2018 Higher Paper 2

① $y = 3 + 2x - x^2$

Area is given by $\int_{-1}^{3} (3 + 2x - x^2)\,dx$

$$= \left[3x + \frac{2x^2}{2} - \frac{x^3}{3}\right]_{-1}^{3} = \left[3x + x^2 - \frac{1}{3}x^3\right]_{-1}^{3}$$

$$= 3(3) + 3^2 - \frac{1}{3}(3)^3 - \left[3(-1) + (-1)^2 - \frac{1}{3}(-1)^3\right]$$

$$= 9 + 9 - 9 \quad - \left(-3 + 1 + \frac{1}{3}\right)$$

$$= 9 - \left(-1\frac{2}{3}\right)$$

$$= \underline{\underline{10\frac{2}{3} \text{ units}^2}}$$

② a) $u.v = (-1)\times(-7) + 4\times 8 + (-3)\times 5$

$$= 7 + 32 - 15$$

$$= \underline{\underline{24}}$$

b) $u.v = |u||v|\cos\theta$

$$\cos\theta = \frac{u.v}{|u||v|} = \frac{24}{\sqrt{26}\sqrt{138}}$$

$$|u| = \sqrt{(-1)^2 + 4^2 + (-3)^2} = \sqrt{1 + 16 + 9} = \sqrt{26}$$

$$|v| = \sqrt{(-7)^2 + 8^2 + 5^2} = \sqrt{49 + 64 + 25} = \sqrt{138}$$

$$\cos\theta = 0{\cdot}4$$

$$\theta = \underline{\underline{66{\cdot}4^\circ}}$$

③ $f(x) = x^3 - 7x - 6$

$f'(x) = 3x^2 - 7$

When $x = 2$, $f'(x) = 3(2)^2 - 7 = 12 - 7 = 5$

A function is increasing for $f'(x) > 0$, so this function is $\underline{\underline{\text{increasing}}}$ as $5 > 0$

4) $-3x^2 - 6x + 7$

$= -3(x^2 + 2x) + 7$

$= -3[(x^2 + 2x + 1) - 1] + 7$

$= -3[(x+1)^2 - 1] + 7$

$= -3(x+1)^2 + 3 + 7$

$= -3(x+1)^2 + 10$

5a) For L_1: midpoint of $PQ = \left(\frac{3+9}{2}, \frac{4+(-2)}{2}\right) = (\overset{a}{6}, \overset{b}{1})$

$m_{PQ} = \frac{4-(-2)}{3-9} = \frac{6}{-6} = -1$

$m_{\perp} = 1$

$y - b = m(x - a)$

$y - 1 = 1(x - 6)$

$y = x - 5$

b)

$3y + x = 25$

$y - x = -5$

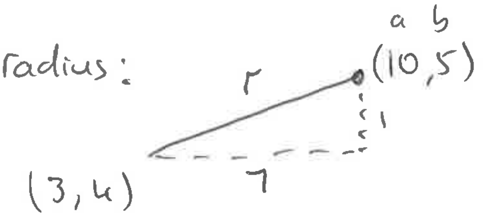
Add $4y = 20$

$y = 5$

If $y = 5$, $5 = x - 5$

$x = 10$ so $C(10, 5)$

c) For radius:



$r^2 = 7^2 + 1^2$

$r^2 = 50$

$(x-a)^2 + (y-b)^2 = r^2$

$(x-10)^2 + (y-5)^2 = 50$

6) $f(x) = 3 + \cos x \qquad g(x) = 2x$

a i) $f(g(x))$

$= f(2x)$

$= \underline{\underline{3 + \cos 2x}}$

ii) $g(f(x))$

$= g(3 + \cos x)$

$= 2(3 + \cos x$

$= \underline{\underline{6 + 2\cos x}}$

b) $3 + \cos 2x = 6 + 2\cos x$

$\cos 2x - 2\cos x - 3 = 0$

$(2\cos^2 x - 1) - 2\cos x - 3 = 0$

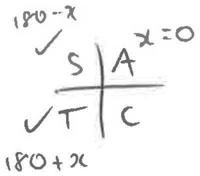
$2\cos^2 x - 2\cos x - 4 = 0$

$(2\cos x + 2)(\cos x - 2) = 0$

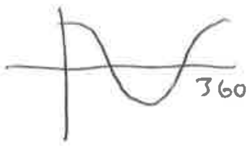
$\cos x = -1 \quad$ or $\quad \cos x = 2$

$x = 180^\circ$

$= \underline{\underline{\pi \text{ radians}}}$



no solutions



7a) i)

2	2	−3	−3	2
		4	2	−2
	2	1	−1	0

no remainder means $(x-2)$ is a factor.

ii) $(x-2)(2x^2 + x - 1)$

$= \underline{\underline{(x-2)(2x-1)(x+1)}}$

b) $u_5 = 2a - 3 \qquad u_{n+1} = au_n - 1$

$u_6 = au_5 - 1 = a(2a-3) - 1 = 2a^2 - 3a - 1$

$u_7 = au_6 - 1 = a(2a^2 - 3a - 1) - 1 = \underline{\underline{2a^3 - 3a^2 - a - 1}}$

as required.

7c) i) $u_7 = u_5$

$$2a^3 - 3a^2 - a - 1 = 2a - 3$$

$$2a^3 - 3a^2 - 3a + 2 = 0$$

From (a) $(a-2)(2a-1)(a+1) = 0$

$$a = 2 \quad \text{or} \quad a = \tfrac{1}{2} \quad \text{or} \quad a = -1$$

For limit to exist $-1 < a < 1$

so $\underline{\underline{a = \tfrac{1}{2}}}$

ii) $u_{n+1} = \frac{1}{2}u_n - 1$

$$L = \tfrac{1}{2}L - 1$$

$$\tfrac{1}{2}L = -1$$

$$\underline{\underline{L = -2}}$$

or $L = \dfrac{b}{1-a} = \dfrac{-1}{1-\frac{1}{2}} = \dfrac{-1}{\frac{1}{2}} = -2$

or if $u_7 = u_5$ then both u_7 and u_5 are equal to the limit

$$u_5 = 2\left(\tfrac{1}{2}\right) - 3 = 1 - 3 = -2$$

8) a) $2\cos x^\circ - \sin x^\circ = k\cos(x-a)^\circ$

$$= k(\cos x \cos a + \sin x \sin a)$$

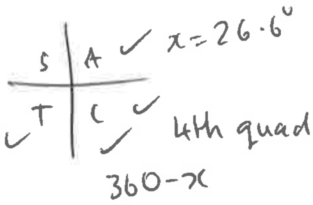
$$= k\cos a \cos x + k\sin a \sin x$$

$k\cos a = 2$

$k\sin a = -1$

$k = \sqrt{2^2 + (-1)^2}$

$= \sqrt{5}$



$$\tan a = \frac{k\sin a}{k\cos a} = -\frac{1}{2}$$

$$a = 360 - 26.6^\circ$$

$$= 333.4^\circ$$

so $\underline{\underline{2\cos x^\circ - \sin x^\circ = \sqrt{5}\cos(x - 333.4)^\circ}}$

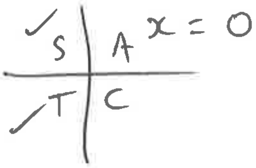
8b) i) $6\cos x^\circ - 3\sin x^\circ = 3(2\cos x - \sin x)$

$= 3(\sqrt{5}\cos(x-333.4)^\circ)$

$= 3\sqrt{5}\cos(x-333.4)^\circ$

minimum value $= \underline{\underline{-3\sqrt{5}}}$

ii) when $\cos(x-333.4)^\circ = -1$



$x - 333.4 = 180$

$x = 513.4^\circ$

For $0 \le x \le 360$, $x = 513.4 - 360 = \underline{\underline{153.4^\circ}}$

(9) $P = 2x + \frac{128}{x}$

$P = 2x + 128x^{-1}$

$P'(x) = 2 - 128x^{-2} = 0$ for S.P

$2 - \frac{128}{x^2} = 0$

$2x^2 - 128 = 0$

$2x^2 = 128$

$x^2 = 64$

$x = \pm 8$

x	$-10 \to -8 \to$			0	$\to 8 \to 10$		
$P'(x)$	$+$	0	$-$		$-$	0	$+$
Slope					$\backslash$	$\_$	$/$

$x=8$ gives minimum value of P.

When $x=8$, $P = 2(8) + \frac{128}{8}$

$= 16 + 16$

$= \underline{\underline{32\text{cm}}}$

(10) $x^2+(m-3)x+m=0$ compare with $ax^2+bx+c=0$

$a=1$, $b=m-3$, $c=m$

$b^2-4ac>0$ for 2 real and distinct roots

$(m-3)^2-4m>0$

$m^2-6m-9-4m>0$

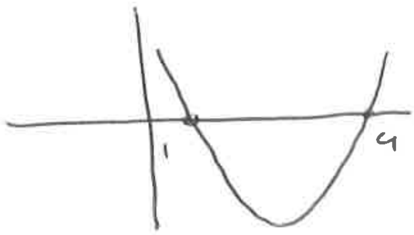
$m^2-10m+9>0$

Sketch the graph of $y=x^2-10x+9$

Set $y=0 \Rightarrow x^2-10x+9=0$

$(x-9)(x-1)=0$

$x=9$ or $x=1$



From graph $\underline{\underline{m<1 \text{ and } m>9}}$

(11) $P=100(1-e^{kt})$

a) $50=100(1-e^{3k})$

$100(1-e^{3k})=50$

$1-e^{3k}=0.5$

$-e^{3k}=-0.5$

$e^{3k}=0.5$

$\log_e e^{3k}=\log_e 0.5$ $\quad (\log_e = \ln)$

$3k\log_e e=\log_e 0.5$ $\quad (\log_e e = 1)$

$3k=\log_e 0.5$

$k=\dfrac{\log_e 0.5}{3}=\underline{\underline{-0.23}}$

b) $P=100(1-e^{-0.23\times 5})=100(1-e^{-1.15})=\underline{\underline{68.3\%}}$

(12) a) i) C_1 has centre $(13,-4)$ $r=10$

ii) $x^2+y^2+14x-22y+c=0$

$(13,-4) \Rightarrow 13^2+(-4)^2+14(13)-22(-4)+c=0$

$169+16+182+88+c=0$

$455+c=0$

$\underline{\underline{c=-455}}$ as required.

b) i) C_2: $x^2+y^2+14x-22y-455=0$

$x^2+y^2+2gx+2fy+c=0$

$2g=14 \quad 2f=-22$

$g=7 \quad f=-11$

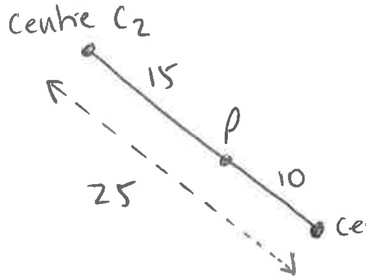
centre $(-7,11)$

$r=\sqrt{g^2+f^2-c}$

$=\sqrt{49+121+455}$

$=\sqrt{625}$

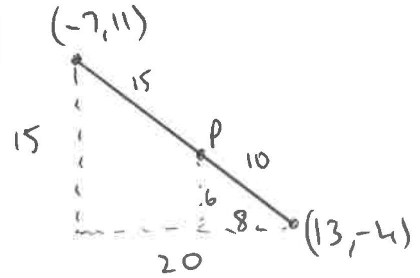
$=25$



P divides C_1 to C_2 in the ratio $2:3$

or P divides C_2 to C_1 in the ratio $\underline{\underline{3:2}}$

ii)



So P is $\underline{\underline{(5,2)}}$

c) C_3: centre $(5,2)$ $r=10$

$\underline{\underline{(x-5)^2+(y-2)^2=100}}$