The Scottish Mathematical Council

www.scot-maths.co.uk

MATHEMATICAL CHALLENGE 2013−2014

Entries must be the unaided efforts of individual pupils.
Solutions must include explanations and answers without explanation will be given no credit.
Do not feel that you must hand in answers to all the questions.

CURRENT AND RECENT SPONSORS OF MATHEMATICAL CHALLENGE ARE
The Edinburgh Mathematical Society, The Maxwell Foundation, Professor L E Fraenkel, The London Mathematical Society and The Scottish International Education Trust.
The Scottish Mathematical Council is indebted to the above for their generous support and gratefully acknowledges financial and other assistance from schools, universities and education authorities.
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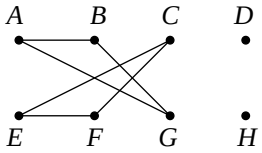
Junior Division: Problems 2

J1. The date of the second Thursday of a particular month is a square number. What is the date of the last Wednesday of that month?
Explain your reasoning.

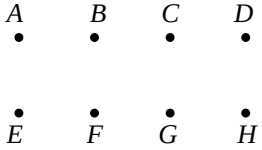
J2. One family outing last summer included an impromptu sports day with five events in which 4, 2 and 1 points were awarded for the first three places in each event. Douglas and John tied with 8 points each. Jackie came next with 7, and Bill and Colin each had 6.
Colin didn't win any event but gained points in three of them. He beat both Bill and Douglas in the 200 metres but was well behind John in the High Jump.
Jackie won the Long Jump, but was well out of the points in the High Jump.
Bill was the only child who gained points in every event, his best effort being in the 100 metres.
Who were the first three children in the 400 metres event and in what order did they finish?
Explain your reasoning.

J3. Triangles are called 'congruent' when they are identical. This means they are the same size and shape, although they can be in different positions. For example, triangles *ABG* and *EFC* are congruent.



Non-congruent triangles must be different is some respect.

How many **non-congruent** triangles can be formed by joining the dots on the grid below?



Explain your reasoning.

SEE OVER FOR QUESTIONS J4 AND J5.

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Mathematical Challenge Problems 2

JUNIOR DIVISION 2013-2014

PLEASE USE CAPITALS TO COMPLETE

SURNAME	
OTHER NAME(S) (underline the one you prefer)	
SCHOOL	
AGE	YEAR OF STUDY S

FOR OFFICIAL USE

Marker

Marks

1	2	3	4	5

Total

– – – – CUT ALONG HERE – – – –

Please write your solutions on A4 paper and staple the above form to them.

PLEASE WRITE YOUR NAME ON EVERY PAGE.

Send your entry directly or through your school to :

David Pritchard

University of Strathclyde, Livingstone Tower,

26 Richmond Street, Glasgow

G1 1XH

For further information on the competition, please see the Information Circular, which has been distributed to all secondary schools. Please contact the local organiser, whose name and address are given above, if you require a further copy.

J4. A block of four postage stamps, with perforations along the joins so that they can be easily separated, have values in pence as shown:

1	2
4	3

Show that it is possible to make every postage value from 1p to 10p using either a single stamp or a number of stamps joined along lines of perforations.

Using a different set of stamp values in the block of four, it is possible to make every postage value from 1p to a higher limit than 10p. Construct an arrangement of stamp values which reaches the highest possible limit.

Are there any other solutions which give this limit? Explain.

J5. A brother and sister, Peter and Fiona, are always thinking about numbers.

On his birthday Peter said “My age is a square number.”

His older sister Fiona said “That's right, but the sum of our ages and the difference of our ages also give squared numbers.”

Peter replied “In three years time, both our ages will be prime numbers.”

Fiona replied “Three years ago, both our ages were also prime numbers.”

What are the ages of Peter and Fiona now?

END OF PROBLEM SET 2

CLOSING DATE FOR RECEIPT OF SOLUTIONS : 14th February 2014

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Middle Division: Problems 2

M1. A block of four postage stamps, with perforations along the joins so that they can be easily separated, have values in pence as shown:

1	2
4	3

Show that it is possible to make every postage value from 1p to 10p using either a single stamp or a number of stamps joined along lines of perforations.

Using a different set of stamp values in the block of four, it is possible to make every postage value from 1p to a higher limit than 10p. Construct an arrangement of stamp values which reaches the highest possible limit.

Are there any other solutions which give this limit? Explain.

M2. A brother and sister, Peter and Fiona, are always thinking about numbers.

On his birthday Peter said "My age is a square number."

His older sister Fiona said "That's right, but the sum of our ages and the difference of our ages also give squared numbers."

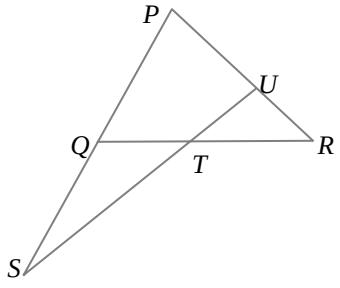
Peter replied "In three years time, both our ages will be prime numbers."

Fiona replied "Three years ago, both our ages were also prime numbers."

What are the ages of Peter and Fiona now?

M3. PQR is any triangle. The side PQ is extended to S where $PQ = QS$. The point U divides the side PR in the ratio 3 : 2. The point T is where the lines QR and SU cross.

Find the ratio $\dfrac{QT}{QR}$.



M4. Rebecca celebrated her graduation by going for a hot air balloon ride. At first the wind blew the balloon $\frac{1}{2}$ a mile due east. Then the balloon was blown $\frac{3}{4}$ of a mile southwest. Finally, it was blown $\frac{1}{2}$ a mile north and then landed. How many miles did the balloon land from the point where it was launched? Express your answer as a decimal to three places.

Do not use a scale drawing.

SEE OVER FOR QUESTION M5.

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Mathematical Challenge Problems 2

MIDDLE DIVISION 2013−2014

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M5. Dots are arranged in a rectangular grid with 4 rows and n columns. Consider different ways of colouring the dots, in which each dot either red or blue. A colouring is 'good' if no four dots of the same colour form a rectangle with horizontal and vertical sides.
Find the maximum value of n for which there is a good colouring.

END OF PROBLEM SET 2

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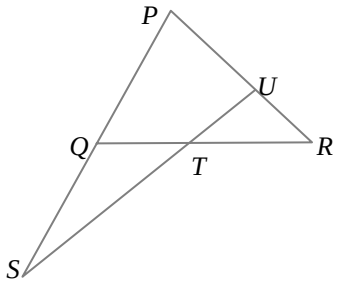
Senior Division: Problems 2

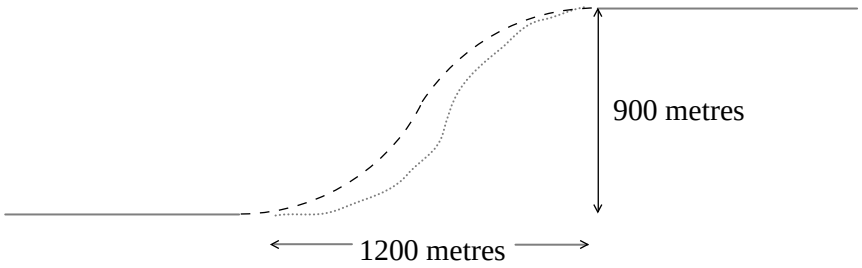
S1. PQR is any triangle. The side PQ is extended to S where $PQ = QS$. The point U divides the side PR in the ratio 3 : 2. The point T is where the lines QR and SU cross.
Find the ratio $\dfrac{QT}{QR}$.



S2. Dots are arranged in a rectangular grid with 4 rows and n columns. Consider different ways of colouring the dots, in which each dot either red or blue. A colouring is 'good' if no four dots of the same colour form a rectangle with horizontal and vertical sides.
Find the maximum value of n for which there is a good colouring.

S3. A triangle has sides of length 4, 7 and 9 units. Find the length of the longest median. **Show your reasoning.**

S4.



Two straight sections of a road, each running from east to west, and located as shown, are to be joined smoothly by a new roadway consisting of arcs of two circles of equal radius. The existing roads are to be tangents at the joins and the arcs themselves are to have a common tangent where they meet. ′Find the length of the radius of these arcs.

SEE OVER FOR QUESTION S5.

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Mathematical Challenge Problems 2

SENIOR DIVISION 2013−2014

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S5. Let

$$f(x) = x^n + a_1x^{n-1} + \ldots + a_n,$$

where $a_1, a_2, \ldots, a_n$ are given numbers. It is given that $f(x)$ can be written in the form

$$f(x) = (x + k_1)(x + k_2)\ldots(x + k_n).$$

By considering $f(0)$, or otherwise, show that $k_1k_2\ldots k_n = a_n$.

Show also that

$$(k_1 + 1)(k_2 + 1)\ldots(k_n + 1) = 1 + a_1 + a_2 + \ldots + a_n$$

and give the corresponding result for $(k_1 - 1)(k_2 - 1)\ldots(k_n - 1)$.

Hence, find the roots of the equation

$$x^4 + 22x^3 + 172x^2 + 552x + 576 = 0,$$

given that they are all integers.

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