Credit Mathematics - Practice Examination D

Please note … the format of this practice examination is different from the current format. The paper timings are different and calculators can be used throughout.

MATHEMATICS
Standard Grade - Credit Level

Time allowed - 2 hours 15 minutes

Read Carefully

1. Answer as many questions as you can.
2. Full credit will be given only where the solution contains appropriate working.
3. You may use a calculator

FORMULAE LIST

The roots of $ax^2 + bx + c = 0$ are $x = \dfrac{-b \pm \sqrt{\left(b^2 - 4ac\right)}}{2a}$

Sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$ *or* $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

Area of a triangle: Area $= \frac{1}{2}ab\sin C$

	KU	RA

1. Solve **algebraically** the following equation

$$5x - 11 = 4(2x + 1)$$ **4**

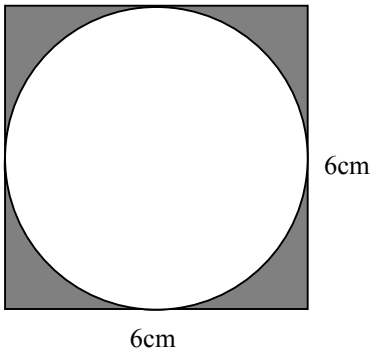
2. A function is defined as $f(x) = 2x^2 - 1$.

(a) Find i) $f(2)$ ii) $f(\sqrt{2})$ **2**

(b) If $f(t) = 17$, find the value(s) of t. **3**

3. Circular tops for cans are often "stamped out" from a square of aluminium.

The circular top for the can across is "stamped out" of a square of side 6 cm.

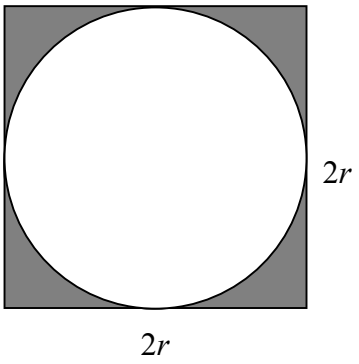


(a) Calculate the area of aluminium discarded for each top (i.e. find the shaded area). **3**

(b) Express this discarded area **as a percentage** of the area of the original square. **2**

(c) Show that, in general, for a circle stamped out of a square of side $2r$, the shaded area, A, is given by

$$A = r^2(4 - \pi)$$ **3**



(d) Hence, show that the percentage, P, of the original area of the square which will be discarded is given by

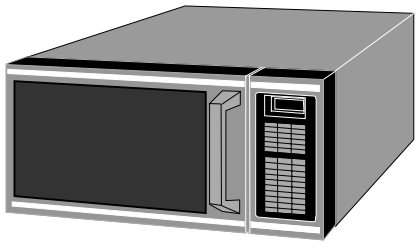
$$P = 25(4 - \pi)$$ **3**

4. The value of an antique grandfather clock is expected to increase each year by 4% of the value it had at the beginning of the previous year.

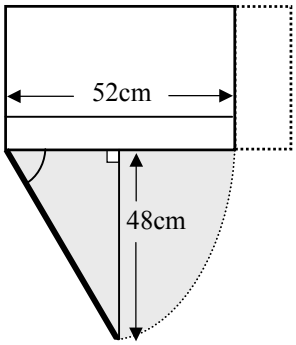
If it was valued at £560 at the **beginning** of 1999, what will its expected value be at the **end** of the year 2001, to the nearest £ ? **3**

	KU	RA

5.



A microwave oven has a 52 cm door which is designed in such a way that the door will swing, at most, 48cm from the oven, as shown in the diagram below.
(viewed from **above**)

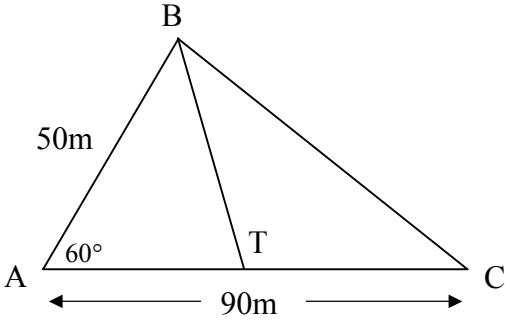


(a) Show that the maximum angle the door can swing through is 67°
(given to the nearest whole number of degrees) **3**

(b) What area will be swept out when the door swings through this angle ? **3**

6. Solve **algebraically** the equation $3p^2 + 2p - 4 = 0$, giving your answers correct to 1 decimal place. **4**

7.



A farmer wishes to test the effect of a new fertilizer on his crops. To compare yields, he splits a triangular piece of land into two equal areas by constructing a fence along BT. Each of the two smaller triangles is now "half" the area of the original triangle.

Given that AB = 50 metres, AC = 90 metres and that the angle BAC = 60°,
calculate the length of AT. **4**

	KU	RA

8. An **accurate** formula for changing degrees Centigrade (C) to degrees Fahrenheit (F) is given as

$$F = 1{\cdot}8C + 32$$

(a) A "rough" method of changing $^\circ C$ into $^\circ F$ is to "double C and then add 30" .

Write down this "rough" method as a **formula**. **1**

(b) There is only one temperature, in $^\circ C$, where the "rough" formula gives **exactly** the same answer as the accurate formula.

Find this temperature **algebraically**. **3**

9.

2 · 5cm

2 · 0cm

" Les Egouts de Paris " is a perfume which is sold in two sizes, as shown opposite.

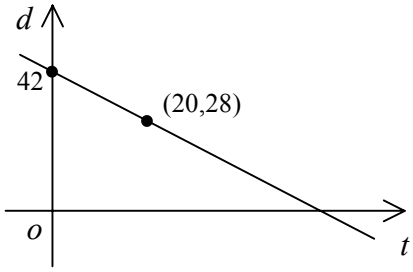
The bottles are mathematically **similar** in shape.

(a) The volume of the larger bottle, which has a base diameter of $2{\cdot}5$ cm, is 100ml.

Calculate the volume of the smaller bottle which has a base diameter of $2{\cdot}0$ cm.
Give your answer correct to the nearest millilitre (ml). **3**

(b) The manufacturers wish to make a 20ml sampler bottle.
If it is to be similar to the other two bottles, find its base diameter to the nearest **millimetre**. **4**

10. Establish the equation of the line opposite in terms of d and t. **4**

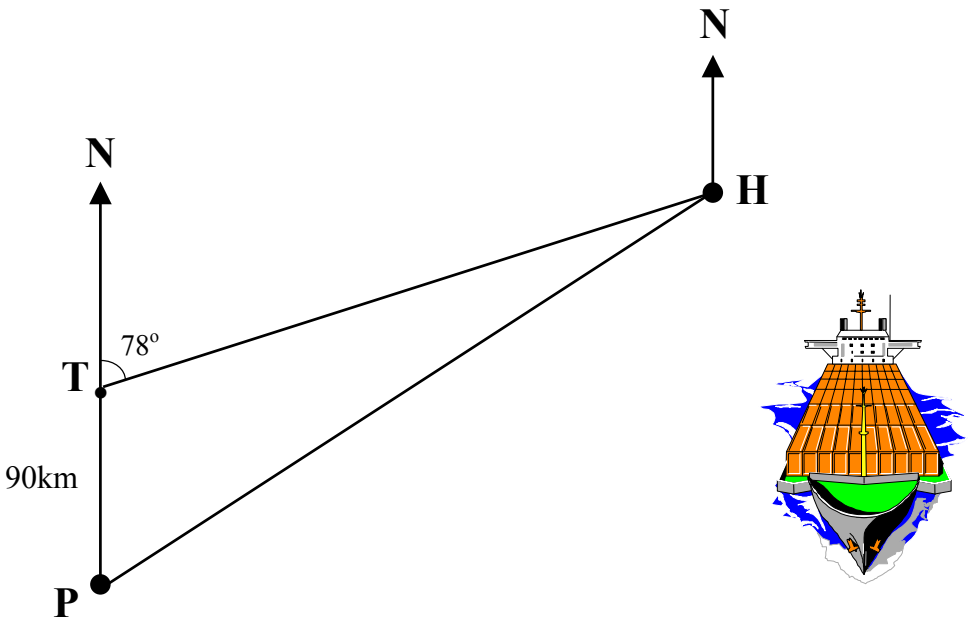


11. Solve the following equation for x, where $0 \le x \le 360^o$.

$$6 + 4\tan x^\circ = 9$$ **3**

	KU	RA

12. A cargo ship sets out from its home port P with a load of machinery, bound for port H, as shown in the diagram.



Because of bad weather, the ship sails due north from P until it reaches point T, a distance of 90km, and then it travels on a bearing of 078° to its destination port H.

If the bearing of P **from** H is 222°, calculate the distance from P to H, giving your answer correct to the nearest kilometre. **4**

13. WESTERN GAS charge their customers for the amount of gas that they use every quarter of a year as follows :

(i) *1.6p per unit for the first 600 units used;*
(ii) *1.1p per unit for any further units used;*
(iii) *a service charge of £8.50 is then added, no matter how many units are used.*

(a) John uses 2800 units of gas in one quarter. Calculate his bill for this usage. **3**

(b) Find a formula for C, the cost in pounds, of using u units of gas, where $u > 600$.
Write the formula in its simplest form. **3**

14. (a) Factorise $2x^2 - 9x + 10$ **1**

(b) Hence simplify $\dfrac{2x^2 - 9x + 10}{x^2 - 4}$ **2**

	KU	RA

15. The value of $\frac{1}{2}\pi$, where $\pi = 3{\cdot}1415.....$, can be shown to have an approximate value which is given by evaluating a series of fractions, as shown in the examples below :

i) $\frac{1}{2}\pi\,[4] = \frac{2}{1}\times\frac{2}{3}\times\frac{4}{3}\times\frac{4}{5} = 1{\cdot}422....$ ii) $\frac{1}{2}\pi\,[5] = \frac{2}{1}\times\frac{2}{3}\times\frac{4}{3}\times\frac{4}{5}\times\frac{6}{5} = 1{\cdot}706....$

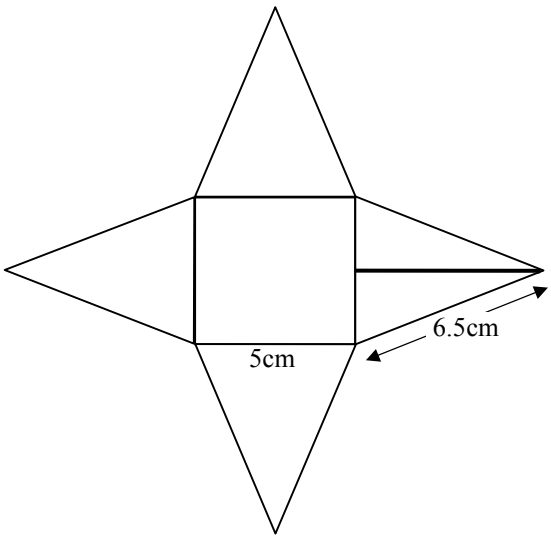
iii) $\frac{1}{2}\pi\,[6] = \frac{2}{1}\times\frac{2}{3}\times\frac{4}{3}\times\frac{4}{5}\times\frac{6}{5}\times\frac{6}{7} = 1{\cdot}462....$

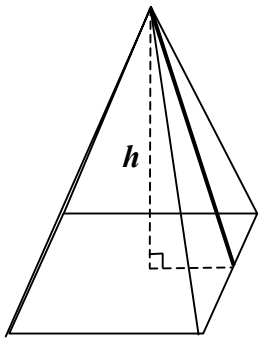
(a) Write down and evaluate the series of fractions for $\frac{1}{2}\pi\,[8]$. **2**

(b) Let n be any **even** number. Write down the last **two** fractions of the series $\frac{1}{2}\pi\,[n]$, in terms of n. **2**

(c) This time let n be any **odd** number. Again, write down the last **two** fractions of the series $\frac{1}{2}\pi\,[n]$, in terms of n. **2**

16.





A lady's hair colourant comes in a package which is a square-based pyramid made of cardboard.

Its net is shown above.

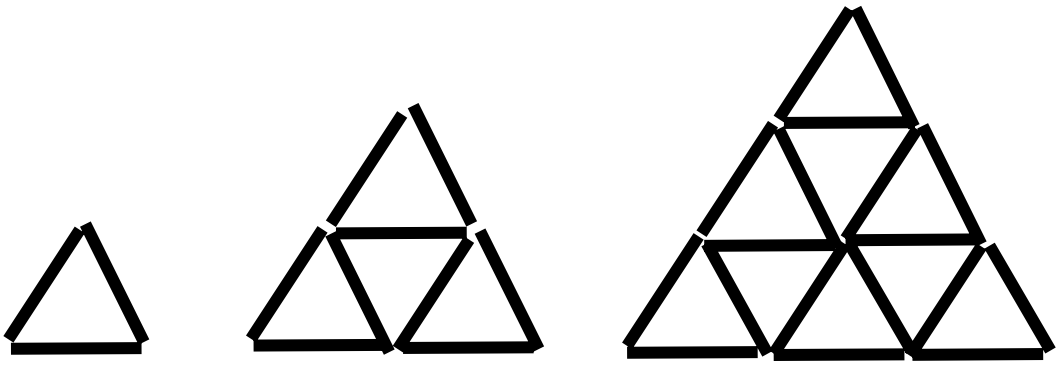
The package has a square base of side 5cm and each of the four isosceles triangles has a sloping edge of length $6{\cdot}5$ cm .

Find the volume of this pyramid , to the nearest cubic centimetre.

[Volume of a pyramid is given as $V = \frac{1}{3}Ah$, where A is the base area and h is the vertical height] **5**

KU	RA

17.



1 LAYER **2 LAYERS** **3 LAYERS**

Clare is using headless matchsticks to make simple triangular patterns, as shown above.

(a) How many matches will she need to construct a similar pattern which has **4 LAYERS** ? **1**

(b) The number of matches, M , required to complete a pattern which has L layers is given by the equation

$$M = aL^2 + bL$$

Find **algebraically** the values of a and b . **5**

[*END OF QUESTION PAPER*]

Credit Mathematics - Practice Exam D — Marking Scheme

1.	For …. $= 8x + 4$	…. (1)		
	For $-15 = 3x$ or $15 = -3x$	…. (2)		
	For $-5 = x$	…. (1)	**[KU 4]**	
2	a) (i) For $f(2) = 7$	…. (1)		
	(ii) For $f(\sqrt{2}) = 3$	…. (1)	**[KU 2]**	
	b) For $2t^2 - 1 = 17$	…. (1)		
	For $t^2 = 9$	…. (1)		
	For <u>two</u> answers $t = -3$ *or* $t = 3$	…. (1)	**[KU 3]**	
3	a) For radius = 3 cm stated / implied	…. (1)		
	For area of circle = 28.24	…. (1)		
	For area of square = 36 **and** shaded area = 7.74sq. cm.	…. (1)	**[KU 3]**	
	b) For 7.74 /36 x 100	…. (1)		
	For %ge = 21.5	…. (1)	**[KU 2]**	
	c) For area of square $= 4r^2$	…. (1)		
	For area of circle $= \pi r^2$	…. (1)		
	For $A = 4r^2 - \pi r^2 = r^2(4 - \pi)$	…. (1)	*[RA 3]*	
	d) For $P = r^2(4 - \pi)/ 4r^2$ x 100	…. (2)		
	For canceling down to $P = 25(4 - \pi)$	…. (1)	*[RA 3]*	
4.	For 1.04 stated / implied	…. (1)		
	For 1.04^3 stated / implied	…. (1)		
	For £630 (or the unrounded £629.92)	…. (1)	**[KU 3]**	
5.	a) For hypotenuse = 52cm, stated / implied	…. (1)		
	For $\sin x = 48/52$	…. (1)		
	For $x = 67°$	…. (1)	**[KU 3]**	
	b) For A = 67 / 360 ×….	…. (1)		
	For 3.14×52^2	…. (1)		
	For Area = 1581cm^2 (accept correct, unrounded answers)	…. (1)	**[KU 3]**	
6	For $a = 3, b = 2, c = -4$, stated / implied	…. (1)		
	For correct substitution of above into quadratic formula (s/i)	…. (1)		
	For $(-2 \pm \sqrt{52}) / 6$	…. (1)		
	For $x = 0.9$ and -1.5	…. (1)	**[KU 4]**	

7. For e.g. Area of Δ ABC = ½ x 50 x 90 x sin60° = 1948.6m^2 (or 1949) .... (1)
For ½ of above area = 974.3 m^2 (or 974.50 .... (1)
For re- using area formula i.e. 974.3 = ½ x 50 x AT x sin60° .... (1)
For AT = 45m .... (1) *[RA 4]*

8. a) For $F = 2C + 30$ or equivalent .... (1) **[KU 1]**

b) For $2C + 30 = 1.8C + 32$ .... (1)
For $0.2C = 2$ .... (1)
For $C = 10$ .... (1) *[RA 3]*
*"C = 10" unsupported by equation work...0/3 ; C = 10" checked in/ into **both** equations..1/3*

9. a) For scale factor = 2.0 / 2.5 = 0.8, stated or implied .... (1)
For $V = 0.8^3$… .... (1)
For $V = 0.8^3$ x 100 = 51ml (Accept 51.2ml) .... (1) **[KU 3]**

b) For e.g. 20 = (S.F.)^3 x 100 .... (1)
For 20 / 100 = (b / 2.5)^3 .... (1)
For 0.58 = b / 2.5 .... (1)
For 1.45cm = b **and** b = 15mm .... (1) *[RA 4]*
Accept legitimate rounding differences leading to 14mm.

10. For $c = 42$ .... (1)
For $m = (42 - 28) \div (0 - 20)$, stated or implied .... (1)
For $m = -0.7$ .... (1)
For $d = -0.7t + 42$ .... (1) **[KU 4]**
N.B. For $y = -0.7x + 42$ **0/1**

11. For $\tan x° = 0.75$ .... (1)
For $x° = 36.9°$ (or 37°) .... (1)
For $x° = 216.9°$ (or 217°) .... (1) **[KU 3]**

12. For deducing that angle THP = 36° .... (1)
For knowing to use sine rule and attempting to substitute values .... (1)
For $\frac{PH}{\sin 102°} = \frac{90}{\sin 36°}$ or equiv. .... (1)

For PH = 149.7km (or 150km) .... (1) **[KU 4]**

13. a) For 600 x 1.6p = 960p (or £9.60) …. (1)
For 2200 x 1.1p = 2420p (or £24.20) …. (1)
For adding service charge of £8.50 to previous charges to get total bill of £42.30 …. (1) *[RA 3]*

b) For appearance of **both** 9.60 **and** 8.50 (1^{st} 600 units + s/c) …. (1)
(or 18.10 -£ sign may be included.)

For " of $(U - 600) \times 0.011$ (no £ sign required) …. (1)
For "tidying up" to give $C = 0.011U + 11.5$ or equiv. …. (1) *[RA 3]*

14. a) For $(2x - 5)(x - 2)$ …. (1)

b) For $(x - 2)(x + 2)$ …. (1)

For simplifying to get $\frac{2x-5}{x+2}$ (Ignore further cancelling) …. (1) **[KU 3]**

15. a) For $\frac{2}{1}\times\frac{2}{3}\times\frac{4}{3}\times\frac{4}{5}\times\frac{6}{5}\times\frac{6}{7}\times\frac{8}{7}\times\frac{8}{9}$ =… …. (1)

For …= 1.48 (or 1.5) …. (1)

b) For $\frac{n}{n-1}\times\frac{n}{n+1}$ (1 mark for each fraction) …. (2)

c) For $\frac{n-1}{n}\times\frac{n+1}{n}$ (1 mark for each fraction) …. (2) *[RA 6]*

16. For Area of base = $25cm^2$ …. (1)
For $s^2 = 6.5^2 - \mathbf{2.5^2}$ where s = sloping height …. (1)
For s = 6cm …. (1)
For vertical height $\boldsymbol{h}$ = 5.5cm (or 5.45cm) …. (1)
For Volume = 1/3 x 25 x 5.5 = $46cm^3$ …. (1) *[RA 5]*

17. a) For 30 matches …. (1)

b) For attempting to use sim. equations …. (1)

For e.g. $3 = a \times 1^2 + b \times 1$ i.e. $3 = a + b$ and then similarly, with e.g. the 2^{nd} diagram $9 = 4a + 2b$ …. (2)

For solving to find $a = 1.5$, $b = 1.5$ (1 mark each) …. (2) *[RA 6]*

Totals	
KU	**RA**
45	**40**