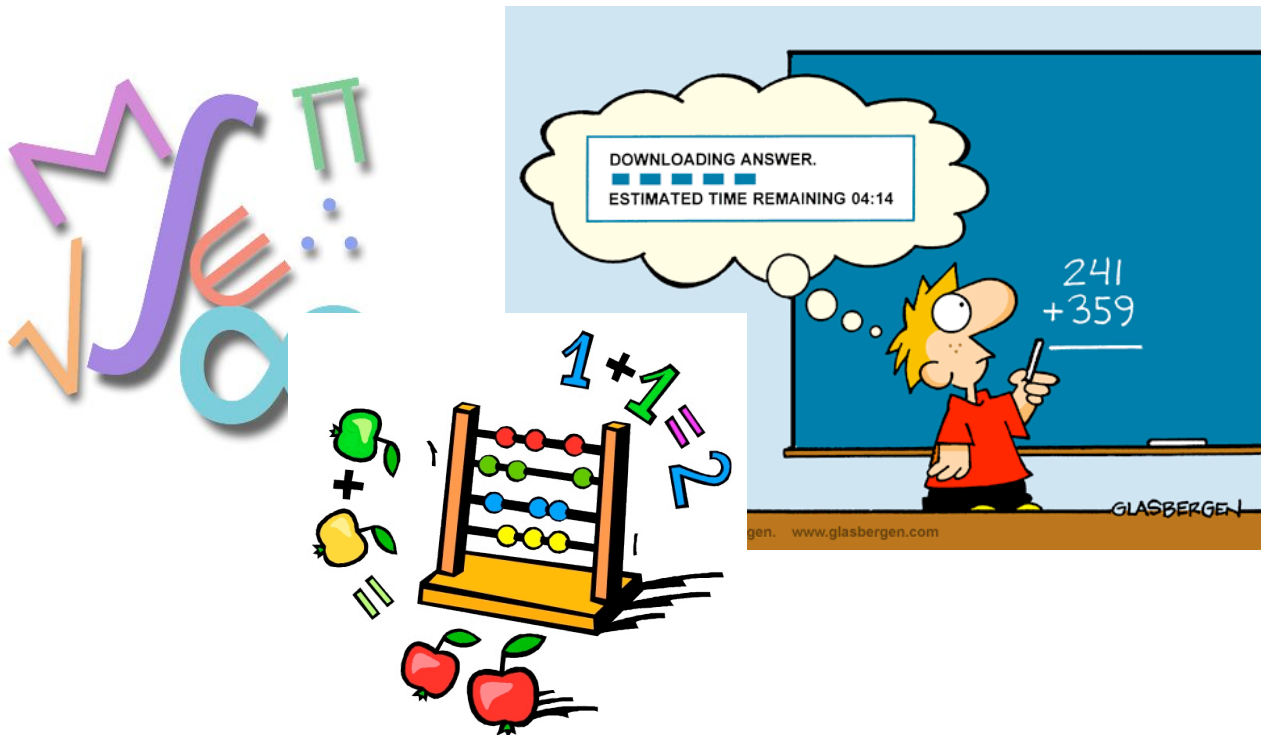


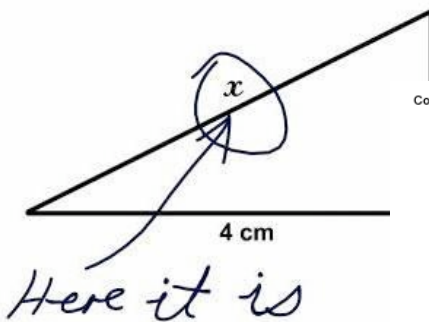
*'Mistakes are the portals to Discovery' James Joyce*

# Parents Guide to helping your child with Higher Maths



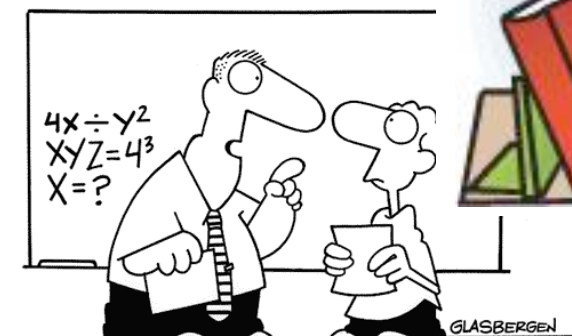
*"The essence of mathematics is not to make simple things complicated, but to make complicated things simple." S. Gudder*

3. Find x.



Ocular Trauma - by 1

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**"Algebra class will be important to you later in life because there's going to be a test six weeks from now."**

*"Arithmetic is being able to count up to twenty without taking off your shoes" M. Mouse*

## Help Your Child With Higher Maths

### Introduction

We've designed this booklet so that you can use it with your child throughout the session, as he/she moves through the Higher course, in order to help them remember key facts and methods. There are separate sections covering the three units of the Higher, as well as one on Credit/Intermediate 2 revision. Obviously there's no point trying to revise Unit 3 work when your child hasn't yet covered it in class, so it's worth giving a rough timetable for the course:

Unit 1 is typically completed by mid-October

Unit 2 is typically completed by January

Unit 3 is typically completed by April

(The section on Credit/Intermediate 2 revision is fair game at any time of the course – your child should already know it all!)

The booklet is not an exhaustive summary of the content of the Higher – a separate booklet is available for that, should you wish to read it! – nor is it meant to replace the set of much more detailed Higher notes which your child should be building up over the session. But the booklet does summarise the key facts and methods which your child will need to be familiar with, if they are to have a chance of passing the Higher. Regular revision of these facts and methods will pay real dividends: it is much better to continually revise throughout the year, than to attempt to “cram” it all in at the last minute.

### How to use the booklet

Your child could use this booklet on their own, but we think it would be much better if they had someone else to “test” them on the content. Basically, all you have to do is read out the question on the left-hand column of the page, and all they have to do is give the correct answer (more or less) which is shown on the right. If you prefer, you could simply show your child the answer and ask them what they think the question is.

So by the end of Unit 1 (mid-October), for example, you could test your child on the entire Unit 1 content. However we think it's much better to revise more regularly than that, so we'd suggest that you find out from your child which section they are on (or have already completed) and test them on that, at reasonably regular intervals. If you can do this then you will be making a real contribution to your child's knowledge of the course. In a sense, you will be helping them to remember their “lines” for the performance that will be the Higher Mathematics examination.

### If you lack confidence in maths yourself

Don't worry! We have tried to explain the terminology and notation as we go along, but if you are in doubt, then either simply show the question to your child instead or ask them to explain to you how to say it. You may find that some of your child's answers differ slightly from what is here: this may not necessarily mean that they are wrong, as different teachers will naturally teach things in slightly different ways. If in doubt, check with your child. If still in doubt, please feel free to get in touch with us at the school.

Thanks in advance for your help, and good luck!

Revision from Credit/Intermediate 2	
What is the Sine Rule?	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
What is the Cosine Rule?	$a^2 = b^2 + c^2 - 2bc \cos A$
What is the formula for the area of a triangle?	$Area = \frac{1}{2} ab \sin C$
How do you solve a quadratic equation?	Make one side zero, then factorise the other (or use the quadratic formula)
How do you factorise a quadratic?	Look for: 1. Common factor 2. Difference of squares 3. Double brackets
What is the quadratic formula?	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
How do you find the solution to a simple trig equation, with solutions from 0 to 360 degrees?	Find the acute angle then use an ASTC diagram to find the solutions (usually two)
How do you write $\sqrt{x}$ as a power of x?	$x^{\frac{1}{2}}$ (x to the power a half)
How do you write $\sqrt[n]{x^m}$ as a power of x? (the nth root of x to the power m)	$x^{\frac{m}{n}}$
What is $x^0$ ?	1
How do you write $x^{-n}$ with a positive power?	$\frac{1}{x^n}$

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Quadratic Functions	
How do you sketch a quadratic curve (parabola)?	<ol style="list-style-type: none"> <li>1. Find the shape – “happy” or “sad”?</li> <li>2. Find the roots (if they exist) – ie. where the curve cuts the x-axis (solve <math>y = 0</math>)</li> <li>3. Find where the curve cuts the y-axis (<math>x = 0</math>)</li> <li>4. Use symmetry to find the turning point (or use differentiation)</li> </ol>
<p>Completing the square: Why do we complete the square?</p> <p>What is the process for completing the square?</p> <p>What form must the expression be in before you can complete the square?</p>	<p>To allow us to make a quick sketch of the parabola, which allows us to find the turning point</p> <p>Identify the x-coefficient Halve it Square it Add it on/take it away</p> <p>OR: expand bracket and equate coefficients</p> <p>Must be <math>x^2 + \dots</math> and not <math>2x^2</math> etc, so take out a common factor if you have to</p>
<p>The discriminant: <b>What is the condition for...</b></p> <p>... equal roots?</p> <p>... two distinct real roots?</p> <p>... real roots?</p> <p>... non-real roots? (or no real roots)</p>	<p><math>b^2 - 4ac = 0</math></p> <p><math>b^2 - 4ac &gt; 0</math></p> <p><math>b^2 - 4ac \geq 0</math></p> <p><math>b^2 - 4ac &lt; 0</math></p>
How do you show that a line is a tangent to a curve?	Substitute the line into the curve and solve the equation to show that there are equal roots (or show that $b^2 - 4ac = 0$ )
What does it mean to say that a quadratic is irreducible?	It cannot be factorised

Polynomials	
How do you show that $x - a$ is a factor of $f(x)$ ?	Use synthetic division (with $a$ ) to show that the remainder is zero, or show that $f(a) = 0$
How do you factorise a cubic?	First find a linear factor, using synthetic division, then factorise the quadratic from the bottom row of the table.
How do you sketch the graph of a polynomial?	<ol style="list-style-type: none"><li>1. Find where the curve crosses the x-axis (<math>y = 0</math>) and the y-axis (<math>x = 0</math>)</li><li>2. Differentiate and solve <math>\frac{dy}{dx} = 0</math> to find the stationary points</li><li>3. Use a nature table to determine nature</li><li>4. Sketch the graph</li></ol>

Integration	
How do you integrate?	Increase the power by one, then divide by the new power
How do you prepare for integration?	Change any roots into powers $x$ must not be on the denominator (bottom) of any fraction Any pairs of brackets should be expanded
When integrating an indefinite integral (one with no limits), what must we always remember?	+C
Why do we integrate?	To find the area under a curve, or to recover $f(x)$ from $f'(x)$
What do we have to remember when the enclosed area is below the x-axis?	The answer will be negative, so we explain this fact and change the answer to positive
What do we have to remember when the area is partly above and partly below the x-axis?	We have to work out the areas separately (one above x-axis, one below) then add
How do we find the area between two curves or a line and a curve? How do we find where the curves meet?	$\int (\text{curve above} - \text{curve below}) dx$ Use $y = y$ and solve
What do we get if we integrate acceleration? What do we get if we integrate speed?	Speed Distance
Compound Angle Formulae	
$\cos(A + B) = ?$ $\cos(A - B) = ?$ $\sin(A + B) = ?$ $\sin(A - B) = ?$	$\cos A \cos B - \sin A \sin B$ $\cos A \cos B + \sin A \sin B$ $\sin A \cos B + \cos A \sin B$ $\sin A \cos B - \cos A \sin B$
When asked to find the exact value of sin, cos or tan, what should you look for? If a right-angled triangle is not involved, what should you do?	Right-angled triangles Try to make an expression up which involves right-angled triangles and exact values you know (eg 30, 45, 60 degrees)

<p>If you are give sin, cos or tan and told that the angle is acute  <math>(0^\circ &lt; x^\circ &lt; 90^\circ, 0 &lt; x &lt; \frac{\pi}{2})</math>, how can you find the other ratios as exact values?</p>	<p>Draw a right-angled triangle, use Pythagoras to find the missing side, then use SOHCAHTOA</p>
<p><math>\sin 2A = ?</math>  <math>\cos 2A = ?</math> (three possible answers)</p>	<p><math>2 \sin A \cos A</math>  <math>\cos^2 A - \sin^2 A</math>  <math>2 \cos^2 A - 1</math>  <math>1 - 2 \sin^2 A</math></p>
<p>How can you expand <math>\cos 4A, \sin 4A</math> etc?          How can you expand <math>\cos 3A</math> etc?</p>	<p>Write as <math>(2A+2A)</math> then expand using the formulae          Write as <math>(2A+A)</math> then expand using the formulae</p>
<p>When solving a trig equation, what two-step process should you follow?</p>	<p>Is it a "straight-forward solve"?          If not, then "double angle solve"</p>
<p>How do you recognise and solve a "straight-forward solve"?</p>	<p>sin, cos or tan appears once only          Solve to find acute angle, then use ASTC</p>
<p>How do you recognise and solve a "double-angle solve"?</p>	<p>Look for a double angle and a single angle (eg <math>2A</math> and <math>A</math>)          Replace the double-angle formula with an appropriate single expression, then make one side zero and factorise in order to solve</p>
<p>What should you always check at the end of a trig question?</p>	<p>Should the answer be given in degrees or radians?</p>



The Circle	
<p>What kind of circle has equation...</p> <p><math>x^2 + y^2 = r^2</math> ?</p> <p><math>(x - a)^2 + (y - b)^2 = r^2</math> ?</p> <p><math>x^2 + y^2 + 2gx + 2fy + c = 0</math> ?</p>	<p>centre (0, 0), radius <math>r</math></p> <p>centre <math>(a, b)</math>, radius <math>r</math></p> <p>centre <math>(-g, -f)</math>, radius <math>\sqrt{g^2 + f^2 - c}</math></p>
<p>How do you find the equation of a circle?</p> <p>Do you need to expand the brackets and tidy up your answer?</p>	<p>Find the centre and radius, then use <math>(x - a)^2 + (y - b)^2 = r^2</math></p> <p>No!!</p>
<p>How can you show that an equation does NOT represent a circle?</p>	<p>Try to find the radius – you should be left with the square root of a negative number, which is impossible, or zero</p>
<p>How do you find where a line meets a circle?</p>	<p>Rearrange the line into the form <math>y =</math> or <math>x =</math> (whichever is easier) then substitute this into the circle and solve</p>
<p>How do you show that a line is a tangent to a circle?</p>	<p>As above – you should find equal roots, ie only one point of contact (alternatively, show that <math>b^2 - 4ac = 0</math>)</p>
<p>How do you show that a line does not meet a circle at all</p>	<p>As above – this time show that there are no real roots, ie <math>b^2 - 4ac &lt; 0</math></p>
<p>How do you find the equation of a tangent to a circle?</p>	<p>Find the gradient of the radius</p> <p>Use <math>m_1 \times m_2 = -1</math> to find the gradient of the tangent</p> <p>Then use <math>y - b = m(x - a)</math></p>
<p>What is a common tangent?</p>	<p>A line which is a tangent to two circles</p>
<p>How do I show that two circles touch externally?</p>	<p>Show that the distance between the two centres is equal to the sum of the two radii</p>
<p>What is meant by congruent circles?</p> <p>What is meant by concentric circles?</p>	<p>Circles that are the same size</p> <p>Circles with the same centre</p>

Vectors	
What is the difference between a vector and a scalar?	A vector has magnitude (size) and direction, whereas a scalar only has magnitude
What is meant by giving a vector in component form?	Writing the answer as a column vector with brackets, eg $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$
How would you write this in i, j, k form?	$a\underline{i} + b\underline{j} + c\underline{k}$
How do you find the magnitude (length) of vector $\underline{u} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$ ?	$ \underline{u}  = \sqrt{a^2 + b^2 + c^2}$
How do you find vector $\overrightarrow{AB}$ ?	$\overrightarrow{AB} = \underline{b} - \underline{a}$
How do you show that two vectors are parallel?	Show that one vector is a multiple of the other
If point P divides AB in the ratio m:n, how do you find the coordinates of P?	Use the Section Formula: $\underline{p} = \frac{1}{m+n}(n\underline{a} + m\underline{b})$ and then write out the coordinates of P (or use ratios to create an equation and solve)
What are the two forms of the scalar (or dot) product?	$\underline{a} \cdot \underline{b} =  \underline{a}  \underline{b} \cos\theta$ (for this version, remember that the vectors must NOT be "nose-to-tail") $\underline{a} \cdot \underline{b} = a_1b_1 + a_2b_2 + a_3b_3$
How do you find the angle between two vectors?	Use the dot product and solve to find $\theta$ - or use formula $\cos\theta = \frac{\underline{a} \cdot \underline{b}}{ \underline{a}  \underline{b} }$
How do you show that two vectors are perpendicular?	Show that $\underline{a} \cdot \underline{b} = 0$
Useful rules: $\underline{a} \cdot \underline{a} = ?$ $\underline{a} \cdot (\underline{b} + \underline{c}) = ?$	$ \underline{a} ^2$ $\underline{a} \cdot \underline{b} + \underline{a} \cdot \underline{c}$

Further Calculus	
What do you get if you differentiate: $\sin x$ ? $\cos x$ ?	$\cos x$ $-\sin x$
What do you get if you integrate: $\sin x$ ? $\cos x$ ?	$-\cos x + c$ $\sin x + c$
What is the chain rule for differentiation? (How do you differentiate $f(g(x))$ ?)	$f'(g(x)) \times g'(x)$ (differentiate around the brackets, then multiply by the derivative of what is inside the brackets)
What do you get if you differentiate: $\sin(ax + b)$ ? $\cos(ax + b)$ ?	$a \cos(ax + b)$ $-a \sin(ax + b)$
What do you get if you integrate: $\sin(ax + b)$ ? $\cos(ax + b)$ ?	$-\frac{1}{a} \cos(ax + b) + c$ $\frac{1}{a} \sin(ax + b) + c$
What you get if you integrate $(ax + b)^n$ ?	$\frac{(ax + b)^{n+1}}{a(n+1)} + c$

The Wave Function	
How do you express $a\cos x + b\sin x$ in the form $k\cos(x \pm \alpha)$ or $k\sin(x \pm \alpha)$ ?	Expand the brackets Equate coefficients Solve to find $k$ (square and add to get $k^2$ ) Solve to find $\alpha$ (divide to get $\tan \alpha$ )
How do you know which quadrant $\alpha$ is in?	Look at the signs for $k\cos \alpha$ and $k\sin \alpha$ - if both are positive then $\alpha$ is acute, otherwise you need to do an ASTC diagram
Given the choice, which version of the wave function should you use?	If it starts with $\cos$ , use $k\cos(x \pm \alpha)$ If it starts with $\sin$ , use $k\sin(x \pm \alpha)$ Use the version which keeps both coefficients positive, if possible
How do you find the maximum or minimum values of a wave function	Think of the graph: when is $\cos$ (or $\sin$ ) at a maximum or minimum, then adjust as necessary
How do you solve $a\cos x + b\sin x = c$ ?	Put the left-hand side into a wave function form, then solve in the usual way
What if the question has $2x$ or $3x$ etc?	You still solve the problems in the usual way with $k$ and $\alpha$ found as before – but at the end you will need to divide any answers to find $x$

Exponential and Logarithmic Functions	
What points does the graph of $y = a^x$ always pass through? (y equals a to the power x)	(0,1) and (1, a)
What points does the graph of $y = \log_a x$ always pass through? (y equals the log of x, base a)	(1,0) and (a,1)
How do you rewrite $y = \log_a x$ in power form?	$x = a^y$
How do you solve an equation where x is the power? (eg $4^x = 10$ )	Take logs of both sides then use log rules to work out $x$
How do you solve a log equation?	Express each side as a single log then "cancel" the logs  Or, get logs to one side and numbers to the other, then rewrite using power form
Log rules: $\log x + \log y = ?$ $\log x - \log y = ?$ $\log x^n = ?$ $\log_a 1 = ?$ $\log_a a = ?$	$\log xy$ $\log \frac{x}{y}$ $n \log x$ 0 1
How do you get log to base e (the natural log) on your calculator?	ln button
How do you get log to base 10 on your calculator?	log button
If the graph of $\log y$ against $\log x$ is a straight line, how do you find $y$ in terms of $x$ ?	$y = kx^n$ The values of $k$ and $n$ can be found from the graph
If the graph of $\log y$ against $x$ is a straight line, how do you find $y$ in terms of $x$ ?	$y = ab^x$ The values of $a$ and $b$ can be found from the graph