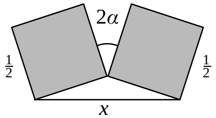
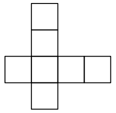
18. The diagram shows two squares, with sides of length $\frac{1}{2}$, inclined at an angle 2α to one another. What is the value of x?



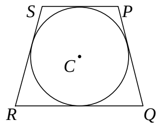
A $\cos\alpha$ B $\frac{1}{\cos\alpha}$ C $\sin\alpha$ D $\frac{1}{\sin\alpha}$ E $\tan\alpha$

19. The numbers 2, 3, 4, 5, 6, 7, 8 are to be placed, one per square, in the diagram shown so that the sum of the four numbers in the horizontal row equals 21 and the sum of the four numbers in the vertical column also equals 21. In how many different ways can this be done?



A 0 B 2 C 36 D 48 E 72

20. In trapezium $PQRS$, $SR = PQ = 25$cm and SP is parallel to RQ. All four sides of $PQRS$ are tangent to a circle with centre C. The area of the trapezium is 600cm^2. What is the radius of the circle?

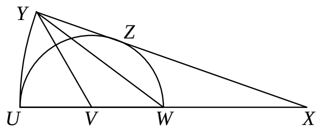


A 7.5cm B 8cm C 9cm D 10cm E 12cm

21. Which of the following numbers does *not* have a square root in the form $x + y\sqrt{2}$, where x and y are positive integers?

A $17 + 12\sqrt{2}$ B $22 + 12\sqrt{2}$ C $38 + 12\sqrt{2}$ D $54 + 12\sqrt{2}$ E $73 + 12\sqrt{2}$

22. A semicircle of radius r is drawn with centre V and diameter UW. The line UW is then extended to the point X, such that UW and WX are of equal length. An arc of the circle with centre X and radius $4r$ is then drawn so that the line XY is a tangent to the semicircle at Z, as shown. What, in terms of r, is the area of triangle YVW?

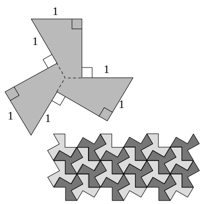


A $\frac{4r^2}{9}$ B $\frac{2r^2}{3}$ C r^2 D $\frac{4r^2}{3}$ E $2r^2$

23. Tom and Geri have a competition. Initially, each player has one attempt at hitting a target. If one player hits the target and the other does not then the successful player wins. If both players hit the target, or if both players miss the target, then each has another attempt, with the same rules applying. If the probability of Tom hitting the target is always $\frac{4}{5}$ and the probability of Geri hitting the target is always $\frac{2}{3}$, what is the probability that Tom wins the competition?

A $\frac{4}{15}$ B $\frac{8}{15}$ C $\frac{2}{3}$ D $\frac{4}{5}$ E $\frac{13}{15}$

24. The top diagram on the right shows a shape that tiles the plane, as shown in the lower diagram. The tile has nine sides, six of which have length 1. It may be divided into three congruent quadrilaterals as shown. What is the area of the tile?



A $\frac{1 + 2\sqrt{3}}{2}$ B $\frac{4\sqrt{3}}{3}$ C $\sqrt{6}$ D $\frac{3 + 4\sqrt{3}}{4}$ E $\frac{3\sqrt{3}}{2}$

25. How many distinct pairs (x, y) of real numbers satisfy the equation $(x + y)^2 = (x + 4)(y - 4)$?

A 0 B 1 C 2 D 3 E 4

UKMT

UK SENIOR MATHEMATICAL CHALLENGE

Tuesday 6 November 2012

Organised by the **United Kingdom Mathematics Trust**

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making financial sense of the future

RULES AND GUIDELINES (to be read before starting)

1. Do not open the question paper until the invigilator tells you to do so.
2. **Use B or HB pencil only**. Mark *at most one* of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option.
3. Time allowed: **90 minutes**.
   No answers or personal details may be entered on the Answer Sheet after the 90 minutes are over.
4. The use of rough paper is allowed.
   Calculators, measuring instruments and squared paper are forbidden.
5. Candidates must be full-time students at secondary school or FE college, and must be in Year 13 or below (England & Wales); S6 or below (Scotland); Year 14 or below (Northern Ireland).
6. There are twenty-five questions. Each question is followed by five options marked A, B, C, D, E. Only one of these is correct. Enter the letter A-E corresponding to the correct answer in the corresponding box on the Answer Sheet.
7. **Scoring rules**: all candidates start out with 25 marks;
   0 marks are awarded for each question left unanswered;
   4 marks are awarded for each correct answer;
   1 mark is deducted for each incorrect answer.
8. **Guessing**: Remember that there is a penalty for wrong answers. Note also that later questions are deliberately intended to be harder than earlier questions. You are thus advised to concentrate first on solving as many as possible of the first 15-20 questions. Only then should you try later questions.

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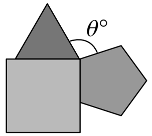
1. Which of the following cannot be written as the sum of two prime numbers?

A 5 B 7 C 9 D 10 E 11

2. The diagram shows an equilateral triangle, a square and a regular pentagon which all share a common vertex. What is the value of θ?



A 98 B 102 C 106 D 110 E 112

3. The price of my favourite soft drink has gone up by leaps and bounds over the past ten years. In four of those years it has leapt up by 5p each year, whilst in the other six years it has bounded up by 2p each year. The drink cost 70p in 2002. How much does it cost now?

A £0.77 B £0.90 C £0.92 D £1.02 E £1.05

4. According to one astronomer, there are one hundred thousand million galaxies in the universe, each containing one hundred thousand million stars. How many stars is that altogether?

A 10^{13} B 10^{22} C 10^{100} D 10^{120} E 10^{121}

5. All six digits of three 2-digit numbers are different. What is the largest possible sum of three such numbers?

A 237 B 246 C 255 D 264 E 273

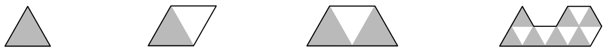
6. What is the sum of the digits of the largest 4-digit palindromic number which is divisible by 15? [Palindromic numbers read the same backwards and forwards, e.g. 7227.]

A 18 B 20 C 24 D 30 E 36

7. Given that $x + y + z = 1$, $x + y - z = 2$ and $x - y - z = 3$, what is the value of xyz?

A -2 B $-\frac{1}{2}$ C 0 D $\frac{1}{2}$ E 2

8. The diagrams below show four types of tile, each of which is made up of one or more equilateral triangles. For how many of these types of tile can we place three identical copies of the tile together, without gaps or overlaps, to make an equilateral triangle?



A 0 B 1 C 2 D 3 E 4

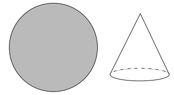
9. Pierre said, "Just one of us is telling the truth". Qadr said, "What Pierre says is not true". Ratna said, "What Qadr says is not true". Sven said, "What Ratna says is not true". Tanya said, "What Sven says is not true".
How many of them were telling the truth?

A 0 B 1 C 2 D 3 E 4

10. Let N be the smallest positive integer whose digits add up to 2012. What is the first digit of $N + 1$?

A 2 B 3 C 4 D 5 E 6

11. Coco is making clown hats from a circular piece of cardboard. The circumference of the base of each hat equals its slant height, which in turn is equal to the radius of the piece of cardboard. What is the maximum number of hats that Coco can make from the piece of cardboard?



A 3 B 4 C 5 D 6 E 7

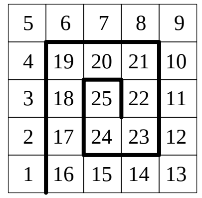
12. The number 3 can be expressed as the sum of one or more positive integers in four different ways:

$$3; \quad 1 + 2; \quad 2 + 1; \quad 1 + 1 + 1.$$

In how many ways can the number 5 be so expressed?

A 8 B 10 C 12 D 14 E 16

13. A cube is placed with one face on square 1 in the maze shown, so that it completely covers the square with no overlap. The upper face of the cube is covered in wet paint. The cube is then 'rolled' around the maze, rotating about an edge each time, until it reaches square 25. It leaves paint on all of the squares on which the painted face lands, but on no others. The cube is removed on reaching the square 25. What is the sum of the numbers on the squares which are now marked with paint?



A 78 B 80 C 82 D 169 E 625

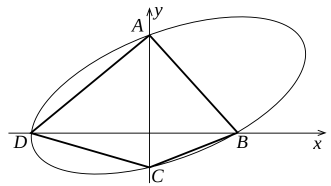
14. Six students who share a house all speak exactly two languages. Helga speaks only English and German; Ina speaks only German and Spanish; Jean-Pierre speaks only French and Spanish; Karim speaks only German and French; Lionel speaks only French and English whilst Mary speaks only Spanish and English. If two of the students are chosen at random, what is the probability that they speak a common language?

A $\frac{1}{2}$ B $\frac{2}{3}$ C $\frac{3}{4}$ D $\frac{4}{5}$ E $\frac{5}{6}$

15. Professor Rosseforp runs to work every day. On Thursday he ran 10% faster than his usual average speed. As a result, his journey time was reduced by x minutes. How many minutes did the journey take on Wednesday?

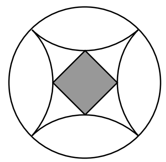
A $11x$ B $10x$ C $9x$ D $8x$ E $5x$

16. The diagram shows the ellipse whose equation is $x^2 + y^2 - xy + x - 4y = 12$. The curve cuts the y-axis at points A and C and cuts the x-axis at points B and D. What is the area of the inscribed quadrilateral $ABCD$?



A 28 B 36 C 42 D 48 E 56

17. The diagram shows a pattern found on a floor tile in the cathedral in Spoleto, Umbria. A circle of radius 1 surrounds four quarter circles, also of radius 1, which enclose a square. The pattern has four axes of symmetry. What is the side length of the square?



A $\dfrac{1}{\sqrt{2}}$ B $2-\sqrt{2}$ C $\dfrac{1}{\sqrt{3}}$ D $\dfrac{1}{2}$ E $\sqrt{2}-1$