Common Language and Methodology for Teaching Numeracy St Ninian's Cluster

Contents

Writing and Reading Numbers

Students need to be encouraged to write figures simply and clearly. Most students are able to write numbers up to a thousand in words, but there are often problems with bigger numbers. It is now common practice to use spaces between each group of three figures in large numbers rather than commas. e.g. 34 000 not 34,000.

In reading large numbers students should apply their knowledge of place value working from the *right* in groups of three digits. So the first group contains hundreds, tens and units. This is repeated in the next group as thousands and the next group as millions. The number is then read from left to right. So 2 084 142 is two million, eighty-four thousand, one hundred and forty-two.

Language of Operations

Students should have an understanding of different terms for the four basic operations as they may encounter a variety of vocabulary in maths word problems. Some students with additional support needs may have difficulty in associating terms with symbols.

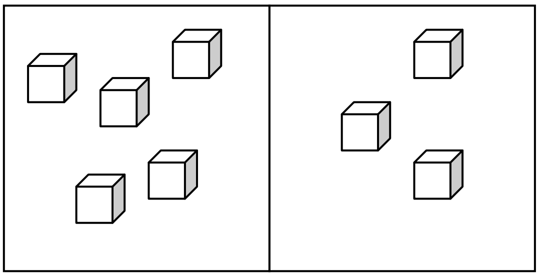
$+$	$-$	$\times$	$\div$
add increase more plus sum total	decrease difference less minus reduce subtract take	multiply of product times	divide quotient share equally per
Early Level language listed in the East Renfrewshire skills framework.			
and	take away		
makes	less than		
more than			
altogether			
one more	*one less*		

Although children should have an understanding of different vocabulary for operations, when teaching formal calculations, teachers should use the following vocabulary:
add, subtract, multiply and divide.

<u>Early Level</u>

Addition

Students gain an understanding of the concept of addition through the use of concrete materials. Through doing this, students should understand and use the language of addition.



- 5 **and** 3 **makes** 8 cubes **altogether**.
- If I **add** 3 more, how many will that **make**?
- We have 5 cubes on this side and 3 cubes on the other side, how many do we have **altogether**?
- Which side has **more than** 4 cubes?

Number bonds to 10

e.g:
1 + 9 = 10
2 + 8 = 10 etc.

Commutative Law

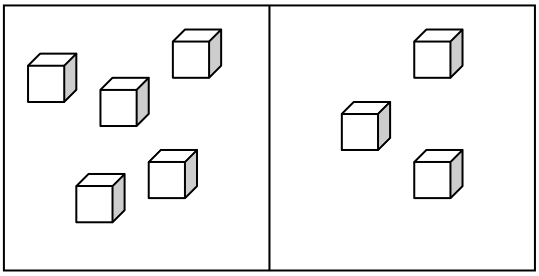
3 + 4 = 4 + 3

Associative Law

2 + 4 + 3 = 6 + 3 = 9
2 + 4 + 3 = 2 + 7 + 9

Subtraction

As with addition, students gain an understanding of the concept of subtraction through the use of concrete materials. Through doing this, students should understand and use the language of subtraction.



- 8 – 3 = 5 (8 **minus** 3 equals 5)
- We have 8 cubes. How many will we have if we **take away** 3?
- Which pile of cubes has **less than** 4?

Order of Operations

When carrying out a series of arithmetic operations students can be confused about the order in which the operations should be done.

e.g. Does $6 + 5 \times 7$ mean 11×7 or $6 + 35$?

Some students will be familiar with the mnemonic BODMAS

Brackets, **O**rder, **D**ivision, **M**ultiplication, **A**ddition, **S**ubtraction

The important *facts* are that brackets are done first, then orders, multiplication and division and finally addition and subtraction.

Example 1	**Example 2**	**Example 3**	**Example 4**
$6 + 5 \times 7$	$(6 + 5) \times 7$	$3 + 4^2 \div 8$	$2 \times 4 - 3 \times 4$
$= 6 + 35$	$= 11 \times 7$	$= 3 + 16 \div 8$	$= 8 - 12$
$= 41$	$= 77$	$= 3 + 2$	$= -4$
		$= 5$	

Some non-scientific calculators will give incorrect answers because they do operations in the order in which they are put into the calculator rather than following the BODMAS order.

It should be highlighted to students that division and multiplication are inter-changeable and so are addition and subtraction. This is particularly important for examples such as the following:

Example

$10 - 3 + 4$	In examples like this, go with the order of the question
$= 11$	i.e. subtract 3 from 10 then add 4.

Calculations

Many students are over-dependent on the use of calculators for simple calculations. Whenever possible they should be encouraged to do calculations either mentally or using pencil and paper methods. Some consideration needs to be given to the ability of the students. It is important to provide opportunities to develop basic numeracy skills. Students with learning difficulties in particular are unlikely to develop basic calculation skills, unless they are encouraged to remember number facts and practise processes. However, it is also necessary to enable students to complete tasks successfully and with some students, this may mean that calculators are needed for relatively simple questions if progress is to be made. Before completing any calculation, students should be expected to estimate an answer to the question. Having completed the calculation, they should then consider whether the answer is similar to the estimate and whether the answer seems reasonable in the context of the question.

Estimate, Calculate, Check

Writing Calculations

Students have a tendency to use the '=' sign incorrectly and write mathematical expressions that do not make sense.

e.g. $3 \times 2 = 6 + 4 = 10 - 7 = 3$ ✗

It is important that all teachers encourage students to write such calculations correctly.

e.g.

$$3 \times 2 = 6$$
$$6 + 4 = 10$$
$$10 - 7 = 3$$

✓

Encourage students to take a separate line for each new calculation.

The '=' sign should only be used when both sides of the equation have the same value. There is no problem with calculations such as:

$34 + 28 = 30 + 20 + 4 + 8 = 50 + 12 = 62$

because each part of the equation has the same value.

Mental Calculations

All students-should be able to carry out the following processes mentally, although some will need further thinking time

- recall addition and subtraction facts up to 20.
- recall multiplication and division facts for multiplication tables up to 10×10

Students should also be encouraged to carry out other calculations mentally using a variety of strategies but there will be significant differences in their ability to do so.

e.g.

$34 + 28 = 34 + 30 - 2$

$163 - 47 = 163 - 50 + 3$

` $124 - 41 = 124 - 40 - 1$

$23 \times 3 = (20 \times 3) + (3 \times 3)$

$68 + 4 = (68 + 2) + 2$

Students should be encouraged to discuss how they might do mental calculations such as these. A variety of strategies should be discussed. Any valid method that produces a correct answer is acceptable.

Pencil and Paper Calculations

All students should be able to use pencil and paper methods for calculations involving simple addition, subtraction, multiplication and division. Some of the less able students will have difficulty with multiplication and division because of lack of familiarity with multiplication tables and multiplication and division by two digit numbers may be a particular problem for these students.

Before doing a calculation, students should be encouraged to have a rough idea of the answer they expect to get. This can be done by rounding numbers and mentally calculating answers to provide an estimated result. After completing the question, they should also be asked to consider whether or not their answer is sensible in the context of the question.

Generally any method of calculation which a student is able to do correctly is appropriate. There is no necessity to stick to particular methods, but it can cause problems if different teachers expect students to carrying out calculations in different ways.

The following methods are used by the majority of students, and are generally taught to those students who do not have a method that they can carry out correctly.

Addition and Subtraction

3 456 + 975

Estimate

3 500 + 1 000 = 4 500

$$\begin{array}{rcccc} & 3 & 4 & 5 & 6 \\ + & {}_{1} & 9_{1} & 7_{1} & 5 \\ \hline & 4 & 4 & 3 & 1 \end{array}$$

6 286 – 4 857

Estimate

6 300 – 4 900 = 1 400

$$\begin{array}{rcccc} & {}^{5}\cancel{6} & {}^{1}2 & {}^{7}\cancel{8} & {}^{1}6 \\ - & 4 & 8 & 5 & 7 \\ \hline & 1 & 4 & 2 & 9 \end{array}$$

Note the position of the numbers when carrying. Small numbers should be written above the line.

Addition and subtraction of decimals are done using the same methods, but students may need to be reminded that it is necessary to ensure that the decimal points must be underneath each other.

Multiplying and Dividing by 10, 100, 1000 . . .

The rule for multiplying by 10 is that each of the digits moves one place to the left. When multiplying by 100 each digit moves two places to the left and so on. In division, the digits move to the right. This rule works for whole numbers and decimals. ***Decimal points do not move***.

e.g.

$23 \times 100 = 2\ 300$

Th	H	T	U
		2	3
2	3	0	0

Zeros are needed to fill the empty spaces in the tens and units columns, otherwise when the number is written without the column headings it will appear as a different number, i.e. 23 instead of 2 300

$3{\cdot}45 \times 10 = 34{\cdot}5$

Th	H	T	U	.	$\frac{1}{10}$	$\frac{1}{100}$
			3	.	4	5
		3	4	.	5	

Zeros are not generally needed in empty columns after the decimal point except in cases where a specified degree of accuracy is required, (significant figures).

$260 \div 10 = 26$

H	T	U	.	$\frac{1}{10}$	$\frac{1}{100}$
2	6	0	.		
	2	6	.		

$439 \div 100 = 4{\cdot}39$

H	T	U	.	$\frac{1}{10}$	$\frac{1}{100}$
4	3	9	.		
		4	.	3	9

Using the rule of adding or removing zeros can be confusing because it only works for some numbers and therefore should be avoided.

Multiplication and Division

357 × 8

Estimate

$350 \times 2 \times 4 = 700 \times 4 = 2\ 800$

$$\begin{array}{r} 3\ \ 5\ \ 7 \\ \times\ \ \ \ {}_{4}\ \ {}_{5}\ \ 8 \\ \hline 2\ \ 8\ \ 5\ \ 6 \end{array}$$

810 ÷ 6

Estimate

$800 \div 5 = 160$

$$\begin{array}{r|l} & 1\ \ \ 3\ \ \ 5 \\ \hline 6 & 8\ \ {}^{2}1\ \ {}^{3}0 \end{array}$$

329 × 42

Estimate
300 × 40 = 12 000

$$
\begin{array}{ccccc}
 & & 3 & 2 & 9 \\
 & & \times & 4_1 & 2 \\
\hline
 & {}_1 & 6_3 & 5 & 8 \\
1 & 3 & 1_1 & 6 & 0 \\
\hline
1 & 3 & 8 & 1 & 8 \\
\hline
\end{array}
$$

1 768 ÷ 52

Estimate
2 000 ÷ 50 = 40

$$
\begin{array}{r|cccc}
 & & & 3 & 4 \\
\cline{2-5}
52 & 1 & 7 & 6 & {}_{\times}8 \\
 & 1 & 5 & 6 & \\
\cline{2-4}
 & & 2 & 0 & 8 \\
 & & 2 & 0 & 8 \\
\cline{3-5}
 & & & & 0 \\
\end{array}
$$

If, when dividing, there is a remainder, the calculation should be continued to give a decimal answer.

e.g.
$$
\begin{array}{r|ccccccc}
 & & 3 & 4 & \cdot & 6 & 2 & 5 \\
\cline{2-8}
8 & 2 & 7 & 7 & \cdot & 0 & 0 & 0 \\
\end{array}
$$

When multiplying decimal numbers:

(*i*) Estimate the answer.
(*ii*) Complete the calculation as if there are no decimal points.
(*iii*) Insert a decimal point in the answer so that there are the same number of digits on the right of the decimal point in the answer as there are in the question.
(iv) Check that the answer is similar to the estimate and, where appropriate, that the answer is reasonable in the context of the question.

Example

2·3 × 4·1 We take out the decimal points and complete a long multiplication

$$
\begin{array}{cccc}
 & & 2 & 3 \\
 & \times & 4 & 1 \\
\hline
 & & 2 & 3 \\
+ & 9 & {}^{1}2 & 0 \\
\hline
 & 9 & 4 & 3 \\
\hline
\end{array}
$$

Each of the numbers (2·3 and 4·1) have 1 decimal place, therefore the answer will have 2 decimal places.

So, 2·3 × 4·1 = 9·43

When dividing a decimal number by a whole number, the calculation is the same as a division without decimal points, with the decimal point in the answer being inserted above the decimal point in the question.

Example

$$\begin{array}{r|l} & 4\cdot 1\;\,3 \\ \hline 4 & 1\,{}^{1}7\cdot 2\;\,4 \end{array}$$

The decimal point should be put in the same place as it is in the question.

Using a Calculator

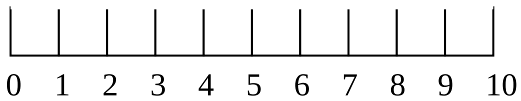
Many students believe that if they use a calculator, the answer they obtain must be correct. It is important that the students are encouraged to make an estimate of the answer they expect to get and, having completed the calculation, compare the answer with the estimate. They also need to consider whether the answer they have got makes sense in the context of the question. Students should always be encouraged to write down the calculation they intend to enter on their calculator rather than simply write down an answer on its own. Exam marks may be lost if only the answer is shown with no indication of how it was arrived at!

Early Level

When comparing numbers students must use and understand the terms:

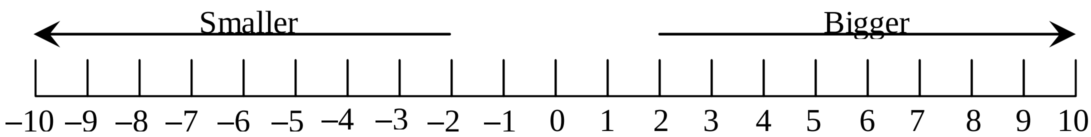
- **smaller/smallest**
- **bigger/biggest**

Students will also use and understand the terms **before** and **after.**



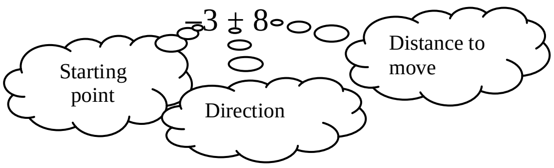
Negative Numbers

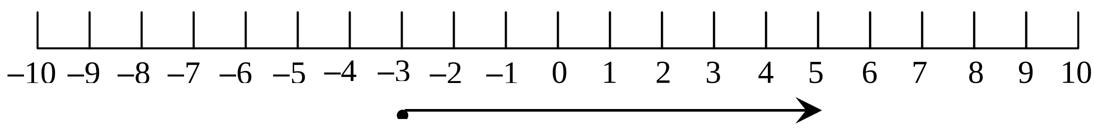
Some students have problems understanding the size of negative numbers and believe that –10 (read as '*negative 10*') is bigger than –5. A number line can be of help when ordering numbers. Moving to the right, numbers getting bigger. Moving to the left, numbers getting smaller.



Students in lower ability groups may have difficulty with calculations involving negative numbers. A number line is useful for addition and subtraction.

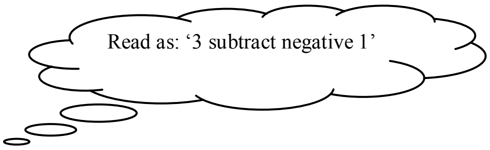
e.g. $-3 + 8$





If there are two signs between the numbers, when the signs are the same, the move is to the right and when the signs are different, the move is to the left. This concept is introduced through pattern recognition as shown overleaf.

$$3 - 3 = 0$$
$$3 - 2 = 1$$
$$3 - 1 = 2$$
$$3 - 0 = 3$$
$$3 - (-1) = 4$$
$$3 - (-2) = 5$$



Care should be taken with the use of language. 'Negative' is used as an adjective to describe the sign of the number. 'Subtract' is used as a verb, indicating the operation.

<u>Example 1</u>

$(-1) + (-4)$
$= (-1) - 4$
$= -5$

<u>Example 2</u>

$5 - (-12)$
$= 5 + 12$
$= 17$

When multiplying positive and negative numbers, if both numbers are positive or both numbers are negative, the answer will be positive. If one of the numbers is positive and the other is negative, the answer will be negative.

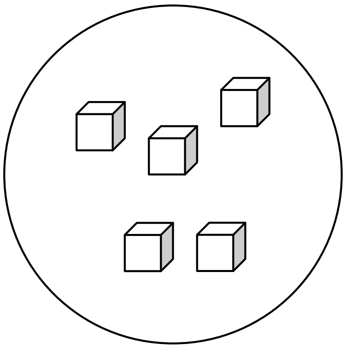
(i) $(-3) \times (-2)$
$= 9$

(ii) $(-8) \div 4$
$= -2$

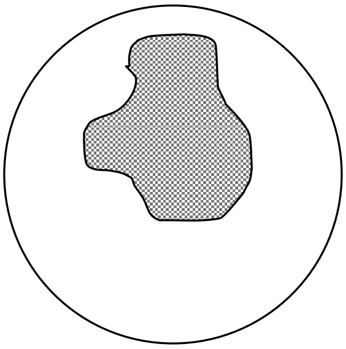
Common Fractions

<u>Early Level</u>

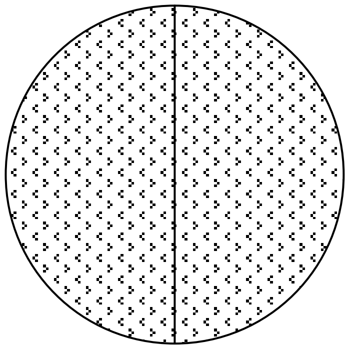
Fractions should be introduced using concrete objects.



Giving out individual items from a set:
- "give a cube to all children at the table"



Split a whole object into smaller parts:
- "share out the dough"



Split a whole object into a specified number of pieces/understand the concept of half:
- "cut the pizza into 2 equal pieces"/ "cut the pizza in half"

From Second Level

Most students should be able to calculate simple fractions of quantities. They should know that to find half of an amount they need to divide by 2. To find a third of something they divide by 3 etc..

They should know that the denominator (bottom number of the fraction) indicates the number of parts the amount is to be divided into and the numerator (top number) indicates the number of parts required.

<u>Example</u>

To find $\frac{3}{4}$ of £24, we start by finding $\frac{1}{4}$ first and then multiply the answer by 3.

Layout: $\frac{1}{4}$ of £24 = £6

$\frac{3}{4}$ of £24 = £6 × 3 = £18

Generally, students should be able to do this type of calculation mentally, but some may need prompting in order to do so. The rule divide by the bottom number and multiply by the top number does work, but if this rule is used without understanding, it can cause problems because students tend to forget which numbers they use to divide or multiply.

Most students should be able to make equivalent fractions by multiplying or dividing the numerator and denominator of a fraction by the same number.

e.g. (*i*) $\frac{3}{4} \xrightarrow{\times 2} \frac{6}{8}$, i.e. $\frac{3}{4} = \frac{6}{8}$ (numerator and denominator $\times 2$)   (*ii*) $\frac{10}{15} = \frac{2}{3}$ (numerator and denominator $\div 3$)

Ratio and Proportion

Students may have difficulty in understanding and using ratio and proportion.

Ratios compare numbers or measurements.

For example, when making pastry the ratio of fat to flour is 1: 2. This means that for every gram of fat used, 2 grams of flour are required, or for every ounce of fat there must be 2 ounces of flour.

Ratios can be simplified in the same way as fractions by dividing all of the parts of the ratio by the same number.

e.g. The ratio 3 : 12 : 6 can be simplified by dividing by 3 giving 1 : 4 : 2

If an amount is to be split in a given ratio, it is first necessary to begin by finding out how many parts are needed.

e.g. Three friends win £900 in a competition. They share it in the ratio 2 : 3 : 4.
How much does each person receive?

The three people have a total of 2 + 3 + 4 = 9 shares.

£900 ÷ 9 = £100 so the amount of each share is £100.

Layout: 9 shares = £900
1 share = £900 ÷ 9 = £100

2 shares = £200
3 shares = £300
4 shares = £400

Ratios are often used to show the scale of a map or plan.

e.g. If the scale of a map is 1: 2 000 this means that 1 centimetre on the map represents a distance of 2 000 centimetres on the ground.

2 000 centimetres = 20 metres

Students may experience difficulties in adapting recipes to produce larger or smaller quantities. They may need practice in modifying recipes and should be encouraged to adopt a method that is suitable for the particular problem.

If a recipe for four people is to be modified to cater for five people it may be easiest to find the quantities required for one person and multiply these by five, (sometimes referred to as the 'unitary method'). In other cases it will be sensible to use other methods to adjust quantities.

e.g. The following recipe will produce 16 flapjacks:
40 grams of margarine
50 grams of sugar
200 grams of rolled oats
100 grams of golden syrup.

If 24 flapjacks are required, it is probably easiest to find the recipe for 8 flapjacks (by halving the given quantities) and add these amounts to the original recipe.

Students should be encouraged to explore different methods in such calculations.

Decimal Notation

All students should be familiar with decimal notation for money although they may use incorrect notation.

✓	✗
£2·35	£2·35p
£4·60	£4·6
£0·25	0·25p
32p	0·32p

Students should also be familiar with the use of decimal notation for metric measures, but sometimes misinterpret the decimal part of the number. They may need to be reminded for example that 1·5 metres is 150 centimetres not 105 centimetres.

Students often read decimal numbers incorrectly e.g. 8·72 is often read as eight point seventy two instead of eight point seven two.

They may also have problems with comparing the size of decimal numbers and may believe that 2·36 is bigger than 2·8 because 36 is bigger than 8. If they need to compare numbers it may help to write all of the numbers to the same number of decimal places e.g. 2·36 and 2·80.

Percentages

Calculating Percentages of a Quantity

Methods for calculating percentages of amounts vary depending on the percentage required. Students should know that fractions, decimals and percentages are different ways of representing part of a whole and know simple equivalents.

e.g. $10\% = \frac{1}{10}$ $12\% = 0{\cdot}12$

Where percentages have simple fraction equivalents, fractions of amounts can be calculated.

e.g. (i) To find 50% of an amount, halve the amount.
(ii) To find 75% of an amount find a quarter of the amount and multiply it by three.

10% of an amount can be calculated by dividing by 10. Amounts that are multiples of 10 can then be calculated from this.

e.g. 30% is $3 \times 10\%$ 5% is half of 10%.

When using a calculator or computer it is usually easiest to think of a percentage as a decimal number and 'of' as '×'.

e.g. 17·5% of £36 = 0·175 × £36 = £6·30

By S3, National 5 students should be able to increase or decrease amounts using one operation. This is particularly useful in spreadsheets.

e.g. (*i*) Increase 52 by 14% $52 \times 1{\cdot}14 = 59{\cdot}28$
(*ii*) Decrease 175 by 30% $175 \times 0{\cdot}70 = 122{\cdot}5$

Multiplying by the percentage and dividing by 100 is a method used by some students but this method is not encouraged, since students often forget which numbers they are multiplying or dividing by and the method can be unnecessarily difficult for some calculations.

Calculating an Amount as a Percentage

The most straightforward way to do this is to think of the problem as a common fraction and then to convert this to a percentage if the fraction is simple. For more complex questions the fraction can be converted to a decimal number and then a percentage.

e.g. (*i*) What is 20 as a percentage of 80? $\frac{20}{80}=\frac{1}{4}=25\%$

(*ii*) What is 43 out of 50 as a percentage? $\frac{43}{50}=\frac{86}{100}=86\%$

(*iii*) What is 123 as a percentage of 375? $\frac{123}{375}=123 \div 375=0\cdot328=32\cdot8\%$

Rounding Numbers

Most students should be able to round numbers to the nearest 10, 100 or 1 000 without much difficulty. They should also be able to round to a given number of decimal places. Rounding to a given number of significant figures is likely to be more difficult, (Third Level/Fourth Level/National 5). When rounding numbers, **digits below 5 round down and digits of 5 or above round up.**

2 3 $\longrightarrow$ 20 (to the nearest ten)

4 5 $\longrightarrow$ 50 (to the nearest ten)

Below 5 round down 5 or more round up

Round 1·32 to 1d.p: 1 · 3 ⋮ 2 1 · 3 (to 1 d.p.)

Draw a dotted line after the number in the decimal place you are rounding to. Circling the number in the next decimal place will help students to decide whether to round up or down.

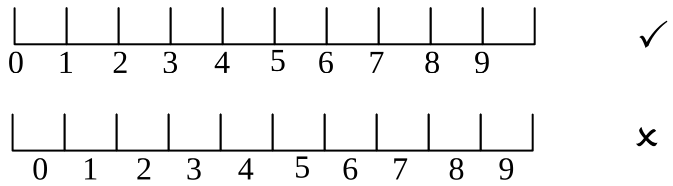
Standard Form (Scientific Notation)

Only students working at Fourth Level towards the end of S2 and National 4 students at the end of their course in S4 are likely to be familiar with standard form or scientific notation, (both terms are in common use). They should be able to use calculators for standard form. Students often have difficulty in interpreting the result of a standard form calculation on some calculators and need to be encouraged to write for example $3\cdot42 \times 10^{8}$ and **not** 3·42 08, (i.e. the calculator display)

Plotting Points and Drawing Graphs

When drawing a diagram on which points are to be plotted, some students will need to be reminded that numbers on the axes are written on the lines not in the spaces.

e.g.



When drawing graphs for experimental data it is customary to use the horizontal axes for the variable which has a regular interval.

e.g. (*i*) In an experiment in which the temperature is taken every 5 minutes the horizontal axis would be used for time and the vertical axis for temperature.

(*ii*) If the depth of a river is measured every metre, the horizontal axis would be used for the distance from the bank and the vertical axis for the depth.

Having plotted points correctly students can be confused about whether they should join the points or draw a line of best fit. Generally when the students have calculated the points from an algebraic expression the points should be joined. If the points have been obtained through experiment, a line of best fit is probably needed. Scatter diagrams are used to compare two sets of data. Points on a scatter diagram should not be joined. A line of best fit is used to indicate the trend of the data.

Length, Mass and Capacity

<u>Early Level</u>

Students will gain an understanding of measure through the use of non-standard units such as hands, cup fulls and dried peas.

Students should describe length using the following language:

- long/longer/longest
- short/shorter/shortest

Students should describe capacity/volume using the following language:

- full and empty
- holds more/most
- holds less/least

* students should be able to understand the concept of volume with regards to measuring liquids and solids.

Students should describe mass/weight using the following language:

- heavy/heavier/heaviest
- light/lighter/lightest

When comparing length, mass and capacity students should also use the term "about the same".

All students should be familiar with metric units of length, mass and capacity but some students will have difficulty with notation and converting from one unit to another.

They should know the following facts:

Length	Mass	Capacity
10 mm = 1 cm	1 000 g = 1 kg	1 000 ml = 1 litre
100 cm = 1 m	1 000 kg= 1 tonne	1 cm^3 = 1 ml
1 000 m = 1 km		

Students are encouraged to develop an awareness of the sizes of units and an ability to make estimates in everyday contexts. They are also expected to consider the appropriateness of their solution to a problem in the particular context of the question.

Time

<u>Early Level</u>

Time vocabulary should include:

- day, night, morning, afternoon
- before and after
- yesterday and tomorrow

Students should understand and use the term "o'clock"

From First Level

All students should be able to tell the time, but some students with additional support needs may experience problems in using analogue clocks and equating analogue and digital times. Some students will need help with problems involving time.

e.g. A recipe says that a cake should be in the oven for 35 minutes. If it is put into the oven at 10.50 am, when will it be ready?

With students who have difficulty this is usually best approached by splitting up the time.

e.g. 10.50 + 10 mins $\rightarrow$ 11.00

11.00 + 25 mins $\rightarrow$ 11.25

Most students should be familiar with 12 and 24 hour clock times, but support may be necessary when interpreting timetables.

Students sometimes use calculators to solve problems involving time and get incorrect answers because there are 60 and not 100 minutes in an hour.

Other versions include four figures without any punctuation.

e.g. 0650 or 1427.

24-hour clock times often appear on digital clocks with a colon between the hours and minutes.

e.g. **<u>06:50</u>** or **<u>14:27</u>**

12-hour clock times are usually written with a **dot** between the hours and the minutes.

e.g. 6.50 am or 2.27 pm.

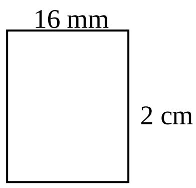
Reading Scales

Some students may have difficulty in reading scales on graphs. They are inclined to assume that one division on the scale represents one unit. It may help if they begin by counting the number of divisions between each number on the scale and then determine what each division of the scale represents.

Area and Volume

Problems can arise when calculating areas and volumes if the lengths given are not in the same units. It is usually easiest to begin by converting all of the units of length to the units that are required for the answer, before doing any calculation.

e.g.



Either

Area = $16 \times 20 = 320$ mm^2

or

Area = $1{\cdot}6 \times 2 = 3{\cdot}2$ cm^2

Note that "cm^2" should be read as "squared centimetres" and **not** "centimetres squared".
This is also the case for cm^3 i.e. "cubic centimetres" not "centimetres cubed".

Fractions

<u>Adding and Subtracting Fractions</u>

Adding and subtracting fractions should be introduced by showing visual examples in the first instance.

Visual examples help pupils to see that when adding fractions with a common denominator, we simply add the numerators together (or subtract them in the case of subtracting fractions).

e.g.

$$\frac{1}{5} \qquad \frac{2}{5} \qquad \frac{3}{5}$$

This also helps to emphasise to the students that the denominator does not change when adding fractions.

When adding fractions with a different denominator, we start by telling students to change the fractions into equivalent fractions so that they have a common denominator. They should realise from the previous examples that once they have a common denominator it is simple to add or subtract the numerators to get the final answer.

Example 1:

$$\frac{1}{4} + \frac{3}{8}$$

$$\times 2$$

$$= \frac{2}{8} + \frac{3}{8}$$

$$= \frac{5}{8}$$

Sometimes it is possible to turn one of the denominators into the other. In this case 8 is the lowest common denominator

Example 2:

$$\frac{1}{2} + \frac{1}{3}$$

$$\times 2 \qquad \times 3$$

$$= \frac{3}{6} + \frac{2}{6}$$

$$= \frac{5}{6}$$

We need to find the lowest common multiple of 2 and 3 in order to find the common denominator. In this case, the lowest common denominator is 6.

Example 3:

$$\frac{1}{4} - \frac{1}{5}$$

$$= \frac{5}{20} - \frac{4}{20}$$

$$= \frac{1}{20}$$

Lowest common multiple of 4 and 5 is 20

When adding and subtracting mixed fractions, we always change them to improper (“top-heavy”) fractions before adding/subtracting.

Example 3:

$$1\frac{1}{3} + 2\frac{2}{5}$$

$$= \frac{4}{3} + \frac{12}{5}$$

$$= \frac{20}{15} + \frac{36}{15}$$

$$= \frac{56}{15}$$

$$= 3\frac{11}{15}$$

Mixed fractions should be changed into improper fractions before finding a common denominator. In this case the lowest common multiple of 3 and 5 is 15.

<u>Multiplying and Dividing Fractions</u>

When multiplying fractions we multiply the numerators and multiply the denominators

e.g.

$$\frac{2}{3} \times \frac{1}{5} = \frac{2 \times 1}{3 \times 5} = \frac{2}{15}$$

$$\frac{3}{4} \times \frac{2}{5} = \frac{3 \times 2}{4 \times 5} = \frac{6}{20} = \frac{3}{10}$$

Simplify where possible

When multiplying mixed fractions, they should be turned into improper fractions first.

When introducing dividing fractions, a discussion should be held about what happens when you divide by a fraction e.g. What happens when you divide by a half?
Students should realise that dividing by a half is the same as multiplying by 2. This can promote further discussion of dividing by a third, a quarter etc.

Once students have the understanding of dividing fractions a quick method can be shown for dividing by any type of fraction.

When dividing fractions

- Flip the fraction you are dividing by upside down
- Multiply the fractions together
- Simplify where possible

e.g. $\frac{1}{3} \div \frac{2}{5}$

$= \frac{1}{3} \times \frac{5}{2}$

$= \frac{5}{6}$