y = x^2 \cos(2x+1)$ (b) $y = \dfrac{2x+1}{\sqrt{2x-1}}$ (c) $y = \dfrac{\sin x}{2+\cos x}$ (d) $y = \cos x^2 \sin 3x$

2. Differentiate f(x) = 3x² + 4x from 1st Principles:

3. Find the gradient of the curve $y = xe^{x-4}$ at the point (4,4).

4. Prove the following identities:

(a) $\sin\theta\tan\theta + \cos\theta = \sec\theta$ (b) $\cos ec\theta - \sin\theta = \cot\theta\cos\theta$

5. Differentiate the following with respect to *x*, simplifying your answers where possible:

(a) $y = \tan 6x$ (b) $y = \tan^5 x$ (c) $y = \cot(2x^2+1)$

(d) $y = \exp(x^2+4)$ (e) $y = e^{-x}\sin x$ (f) $y = e^x \ln x$

Homework 3

MAC – Differentiation 2

1. If $y = 4x^2 - 3x + 1$, show that $y\frac{d^2y}{dx^2} + \frac{dy}{dx} - 8y + 3 = 8x$.

2. If $f(x) = 2x^3 + 5x$, find the value of a such that $f''(a) = 36$

3. Differentiate the following with respect to x :

(a) $y = \ln(ax^2 + bx + c)$ (b) $\ln(x^2 e^x)$ (c) $y = \sec(4x^2 + 1)$

(d) $y = \cos ec 6x$ (e) $y = \frac{\ln x}{e^x}$ (f) $y = \ln(\sec x)$

4. A function is defined by: $f(x) = \frac{3x}{x-2}$, $x \neq 2$. Show that f(x) is always decreasing.

5. Differentiate each of the following with respect to x:

(a) $y = \tan^{-1} 3x$ (b) $y = x sin^{-1} x$

6. Find $\frac{dy}{dx}$ in terms of x and y for each of the following:

(a) $y^2 - x^2 = 12$ (b) $x^2 + xy + y^2 = 7$.

Homework 4

MAC – Differentiation 3

1. Find the equation of the tangent to the curve $3x^2 + 5y^2 = 17$ at the point (-2,1).

2. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ when $xy + y^2 = 1$.

3. Use logarithmic differentiation to find $\frac{dy}{dx}$ of $y = 2x^x$.

4. Find $\frac{dy}{dx}$ in terms of *t* for $x = t^2 + 6$ and $y = 4t^3$

5. A curve is defined by the parametric equations $x = \frac{1}{t+1}$, $y = 4t$.
Find the equation of the tangent to this curve at the point where $t = 1$.

6. Find $\frac{d^2y}{dx^2}$ in terms of *t* for the parametric equations: $x = 3\cos t$, $y = 3\sin t$.

7. A spherical balloon is being inflated at a rate of 240cm^3 per second.
At what rate is the radius increasing when it is equal to 8cm?

Homework 5

MAC – Integration 1

1. Integrate the following with respect to the relevant variable:

(a) $\int 6x^{-3} - 2 + 3x^2 dx$ (b) $\int \frac{3-x^5}{x^3} dx$ (c) $\int \frac{dt}{\sqrt{5-2t}}$

2. Integrate the following with respect to *x*:

(a) $\int \frac{dx}{x+3}$ (b) $\int e^{3x-1} dx$ (c) $\int \left(\frac{3}{x-1} - \frac{4}{x-2} \right) dx$

(d) $\int \frac{dx}{1-x}$ (e) $\int \frac{x^2}{x+1} dx$ (f) $\int e^{x/2} dx$

3. Integrate each of the following with respect to *x*:

(a) $\int \frac{x+5}{x+2} dx$ (b) $\int \sec^2(2x+1) dx$ (c) $\int \cos ec^2(3-5x) dx$

4. By using a suitable substitution or otherwise, integrate each of the following:

(a) $\int \frac{6x+5}{3x^2+5x+1} dx$ (b) $\int \frac{x}{2x^2+3} dx$ (c) $\int \frac{e^x}{e^x+1} dx$.

Homework 6

MAC – Integration 2

1. By using the substitution $x = 2\sin t$, show that $\int_1^{\sqrt{3}} \frac{1}{\sqrt{4-x^2}}\,dx = \frac{\pi}{6}$.

2. Find the following integrals :

(a) $\int \frac{1}{\sqrt{64-x^2}}\,dx$ (b) $\int \frac{1}{49+9x^2}\,dx$ (c) $\int_{-2}^{2} \frac{1}{4+x^2}\,dx$

3. Integrate the following, using partial fractions :

(a) $\int \frac{22-x}{(2+x)(4-x)}\,dx$ (b) $\int \frac{2x^2+x+3}{(x+1)^2(3-x)}\,dx$

4. Use integration by parts to integrate the following:

(a) $\int xe^{-x}dx$ (b) $\int x^2 sin^{-1}x dx$

(c) $\int_0^{\frac{\pi}{2}} xsinxdx$

Homework 7

MAC – 1^{st} Order Differential Equations

1. Solve each differential equation i.e find y in terms of x:

(a) $2y\dfrac{dy}{dx} = 5x$ (b) $3\dfrac{dy}{dx} = 4x(y-2)$

2. Given that $x^2 e^y \dfrac{dy}{dx} = 1,$ and y = 0 when x = 1

Find y in terms of x.

3. Mildew hits a crop of corn in a field. Its spread can be modelled by $\dfrac{dP}{dt} = kP(100-P)$ where P is the percentage of the field affected in *t* days. When $t=0, P=1.$ When $t=5, P=60.$

(a) Express P in terms of *t.*

(b) Estimate the time it will take for 80% of the crop to be affected

4. Solve these for y:

(a) $\dfrac{dy}{dx} + \dfrac{y}{x} = 4x^2$ (b) $x\dfrac{dy}{dx} - 2y = \sqrt{x}$

5. Solve $(x+1)\dfrac{dy}{dx} - 3y = (x+1)^4$ given that y = 16 when x = 1 expressing your answer in the form y = f(x)

Homework 8

MAC – 2nd Order Differential Equations

1. Find the general solutions to these 2nd order ODES.

(a) $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$

(b) $\frac{d^2y}{dx^2} + 8\frac{dy}{dx} + 16 = 0$

(c) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$

2. Obtain the general solution of $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = e^{2x}$

3. Obtain the general solution of $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2x^2$ **(7)**

Given that y = ½ and $\frac{dy}{dx} = 1$ when x = 0, find the particular solution **(3)**