Credit Mathematics - Practice Examination C

Please note … the format of this practice examination is different from the current format. The paper timings are different and calculators can be used throughout.

MATHEMATICS
Standard Grade - Credit Level

Time allowed - 2 hours 15 minutes

Read Carefully

1. Answer as many questions as you can.
2. Full credit will be given only where the solution contains appropriate working.
3. You may use a calculator

FORMULAE LIST

The roots of $ax^2 + bx + c = 0$ are $x = \dfrac{-b \pm \sqrt{\left(b^2 - 4ac\right)}}{2a}$

Sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule: $a = b^2 + c^2 - 2bc\cos A$ *or* $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

Area of a triangle: Area $= \frac{1}{2}ab\sin C$

KU	RA

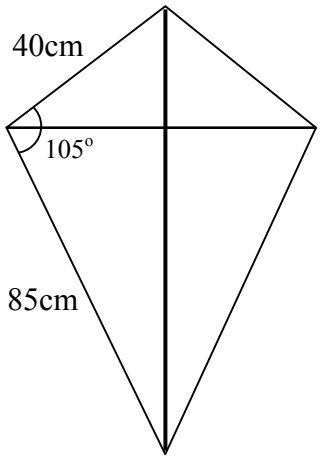
1. Solve the following inequality

$$5(2t - 1) \geq 4t - 23.$$

2. By the end of each week, a garden pond has lost 4% of the volume it had at the beginning of that week. If its volume at the beginning of *week 1* was 26,000 litres, and it continues to lose 4% of its previous volume per week, how many litres will it have by the end of the fourth week ? (Answer to the nearest 100 litres).

3. A man is building a kite for his son. Its sides are 40 cm and 85 cm long and the angle between these two sides is 105°, as shown.
He finds a rod one metre long and intends to use it for the long diagonal of the kite.

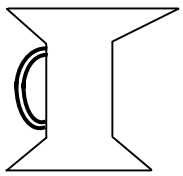
Will this rod be long enough ?

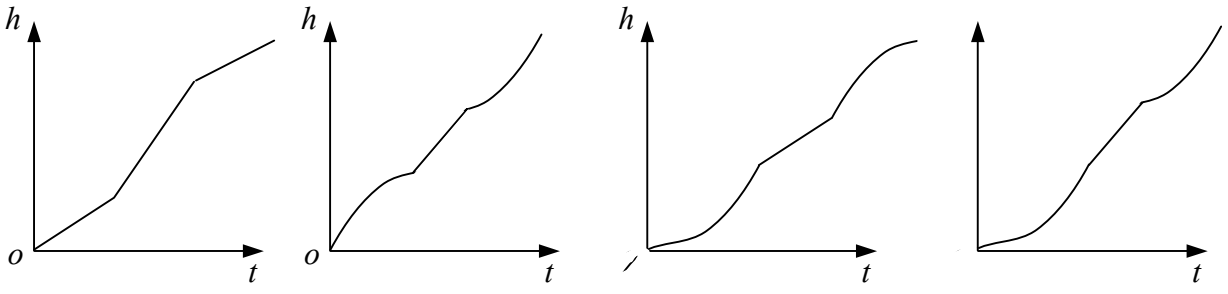


4. On his birthday, Albert decides to buy sweets for himself and his 6 friends at the school tuckshop.
When he buys 5 Venus bars and 2 Tropics, he receives 77 pence change from £3.
The next day, his friend Fred, decides to return Albert's generosity. When Fred buys 4 Venus bars and 3 Tropics, he receives 81 pence change from £3.

Find the cost of each sweet.

5. The empty water jug shown across is being filled with water at a constant rate. Which of the four graphs below best show how the water level, h, is changing with time, t ?
Explain your answer fully.



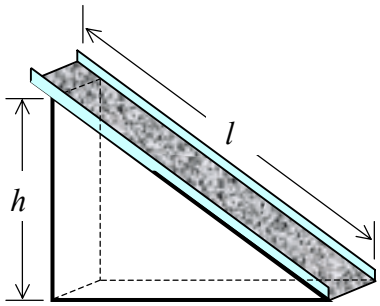


KU	RA

6. Solve the equation $2x^2 - 2x - 7 = 0$, giving your answers to 1 decimal place.

7. Given that the earth is a sphere of circumference 40,000 kms and that the speed of light is 3 x 10^5 km per second, calculate how many times light can travel round the earth in one hour. Give your answer in scientific notation.

8. The time, T seconds, taken for a piece of luggage to slide down the luggage chute of an aeroplane varies directly as the length, l metres, of the chute and inversely as the square root of h, the height in metres of one end of the chute above the other.

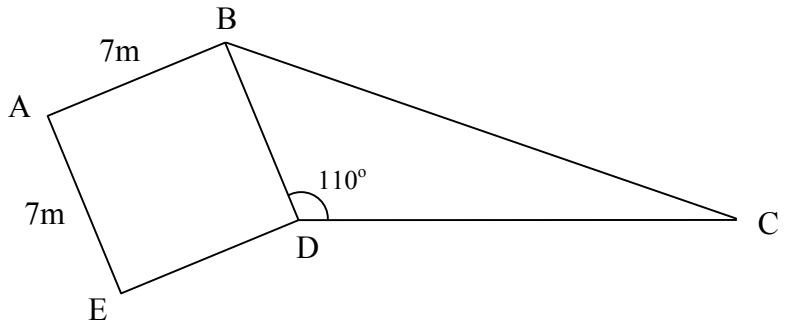


If it takes 14 seconds for a suitcase to travel down a 20 metre chute with one end $6 \cdot 25$ metres above the other :

(a) Find an equation connecting T , h and l.

(b) How long will it take for a suitcase to slide down a chute 15 metres long if $h = 2 \cdot 25$ metres ?

9.



The outline of a children's play area, which is not drawn to scale, is shown above.
It consists of a square of side 7 metres and an obtuse angled triangle.
Angle BDC = 110°.

Given that the area of the triangle is equal to the area of the square, find the length of DC.

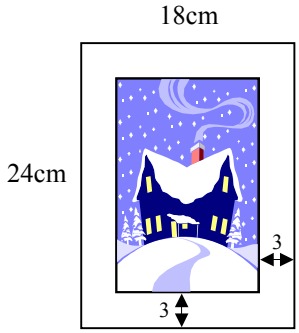
10. (a) Factorise $2a^2 - 11a + 12$

(b) Solve $\frac{x-2}{5} = \frac{2-x}{4}$, for x .

(c) Change the subject of the formula $P = 2\pi r^2 + t$ to r .

	KU	RA

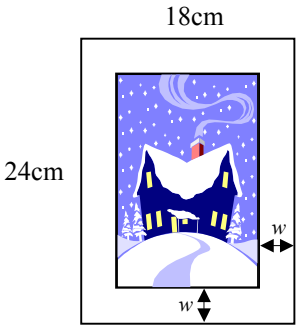
11. A rectangular frame, 24 cm x 18 cm , contains a rectangular photograph. The frame gives the photograph a border of 3 cm all around, as shown opposite.



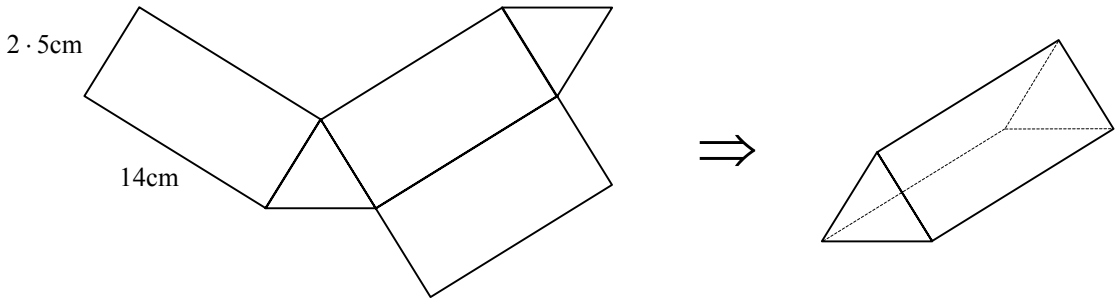
(a) The length – to – breadth ratio of the frame is $\frac{24}{18}$.

By considering the length – to – breadth ratio of the photograph, show that the frame is **not** similar to the photograph. **2**

(b) Show that, if there is a border of width w centimetres around the photograph, then the photograph and the frame can **never** be similar. **4**



12.



The diagram above shows the net of a solid along with the 3-dimensional solid formed by this net.
Calculate the volume of the solid, given that its net is formed by three equal rectangles, each 14 cm x $2 \cdot 5$ cm , and two equilateral triangles. **5**

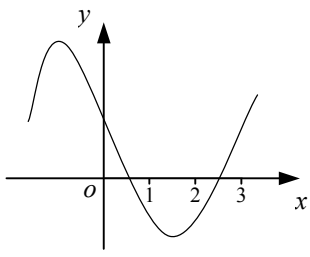
13. A function is defined by $f(x) = \sqrt{3x + 4}$ where $x \geq -\frac{4}{3}$.

(a) Find the values of (i) $f(-1)$ (ii) $f(15)$. **2**

(b) If $f(t) = 16$, find the value of t . **3**

14. The diagram opposite shows part of the graph of

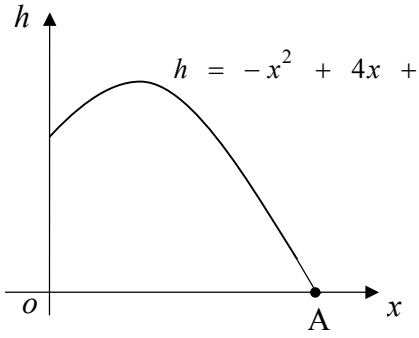
$$y = x^3 - 6x + 3 .$$



Given that this curve has a root between 2 and 3 , find this root correct to 1 decimal place. **3**

	KU	RA

15.



When a test rocket is fired, its trajectory follows the shape of a parabola.
When this flight path is transferred to a coordinate diagram, as shown above, then its path is described by the formula

$$h = -x^2 + 4x + 12$$

(a) By establishing the coordinates of A, state how far the rocket has travelled horizontally from its starting point. (On the x – axis, 1 unit = 500 metres). **3**

(b) Find the maximum height of the rocket. (On the h – axis, 1 unit = 100 metres). **3**

16. The sum of a series of numbers of the form $1 + x + x^2 + x^3 + \ldots.. + x^{n-2} + x^{n-1}$ can be found by using the formula

$$1 + x + x^2 + x^3 + \ldots.. + x^{n-2} + x^{n-1} = \frac{x^n - 1}{x - 1}$$

Example Find the sum of $1 + 2 + 2^2 + 2^3 + 2^4 + 2^5$ by using the above formula.

Solution $1 + 2 + 2^2 + 2^3 + 2^4 + 2^5 = \frac{2^6 - 1}{2 - 1} = \frac{64 - 1}{1} = 63$

(a) Use the method above to find the sum of $1 + 3 + 3^2 + 3^3 + \ldots.. + 3^7$. **2**

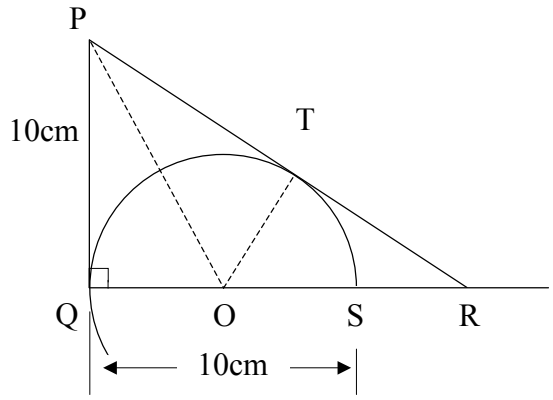
(b) The sum $1 + 3 + 3^2 + 3^3 + \ldots.. + 3^{n-2} + 3^{n-1} = \frac{3^p - 1}{3 - 1}$.
Write down p in terms of n. **1**

(c) Hence, by considering your answers to both (a) and (b) above, show that

$$3^8 + 3^9 + 3^{10} + \ldots.. + 3^{n-2} + 3^{n-1} = \frac{3^n - 6561}{2}.$$ **3**

KU	RA

17.



In the diagram above, two tangents are drawn from the point P to the semi-circle centre O. The semi-circle has a diameter of 10 cm. Angle PQR = 90° and PQ = 10 cm.

(a) Calculate the size of angle POQ , giving your answer correct to one decimal place. **3**

(b) Hence, or otherwise, find the size of angle PRQ. **2**

(c) Show that, if angle POQ = y and angle PRQ = x , then $x = 2y - 90$. **3**

18. *Diagram 1* shows the outline of an ordinary cassette.
It consists of two identical "ring" shapes, each formed by two concentric circles with the outer circle having a radius of 12 *mm* as shown.
The cassette tape is attached to the left hand ring and has a width of 8 *mm*.

diagram 1

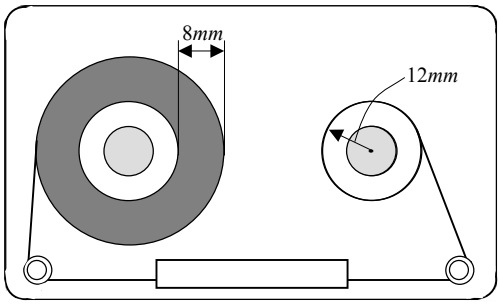
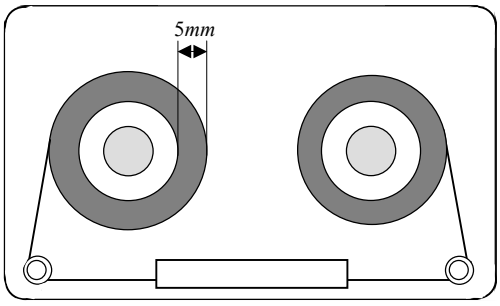


diagram 2



(a) Show that the area of tape in diagram 1 is 256π square millimetres. **3**

(b) After the tape has been playing for some time, the 8 mm thick band of tape (on the left hand ring) has been reduced to 5 mm. (see *diagram 2*)

Find, in terms of π , the area of tape now attached to the left hand ring. **2**

(c) When the tape on the left hand ring is 8 mm broad, it has a playing time of 45 minutes.

Calculate the number of minutes it takes for the tape to reduce its width to 5 mm. (Assume that the time played is proportional to the area of tape used) **3**

[*END OF QUESTION PAPER*]

Credit Mathematics - Practice Exam C Marking Scheme

1. For $10t - 5 \geq 4t - 23$ ..... (1)
For $6t \geq -18$ ..... (2)
For $t \geq -3$ ..... (1) **[KU 4]**

2. For 0.96 ..... (1)
For $(0.96)^4$ (or equiv.) ..... (1)
For 22 100 *litres* (ignore roundings) ..... (1) **[KU 3]**

3. For knowing to use Cosine Rule ..... (1)
For $d^2 = 40^2 + 85^2 - 2 \times 40 \times 85 \times \cos 105$ ..... (1)
For $= 1600 + 7255 - (-1760)$ ..... (1)
For $= 10585$ ..... (1)
For $d = 102.9$cm and " No " ..... (1)
Accept any reasonable roundings. **[KU 5]**

4. For ... $5v + 2t = 223$ and $4v + 3t = 219$ (or equiv.) ..... (1)
For $15v + 6t = 669$ (or equiv.) ..... (1)
For $8v + 6t = 438$ ..... (1)
For $7v = 231$ ..... (1)
For $v = 33$ pence ..... (1)
For $t = 29$ pence ..... (1) **[*RA 6*]**

5. For answer .... Graph C ..... (1)
For explaining how each of the 3 parts of the jug are related to the 3 respective parts of the graph ..... (3) **[*RA 4*]**

6. For knowing to use the quadratic formula ..... (1)
For calculating discriminant 7.75 (or equiv.) ..... (1)
For correct sub. to $x = (2 \pm 7.75) \div 4$ (or equiv) ..... (1)
For 2.44 and -1.44 ..... (1)
For 2.4 and -1.4 ..... (1) **[KU 5]**

7. Give **any** correct version ..... 4 marks. For example :
For changing to km/h i.e. 10800×10^5 km/h ..... (1)
For **dividing** ..... (1)
For 27 000 ..... (1)
For $2 \cdot 7 \times 10^4$ times ..... (1) **[KU 4]**

8. (a) For $T = kL / \sqrt{h}$ ..... (1)
For $14 = k \times 20 / \sqrt{6.25}$ ..... (1)
For $k = 1.75$ ..... (1) **[KU 3]**
(b) For $T = 1.75 \times 15 / \sqrt{2.25}$ ..... (1)
For $T = 17.5$ secs ..... (1) **[KU 2]**

9. For area of square = 49 ..... (1)
For area of triangle = 0.5 x 7 x DC x sin 110° ..... (1)
For = 3.3 x DC ..... (1)
For DC = 14.9 m ..... (1) **[*RA 4*]**

10. (a) For $(2a - 3)$ and $(a - 4)$ ..... (2) **[KU 2]**
(b) For $4(x - 2) = 5(2 - x)$ ..... (1)
For $4x - 8 = 10 - 5x$ ..... (1)
For $9x = 18$ ..... (1)
For $x = 2$ ..... (1) **[KU 4]**
(c) For $P - t = 2\pi r^2$ ..... (1)
For $(P - t) / 2\pi = r^2$ ..... (1)
For $r = \sqrt{(P - t / 2\pi)}$ ..... (1) **[KU 3]**

11. (a) For photo ratio of 18 / 12 or 1.5 ..... (1)
For noting that $18/12 \neq 24/18$ so, no similarity ..... (1) **[KU 2]**
(b) For $24 - 2w$ and $18 - 2w$ ..... (1)
For $\frac{24-2w}{18-2w} = \frac{4}{3}$ (or equivalent equ. const.) ..... (1)
For cross mult. to solve (or equiv.) ..... (1)
For $w = 0$ and conclusion ..... (1) **[*RA 4*]**

12. For Area = $\frac{1}{2}ab\sin C$ (or equiv.) ..... (1)
For = 0.5 x 2.5 x 2.5 x Sin 60 ..... (1)
For = 2.7 sq. cm ..... (1)
For V = 2.7 x 14 ..... (1)
For V = 37.8 cm^3 ..... (1) **[*RA 5*]**

13. (a) For $f(-1) = 1$ and $f(15) = 7$ ..... (2) **[KU 2]**
(b) For $16 = \sqrt{(3t + 4)}$ ..... (1)
For $256 = 3t + 4$ ..... (1)
For $84 = t$ ..... (1) **[KU 3]**

14. For organised approach e.g. trying ... $f(2.0)$, $f(2.1)$, etc. ..... (1)
For $2.1 \leq x \leq 2.2$ ..... (1)
For $x = 2.1$ to one decimal place ..... (1) **[KU 3]**

15. (a) For $(-x - 2)(x - 6) = 0$ ..... (1)
For $x = -2$ or $x = 6$ ..... (1)
For A is (6,0) and distance = 3000m ..... (1) **[KU 3]**
(b) For e.g. axis of symmetry is $x = 2$ ..... (1)
For $h = 16$ when $x = 2$ ..... (1)
For the maximum height is 1600m ..... (1) **[KU 3]**

16. (a) For $1 + 3 +$ $3 - 1)$ ..... (1)
For ..... (1) **[** *RA 2* **]**
(b) For $p =$ ..... (1) **[** *RA 1* **]**
(c) For realising that the solution involves the subtraction of
$(3^n - 1) / (3 - 1) - 3280$ [from (a) and (b)] ..... (1)
For $3^8 + 3^9 + {}^{10} + \ldots + 3^{n-2} + 3^{n-1}$
..... $= (3n - 1) / 2 - 3298$ ..... (1)
For reaching the final answer of $(3^n - 6561) / 2$ ..... (1) **[** *RA 3* **]**

17. (a) For OQ = 5 and using tangent ..... (1)
For tan POQ = 10 / 5 (or 2) ..... (1)
For angle POQ = 63.4° ..... (1) **[** *RA 3* **]**
(b) For angle OPQ = angle OPR (kite !) ..... (1)
For angle PRQ = 36. 8° ..... (1) **[** *RA 2* **]**
(c) For angle QPO = 90° - y ..... (1)
For e.g. $90° + 2 (90 - y)° + x° = 180$ ..... (1)
For $x = 2y - 90$ ..... (1) **[** *RA 3* **]**

18. (a) For Area of circle (large) = 400π ..... (1)
For Area of circle (small) = 144π ..... (1)
For Area of tape 256π ..... (1) **[** *RA 3* **]**
(b) For Area of circle (larger) = $289\ \pi$ ..... (1)
For new area of tape = 145π (i.e. $289\pi - 144\pi$) ..... (1) **[** *RA 2* **]**
(c) For tape used up during playing = 111π ..... (1)
For e.g. correct ratio of tape used up i.e. $111\pi / 256\ \pi$ ..... (1)
For $111\pi / 256\ \pi$ x 45 mins = 19.5 mins ..... (1) **[** *RA 3* **]**
(If 145π is used in ratio for ans. of 25.5 minutes , 2 out of 3 marks)

* (*If* $\pi = 3.14$ *is used for multiplying / division in a correct manner* … **1 mark off**.)

	K U	**R A**
Totals	**51**	*45*