# Assessing my own progress in

# Maths

## S6 Advanced Higher Course

Mathematics Department – Biggar High School

**Name:** **Class:**

# 1 Unit 1 (The Binomial Theorem)

**a** Know and use the notation n! and $\begin{pmatrix} n \\ r \end{pmatrix}$.

- I understand this ☐
- I need to revise ☐

**b** Know the results $\begin{pmatrix} n \\ r \end{pmatrix} = \begin{pmatrix} n \\ n-r \end{pmatrix}$ and $\begin{pmatrix} n \\ r-1 \end{pmatrix} + \begin{pmatrix} n \\ r \end{pmatrix} = \begin{pmatrix} n+1 \\ r \end{pmatrix}$.

- I understand this ☐
- I need to revise ☐

**c** Know how to use Pascal's triangle to expand a bracket to a power.

- I understand this ☐
- I need to revise ☐

**d** Know and be able to use the binomial theorem $(a+b)^n = \sum_{r=0}^{n} \begin{pmatrix} n \\ r \end{pmatrix} a^{n-r} b^r$

for $r, n \in N$ to evaluate specific terms in a binomial expansion.

- I understand this ☐
- I need to revise ☐

**e** Be able to express a rational fraction with denominator less than or equal to degree 3 in its partial fractions.

- I understand this ☐
- I need to revise ☐

# 2 Unit 1 (Differentiation)

**a** Know the definitions of $\operatorname{cosec} x$, $\sec x$ and $\cot x$ and the formulae

$1 + \tan^2 x = \cot^2 x$, $1 + \cot^2 x = \operatorname{cosec}^2 x$.

- I understand this ☐
- I need to revise ☐

**b** Be able to use the product rule including examples involving the chain rule.

- I understand this ☐
- I need to revise ☐

**c** Be able to use the quotient rule including examples involving the chain rule.

- I understand this ☐
- I need to revise ☐

**d**  Know 2 notations for higher derivatives.

- I understand this ☐
- I need to revise ☐

**e**  Be able to use the derivatives of $\tan x, \operatorname{cosec} x, \sec x$ and $\cot x$.

- I understand this ☐
- I need to revise ☐

**f**  Be able to use the derivatives of $e^x$ and $\ln x$.

- I understand this ☐
- I need to revise ☐

**g**  Know how to differentiate from first principles.

- I understand this ☐
- I need to revise ☐

**h**  Understand application of differentiation to rectilinear motion.

- I understand this ☐
- I need to revise ☐

**i**  Understand application of differentiation to maxima and minima.

- I understand this ☐
- I need to revise ☐

## 3  Unit 1 (Integration)

**a**  Be able to recognise all previous work on differentiation when applied in reverse to integration.

- I understand this ☐
- I need to revise ☐

**b**  Be able to use the integral of $\dfrac{f'(x)}{f(x)}$.

- I understand this ☐
- I need to revise ☐

**c**  Be able to use the integral of $f'(x)\left[f(x)\right]^n$.

- I understand this ☐
- I need to revise ☐

**d**  Be able to find an integral using a trigonometric substitution.

- I understand this ☐
- I need to revise ☐

**e**  Be able to find an integral using an algebraic substitution.

- I understand this ☐
- I need to revise ☐

**f** Use integration to evaluate the area between a curve and the x or y axis.

- I understand this ☐
- I need to revise ☐

**g** Use integration to evaluate a volume of revolution.

- I understand this ☐
- I need to revise ☐

## 4  Unit 1 (Properties of Functions)

**a** Be able to find critical (stationary) points in order to sketch $y = f(x)$.

- I understand this ☐
- I need to revise ☐

**b** Know how to find vertical and non-vertical asymptotes in order to sketch $y = f(x)$.

- I understand this ☐
- I need to revise ☐

**c** Know how to find points of inflection in order to sketch $y = f(x)$.

- I understand this ☐
- I need to revise ☐

**d** Know how to sketch the functions $\dfrac{1}{f(x)}$, $\left| f(x) \right|$ and $f^{-1}(x)$.

- I understand this ☐
- I need to revise ☐

**e** Know if a function is odd, even, or neither.

- I understand this ☐
- I need to revise ☐

**f** Know the functions $\sin^{-1} x,\ \cos^{-1} x$ and $\tan^{-1} x.$

- I understand this ☐
- I need to revise ☐

## 5  Unit 1 (Systems of Linear Equations)

**a** Be able to solve a $3 \times 3$ system of linear equations using Gaussian elimination on an (augmented) matrix.

- I understand this ☐
- I need to revise ☐

**b** Be able to recognise the geometrical meaning of the solution of a $3 \times 3$ system of linear equations, including cases of unique solution, no solution (inconsistency) and an infinite number of solutions.

- I understand this ☐
- I need to revise ☐

## 6 Unit 2 (Further differentiation)

**a** Be able to use the derivatives of $\sin^{-1} x$, $\cos^{-1} x$ and $\tan^{-1} x$.

- I understand this  ☐
- I need to revise  ☐

**b** Be able to differentiate a function using logarithmic differentiation.

- I understand this  ☐
- I need to revise  ☐

**c** Be able to find first and second derivatives for a function defined implicitly.

- I understand this  ☐
- I need to revise  ☐

**d** Understand how a function can be defined parametrically.

- I understand this  ☐
- I need to revise  ☐

**e** Be able to find first and second derivatives for functions defined by parametric equations and apply this to motion in a plane.

- I understand this  ☐
- I need to revise  ☐

**f** Be able to find a rate of change in various types of problem questions.

- I understand this  ☐
- I need to revise  ☐

## 7 Unit 2 (Further integration)

**a** Know the integrals of $\dfrac{1}{\sqrt{1-x^2}}$ , $\dfrac{1}{1+x^2}$ .

- I understand this  ☐
- I need to revise  ☐

**b** Be able to find and evaluate integrals of the form $\dfrac{1}{\sqrt{a^2-x^2}}$, $\dfrac{1}{a^2+x^2}$ .

- I understand this  ☐
- I need to revise  ☐

**c** Be able to integrate using partial fractions.

- I understand this  ☐
- I need to revise  ☐

**d** Be able to integrate by parts using one or repeated applications.

- I understand this  ☐
- I need to revise  ☐

**e** Know the definition of differential equation and the meaning of the terms order, separable, general solution, arbitrary constant, particular solution, initial condition.

- I understand this ☐
- I need to revise ☐

**f** Be able to solve first order differential equations using separation of variables.

- I understand this ☐
- I need to revise ☐

**g** Be able to apply solution of a separable first order differential equation to a problem

- I understand this ☐
- I need to revise ☐

## 8 Unit 2 (Complex Numbers)

**a** Know the definition of $i$ as $i = \sqrt{-1}$.

- I understand this ☐
- I need to revise ☐

**b** Know the notation $z = a + ib$, $a, b \in R$, and the terms real part, imaginary part and purely imaginary.

- I understand this ☐
- I need to revise ☐

**c** Perform algebraic operations on complex numbers: equality (equating real and imaginary parts), addition, subtraction, multiplication, and division.

- I understand this ☐
- I need to revise ☐

**d** Evaluate the modulus, argument and complex conjugate of a complex number.

- I understand this ☐
- I need to revise ☐

**e** Convert between Cartesian and polar form.

- I understand this ☐
- I need to revise ☐

**f** Be able to multiply and divide complex numbers in polar form.

- I understand this ☐
- I need to revise ☐

**g** Know all the properties of complex conjugates, including the relationship to polynomials with real coefficients.

- I understand this ☐
- I need to revise ☐

**h** Be able to solve polynomials equations with real coefficients.

- I understand this ☐
- I need to revise ☐

**i** Solve equations involving a complex variable knowing when to solve directly and when to solve by equating real and imaginary parts.

- I understand this ☐
- I need to revise ☐

**j** Know and use de Moivre's theorem with positive integer indices and fractional indices.

- I understand this ☐
- I need to revise ☐

**k** Apply de Moivre's theorem to a multiple angle trigonometric formulae.

- I understand this ☐
- I need to revise ☐

**l** Know the fundamental theorem of Algebra and be able to use de Moivre's theorem to find the n$^{th}$ roots of unity.

- I understand this ☐
- I need to revise ☐

**m** Be able to solve a range of problems involving complex numbers.

- I understand this ☐
- I need to revise ☐

## 9 Unit 2 (Sequences and series)

**a** Know the definition of an arithmetic sequence and series, and be able to use the formulae $u_n = a + (n-1)d$ and $S_n = \frac{1}{2}n[2a + (n-1)d.]$

- I understand this ☐
- I need to revise ☐

**b** Know the definition of a geometric sequence and series, and be able to use the formulae $u_n = ar^{n-1}$ , $S_n = \frac{a(1-r^n)}{1-r}$ and $S_\infty = \frac{a}{1-r}$.

- I understand this ☐
- I need to revise ☐

**c** Know the notation and be able to use the formula $\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$.

- I understand this ☐
- I need to revise ☐

**d** Be able to solve a range of problems involving sequences and series.

- I understand this ☐      • I need to revise ☐

## 5 Unit 2 (Elementary Number Theory and Methods of Proof)

**a** Know the terms natural number, whole number, integer, rational number, irrational number, real number, prime number and composite number.

- I understand this ☐      • I need to revise ☐

**b** Understand and make use of the notations $\forall, \exists, \Rightarrow, \Leftarrow$ and $\Leftrightarrow$.

- I understand this ☐      • I need to revise ☐

**c** Know the corresponding terminology "implies", "implied by" and "equivalence", if and only if, necessary and sufficient.

- I understand this ☐      • I need to revise ☐

**d** Know the requirements of a proof, in particular when a formal proof is required and when a counter-example is appropriate.

- I understand this ☐      • I need to revise ☐

**e** Know how to find the negation and converse of a statement.

- I understand this ☐      • I need to revise ☐

**f** Be familiar with the modulus notation and be able to solve inequalities.

- I understand this ☐      • I need to revise ☐

**g** Be able to use proof by contradiction in appropriate examples.

- I understand this ☐      • I need to revise ☐

**h** Be able to solve simple problems involving division.

- I understand this ☐      • I need to revise ☐

**i** Be able to prove appropriate results by induction.

- I understand this ☐      • I need to revise ☐

## 11 Unit 3 (Vectors)

**a** Know the meaning of the terms position vector, unit vector and be able to use the distance formula and section formula.

- I understand this ☐      • I need to revise ☐

**b** Know the equations of simple lines and planes.

- I understand this ☐ • I need to revise ☐

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**c** Know the meaning of the term direction vector, be able to use this to find the 3 different forms of the equation of a line, and also know the term direction cosines.

- I understand this ☐ • I need to revise ☐

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**d** Be able to find the scalar product of 2 vectors and be able to use it to find the angle between 2 lines.

- I understand this ☐ • I need to revise ☐

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**e** Know the form of the equation of a plane and its normal vector.

- I understand this ☐ • I need to revise ☐

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**f** Be able to find the point where a line intersects a plane.

- I understand this ☐ • I need to revise ☐

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**g** Be able to find the angle between two planes, and the angle between a line and a plane.

- I understand this ☐ • I need to revise ☐

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**h** Be able to find the distance of a point from a plane, and the distance between 2 planes.

- I understand this ☐ • I need to revise ☐

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**i** Be able to use the definition of a vector product and the properties of a vector product.

- I understand this ☐ • I need to revise ☐

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**j** Be able to use the definition of the scalar triple product and the properties of the scalar triple product.

- I understand this ☐ • I need to revise ☐

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**k** Be able to find the equation of a plane given suitable information.

- I understand this ☐ • I need to revise ☐

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**l** Be able to solve a range of problems in 3 dimensional coordinate geometry.

- I understand this ☐ • I need to revise ☐

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## 12 Unit 3 (Matrix algebra)

**a** Know the definition of a matrix, when matrices are equal, the definition of the transpose of a matrix and the zero matrix.

- I understand this ☐
- I need to revise ☐

**b** Perform matrix operations addition, subtraction. multiplication by scalar

- I understand this ☐
- I need to revise ☐

**c** Know when a matrix product exists and be able to multiply 2 matrices when the product exists. Know also the associated properties of matrix multiplication.

- I understand this ☐
- I need to revise ☐

**d** Be able to deal with a range of problems involving matrix multiplication.

- I understand this ☐
- I need to revise ☐

**e** Be able to calculate the determinant of $2 \times 2$ and $3 \times 3$ matrices, and know what is meant by a singular and non-singular matrix. Know also the properties of the determinant of a matrix.

- I understand this ☐
- I need to revise ☐

**f** Be able to deal with a range of problems involving **e**.

- I understand this ☐
- I need to revise ☐

**g** Know what is meant by the inverse of a matrix, be able to find the inverse of a $2 \times 2$ matrix and be able to use the inverse in problems.

- I understand this ☐
- I need to revise ☐

**h** Know the conditions for an inverse to exist and the term adjugate.

- I understand this ☐
- I need to revise ☐

**i** Be able to find the inverse of a $3 \times 3$ matrix.

- I understand this ☐
- I need to revise ☐

**j** Know the role of the inverse matrix in solving linear systems.

- I understand this ☐
- I need to revise ☐

**k** Be able to use induction within a matrix context.

**l** Know the matrices associated with special geometric transformations.

    • I understand this ☐          • I need to revise ☐

**m** Be able to solve a range of problems associated with matrix transformations.

    • I understand this ☐          • I need to revise ☐

## 13 Preliminary Examination

The Preliminary Examination will take place in late January or early February and will cover topics 1 to 12 except topic 11. Note that the topics in this booklet are listed in syllabus order but topic 12 must be done before topic 11 since $3 \times 3$ determinants from topic 12 are required in finding a vector product in topic 11.

## 14 Unit 3 (Further sequences and series)

**a** Know the term power series and MacLaurin's Theorem.

    • I understand this ☐          • I need to revise ☐

**b** Know the special MacLaurin series expansions of the functions $\sin x,\ \cos x,\ e^x,\ \ln(1+x),\ \tan^{-1} x$ and $(1+x)^n$, and the domains of validity of each one.

    • I understand this ☐          • I need to revise ☐

**c** Be able to find power series of simple functions using MacLaurin's Theorem to a stated number of terms.

    • I understand this ☐          • I need to revise ☐

**d** Use the series of **b (or possibly c)** to find the MacLaurin expansions for simple composites such as $e^{2x}$ or other series associated with them.

    • I understand this ☐          • I need to revise ☐

**e** Know what is meant by an iterative formula and its fixed-point.

    • I understand this ☐          • I need to revise ☐

**f** Use iterative schemes of the form $x_{n+1} = g(x_n)$, to solve equations where $x = g(x)$ is a rearrangement of the original equation.

• I understand this ☐          • I need to revise ☐

**g** Know the condition for convergence for **f** and the meaning of the terms first and second order iterative process.

• I understand this ☐          • I need to revise ☐

**h** Be able to use Newton's method for finding the approximate root of an equation.

• I understand this ☐          • I need to revise ☐

## 15 Unit 3 (Further differential equations)

**a** Be able to solve first order linear differential equations using an integrating factor.

• I understand this ☐          • I need to revise ☐

**b** Be able to do **a** above for the general case or for initial value problems.

• I understand this ☐          • I need to revise ☐

**c** Know the meaning of the terms: second order linear differential equation with constant coefficients, homogeneous, non-homogeneous, auxiliary equation, complementary function, and particular integral.

• I understand this ☐          • I need to revise ☐

**d** Solve second order homogeneous ordinary differential equations in the three cases where the roots of the auxiliary equation are

(*i*)      real and distinct
(*ii*)     equal
(*iii*)    complex.

• I understand this ☐          • I need to revise ☐

**e** Solve second order non homogeneous ordinary differential equations with constant coefficients using the auxiliary equation and finding the complementary function and the particular integral method.

• I understand this ☐          • I need to revise ☐

**f** Find particular solutions to second order linear differential equations

• I understand this ☐          • I need to revise ☐

## 16  Unit 3 (Further number theory and further methods of proof)

**a**  Revise the terms necessary condition, sufficient condition, if and only if, converse, negation, and be able to state the contrapositive of a statement.

- I understand this  ☐
- I need to revise  ☐

**b**  Be able to use the contrapositive to prove a statement.

- I understand this  ☐
- I need to revise  ☐

**c**  Be able to use induction in some 'unusual' examples.

- I understand this  ☐
- I need to revise  ☐

**d**  Be able to use $\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$, $\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$ and $\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$ in proving other results,

- I understand this  ☐
- I need to revise  ☐

**e**  Know the division theorem and the notation for divisibility.

- I understand this  ☐
- I need to revise  ☐

**f**  Know the meaning of the integral part and the fractional part of a number. Know also the terms l.c.m. and g.c.d. (h.c.f.)

- I understand this  ☐
- I need to revise  ☐

**g**  Be able to use Euclid's algorithm to find the greatest common divisor (g.c.d.) of two positive integers.

- I understand this  ☐
- I need to revise  ☐

**h**  Know how to express the g.c.d. of 2 integers as a linear combination of the original 2 integers.

- I understand this  ☐
- I need to revise  ☐

**i**  Be able to cope with a range of questions involving divisibility including the use of Euclid's Algorithm, numbers which are relatively prime and also the prime property of primes.

- I understand this  ☐
- I need to revise  ☐

**j** Be able to use number bases other than ten.

- I understand this ☐
- I need to revise ☐

**k** Be able to deal with some 'unusual' questions requiring a direct proof.

- I understand this ☐
- I need to revise ☐

## 17 Final Revision

The SQA Advanced Higher Examination is based on the above syllabus. It is essential that you complete as many past papers as possible. If possible, try to do some additional questions from your sheets of miscellaneous examples.