2017 Higher Paper 2

① a) midpoint of BC $= \left(\frac{3+9}{2}, \frac{0+(-2)}{2}\right) = (6, -1)$ (a, b)

$m_{BC} = \frac{-2-0}{9-3} = \frac{-2}{6} = -\frac{1}{3}$

$\therefore m_{\perp} = 3$

$y - b = m(x - a)$

$y + 1 = 3(x - 6)$

$y + 1 = 3x - 18$

$y = 3x - 19$

b) $m = \tan\theta$

$m_{AB} = \tan 45^\circ = 1$ (a, b) $(3, 0)$

$y - b = m(x - a)$

$y - 0 = 1(x - 3)$

$y = x - 3$

c) $3x - 19 = x - 3$

$2x = 16$

$x = 8$

$y = 8 - 3 = 5$

$(8, 5)$

(2) a)

1	2	-5	1	2
		2	-3	-2
	2	-3	-2	0

no remainder means $(x-1)$ is a factor of $f(x)$.

b) $(x-1)(2x^2-3x-2)=0$

$(x-1)(2x+1)(x-2)=0$

$x=1$ or $x=-\frac{1}{2}$ or $x=2$

(3) $(x-2)^2+(y-1)^2=25$, $y=3x$

$(x-2)^2+(3x-1)^2=25$

$x^2-4x+4+9x^2-6x+1-25=0$

$10x^2-10x-20=0$

$10(x^2-x-2)=0$

$(x-2)(x+1)=0$

$x=2$ or $x=-1$

when $x=2$, $y=6$ $(2,6)$

$x=-1$, $y=-3$ $(-1,-3)$

(4) a) $3x^2+24x+50$

$=3(x^2+8x)+50$

$=3[(x^2+8x+16)-16]+50$

$=3[(x+4)^2-16]+50$

$=3(x+4)^2+2$.

4b) $f(x) = x^3 + 12x^2 + 50x - 11$

$f'(x) = \underline{\underline{3x^2 + 24x + 50}}$

c) $f'(x) = 3(x+4)^2 + 2$

$f(x)$ is increasing when $f'(x) > 0$

$3(x+4)^2 + 2$ is always greater than 0 for all values of x, so $f(x)$ is always increasing.

(5) a) $\overrightarrow{PQ} = \overrightarrow{PR} + \overrightarrow{RQ}$

$= 9i + 5j + 2k + (-12i - 9j + 3k)$

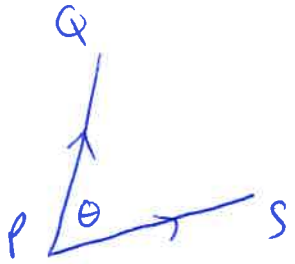
$= \underline{\underline{-3i - 4j + 5k}}$

b) $\overrightarrow{PS} = \overrightarrow{PR} + \frac{2}{3}\overrightarrow{RQ}$

$= 9i + 5j + 2k + (-8i - 6j + 2k)$

$= \underline{\underline{i - j + 4k}}$ as required.

c)



$\cos\theta = \dfrac{PQ \cdot PS}{|PQ||PS|}$

$\cos\theta = \dfrac{-3 + 4 + 20}{\sqrt{50}\sqrt{18}} = \dfrac{21}{30}$

$\therefore \theta = \underline{\underline{45.6^\circ}}$

(6) $5\sin x - 4 = 2\cos 2x$

$2\cos 2x - 5\sin x + 4 = 0$

$2(1 - 2\sin^2 x) - 5\sin x + 4 = 0$

$-4\sin^2 x - 5\sin x + 6 = 0$

$4\sin^2 x + 5\sin x - 6 = 0$

$(4\sin x - 3)(\sin x + 2) = 0$

$\sin x = \frac{3}{4}$ or $\sin x = -2$

no solutions

$x = 48.6°$ or $131.4°$ (S ✓ | A ✓ / T | C)

$x = \frac{48.6}{180}\pi$ or $\frac{131.4\pi}{180}$

$x = 0.85$ or 2.29 radians.

(7) a) $y = 6x - 2x^{3/2}$

$\frac{dy}{dx} = 6 - 3x^{1/2} = 0$ for S.P

$-3x^{1/2} = -6$

$x^{1/2} = 2$

$x = 4$

b) when $x = 4$, $y = 6(4) - 2(4)^{3/2} = 24 - 16 = 8$

when $x = 1$, $y = 6(1) - 2(1)^{3/2} = 6 - 2 = 4$

when $x = 9$, $y = 6(9) - 2(9)^{3/2} = 54 - 54 = 0$

max value of y is 8, min value of y is 0

(8) a) $u_{n+1} = ku_n - 20$

$u_0 = 5$

$u_1 = k(u_0) - 20$

$\Rightarrow u_1 = 5k - 20$

$u_2 = ku_1 - 20$

$u_2 = k(5k-20) - 20$

$u_2 = \underline{5k^2 - 20k - 20}$ as required.

b) $u_2 < u_0$

$5k^2 - 20k - 20 < 5$

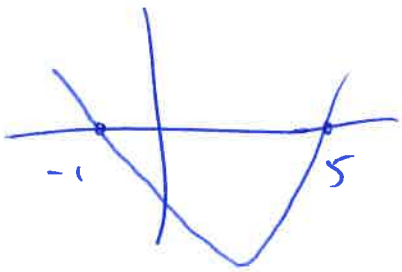
$5k^2 - 20k - 25 < 0$

$5(k^2 - 4k - 5) < 0$

Let $k^2 - 4k - 5 = 0$

$(k-5)(k+1) = 0$

$k = 5$ or $k = -1$



From graph $\underline{-1 < k < 5}$

(9) $y = mx + c$

$\log_2 y = \frac{1}{4}\log_2 x + 3$

$\log_2 y = \log_2 x^{1/4} + \log_2 8$

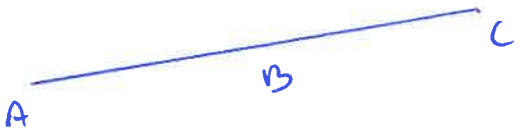
$\log_2 y = \log_2 8x^{1/4}$

$y = 8x^{1/4}$

$\underline{k = 8, \quad n = \frac{1}{4}}$

$\log_2 \boxed{8} = 3$

(10) a) $A(-7,-2)$, $B(2,1)$ $C(17,6)$



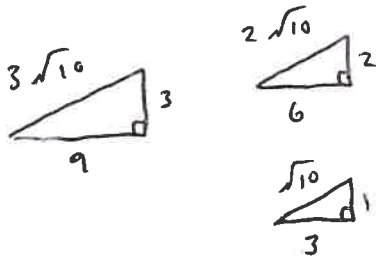
$$m_{AB} = \frac{1--2}{2--7} = \frac{3}{9} = \frac{1}{3}$$

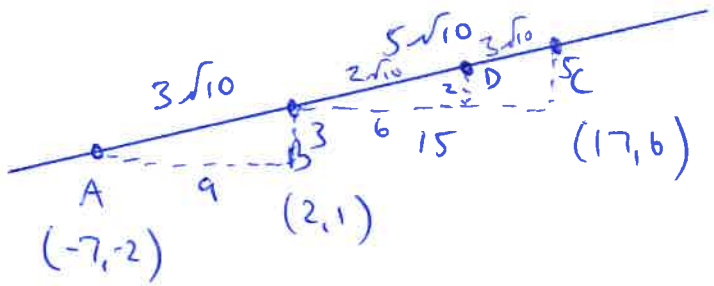
$$m_{BC} = \frac{6-1}{17-2} = \frac{5}{15} = \frac{1}{3}$$

$m_{AB} = m_{BC}$ so AB is parallel to BC. As B is a common point, A, B and C are collinear.

b) $r_A = \sqrt{10}$, $r_B = 2r_A = 2\sqrt{10}$, $r_C = r_A + r_B = 3\sqrt{10}$

$\therefore r_D = 2r_C = \underline{6\sqrt{10}}$





~~D(8,3)~~ D(8,3)

$(x-8)^2 + (y-3)^2 = (6\sqrt{10})^2$

$\Rightarrow \underline{(x-8)^2 + (y-3)^2 = 360}$

(From B, D is 6 along, 2 up)

(11) a) $\dfrac{\sin 2x}{2\cos x} - \sin x \cos^2 x = \sin^3 x$

$\sin^2 x + \cos^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

LHS $\dfrac{2\sin x \cos x}{2\cos x} - \sin x (1 - \sin^2 x)$

$= \sin x - \sin x + \sin^3 x$

$= \underline{\sin^3 x}$ as required

b) Let $y = (\sin x)^3$

$\dfrac{dy}{dx} = 3(\sin x)^2 \cos x$

$= \underline{3\cos x \sin^2 x}$