2018 Higher Paper 1

(1) midpoint of PQ $= \left(\frac{-2+4}{2}, \frac{4+0}{2}\right) = (1, 2)$

$$m_{\text{median}} = \frac{6-2}{3-1} = \frac{4}{2} = 2$$

$\underset{a\ \ b}{(3, 6)}$

$$y - b = m(x-a)$$

$$y - 6 = 2(x-3)$$

$$y - 6 = 2x - 6$$

$$\underline{\underline{y = 2x}}$$

(2) $g(x) = \frac{1}{5}x - 4$

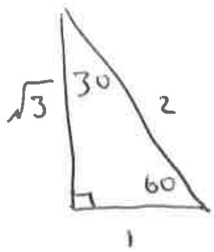
Let $y = \frac{1}{5}x - 4$

$$\frac{1}{5}x = y + 4$$

$$x = 5y + 20$$

so $\underline{\underline{g^{-1}(x) = 5x + 20}}$

(3) $h(x) = 3\cos 2x$



$$h'(x) = 3(-2\sin 2x) = -6\sin 2x$$

$$h'\left(\tfrac{\pi}{6}\right) = -6\sin\frac{2\pi}{6} = -6\sin\frac{\pi}{3} = -6\frac{\sqrt{3}}{2} = \underline{\underline{-3\sqrt{3}}}$$

$\frac{\pi}{3} = 60°$

(4) $x^2 + y^2 - 12x - 6y - 23 = 0$

$x^2 + y^2 + 2gx + 2fy + c = 0$

$2g = -12 \qquad 2f = -6$

$g = -6 \qquad f = -3$

centre $(6, 3)$

$\underset{a\ \ b}{(8, -5)}$

$$m_{\text{rad}} = \frac{3 - -5}{6-8} = \frac{8}{-2} = -4$$

$m_\perp = \frac{1}{4}$

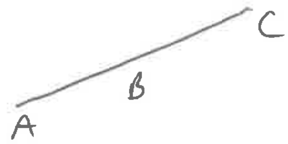
$$y - b = m(x-a)$$

$$y + 5 = \frac{1}{4}(x-8)$$

$$y + 5 = \frac{1}{4}x - 2$$

$$\underline{\underline{y = \frac{1}{4}x - 7}}$$

(5) a) $A(-3,4,-7)$ $\quad B(5,t,5)$ $\quad C(7,9,8)$



$$\overrightarrow{AB} = b - a = \begin{pmatrix} 5 \\ t \\ 5 \end{pmatrix} - \begin{pmatrix} -3 \\ 4 \\ -7 \end{pmatrix} = \begin{pmatrix} 8 \\ t-4 \\ 12 \end{pmatrix}$$

$$\overrightarrow{BC} = c - b = \begin{pmatrix} 7 \\ 9 \\ 8 \end{pmatrix} - \begin{pmatrix} 5 \\ t \\ 5 \end{pmatrix} = \begin{pmatrix} 2 \\ 9-t \\ 3 \end{pmatrix}$$

so $\overrightarrow{AB} = 4\overrightarrow{BC}$

$\dfrac{\overrightarrow{AB}}{\overrightarrow{BC}} = \dfrac{4}{1}$ ratio is $\underline{\underline{4:1}}$

b) $t - 4 = 4(9-t)$

$t - 4 = 36 - 4t$

$5t = 40$

$\underline{\underline{t = 8}}$

(6) $\log_5 250 - \frac{1}{3}\log_5 8$

$= \log_5 250 - \log_5 8^{1/3}$

$= \log_5 250 - \log_5 2$

$= \log_5 \frac{250}{2}$

$= \log_5 125$

$= \underline{\underline{3}}$ $\quad$ (as $5^3 = 125$)

(7) $y = x^3 - 3x^2 + 2x + 5$

a) when $x = 0,\ y = 5$ $\quad \underline{\underline{P(0,5)}}$

b) $\frac{dy}{dx} = 3x^2 - 6x + 2$

when $x = 0,\ \frac{dy}{dx} = 2$ $\quad \overset{a\ \ b}{(0,5)}$

$$y - b = m(x - a)$$
$$y - 5 = 2(x - 0)$$
$$\underline{\underline{y = 2x + 5}}$$

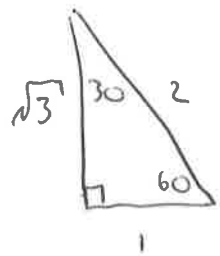
c) $x^3 - 3x^2 + 2x + 5 = 2x + 5$

$$x^3 - 3x^2 = 0$$
$$x^2(x - 3) = 0$$
$$x^2 = 0 \quad \text{or} \quad x - 3 = 0$$
$$x = 0 \quad \text{or} \quad x = 3$$

when $x = 3,\ y = 2(3) + 5 = 11$ $\quad \underline{\underline{Q(3,11)}}$

(8) $y - \sqrt{3}x + 5 = 0$

$$y = \sqrt{3}x - 5$$



$m = \tan\theta$

$\tan\theta = \sqrt{3}$ $\quad \left(\frac{\sqrt{3}\ O}{1\ A}\right)$

$\underline{\underline{\theta = 60^\circ}}$

(9) a) $\overrightarrow{BC} = -t + u = \underline{\underline{u - t}}$

b) $\overrightarrow{MD} = \frac{1}{2}\overrightarrow{BC} - \overrightarrow{AC} + \overrightarrow{AD}$

$= \frac{1}{2}(u-t) - u + v$

$= \frac{1}{2}u - \frac{1}{2}t - u + v$

$= \underline{\underline{-\frac{1}{2}u - \frac{1}{2}t + v}}$

or $\overrightarrow{MD} = \overrightarrow{MB} + \overrightarrow{BA} + \overrightarrow{AD}$

$= +\frac{1}{2}CB + BA + AD$

$= -\frac{1}{2}(u-t) - t + v$

$= -\frac{1}{2}u + \frac{1}{2}t - t + v$

$= -\frac{1}{2}u - \frac{1}{2}t + v$

(10) $\frac{dy}{dx} = 6x^2 - 3x + 4 \qquad \overset{x\ \ y}{(2, 14)}$

$y = \int (6x^2 - 3x + 4)\,dx$

$y = \frac{6x^3}{3} - \frac{3x^2}{2} + 4x + C$

$14 = 2(2)^3 - \frac{3}{2}(2)^2 + 4(2) + C$

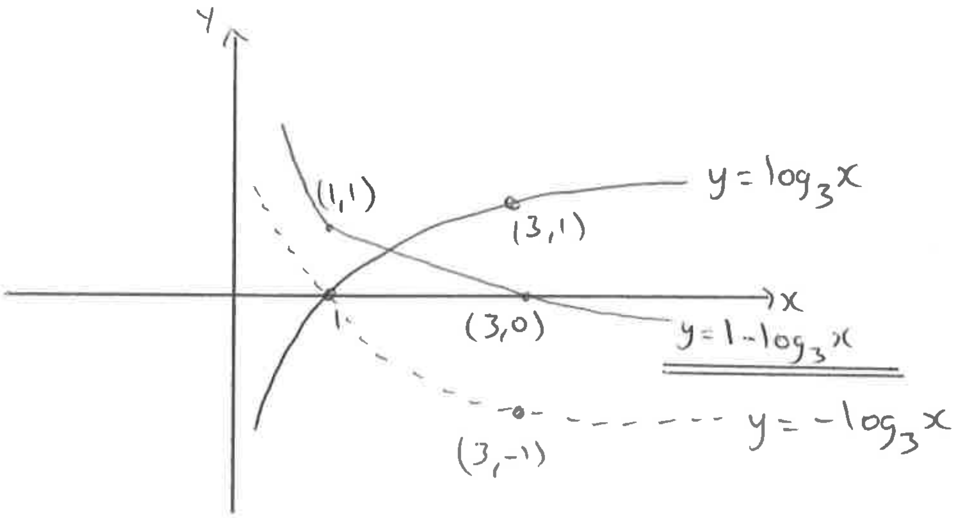
$14 = 16 - 6 + 8 + C$

$14 = 18 + C$

$C = -4$

so $\underline{\underline{y = 2x^3 - \frac{3}{2}x^2 + 4x - 4}}$

(11) a)



reflect in x axis then move up one unit.

11b) $\log_3 x = 1 - \log_3 x$

$2\log_3 x = 1$

$\log_3 x^2 = 1$

$x^2 = 3^1$

$\underline{\underline{x = \sqrt{3}}}$

12) $a = 4i - 2j + 2k \qquad b = -2i + j + pk$

a) $2a + b = 2\begin{pmatrix} 4 \\ -2 \\ 2 \end{pmatrix} + \begin{pmatrix} -2 \\ 1 \\ p \end{pmatrix}$

$= \begin{pmatrix} 8 \\ -4 \\ 4 \end{pmatrix} + \begin{pmatrix} -2 \\ 1 \\ p \end{pmatrix}$

$= \underline{\underline{\begin{pmatrix} 6 \\ -3 \\ 4+p \end{pmatrix}}}$

b) $|2a+b| = 7$

$\sqrt{6^2 + (-3)^2 + (4+p)^2} = 7$ square both sides

$36 + 9 + p^2 + 8p + 16 = 49$

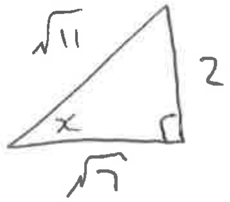
$p^2 + 8p + 61 - 49 = 0$

$p^2 + 8p + 12 = 0$

$(p+6)(p+2) = 0$

$\underline{\underline{p = -6 \text{ or } p = -2}}$

(13)



$$\sqrt{(\sqrt{11})^2 - 2^2}$$
$$= \sqrt{11-4}$$
$$= \sqrt{7}$$

$$\sin x = \frac{2}{\sqrt{11}}$$

$$\cos x = \frac{\sqrt{7}}{\sqrt{11}}$$

a) i) $\sin 2x = 2\sin x \cos x$

$$= 2 \times \frac{2}{\sqrt{11}} \times \frac{\sqrt{7}}{\sqrt{11}}$$

$$= \underline{\underline{\frac{4\sqrt{7}}{11}}}$$

ii) $\cos 2x = \cos^2 x - \sin^2 x$

$$= \left(\frac{\sqrt{7}}{\sqrt{11}}\right)^2 - \left(\frac{2}{\sqrt{11}}\right)^2$$

$$= \frac{7}{11} - \frac{4}{11}$$

$$= \underline{\underline{\frac{3}{11}}}$$

b) $\sin 3x = \sin(2x + x) = \sin 2x \cos x + \cos 2x \sin x$

$$= \frac{4\sqrt{7}}{11} \times \frac{\sqrt{7}}{\sqrt{11}} + \frac{3}{11} \times \frac{2}{\sqrt{11}}$$

$$= \frac{28}{11\sqrt{11}} + \frac{6}{11\sqrt{11}}$$

$$= \underline{\underline{\frac{34}{11\sqrt{11}}}}$$

7.

(14) $$\int_{-4}^{9} \frac{1}{\sqrt[3]{(2x+9)^2}}\,dx = \int_{-4}^{9} \frac{1}{(2x+9)^{2/3}}\,dx = \int_{-4}^{9} (2x+9)^{-2/3}\,dx$$

$$= \left[\frac{(2x+9)^{1/3}}{\frac{1}{3}\times 2} \right]_{-4}^{9} = \left[\frac{3}{2}(2x+9)^{1/3} \right]_{-4}^{9}$$

$$= \frac{3}{2}\left[(2(9)+9)^{1/3} - (2(-4)+9)^{1/3} \right]$$

$$= \frac{3}{2}\left[27^{1/3} - 1^{1/3} \right]$$

$$= \frac{3}{2}(3-1)$$

$$= \underline{\underline{3}}$$

(15) $(x+4)$ is a factor so $x=-4$ is a root

$x=2$ is a repeated root so curve turns on x axis at $x=2$

$f'(-2)=0$ so stationary point at $x=-2$

$f'(x)>0$ so curve is sloping up from left to right (increasing) as it crosses the y axis